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LUMINOSITY ENHANCEMENT BY FOCUSING AND COLLIDING BEAMS IN A PLASMA*

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ABSTRACT

The potential gain of luminosity by both focusing and colliding e^+e^- beams in an underdense plasma is investigated. We first suggest the possibility of creating the plasma by the beam self-induced "tunneling ionization" of a gas. We then study the beam optics in the continuous focusing environment provided by the plasma, and the subsequent e^+e^- beam-beam interaction in the same plasma. When applied to a range of beam parameters based upon the present SLC conditions, we find that a gain by more than a factor 10 in luminosity is possible. The sensitivity of various non-ideal situations is discussed.

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Since the proposal of the self-focusing plasma lens,[1]—which uses the transverse wake field of a bunched relativistic charged-particle beam in a plasma, and promises a very strong focusing—there has been substantial progress on the concept. On the theory side, the aberrations from the nonlinear focusing force in an overdense plasma (i.e., the beam peak density n_b is much less than the ambient plasma density n_p) has been studied in detail.[2] More recently, inspired by the concern over the potential backgrounds due to the high concentration of plasma ions near the interaction region in a collider, underdense plasma lens[3,4] and its induced asymmetric beam-beam interactions was investigated.[3] The existence and the behavior of this self-focusing effect has been experimentally verified at the Argonne National Laboratory[5] and in Japan.[6]

Although the thick-lens corrections to the thin-lens assumption of plasma lenses have been calculated earlier,[2,3] the plasma lens concepts have been limited to be a discrete focusing element (the focal length is assumed to be much longer than the plasma lens thickness), before the proposal of the adiabatic focusing concept.[7] It is suggested that by adiabatically focusing the beam and by avoiding the drift space between the lens and the focal point, the well-known Oide limit[8] can be avoided. It is evident that in this scheme, the beams have to collide within the plasma, but the issue was not discussed in Ref. 7.

For the maximum gain of luminosity at the expense of somewhat larger backgrounds, it is conceivable to both focus and collide the e^+e^- beams in a plasma. In this setting, the incoming beam is intense enough to trigger ionizations that turns a gas into a plasma. This is particularly attractive in that an externally induced plasma at high densities is nontrivial. The possibility that the high-energy, high-intensity beam induces both self-ionization and self-focusing greatly simplifies the scheme, thus making the practical application for high-energy experiments more plausible. Notwithstanding, in this approach the beam-beam interaction inside the plasma still provides an effective disruption, which further pinches the beams.

There are basically two ionization mechanisms that can be provided by a high intensity, high energy beam. First, there is the collisional ionization, where an individual beam particle ionizes the atom by a virtual photon exchange. The cross section can be estimated via the photo-ionization cross section, using the Weiszacker-Williams spectrum. For hydrogen atoms ionized by a 50 GeV electron, $\sigma_i \sim 0.22$ Mb. The fraction of atoms that can be ionized through this mechanism by an incoming beam with N particles and size σ_r is $R_i = N\sigma_i/4\pi\sigma_r^2$. For the Stanford Linear Collider (SLC) beams, $\sigma_r \sim 1\mu$m and $N \sim 10^{10}$, so R_i is only of the order of a few percent, which is far from saturation. One therefore needs to have a gas which is $1/R_i$ times denser to provide the necessary amount of plasma. This is not desirable from the backgrounds point of view. In addition, the nonsaturation of ionization also causes the tail of the beam to encounter a higher concentration of plasma than that seen by the head of the beam. This degrades the beam optics.

There is another ionization mechanism that relies on the collective field of the beam. When an external electric field is strong enough so that the atomic Coulomb potential is sufficiently distorted, there is a finite probability that the bound state electron can tunnel through the potential barrier and become free. For hydrogen atoms, the ionization probability (per unit time) is given by[9]:

$$W = 4\frac{\alpha^5 c}{\lambda_c{}^2}\,\frac{mc^2}{eE}\,\exp\left\{-\frac{2}{3}\frac{\alpha^3}{\lambda_c}\,\frac{mc^2}{eE}\right\} \quad , \tag{1}$$

where E is the external electric field. The coefficient in the exponent is $(2/3)(\alpha^3/\lambda_c)$ $(mc^2/eE) \simeq 34.1$ eV/Å. It is interesting to note that the ionization probability is already substantial, way before the exponent reaches a value of the order unity, due to the typical largeness of the non-exponential part. For example, an external field of 3.41 eV/Å would give $W \simeq 1.15 \times 10^{14}$ sec^{-1}. Under this condition, the ionization will be saturated within 10 femto-seconds. In fact, it can be shown for a field strength larger than $eE_{th} = 3.72$ eV/Å, where the ground state binding energy is above the potential barrier, that, even classically, the electron can escape from the atom.

