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HEAVY QUARK CORRELATIONS IN e^+e^- ANNIHILATIONS

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ABSTRACT

In heavy quark jets the quark mass acts as a regulator of collinear singularities, making the quark momentum an infra-red safe variable in perturbative QCD. This allows a direct comparison of measured heavy hadron momentum spectra with perturbative calculations. We exploit the factorisation of heavy quark fragmentation to derive QCD predictions for momentum correlations between heavy hadrons produced in e^+e^- annihilations. We study the practical feasibility and model sensitivity of our approach using Monte Carlo simulations. Higher order perturbative corrections and contributions from non-perturbative effects are found to be at the level of 10%.

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1. Introduction

The rules for applying perturbative QCD to hadron production in hard collisions are well known.[1] Only quantities which do not depend on small dimensionful parameters of $\mathcal{O}(\Lambda_{QCD})$ may be assumed to be 'infra-red safe', *i.e.* only such quantities are insensitive to confinement effects. In particular, no dependence on light quark masses or on soft gluon momenta is allowed in a perturbatively calculated quantity which is to be compared with experimental data. For cross-sections involving individual hadrons in the initial or final states, the QCD factorisation theorem[2] shows that at leading twist, *i.e.* up to terms suppressed by inverse powers of the hard scale, all infra-red sensitive terms may be factored into universal structure and fragmentation functions. These functions are characteristic of the hadrons involved, but independent of the hard process. In practice, most experimental tests of QCD depend on the factorisation theorem.

In the case of heavy (charm, bottom) quark production, the mass itself acts as an infra-red cutoff. An explicit dependence on the quark mass m_Q then is no longer an obstacle to the observability of a cross-section, given that $m_Q \gg \Lambda_{QCD}$. Thus the fragmentation function of a heavy quark is calculable in perturbative QCD, up to power corrections in Λ_{QCD}^2/m_Q^2.[3,4] The hadron containing the heavy quark should move nearly in the direction of the quark and carry most of its energy, leaving relatively little energy to accompanying light hadrons. This is in fact experimentally observed in charm and bottom quark hadronisation.[5]

From a practical point-of-view, an essential difference between the phenomenology of heavy and light quarks is that, to a very good approximation, no heavy quarks are produced in non-perturbative hadronisation processes. For example, whereas the quarks of most final state pions were produced in soft processes, we may be confident that a B meson harbors a bottom quark from the hard part of the collision. Moreover, the strong correlation between the heavy hadron and quark momenta suggests that hadronisation corrections to QCD predictions of the quark distributions are relatively smaller for heavy quarks. Here we shall in fact remove most of the remaining dependence on the hadronisation process by means of the QCD factorisation theorem. Since the fragmentation of the heavy quark is universal, *i.e.* independent of how the heavy quark was produced, we can predict correlations between heavy hadron momenta which are in principle insensitive to the form of the hadronisation distribution. We verify that the momentum distributions of heavy hadrons in standard Monte Carlo programs agree closely with the predictions of perturbative QCD and factorisation. Hence model-dependent corrections are reasonably small.

Heavy quark production in e^+e^- annihilation provides a particularly clean test of QCD. Exact matrix elements, including quark mass effects, are known[6,7] to $\mathcal{O}(\alpha_s)$.* From an experimental point-of-view, the abundant production of heavy quarks in e^+e^- annihilations, coupled with novel opportunities to observe directly the heavy hadron decays using vertex detectors and particle identification, offers

* Currently $\mathcal{O}(\alpha_s^2)$ matrix elements are available for massless quarks only.

possibilities to reconstruct the hadron momenta with good statistics. Measuring single heavy hadron distributions allows a determination of the non-perturbative hadronisation distribution of the quark. Events in which two heavy hadrons are detected then provide interesting QCD tests of the angular and energy correlations of the quarks.

In section 2 we derive predictions for the double inclusive heavy hadron momentum distributions in e^+e^- annihilations using the QCD factorisation theorem and the single inclusive hadron distributions. In Section 3 we use a Monte Carlo simulation to illustrate our method and to investigate the influence of non-perturbative, and higher order perturbative, effects on our prediction. A summary and conclusions are given in Section 4.

2. Perturbative QCD Calculations

2.1 Single Moments

Consider, then, the production of heavy quarks in e^+e^- collisions according to QCD.

Let us first assume that the momentum of only one of the heavy hadrons in the event is measured.* Integrating over the production angles we are left with a single variable, which we take as the energy fraction $z = 2E_H/E_{CM}$ of the hadron H. According to the QCD factorisation theorem,[2] the cross-section for the inclusive reaction $e^+e^- \to H(z) + X$ is given by the production cross section $\hat{\sigma}(e^+e^- \to Q(x) + X)$ of the heavy quark with energy fraction $x = 2E_Q/E_{CM}$, convoluted with the quark fragmentation function $D(z/x, \mu)$,

$$\frac{1}{\sigma}\frac{d\sigma}{dz} = \int \frac{dx}{x} D(\frac{z}{x}, \mu) \frac{1}{\hat{\sigma}}\frac{d\hat{\sigma}}{dx} \tag{1}$$

Taking moments of the hadron energy spectrum we thus get

$$D_k \equiv \int dz z^{k-1} \frac{1}{\sigma}\frac{d\sigma}{dz} = M_k P_k \tag{2}$$

where M_k and P_k are, respectively, moments of the quark fragmentation and production distributions. These are distinguished by the momentum scales of the processes they include. All processes involving momentum scales larger than some factorisation scale μ are included in the production moments P_k. Conversely, all dependence on the soft hadronisation and harder processes up to the scale μ appears in the fragmentation moments M_k. In an exact (all orders) calculation, the μ dependences of M_k and P_k will cancel each other, making their product D_k independent of μ. In any practical calculation, however, there will be a residual dependence of D_k on μ due to neglected orders of perturbation theory.

