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Strange Form Factors, the σ-Term and Strange Quark Distributions in the Gross–Neveu Model

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ABSTRACT

For $N_c \rightarrow \infty$ the structure function of physical fermions in the Gross-Neveu model is of pure valence type. Nevertheless several strange form factors of u-quarks, including the induced magnetic moment and the σ-term acquire substantial $\mathcal{O}(N_c^0)$ contributions from virtual $\bar{s}s$ pairs. Those arise from matrix elements which are off-diagonal in Fock space, *i.e.* they do not have a simple interpretation in terms of the quark distributions. It is shown that, under certain circumstances, a similar behavior in QCD_{3+1} is conceivable.

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Recently, there has been a growing discussion about the "strangeness content" of the nucleon (for a review, see e.g. Ref[1]). Although $\bar{s}s$ pairs carry less than 5% of the nucleon momentum[2] it has been suggested that they contribute significantly to the nucleon spin[#1] and almost cancel the spin carried by u and d quarks. Another surprising result that emerged from an analysis of the low energy πN forward scattering amplitude[3] seems to require a large value for the strange scalar "charge" or σ-term $m_s \langle N \,|\, \bar{s}s \,|\, N \rangle$ of the nucleon.

In this note I will show that a large strange σ-term together with small $\bar{s}s$ structure functions does not necessarily mean a contradiction. Although this will be demonstrated on the basis of the Gross-Neveu model[4] in 1+1 dimensions, I will argue that *a priori* something similar might happen in QCD_{3+1}. Based on the observation that $\bar{s}s$ is twist-3 whereas structure functions are measured by twist-2 operators it has been already emphasized that such a behaviour is conceivable.[5] The example to be studied here will help to understand the underlying physics of large, strange σ-terms combined with a suppression of strange quarks in the structure function.

In some simple field theories (like free fermions or QCD_{1+1}) $m_s \langle N \,|\, \bar{s}s \,|\, N \rangle$ has a direct parton model interpretation, *i.e.* one can express the strange scalar form factor in the limit of zero momentum transfer (the scalar charge) in terms of the strange quark distribution functions $s(x_{bj})$ and $\bar{s}(x_{bj})$. In these theories the constraint equation for the left-handed fields takes the form

$$\sqrt{2}\, i\, \partial^+ \psi_L = m \psi_R \tag{1}$$

#1 This interpretation of the EMCII-effect is still controversial.

i.e. it is linear and thus

$$\bar{s}s = s_L^+ s_R + s_R^+ s_L = \sqrt{2}\, m\, s_R^+ (i\partial^+)^{-1} s_R \ . \tag{2}$$

For vanishing q^+ (q^μ being the momentum transfered by the $\bar{s}s$ insertion), where pair creation is suppressed at the vertex, the matrix elements of $\bar{s}s$ become diagonal in Fock space yielding[6]

$$m\, \langle N \,|\, \bar{s}s \,|\, N \rangle = \sqrt{2}\, m^2 \int_0^1 \frac{dx}{x} \left(s(x) + \bar{s}(x) \right) . \tag{3}$$

For these simple theories a large strange σ-term will in general imply a large strange quark distribution function and vice versa.[#2] In most field theories the simple constraint equation (1) will be replaced by more complicated relations. For example in gauge theories in more than two dimensions

$$\sqrt{2}\, i\, \partial^+ \psi_- = \left[\alpha_\perp \cdot (i\partial_\perp - gA_\perp) + \gamma^0 m \right] \, \psi_+ \tag{4}$$

or in Yukawa theories

$$\sqrt{2}\, i\, \partial^+ \psi_- = \left[i\alpha_\perp \cdot \partial_\perp + \gamma^0 \left(m - g_s \phi_s - i\gamma_5 g_p \phi_p \right) \right] \, \psi_+ \tag{5}$$

(where $\psi_\pm = \Lambda_\pm \psi = \frac{\gamma^0 \gamma^\pm}{\sqrt{2}} \psi$ and ϕ_s/ϕ_p are scalar/pseudoscalar fields). Here $\bar{\psi}\psi$ contains interaction terms (cubic in the fields) which in general implies for the matrix elements—even for zero p^+ transfer—contributions from off-diagonal terms in Fock space.[7] This makes it rather difficult (if not impossible) to relate $\langle N \,|\, \bar{s}s \,|\, N \rangle$ to the strange quark distribution functions.

