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SHIELDED COHERENT SYNCHROTRON RADIATION AND ITS POSSIBLE EFFECT IN THE NEXT LINEAR COLLIDER*

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Abstract

Shielded coherent synchrotron radiation is discussed in two cases: (1) a beam following a curved path in a plane midway between two parallel, perfectly conducting plates, and (2) a beam circulating in a toroidal chamber with resistive walls. Wake fields and the radiated energy are computed with parameters for the high-energy bunch compressor of the Next Linear Collider (NLC).

I. INTRODUCTION

We consider a bunch of particles following a curved path. On first sight, one would expect the particles to radiate coherently at wavelengths comparable to the bunch size or larger. A closer examination reveals that the long-wavelength part of the synchrotron spectrum will be strongly suppressed in the presence of shielding, for instance the conducting surfaces of the vacuum chamber in an accelerator. This long-wavelength cutoff is not the usual wave-guide cutoff, comparable to the pipe diameter, but has a value that depends on the radius of curvature of the orbit as well as the pipe diameter, and is typically much smaller than the wave-guide cutoff. In fact, appreciable radiation under shielding is expected only in an interval of $\lambdabar = \lambda/2\pi$ given by $\lambdabar_0 \leq \lambdabar \leq \lambdabar_1$, where $\lambdabar_0$ is near the longitudinal bunch length σ, and

$$\lambdabar_1 \approx \frac{h}{\pi}\left(\frac{h}{R}\right)^{1/2} \quad , \tag{1}$$

where R is the orbit radius and h is the transverse size of the chamber (see Refs. [1] and [2]). It follows that shielding can be important in typical experimental situations. In electron storage rings one usually has $\lambdabar_1 < \lambdabar_0$, so that the shielding totally suppresses coherent radiation. Using a short bunch from a linac, and an apparatus with small R and large h, Nakazato et al. [3], observed coherent radiation. They verified a quadratic dependence on the number N of particles in the bunch, and checked coherence by an interference experiment. This observation is quantitatively consistent with the theory of shielding, as is the failure to see coherent radiation in earlier experiments at storage rings.

In this paper we give a brief review of the theory of shielding and an application to the NLC. Further details can be found in Refs. [4] and [2].

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2. IMPEDANCE, ENERGY LOSS, AND WAKE FIELD

We work in cylindrical coordinates (r,θ,z) and treat a rigid bunch with centroid moving on a circular orbit in the plane $z=0$. We are actually interested in more general orbits, for instance straight paths joined to arcs of circles, and a bunch that gradually changes shape. These situations will be treated in more detail in later work; for the present we use provisional approximations derived from the simpler theory. The charge density is assumed to have the form

$$\rho(r,\theta,z,t) = q\lambda(\theta-\omega_o t)f(r,z) \quad , \tag{2}$$

where ω_o is the revolution frequency and

$$\int_0^{2\pi}\lambda(\theta)d\theta = 1 \quad , \qquad \int r dr \int dz f(r,z) = 1 \quad . \tag{3}$$

The corresponding current density is $J_\theta = \omega_o r\rho$. The assumption of coherence enters at this point, since the atomic nature of the beam is ignored, and the continuously distributed charge radiates as a whole.

Since we are interested primarily in longitudinal effects, we make an average of the longitudinal electric field over r and z, weighted with the transverse charge distribution of the beam. We work with a Fourier analysis of the averaged field,

$$\begin{aligned} E_\theta(\theta,t) &= \int r dr \int dz\ f(r,z)\ E_\theta(r,\theta,z,t) \\ &= \int_{-\infty}^{\infty} d\omega\ e^{-i\omega t}\sum_{n=-\infty}^{\infty} e^{in\theta}\ E_\theta(n,\omega) \quad . \end{aligned} \tag{4}$$

The Fourier transform of the line density $\lambda(\theta-\omega_o t)$ is $\lambda_n\delta(\omega-\omega_o n)$, and the corresponding transform of the current is

$$\begin{aligned} I_\theta(n,\omega) &= \int dr \int dz\ J_\theta(r,n,z,\omega) \\ &= \omega_o q\ \lambda_n\ \delta(\omega-\omega_o n) \quad . \end{aligned} \tag{5}$$

If the environment of the beam has no longitudinal inhomogeneity, as in the models treated below, then there exists a complex function $Z(n,\omega)$, the longitudinal coupling impedance, such that

$$-2\pi R\, E_\theta(n,\omega) = Z(n,\omega)\ I_\theta(n,\omega) \quad . \tag{6}$$

This impedance accounts both for curvature of the orbit and for conductors in the environment of the beam.