* For simplicity, we consider a single type of heavy hadron only. If several types of mesons or baryons are detected, our analysis can be carried out separately for each hadron species.

The factorisation scale μ is thus in principle arbitrary, provided it is large enough for perturbation theory to be valid. In our case we could actually take $\mu = m_Q$, the heavy quark mass. However, the QCD calculation of the production moments P_k contains potentially large logarithms of the form $\log(Q^2/\mu^2)$, where Q is the scale of the hard interaction (the c.m. energy in e^+e^- annihilations). A reliable calculation at high energies then requires one to sum such logarithms to all orders. This can be done using the standard evolution equations[8] to leading order in the large logarithms. The next-to-leading order resummation for heavy quark fragmentation is now also available.[4]

Alternatively, we may choose a larger value for μ, making a low order calculation of the production process more reliable. This is the approach we shall adopt here. We take the jet measure[9] $y_{cut} = 0.02$, which at the Z^o pole corresponds to a b-quark-gluon invariant mass $\mu \simeq 13.8$ GeV. With such a large value of μ, our results should be relatively insensitive to higher order perturbative corrections. Correspondingly, the moments M_k that will be obtained from the data will determine the heavy quark fragmentation function at the rather large scale μ. Their dependence on μ should be approximately given by the evolution equations. Note also that for such a large value of μ the ensuing calculation need not be restricted to heavy quarks, as the quark mass is no longer required as a regulator of singularities (see *e.g.* Ref. 10).

The production moments P_k of Eq. (2) are obtained from the single heavy quark inclusive cross-section in QCD

$$P_k = \int dx\, x^{k-1} \frac{1}{\hat{\sigma}} \frac{d\hat{\sigma}}{dx} \tag{3}$$

At $\mathcal{O}(\alpha_s)$ only a single quark pair is produced in the basic processes ($e^+e^- \to Q\overline{Q}$, $Q\overline{Q}g$). The cross-sections for these reactions are given in Ref. 7 for arbitrary quark masses m_Q*. We took the b,c-quark mass values to be 5.0 and 1.5 GeV respectively, and the QCD scale parameter to lowest order, Λ_{QCD}, to be 0.1 GeV. We denote the angle between the heavy quark momenta by ψ, and assume that it is directly measurable as the angle between the heavy hadron momenta. When ψ is appreciably different from π the gluon emission is hard and gives rise to a third jet.**

* We verified numerically that the expressions of Nilles and of Jerzák *et al.* in Ref. 7 agree with each other.

** In the particular case of $\psi = 0$ the hard gluon balances two collinear heavy quark jets. It would be interesting to study whether the heavy quarks hadronise independently of each other even in this case of coalescing jets. Two collinear quarks may act coherently as an effective singlet or octet colour charge. Differences in the hadronisation of different colour charges have been discussed in Ref. 11. Higher twist effects of $\mathcal{O}(\Lambda^2_{QCD}/m^2_Q)$ can break factorisation; one clear signal of this would be quarkonium formation in the $\psi = 0$ configuration.

The singularity of Eq. (3) at $x = 1$ is regulated at the scale μ by taking all $Q\overline{Q}g$ configurations for which the gluon effective mass with one of the quarks is less than μ to be part of the two-jet cross-section (which contributes at $x = 1$). Numerical values of the first few moments P_k, for the case of b- and c-quark production on the Z^0 resonance, are given in Table 1. The fragmentation moments M_k may be determined from Eq. (2), once data on inclusive heavy hadron distributions are available. (In Section 3 we demonstrate the method using Monte Carlo data).

Table 1: Values of P_k calculated from Eq. (3) to $\mathcal{O}(\alpha_s)$ in perturbative QCD for b- and c-quark production.

Order k	P_k (b)	P_k (c)
1	1.000	1.000
2	0.925	0.919
3	0.877	0.869
4	0.842	0.832

2.2 Double Moments

Assuming that the fragmentation moments M_k have thus been determined, consider next the case when the momenta of two heavy hadrons in the event, H and $\overline{H}$, are measured. Again integrating over the orientation with respect to the beam direction we have three variables, which we take as the hadron energy fractions $z_1 = 2E_H/E_{CM}$, $z_2 = 2E_{\overline{H}}/E_{CM}$ and the angle ψ between the two momenta. Analogously to Eq. (1), the probability distribution of z_1, z_2 and ψ is

$$\begin{aligned} P(z_1, z_2; \psi) &\equiv \frac{1}{\sigma(H\overline{H})} \frac{d^3\sigma}{dz_1 dz_2 d\cos\psi} \\ &= \int_{z_1}^{1} \frac{dx_1}{x_1} D\left(\frac{z_1}{x_1}, \mu\right) \int_{z_2}^{1} \frac{dx_2}{x_2} D\left(\frac{z_2}{x_2}, \mu\right) P_{QCD}(x_1, x_2; \psi) \end{aligned} \tag{4}$$

where $\sigma(H\overline{H})$ is the total heavy hadron production cross section, x_1 and x_2 are the energy fractions carried by the Q and $\overline{Q}$ quarks, and $P_{QCD}(x_1, x_2; \psi)$ is the (normalized) heavy quark production probability, which also depends on μ.