#2 Here possible divergences for $x \to 0$ are neglected since, for $m \neq 0$, these do not occur in QCD_{1+1} or for free fermions.

As an example for the latter case (nonlinear relation between ψ_+ and ψ_-) we will consider a generalized Gross-Neveu model[4] described in the Lagrangian

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m_0)\psi + \frac{g^2}{2N_c}\left[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\tau\psi)^2\right] \tag{6}$$

where ψ is a N_c-spinor in color space and a two-spinor in flavor space. For $m_0 = 0$ this Lagrangian is invariant under[#3] $U_V(1) \times SU_L(2) \times SU_R(2)$ transformations—a symmetry which is, for $N_c \to \infty$, spontaneously broken down to $U_V(2)$. The model is asymptotically free (important if we want to "measure" deep inelastic structure functions) and describes non-confined fermions. We will use the model to develop some intuitive understanding about the virtual $q\bar{q}$ cloud surrounding valence quarks in strong interactions. Besides being chirally symmetric one major advantage of the Gross-Neveu model, compared to conventional pion or kaon cloud models[8], is a consistent description of meson and quark degrees of freedom. In meson cloud models the mesons appear as elementary particles with substructure, i.e. one faces the possibility of double counting in higher loop corrections. Although there is a pion in the Gross-Neveu model it appears as a quark-antiquark bound state rather than an elementary particle. Since the Gross-Neveu model can be formulated entirely in terms of quark degrees of freedom the problem of double counting is avoided.

It should be emphasized that the term "color" in the Gross-Neveu model has *a priori* little to do with the color in QCD. In particular, the $1/N_c$ expansion used here might give rise to results which are totally different in a $1/N_c$ expansion in QCD. Here we consider the $1/N_c$-expansion only as an ordering principle for the perturbation series. The term "color" is used only for convenience.

#3 In addition to the $SU_V(N_c)$ symmetry.

In the following we will label the two flavor states up (u) and strange (s); *i.e.* for the sake of simplicity, down quarks will be omitted from the discussion. An extension of the model to three or more flavors is possible and straightforward[4] though this would require the 't Hooft determinantal interaction[9] to break $U_A(1)$. Furthermore u and s quarks will be assumed to be degenerate. The main conclusion in this work, which are anyway qualitative, would be the same in a scenario with explicitly broken flavor symmetry and/or more flavors.

All observables will be computed in the framework of a $1/N_c$-expansion up to the first nontrivial contributions from s-quarks.

Let us consider the structure function of a physical (= dressed) up quark. At $\mathcal{O}(N_c^0)$ the only modifications of the quark propagator arise from tadpole graphs. (Fig. 1). Those diagrams do not give any contribution to structure functions for $x_{bj} \neq 0$, since in Pauli-Villars regularization[10]

$$\int dp^- \frac{1}{p^2 - m^2} = 0 \qquad (p^+ \neq 0) \ . \tag{7}$$

Thus the physical structure functions of an up quark read to $\mathcal{O}(N_c^0)$

$$\begin{aligned} u(x) &= \delta(x-1) \\ s(x) &= \bar{s}(x) = \bar{u}(x) = 0 \ . \end{aligned} \tag{8}$$

To order N_c^{-1} the fermions are dressed by chains of bubbles (Fig. 2). Canonical quantization on a x^+ = const. surface (*i.e.* so-called light-cone or light-front quantization) yields to $\mathcal{O}(N_c^{-1})$ (=leading reder in N_c^{-1}) for the structure functions

$$s(x) = N_c \int_0^{1-x} dy \, |\psi_{s\bar{s}u}(x,y)|^2$$
$$\bar{s}(y) = N_c \int_0^{1-y} dx \, |\psi_{s\bar{s}u}(x,y)|^2 \tag{9}$$

where

$$\psi_{s\bar{s}u}(x,y) = \frac{1}{N_c} \frac{D_\eta(q^2)\left(1-\frac{1}{x}\right)\left(\frac{1}{y}+\frac{1}{1-x-y}\right) - D_\sigma(q^2)\left(1+\frac{1}{x}\right)\left(\frac{1}{y}-\frac{1}{1-x-y}\right)}{1-\frac{1}{x}-\frac{1}{y}-\frac{1}{1-x-y}} \tag{10}$$