If E_θ is the field due to the bunch, including effects of nearby conductors, then the radiated power is

$$\frac{dU}{dt} = -(q\omega_o)^2 \sum_{n=-\infty}^{\infty} |\lambda_n|^2\ \mathrm{Re}\ Z(n,n\omega_o) \quad . \tag{7}$$

This result is obtained by computing the work done per unit time on an infinitesimal element of charge by the field E_θ, then integrating over all elements [4].

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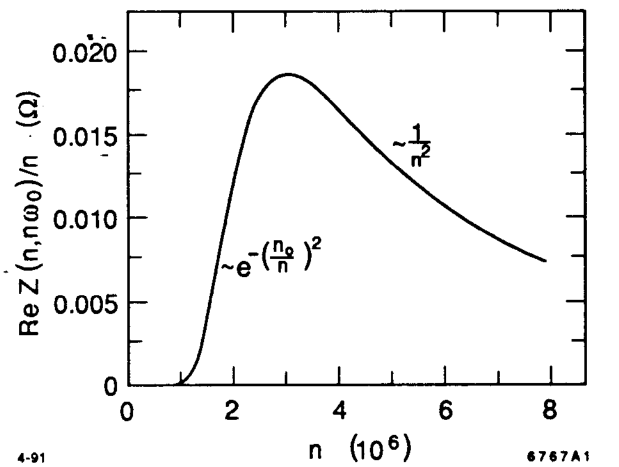

Figure 1. $\mathrm{Re}\,Z(n,n\omega_0)/n$ versus n for the parallel-plate model, $R = 149.5$ m, $h = 2$ cm.

The wake field $E_\theta[\omega_o(t+\tau),t]$ is defined as the field on the trajectory at an angular distance $\omega_o\tau$ from the bunch center. We take τ to be positive for a point in front of the bunch. The wake field is expressed by

$$\begin{aligned}\mathcal{V}(\omega_o\tau) &= -2\pi R\, E_\theta[\omega_o(t+\tau),t] \\ &= \omega_o q \sum_{n=-\infty}^{\infty} \exp[in\omega_o\tau]\ \lambda_n\ Z(n,n\omega_o)\ .\end{aligned} \quad (8)$$

The function $\mathcal{V}$ is the *wake voltage per turn*. A positive value of $\mathcal{V}(\omega_o\tau)$ means energy loss by particles at a distance $\omega_o\tau$ from the center of the bunch.

3. COUPLING IMPEDANCES FOR TWO MODELS

In order to state the impedance in a convenient analytic form, we choose a simple transverse beam profile,

$$f(r,z) = W(r)H(z)\ , \qquad W(r) = \delta(r-R)/R\ , \quad (9)$$

where $H(z)$ is a rectangular step function of width δh, symmetric about $z = 0$. More general beam profiles can be accomodated without much difficulty.

In our first model we have infinite, parallel, perfectly conducting plates, separated by a distance $h = 2g$. The beam circulates in the median plane between the plates. The impedance is computed by using a Fourier analysis in z to meet boundary conditions on the plates. The result is expressed in terms of Bessel functions of high order, that must be evaluated by appropriate asymptotic expansions [4,2]. Using the Olver expansions to lowest order, we find

$$\frac{\mathrm{Re}\,Z(n,n\omega_o)}{n} = 2Z_o\left[\frac{\pi R}{hn}\right]^2 \exp\left[-\frac{2}{3n^2}\left(\frac{\pi R}{h}\right)^3\right]\ , \quad (10)$$

the value being in ohms with $Z_o = 120\pi\ \Omega$; the bending radius is R. Here only the dominant term (axial mode number $p = 1$) has been included, and the vertical size of the beam is much less than h. This simple formula, which seems not to have been noticed previously, gives an adequate representation of the exact result; a plot of the latter is shown in Fig. 1.

The maximum value of expression (10) is

$$\frac{720}{e}\,\frac{g}{R} \approx 265\,\frac{g}{R}\ , \quad (11)$$

in agreement with Faltens and Laslett [1]. The maximum occurs at $n = \pi 2^{1/2}(R/h)^{3/2}$, and the function cuts off exponentially below the threshold $n_o = \pi(R/h)^{3/2}$. Expressed in terms of wavelength, this threshold is $\lambdabar_1$ of Eq. (1).

Table 1. Energy losses for four versions of the NLC bunch compressor.