Multiplying Eq. (4) by $z_1^{k-1} z_2^{l-1}$ and integrating over z_1 and z_2 we get the double moments

$$D_{kl}(\psi) \equiv \int_0^1 dz_1 dz_2 z_1^{k-1} z_2^{l-1} P(z_1, z_2; \psi) = M_k M_l P_{kl}(\psi) \tag{5}$$

where, in QCD

$$P_{kl}(\psi) = \int dx_1 dx_2 x_1^{k-1} x_2^{l-1} P_{QCD}(x_1, x_2; \psi) \tag{6}$$

Eq. (5) is our principal result. Having determined the fragmentation moments M_k from the single hadron distributions D_k (2), and knowing the numerical values of the double moments P_{kl} from QCD, we have an absolute prediction for the measurable double heavy hadron moments D_{kl} for all values of k, l and ψ.

In order to calculate the moments P_{kl}, we note that due to momentum conservation in the $Q\overline{Q}g$ final state there is at $\mathcal{O}(\alpha_s)$ a kinematic relation between the variables x_1, x_2 and ψ:

$$P_{QCD}(x_1, x_2; \psi) = P_{QCD}(x_1; \psi)\delta(x_2 - x_2(x_1, \psi)) \tag{7}$$

$$x_2(x_1, \psi) = \frac{(2 - x_1)(2 - 2x_1 + \rho) \mp v_1 x_1 \cos\psi \sqrt{4(1 - x_1)^2 - v_1^2 x_1^2 \rho \sin^2\psi}}{(2 - x_1)^2 - v_1^2 x_1^2 \cos^2\psi} \tag{8}$$

where $s = E_{CM}^2$, $\rho = 4m_Q^2/s$ and the quark velocity $v_i^2 = 1 - \rho/x_i^2$. Care has to be taken to include all physical configurations of the $Q\overline{Q}g$ state in the integral of Eq. (6), corresponding to solutions (8) that conserve energy and momentum. For $\cos\psi > 0$ the full integration range is

$$x_1(v_1 = 0) = \sqrt{\rho} \leq x_1 \leq \frac{2 - 2\sqrt{\rho} + \rho}{2 - \sqrt{\rho}} = x_1(v_2 = 0) \tag{9}$$

with the upper (negative) sign chosen in front of the square root in (8). However, for $\cos\psi < 0$ one must include, in addition to the range (9), again with the upper sign in front of the square root, also the range

$$x_1(v_2 = 0) \leq x_1 \leq \frac{4 - \sqrt{\rho \sin^2\psi(4 - 4\rho + \rho^2 \sin^2\psi)}}{4 - \rho \sin^2\psi} \tag{10}$$

where the upper limit is determined by the vanishing of the square root in (8). Furthermore, in the range (10) *both* the upper and lower signs in (8) give physical configurations, and thus contribute to the δ-function of Eq. (7). In the limit of vanishing quark masses ($\rho \to 0$) the range (10) shrinks to the point $x_1 = 1$.

The expression for $P_{QCD}(x_1; \psi)$ is obtained by multiplying the normalized cross-section $\hat{\sigma}^{-1} d\hat{\sigma}/dx_1 dx_2$ of Ref. 7 by the Jacobian $dx_2/d\cos\psi$. In Fig. 1(a) we plot the first moment P_{11} in the case of $b\bar{b}$ and $c\bar{c}$ production at the Z^0 resonance. The shape of the moment distribution, *i.e.* the peak at $\psi = \pi$ and the long tail out to low values of ψ, reflects the Bremsstrahlung spectrum for gluon emission. The calculation was not extended to the region $cos\psi < -.9$, where two-jet events begin to dominate and a smearing of the analytic calculation would be necessary. The slight dominance of the charm quark over the bottom quark distribution in Fig. 1(a) at smaller values of ψ reflects the relative enhancement of gluon radiation for lighter quarks. For clarity of display on a linear scale we choose to normalise the higher $(k, l > 1)$ moments by P_{11}; these are plotted in Fig. 1(b). As can be seen, these normalised moments also have a significant dependence on ψ.

The QCD prediction for the double moments $D_{kl}(\psi)$ of the heavy hadron energy distribution in Eq. (5) can now be directly compared with data at all angles ψ, using the moments M_k determined from the single hadron inclusive data. As in the case of the single moments D_k of (2), the factorisation theorem guarantees that although the fragmentation moments M_k and the double moments P_{kl} depend on the factorisation scale μ, the physically measurable double hadron moments D_{kl} do not. In our $\mathcal{O}(\alpha_s)$ calculation, however, the P_{kl} are evaluated at lowest non-vanishing order and are thus independent of μ. Therefore, there will be a residual μ-dependence in the prediction of D_{kl} due to omitted higher order terms.

3. Monte Carlo Studies

In order to gain further insight into the physical significance of the moment calculations we use a Monte Carlo simulation and address the following issues:

1. The feasibility of our method from an empirical point-of-view. Within this context the Monte Carlo data merely represent, in an idealised way, a dataset from a real experiment, and the exact details of the simulation are not important.

2. The size of presently uncalculable non-perturbative effects due to hadronisation of the quarks. In comparing any QCD prediction with experimental data comprising hadronic final states, it is usually necessary to correct the data for the effects of hadronisation, such that the comparison is valid. Those variables for which this correction is small are experimentally preferred, as they provide the most reliable and most sensitive tests of QCD.

3. The size of higher order QCD corrections to the $\mathcal{O}(\alpha_s)$ calculations presented here. We estimate these using a Monte Carlo model that incorporates $\mathcal{O}(\alpha_s^2)$ matrix elements for massless quarks, as well as an all-order leading logarithm approximation (LLA)-type calculation.

We emphasise, however, that the results presented in Section 2 are model-independent.