is the wave function of the $s\bar{s}u$-states. Here

$$q^2 = -M_F^2 \frac{(1-x)^2}{x} \tag{11}$$

where M_F is the physical fermion mass.#4 The functions D_η and D_σ act like form factors in the pseudoscalar and scalar channel. They arise from summing up the infinite chain of $q\bar{q}$ bubbles and can be interpreted as effective meson propagators. Explicitly one finds[4]

$$D_\eta^{-1}(q^2) = \mu_\eta^2 \int_0^1 dz \, \frac{1}{M_F^2 - z(1-z)\mu_\eta^2} - q^2 \int_0^1 dz \, \frac{1}{M_F^2 - z(1-z)q^2} \tag{12}$$

$$D_\sigma^{-1}(q^2) = \mu_\eta^2 \int_0^1 dz \, \frac{1}{M_F^2 - z(1-z)\mu_\eta^2} + (4M_F^2 - q^2) \int_0^1 dz \, \frac{1}{M_F^2 - z(1-z)q^2} \, . \tag{13}$$

Here μ_η is the physical mass of the pseudoscalar $\bar{u}s$ meson (which is, besides M_F,

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#4 Note that we have already made use of the fact that $s(x)$ and $\bar{s}(x)$ are independent of the total p^+ of the fermion when we expressed the results in terms of wavefunctions which have total $p^+ = 1$.

the only free parameter in the renormalized theory). Note that (for $\mu_\eta^2 > 0$) there is no bound state in the scalar channel.

The other type of observables we are interested in are the various strange form factors[11]

$$\langle p' | \bar{s}\gamma^\mu s | p \rangle = \bar{u}(p') \left[\gamma^\mu F_1^{\bar{s}s}(q^2) + \frac{i}{2M_F} \sigma^{\mu\nu} q_\nu F_2^{\bar{s}s}(q^2) \right] u(p) \tag{14}$$

$$\widehat{m}_s \langle p' | \bar{s}s | p \rangle = \bar{u}(p')\, u(p)\, F_s^{\bar{s}s}(q^2) \,, \tag{15}$$

where $\widehat{m}_s$ is the current quark mass defined via $q^\mu \bar{\psi}\tau_z\gamma_\mu\gamma_5\psi = -2\widehat{m}_s\bar{\psi}\tau_z\gamma_5\psi$.#5

Most surprisingly the Pauli and scalar form factors have nonzero $\mathcal{O}(N_c^0)$ contributions

$$F_2^{\bar{s}s}(q^2) = 2M_F^2\, D_\eta(q^2) \tag{16}$$

$$F_s^{\bar{s}s}(q^2) = \mu_\eta^2 \frac{M_F}{2} \int_0^1 dx \frac{1}{M_F^2 - x(1-x)\mu_\eta^2} D_\sigma(q^2) = \frac{M_F}{2} D_\eta^{-1}(0)\, D_\sigma(q^2) \,. \tag{17}$$

There are no contributions to $F_1(q^2)$ from $\bar{s}s$ pairs to this order in N_c^{-1}

$$F_1^{\bar{s}s}(q^2) = 0. \tag{18}$$

Note that F_2 and F_s have a nonzero $\mathcal{O}(N_c^0)$ contribution even for $q^2 = 2q^+q^- = 0$. Thus, even though $s\bar{s}$ pairs are not present in the structure function to $\mathcal{O}(N_c^0)$ (see eqs.(9)(10)), neither the strange σ-term nor the induced $\sigma_{\mu\nu}$-coupling show this $1/N_c$-suppression.

#5 Note that the matrix elements of $\bar{\psi}\psi$ as well as those of $\bar{\psi}\gamma_5\psi$ are cutoff dependent as is $\widehat{m}_s$. The leading cutoff dependence of $\bar{\psi}\psi$ and $\bar{\psi}\gamma_5\psi$ are the same. Since the matrix elements of $\widehat{m}_s\bar{\psi}\tau_z\gamma_5\psi$ are by definition finite, the same will be true for the matrix elements of $\widehat{m}_s\bar{\psi}\psi$.