Version	1	2	3	4
R (m)	84.218	213.6	149.5	106.8
$\Delta\theta$ (degrees)	10.89	180	180	180
σ_i (10^{-6} m)	460	460	460	460
σ_f (10^{-6} m)	37	86	61	44
ΔU plates (MeV)	0.176	0.164	0.827	2.08
ΔU torus (MeV)	0.123	0.0121	0.337	1.30
ΔU incoherent (MeV)	2.19	14.2	20.4	28.5

In the second model the vacuum chamber is a torus of rectangular cross section. The cross section has height $h = 2g$ and width w. In this model we take the walls to be resistive. The expression for the impedance is given in Refs. [2] and [4]. Unlike the parallel plates, this closed chamber has resonances, and the impedance is negligible at frequencies below the lowest mode that is synchronous with the beam. This mode occurs at a value of n roughly equal to (actually, somewhat higher than) the threshold n_o for the parallel plate model.

4. NUMERICAL EXAMPLES OF WAKE FIELD AND LOSS

For numerical calculations, we take a Gaussian bunch of rms length σ; its spectral density, expressed as a function of $\lambdabar$, is

$$|\lambda_n|^2 = \frac{1}{(2\pi)^2}\exp\left[-\left(\frac{\sigma}{\lambdabar}\right)^2\right]\ . \quad (12)$$

For the parallel plate model, the spectral density of radiated power will be proportional to the product of Eq. (12) and Re Z from Eq. (10).

We illustrate with values of σ and R from four different conceptual designs for the NLC high-energy (16.2 GeV) bunch compressor [5]. Table I shows the compressor parameters, including the initial and final bunch lengths, σ_i and σ_f, and the total deflection angle $\Delta\theta$ of the compressor arc. We take $h = 2$ cm for the parallel plate model, and $h = w = 2$ cm for the toroidal model. The walls of the torus have the conductivity of aluminum.

To estimate the total energy loss in the arc, we assume that $dU/d\theta$, the instantaneous energy loss per unit angle of deflection at a particular bunch length, is the same as in our steady-state model running at the same (but fixed) bunch length. Repeating the steady-state calculation for many bunch lengths, we find a curve of $dU/d\theta$ versus σ. Since σ decreases in the compressor almost linearly with θ, this is equivalent to knowing $dU/d\theta$ as a function of θ. Integration with respect to θ produces the figures for total energy loss shown in Table 1. The values are in MeV per particle, supposing that the bunch contains $N = 2\times 10^{10}$ electrons. For comparison, the values of the incoherent radiated energy are listed. For that, we assume that $dU/d\theta$ for each electron has the well-known value for steady state rotation on a full circle.

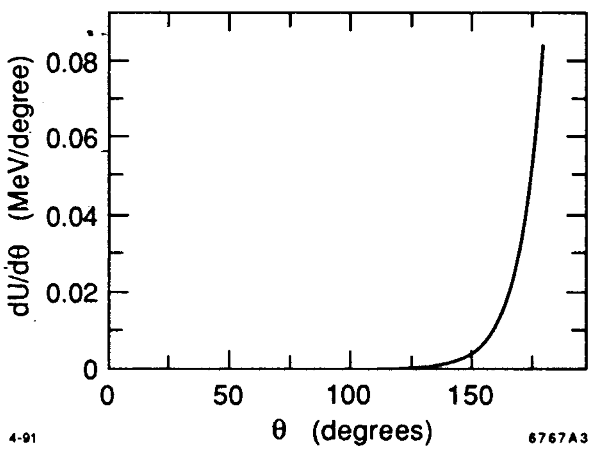

Figure 2. $dU/d\theta$ for the parallel-plate model in Version 3 of the bunch compressor..

The curve of $dU/d\theta$ for Version 3 of the compressor is shown in Fig. 2, for the parallel-plate model. In all four versions, the energy loss is sharply concentrated near the end of the arc, since elsewhere the impedance is too small at wavelengths within the bunch spectrum $|\lambda_n|^2$. The corresponding curve for the resistive toroidal chamber has a similar form, but is concentrated still closer to the end of the arc, due to the higher threshold of the impedance. The high-threshold effect is especially pronounced in Version 2 of the compressor, which has relatively large values of R and σ_f.

In Fig. 3 we show the wake voltage per turn, as defined by Eq. (8), for the parallel-plate problem and Version 3 of the compressor. Particles within one σ of the bunch center lose energy, whereas those around two σ on either side gain energy. The peak wake voltage per unit length is comparable to typical wakes in the SLAC linac structure, which amount to a few volts per picocoulomb over a 3.5 cm cell.