For our purposes we use the Lund Monte Carlo for e^+e^- annihilation, JETSET 6.3.[12] This program combines perturbative QCD calculations of partonic states with a phenomenological hadronisation scheme, to give hadron-level final states which may thus be compared with real hadronic final states produced at e^+e^- colliders. Furthermore, this model is known[13] to provide an excellent description of the experimental data at c.m. energies across the whole range spanned by the PETRA/PEP, TRISTAN and SLC/LEP colliders, namely between 14 and 91 GeV. Other Monte Carlo event generators (see *e.g.* Ref. 14) also provide good descriptions of the experimental data and could have been included in our study. However, the particular advantage of JETSET is that it includes different options for the perturbative QCD calculation of the partonic states, namely matrix elements to $\mathcal{O}(\alpha_s)$ or $\mathcal{O}(\alpha_s^2)$, or a LLA + $\mathcal{O}(\alpha_s)$ 'parton shower'. The $\mathcal{O}(\alpha_s)$ matrix element option allows us to compare directly our analytic calculations of the single and double moments at this order with the moments calculated from the Monte Carlo events, thereby illustrating the feasibility of the method and providing an estimate of the size of non-perturbative hadronisation corrections. The $\mathcal{O}(\alpha_s^2)$ matrix element and LLA + $\mathcal{O}(\alpha_s)$ options enable us to estimate the influence of higher order corrections to our $\mathcal{O}(\alpha_s)$ calculations.

3.1 $\mathcal{O}(\alpha_s)$ Calculations

We first generate heavy flavour events using the cross-sections[6] implemented in JETSET to $\mathcal{O}(\alpha_s)$ in QCD, at a c.m. energy of 91.18 GeV, equal[15] to the Z^0 mass.* We set the b-quark mass to 5.0 GeV; the QCD scale parameter, Λ_{QCD}, to 0.1 GeV and y_{cut} to 0.02 to match the values used in the analytic calculation; all other parameters in the Monte Carlo were left at their default values.[12] Roughly 500 000 events of the type $e^+e^- \rightarrow b\bar{b}$ were then generated. The moments D_k (Eq. (2)) were evaluated for the two cases: a) using the z values of the b-quarks, b) using the z values of the b-hadrons** produced after parton fragmentation according to the Lund string model. These moments were then compared with our calculated values for P_k to obtain the fragmentation moments M_k; the results are given in Table 2.

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* Quark mass effects are fully taken into account for the direct channel photon contribution, but only approximately for Z^o exchange.[16] Therefore, for the purposes of comparison, we ran the Monte Carlo program in 'QED-mode', and repeated the analytic moment calculations for QED only. From a practical point-of-view, the neglect of the electroweak terms is not an important issue for the normalized cross sections we consider here, and affects the numerical values of the single moments P_k (Table 1) at the fourth (fifth) decimal place for b (c)-quarks respectively. This approximation does not affect our conclusions at all.

** We consider all b-hadrons inclusively and do not distinguish between species of mesons or baryons.

Table 2: Single moment values: The quantity D_k (Eq. (2)) was calculated at both the quark and hadron levels from the Monte Carlo data for b and c events. This quantity was then compared with the QCD-calculated production moment P_k to obtain the fragmentation moment M_k.

	b-events		c-events	
Order k	M_k^{quark} (M.C.)	M_k^{hadron} (M.C.)	M_k^{quark} (M.C.)	M_k^{hadron} (M.C.)
2	1.002	0.810	1.004	0.729
3	1.004	0.676	1.007	0.558
4	1.006	0.575	1.009	0.441

We choose the normalisation such that the first order moments ($k = 1$) are unity. The second column in Table 2 shows that the moments determined at the quark level from the Monte Carlo agree with the analytic QCD values P_k to better than 0.6%. The third column shows that effects below the scale μ on the single moment distribution are large, as expected. In JETSET such effects are due entirely to the phenomenological hadronisation scheme. However, in principle all of this hadronisation information is absorbed into the values of M_k, which can now be substituted into Eq. (5) to test the double moment QCD prediction.

We repeated this analysis for the case of charmed quarks, using the value 1.5 GeV for the quark mass in both analytic and Monte Carlo calculations. For the sake of completeness we give the results in Table 2. They are qualitatively very similar to the b-quark case, but we note that the hadronisation effects are somewhat larger (*i.e.* M_k^{hadron} is numerically smaller), as one would have naively expected since the c-quark typically carries a lower fraction of the beam energy than the b-quark.

Using the Monte Carlo datasets, the double moments $D_{kl}(\psi)$ (Eq. (5)) were calculated both at the quark and hadron levels. The values of M_k shown in Table 2 were used to derive the quantities $P_{kl}^{quark}(\psi)$, $P_{kl}^{hadron}(\psi)$. First we compare $P_{kl}^{quark}(\psi)$ with the analytic results for $P_{kl}(\psi)$ of Fig. 1. As an example, $P_{11}^{b-quark}(\psi)$ is shown in Fig. 1a and is in excellent agreement with $P_{11}(\psi)$ for b-quarks.* The agreement for all moments ($1 \leq k, l \leq 3$) is found to be similarly good.

Next we study the effects of hadronisation on the factorisation relation (5) by calculating the ratio $P_{kl}^{quark}(\psi)/P_{kl}^{hadron}(\psi)$. This quantity turns out to be qualitatively similar for all moments. We plot it for (k, l) = (1,1) in Fig. 2(a); deviations from unity are seen at the level of 10–15%, and vary smoothly with

* As the theoretical curve was calculated with full electroweak effects and the Monte Carlo data were generated with QED effects only we expect the two to differ by about 1%, which cannot be distinguished from the logarithmic plot in Fig. 1a.