In order to understand the different powers of $1/N_c$ we will evaluate the N_c-dependence of form factors and structure functions in ordinary perturbation theory (Fig. 3). A measurement of the quark distribution always involves only such operators which are diagonal in Fock space (no pair creation at the insertion of the external probe = "cross"). Thus the $\bar{s}s$ pair in Fig. 3a cannot be annihilated by the measurement but only through a subsequent interaction. Therefore $\bar{s}(x)$ and $s(x)$ are of order $\alpha^2 \cdot N_c$ (the factor N_c arises from the quark loop) where $\alpha = g^2/N_c$;[#6] *i.e.* or order $1/N_c$.

Form factor measurements are quite different in this respect. If the operator is inserted with $q^+ \neq 0$ (which is necessary in 1+1 dimensions if one wants to have $q^2 = 2q^+q^- \neq 0$) terms diagonal and off-diagonal in Fock space contribute (Fig.3b). Although the latter usually vanish for F_1 for $q^+ \to 0$ in general they do not vanish for F_2 or F_s because these involve so-called bad currents.[12]

The diagonal terms (Fig. 3c) are also of order $1/N_c$ (like the quark distribution) but the off-diagonal terms do not need another interaction to create the $s\bar{s}$ pair (it is created at the insertion). Thus the form factors are (only the leading contribution is considered) of the order $\alpha \cdot N_c$, *i.e.* of $\mathcal{O}(N_c^0)$.

One can express this result also in a different way. The $\bar{s}s$ component of a u-quark wave function is of the order $1/N_c$. Thus operators which are diagonal in Fock space and measure the $\bar{s}s$-content will be of the order $N_c|\psi_{s\bar{s}u}|^2 \sim 1/N_c$ (there is always an extra N_c multiplying the matrix element because of the sum over the colors of the s-quarks). In contrast off-diagonal operators are linear in

#6 In perturbation theory one chooses g independent of N_c, in order to have finite $q\bar{q}$ scattering amplitudes for $N_c \to \infty$.

$\psi_{s\bar{s}u}$, *i.e.* of the order $N_c \cdot \psi_{s\bar{s}u} \sim 1$.[#7]

The main question arising at this point is whether there is a similar effect possible in QCD_{3+1}.[#8] The answer is in principle yes although the importance of such an effect is not obvious. A light-cone expansion of the matrix elements of the operator $\bar{s}s$, very similar to the expansion of the matrix element of $g_2(x)$ in Ref.[13] , will (even for $q^+ = 0$) contain operator products of the form $b_s d_s a^+$ (Fig. 4) which annihilate a $s\bar{s}$ pair at the $\bar{s}s$ insertion and replace it by a gluon. Now let us imagine a situation where there is a large gluonic admixture to the proton wavefunction but a small $s\bar{s}$ admixture[#9]

$$|\psi_{qqq}| \approx |\psi_{qqqg}| \gg |\psi_{qqq\bar{s}s}| \ . \tag{19}$$

In this case the main contributions to $\sigma_s = \widehat{m}_s \langle N \,|\, \bar{s}s \,|\, N\rangle$ will come from the off-diagonal terms, *i.e.*

$$\frac{\sigma_s}{m_s} \propto |\psi_{qqqg} \cdot \psi_{qqq\bar{s}s}| \gg s(x) \propto |\psi_{qqq\bar{s}s}|^2 \ . \tag{20}$$

The explicit Fock space expansion of these matrix elements is quite lengthy, since this involves twist three operators and will be omitted here. It could be obtained by differentiating the light-cone Hamiltonian of QCD.[14] The typical structure of such an expansion would be similar to the expansion of $g_2(x)$ in Ref. 13.

#7 A quite similar pattern emerges in $\mathrm{QCD}_{1+1}(N_c \to \infty)$ where to $\mathcal{O}(N_c^0)$ the structure function of mesons is pure valence $\bar{q}q$ while form factors are nevertheless modified by "vector meson dominance"-type corrections of order N_c^0.[10] Intuitively one attributes such kinds of corrections with a $q\bar{q}$-cloud around the valence quark which is in this case a questionable interpretation. However, there are no flavor mixing effects to leading order $1/N_c$ in QCD_{1+1}.

#8 Here we neither assume $N_c \to \infty$ nor do we associate N_c in QCD_{3+1} with the value of N_c in the Gross-Neveu model above.

#9 This picture is probable not too far from reality.