Corresponding results for the toroidal model are displayed in Fig. 4. Within two σ of the center the behavior is similar, although the voltage is somewhat lower due to the higher threshold of the torus impedance. The persistent oscillations well beyond the bunch length can be understood if we refer to the limiting case of infinite conductivity. As that limit is approached the peak of $\mathrm{Re}\, Z(n, n\omega_o)$ narrows to a delta function, and the wake field has the form $\exp[in\omega_o\tau]$ with essentially a single value of n. In fact, the oscillations in Fig. 4 have almost exactly the period of such a single exponential, if we choose n to have the value at the resonance peak of the impedance. For further remarks see Ref. [4].

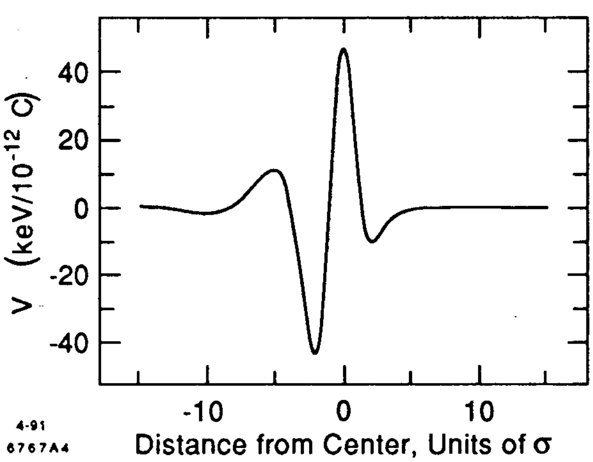

Figure 3. Wake voltage per turn for the parallel-plate model, $R = 149.5$ m, $h = 2$ cm, bunch length $\sigma = 61$ μm.

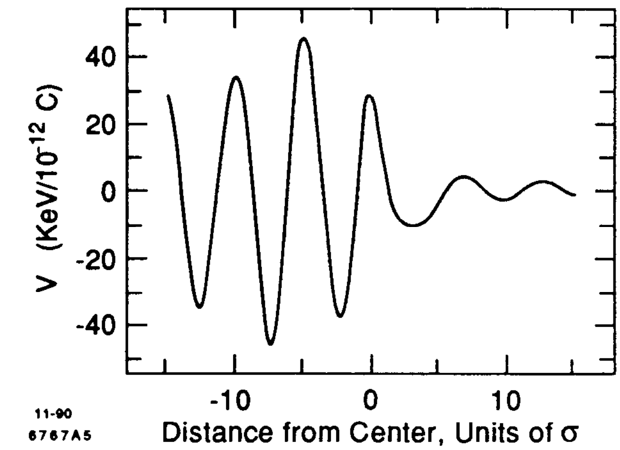

Figure 4. Wake voltage per turn for the toroidal chamber, $R = 149.5$ m, $h = w = 2$ cm, bunch length $\sigma = 61$ μm.

5. OUTLOOK

There are a number of problems in this subject that remain to be investigated. For instance: (i) evolution of the phase-space distribution under the wake field; (ii) transverse impedances; (iii) possible microwave instabilities in rings due to curvature effects; (iv) polarization and angular distribution of coherent radiation; (v) accurate modelling of vacuum chambers for theory of bunch profile measurement through coherent radiation; etc.

REFERENCES

[1] A. Faltens and L.J. Laslett, Part. Accel. **4**, 152 (1973).

[2] R.L. Warnock and P. Morton, Part. Accel. **25**, 113 (1990).

[3] T. Nakazato, M. Oyamada, N. Niimura, S. Urasawa, O. Konno, A. Kagaya, R. Kato, T. Kamiyama, Y. Torizuka, T. Nanba, Y. Kondo, Y. Shibata, K. Ishi, T. Ohsaka, and M. Ikezawa, Phys. Rev. Lett. **63**, 1245 (1989); *Proc. Fourth Advanced ICFA Beam Dynamics Workshop, 1990* (KEK, Tsukuba).

[4] R. L. Warnock, ibid.,pp. 151–160 and 30–35; available as SLAC-PUB-5375 and SLAC-PUB-5417.

[5] S. A. Kheifets, R. D. Ruth, and T. H. Fieguth, *Proc. Int. Conf. High Energy Accel.*, Tsukuba, Japan, 1990; available as SLAC-PUB-5034.