ψ. It can be seen that hadronisation tends to shift the heavy hadrons to a more back-to-back configuration than was the case for the quarks. This may be understood simply as a consequence of the Lund string hadronisation scheme, in which particles are Lorentz-boosted towards the gluon jet in a $q\bar{q}g$ event; this 'string effect' is well-known experimentally.[17] In the transformation from heavy quarks to heavy hadrons, such changes in the angle ψ are not absorbed into the fragmentation moments M_k, which are constructed to take into account energy changes only. Hence we are not surprised that, within the context of this particular Monte Carlo model, the prediction (5) is violated by up to 15%.

We now investigate the normalised moments $P_{kl}(\psi)/P_{11}(\psi)$ (Fig. 1(b)) and calculate the double ratios:

$$\frac{[P_{kl}^{quark}(\psi)/P_{11}^{quark}(\psi)]}{[P_{kl}^{hadron}(\psi)/P_{11}^{hadron}(\psi)]} \quad \text{for } k,l > 1. \tag{11}$$

To the extent that $[P_{kl}^{quark}(\psi)/P_{kl}^{hadron}(\psi)]$ is the same for all k, l, one would expect the ratios (11) to be close to unity across the whole range of ψ. This is illustrated in Fig. 2(b,c) for the two cases $(k,l) =$ (2,1) and (3,3): the (2,1) ratio is indeed consistent with unity to better than a few per cent for all ψ; the (3,3) ratio deviation is of the order of 5%.

The normalised moments thus appear to be somewhat less sensitive to non-perturbative effects, which may make them more useful as quantities for testing perturbative QCD. In particular the normalised (2,1) and (3,1) moments show hadronisation effects smaller than 3%. This degree of insensitivity to hadronisation is certainly comparable with other commonly used variables such as jet ratios[18] or energy-energy correlations.[19]

3.2 Estimation of Higher Order Effects

It has been known for many years that at PETRA/PEP c.m. energies, $\sqrt{s} \simeq$ 30 GeV, effects are observable which can only be described by QCD when higher order terms beyond $\mathcal{O}(\alpha_s)$ in perturbation theory are taken into account. The most striking example is the observation[20] of events containing four (or more) hadronic jets, a topology which, by definition, is not predicted at $\mathcal{O}(\alpha_s)$. At the much higher SLC/LEP, and eventually LEP-2, energies, a QCD calculation to $\mathcal{O}(\alpha_s)$ will provide an even less satisfactory description of jet rates and event topologies. It is therefore interesting to consider the influence of higher order corrections to the $\mathcal{O}(\alpha_s)$ QCD calculations of the moments presented in Section 2.

In the absence of higher order calculations including quark mass effects, we utilise existing $\mathcal{O}(\alpha_s^2)$ and LLA + $\mathcal{O}(\alpha_s)$ calculations for massless quarks. Furthermore, we make use of the fact that these calculations are implemented in the JETSET Monte Carlo, which we use to generate events at the parton level. Hence the values of the QCD moments P_k and $P_{kl}(\psi)$ of Eqs. (3), (6) can be obtained from the Monte Carlo. Since we wish to study only the influence of higher orders on the moment calculations, we should, for the sake of consistency, set the quark masses to zero in the $\mathcal{O}(\alpha_s)$ matrix elements in the Monte Carlo* and repeat the calculations of the previous section. As we perform the $\mathcal{O}(\alpha_s)$ calculation down to the scale $\mu \simeq 13.8$ GeV, well above the b and c-quark masses, we do not expect a significant difference between the massive and massless cases. We verified that this indeed is the case.

We generated approximately 500 000 $e^+e^- \rightarrow b\bar{b}$ events and performed the moment calculations from the parton-level Monte Carlo data for each of the following cases:

(*i*) $\mathcal{O}(\alpha_s)$ matrix element, taking the quarks to be massless.

(*ii*) $\mathcal{O}(\alpha_s^2)$ matrix element**.

(*iii*) LLA + $\mathcal{O}(\alpha_s)$ 'parton shower'.**

Λ_{QCD} was fixed at 0.1 GeV in all cases; y_{cut} was fixed at 0.02 for (i) and (ii); for (iii) the parton virtuality cutoff Q_0[12] was set to 1.0 GeV, which corresponds to $\mu \simeq 5.0$ GeV. The results for the single moments P_k are shown in Table 3.

Table 3: Values of P_k (Eq. (3)) for b-quarks derived from Monte Carlo perturbative QCD calculations for massless quarks.

Order k	$\mathcal{O}(\alpha_s)$	$\mathcal{O}(\alpha_s^2)$	LLA + $\mathcal{O}(\alpha_s)$
2	0.924	0.916	0.841
3	0.877	0.864	0.748
4	0.842	0.826	0.684

From Table 3 one sees a tendency for the P_k to decrease as the order of the QCD calculation increases, which simply reflects the softening of the heavy quark momentum spectrum due to gluon emission. The relatively small difference between $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\alpha_s^2)$ results is due to the fact that with $y_{cut} = 0.02$ ($\mu \simeq$ 13.8 GeV), a second gluon is hard enough to be resolved in only 3% of events. By contrast, the LLA + $\mathcal{O}(\alpha_s)$ calculation evolves down to $\mu \simeq 5.0$ GeV and results in an average of 2.9 soft partons per event, so the P_k's are considerably lower.

In Fig. 3 we show the QCD moment $P_{11}(\psi)$, calculated using the Monte Carlo program at $\mathcal{O}(\alpha_s^2)$, divided by the same moment at $\mathcal{O}(\alpha_s)$. The higher order corrections tend to make the quark configuration less back-to-back. This is

* A parameter is provided for this purpose in the JETSET program.