In the above discussion we concentrated on the scalar form factor (15). In the Gross-Neveu model similar results and conclusion hold for the magnetic form factor $F_2^{\bar{s}s}$ (Eq. (14)). However, for the magnetic moment of quarks in QCD_{3+1} such contributions from off-diagonal Fock space seem to vanish for $q^+ \to 0$ if one uses a γ^+ insertion, i.e. a good current[12] , to measure F_2.

From the example studied here one should learn several lessons. First of all the question "is there a lot of strangeness in the proton" is *a priori* not well posed. The operator used to measure the content of strangeness might be crucial. Although our $N_c \to \infty$ limit is somewhat extreme, a similar behavior is certainly possible in QCD_{3+1}. Secondly, although many operators simplify in the light-cone analysis for $q^+ \to 0$ there can still be a complicated operator left over in this limit. In particular, pair creation and/or many-body effects might be important.—or even dominant. It is however not yet clear how important such effects are in QCD_{3+1}. If they are important this would mean that many intuitive pictures, like the σ-term of a hadron originating from an intrinsic admixture of $\bar{s}s$ pairs, have to be modified.

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Figure Caption

*Fig.*1 Typical $\mathcal{O}(N_c^0)$ contributions to the propagator of a u-quark.

*Fig.*2 Typical $\mathcal{O}(N_c^{-1})$ contribution to the u-quark propagator.

*Fig.*3 Perturbative (light-cone time ordered) diagrams contributing to a.) the strange structure function (only the contribution to the $\bar{s}$-distribution is shown), and b.), c.) the formfactors. In c.) the external photon could also be attached to the s-quark.

*Fig.*4 Typical diagram yielding a contribution to the strange sigma term in a nucleon via terms which are off-diagonal in Fock-space.

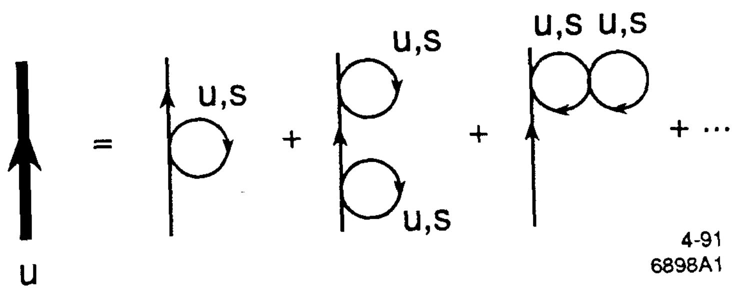

Fig. 1

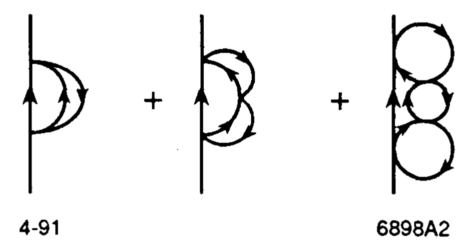

Fig. 2

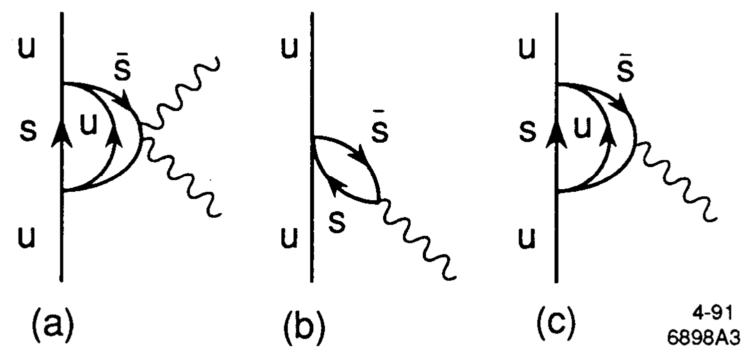

Fig. 3

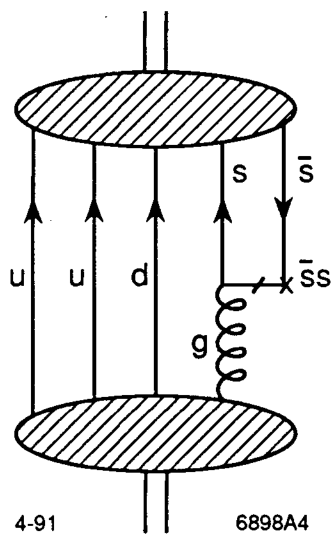

Fig. 4