** In these calculations a secondary pair of heavy quarks can be produced from the splitting of a massive gluon. For b-quarks we find this to occur in less than 0.1% of events and we do not include these secondary quarks in our moment calculations.

expected, as the two gluons at $\mathcal{O}(\alpha_s^2)$ tend to be radiated in the same hemisphere because of coherence.[21] Also given is the same ratio using the all-order leading logarithm Monte Carlo, which shows a somewhat larger correction (but at a smaller value of the factorisation scale $\mu \simeq 5$ GeV). For both of these cases we repeated the investigation of hadronisation effects, by calculating the moments at both quark and hadron levels, and found results similar to those in Section 3.1 for the $\mathcal{O}(\alpha_s)$ matrix elements. Overall, the higher order corrections thus tend to be of similar magnitude (10–20%) as the hadronisation effects.

We also investigated the dependence of the double moment predictions on the value of Λ_{QCD}. In the $\mathcal{O}(\alpha_s^2)$ case we varied $\Lambda_{\overline{MS}}$ from 0.1 to 0.5 GeV and found, for P_{11}, an increase of 35% in the region $-0.9 \leq \psi \leq 0.8$, which is considerably larger than the $\mathcal{O}(\alpha_s^2)$ corrections themselves. Thus α_s can be determined from the shape of the double moment distributions. Such measurements of α_s in heavy flavour events would be particularly useful as a test of the independence of the strong coupling on the quark flavour, for which there is currently little direct experimental evidence.[22]

4. Summary and Conclusions

We proposed and studied QCD predictions for heavy hadron momentum correlations in e^+e^- annihilations. As is true also of other QCD tests, such a study of the heavy quark sector is important, as it might reveal effects which are washed out when summing over all flavours. Using the factorisation between the heavy quark production and fragmentation distributions, and assuming that no heavy quarks are produced in the soft hadronisation process, we calculated the double inclusive heavy hadron distributions from the single inclusive ones and QCD perturbation theory. These predictions are model independent, and can be used as a test of QCD.

Corrections to our results can arise from higher orders in perturbation theory as well as from hadronisation effects, which are power suppressed in the hard scale. At the present time, full QCD calculations including quark mass effects are available only at $\mathcal{O}(\alpha_s)$. In principle, the hard scale may be chosen to be the mass of the heavy quark. At SLC/LEP energies, logarithms of the large ratio E_{cm}/m_Q can then make higher order terms significant. To avoid this we chose a relatively large factorisation scale $\mu \sim 14$ GeV.

We investigated the practical feasibility of our approach, as well as the magnitude of the corrections, using a QCD Monte Carlo program. The experimental difficulties in reconstructing the heavy hadron momenta were not addressed, as they are detector dependent. An accurate (10%) test of the correlations in all momentum configurations requires on the order of 10^5 heavy hadron events. Configurations in which the heavy hadrons are more nearly back-to-back are more frequent, however, and can be studied with less statistics.

We found deviations from the predicted momentum correlations at the level of 10–15% in the Monte Carlo heavy hadron 'data'. Since the program used the same $\mathcal{O}(\alpha_s)$ matrix element as in the perturbation theory calculation, this deviation was found to be due to hadronisation effects, *i.e.* to the Lund string fragmentation model employed. We note, however, that normalising by the lowest order moment reduces the sensitivity to these effects, which practically vanish completely in the case of the normalised (2,1) and (3,1) moments, making these quantities especially reliable for testing our predictions. We also studied higher order perturbative effects by using an $\mathcal{O}(\alpha_s^2)$ matrix element Monte Carlo, as well as an all-order shower program. These simulations were restricted to massless quarks, but at the relatively high factorisation scale we used the mass effect is not very important. For the double moments we found the higher order perturbative corrections to be at the level of 10%. The shape of the double moment distributions is also sensitive to the value of Λ_{QCD} and could therefore be used to measure α_s experimentally.

In conclusion, we note that the method presented for testing QCD in heavy quark production seems to be practical, assuming that a sufficient number of heavy hadron pair momenta can be reconstructed in the data. The influence of higher order perturbative and non-perturbative corrections appears moderate, on a par with those encountered, *e.g.* in energy-energy correlations. Higher order perturbative calculations, including quark mass effects, would make the predictions more accurate.

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FIGURE CAPTIONS

Fig. 1: (a) The $\mathcal{O}(\alpha_s)$ perturbative QCD calculation of the double moment $P_{11}(\psi)$ for b-quarks (solid curve) and c-quarks (dashed curve). Also shown (dotted histogram) is the quark-level JETSET Monte Carlo calculation to $\mathcal{O}(\alpha_s)$ for massive b-quarks.

(b) The $\mathcal{O}(\alpha_s)$ perturbative QCD calculations of the normalised double moments $P_{kl}(\psi)/P_{11}(\psi)$ $(k, l > 1)$ for b-quarks (solid curve) and c-quarks (dashed curve).

Fig. 2: JETSET Monte Carlo calculations to $\mathcal{O}(\alpha_s)$ for massive b-quarks comparing results at the quark and hadron levels. The ratios shown would equal unity if the factorisation relation (5) were exactly valid.

(a) Solid histogram: the ratio of double moments $P_{11}(\psi)^{quark}/P_{11}(\psi)^{hadron}$.

(b) Dashed histogram: the ratio of normalised double moments $[P_{21}(\psi)/P_{11}(\psi)]^{quark}/[P_{21}(\psi)/P_{11}(\psi)]^{hadron}$.

(c) Dotted histogram: the ratio of normalised double moments $[P_{33}(\psi)/P_{11}(\psi)]^{quark}/[P_{33}(\psi)/P_{11}(\psi)]^{hadron}$.

Fig. 3: Monte Carlo QCD calculations of higher order corrections to the $\mathcal{O}(\alpha_s)$ predictions for the double moments $P_{11}(\psi)$; all calculations are for massless quarks. Solid curve: ratio of $\mathcal{O}(\alpha_s^2)$ to $\mathcal{O}(\alpha_s)$ calculations. Dashed curve: ratio of LLA + $\mathcal{O}(\alpha_s)$ to $\mathcal{O}(\alpha_s)$ calculations.

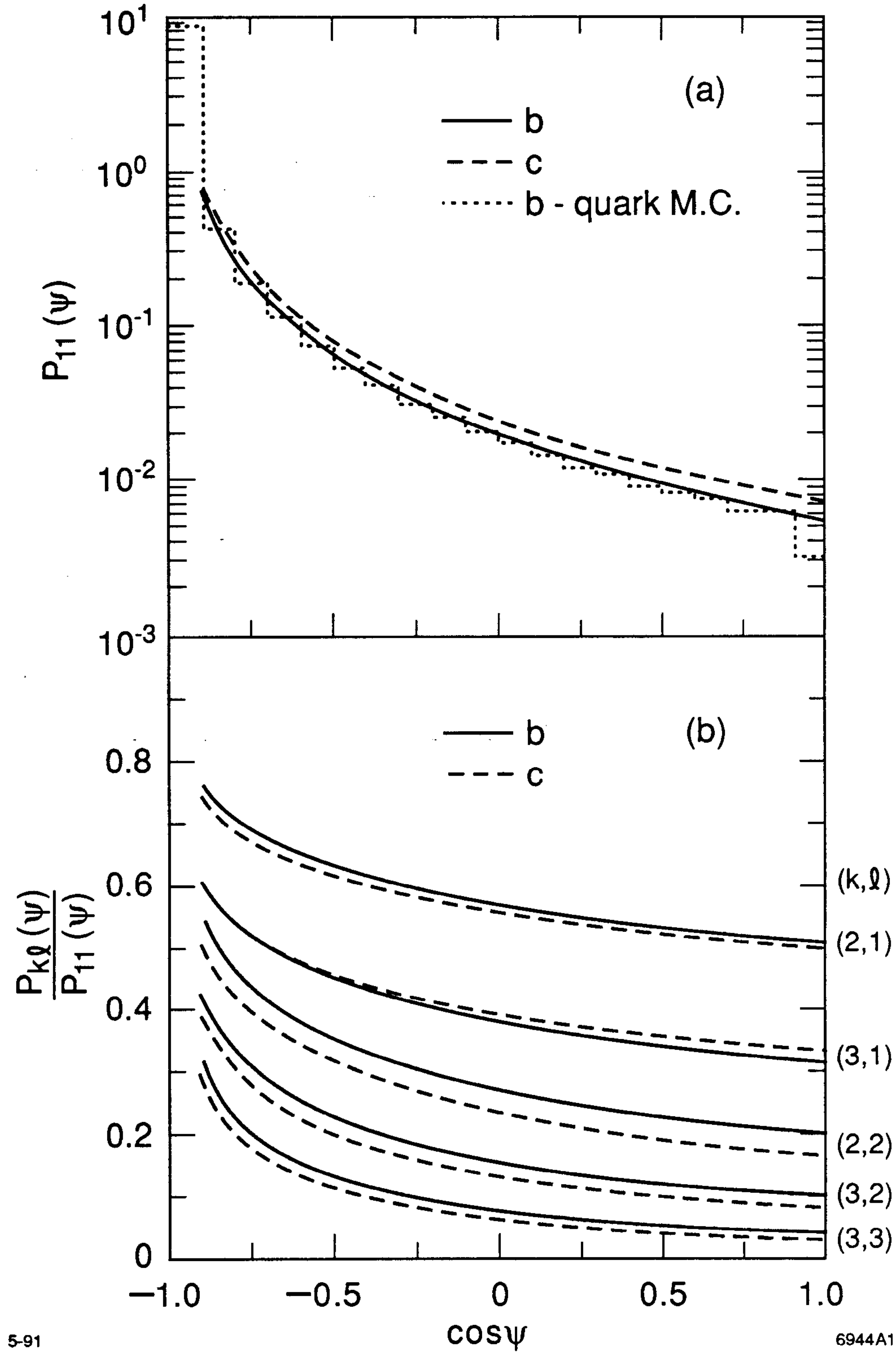

(a)
b
c
b - quark M.C.
$P_{11}(\psi)$
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
(b)
b
c
$\frac{P_{k\ell}(\psi)}{P_{11}(\psi)}$
(k,ℓ)
(2,1)
(3,1)
(2,2)
(3,2)
(3,3)
0.8
0.6
0.4
0.2
0
−1.0
−0.5
0
0.5
1.0
cosψ
5-91
6944A1

Figure 1

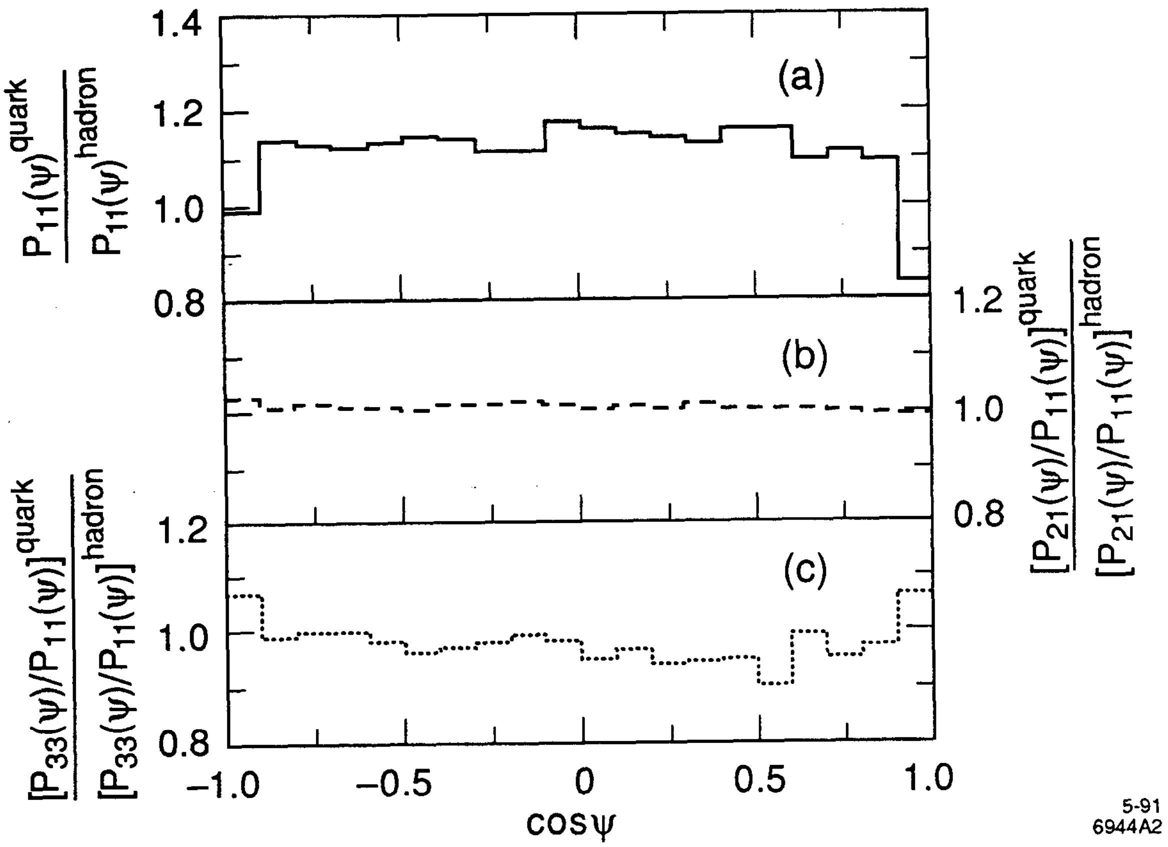
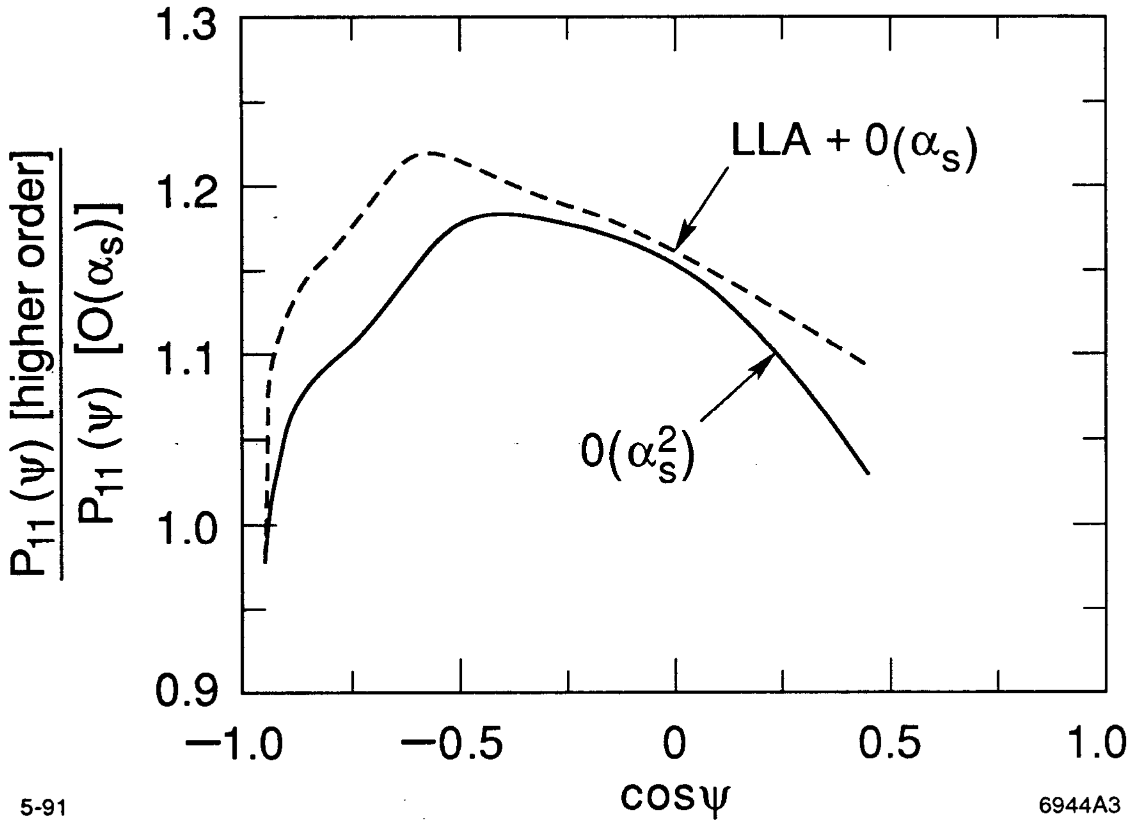


Figure 2



1.3
1.2
1.1
1.0
0.9
−1.0
−0.5
0
0.5
1.0
cosψ
P11 (ψ) [higher order]
P11 (ψ) [O(αs)]
LLA + 0(αs)
0(αs2)
5-91
6944A3

Figure 3