The maximum collective electric field strength in a bi-Gaussian beam can be calculated to be $eE_{\max}/\mathrm{mc}^2 \simeq r_e N/2\sigma_z\sigma_r$, where r_e is the classical electron radius. A maximum field strength of 3.72 eV/Å corresponds to, for example, a beam of $N = 3 \times 10^{10}, \sigma_z = 0.4$ mm, and $\sigma_r = 1.4$ μm. This is within the range of the SLC parameters. We shall thus assume in the following discussions that the beam is effectively seeing a plasma with constant density created by the self-induced tunneling ionization. The subtleties involved in the real situation concerning the (r, z) dependence of the beam collective field will be discussed at the end.

The physics of a high-energy beam focused by an underdense plasma has been addressed in Refs. 3 and 4. Assuming that the ion is infinitely heavy, then an underdense plasma reacts to an electron beam by total rarefaction of the plasma electrons inside the beam volume, producing a uniform ion column of charge density en_p. This uniform column produces linear, nearly aberration-free focusing. Simulations have shown that $n_b \sim 2n_p$ is needed to produce linear focusing over most of the bunch.[4]

We start with the the familiar third-order linear differential equation for the β-function,

$$\beta''' + 4K\beta' + 2K'\beta = 0 \quad . \tag{2}$$

In the underdense plasma regime, the focusing strength K is determined by the density of the plasma: $K = 2\pi r_e n_p/\gamma$. Let the plasma density be determined by the initial beam density with a ratio η, $n_p = \eta n_{b0}$. (In practice we will choose $\eta \sim 1/2$ to ensure the underdense condition.) Assuming a cylindrically symmetric bi-Gaussian beam-density profile $\rho_b = n_b e^{-r^2/2\sigma_r^2} e^{-z^2/2\sigma_z^2}$, the peak beam density can be derived: $n_b = N/(2\pi)^{3/2}\sigma_r^2\sigma_z$. In terms of the the initial beam size, $\sigma_{r0}^2 = \beta_0 \epsilon_n \gamma^{-1}$, we can write

$$K = \frac{\eta N r_e}{\sqrt{2\pi}\ \beta_0 \epsilon_n \sigma_z} \equiv \frac{\zeta}{\beta_0} \quad , \tag{3}$$

where ϵ_n is the normalized emittance. Here, we also introduce the *phase space density* ζ, which measures the beam density in the three-dimensional beam volume of r, r', z, and plays a central role in the physics of the self-focusing plasma lens.

To solve Eq. (2), we first integrate through the δ-function in K' at the start of the lens, and obtain $\Delta\beta'' = -2K\beta_0$. The other two initial conditions are the continuity requirements $\beta' = \beta_0'$ and $\beta = \beta_0$. Also note that $\beta_0'' = 2/\beta_0^*$ just before the lens, where β_0^* is the value at the waist that would be formed in the absence of the lens. The equation of motion is then $\beta'' + 4K\beta = 2/\beta_0^* + 2\zeta$, and we obtain[3]

$$\beta = \frac{\beta_0}{2} + \frac{1}{2K\beta_0^*} + \left(\frac{\beta_0}{2} - \frac{1}{2K\beta_0^*}\right)\cos[\nu(s-s_0)] + \frac{2s_0}{\nu\beta_0^*}\sin[\nu(s-s_0)] \quad , \tag{4}$$

where $\nu = 2\sqrt{K}$. This solution demonstrates an oscillatory behavior when without damping effects. We further assume that $\beta_0 = \beta_0^*$ and $s_0 = 0$. To minimize the backgrounds, we look for the next waist at $\sin(\nu s^*) = 0$, then the path length is

$$s_-^* = \frac{\pi}{\nu} = \frac{\pi}{2\sqrt{K}} \quad . \tag{5}$$

The corresponding β^* is

$$\beta_-^* = \frac{1}{K\beta_0^*} = \frac{1}{\zeta} \quad . \tag{6}$$

To appreciate the results, we note that $\zeta = 10$ cm^{-1} corresponds to the beam parameters of $N = 2.84 \times 10^{10}$, $\sigma_z = 0.4$ mm, and initial normalized emittance

$\epsilon_n = 4 \times 10^{-3}$ cm, if $\eta = 1/2$ is assumed. This will give $\beta^*_- = 1$ mm. With an initial $\beta^*_0 = 5$ mm, the corresponding path length is $s^*_- \simeq 3.54$ mm. If all parameters are fixed except $\sigma_z = 0.2$mm, then $\zeta = 20\ \text{cm}^{-1}$. In this case, the β-function is reduced by a factor 10 to $\beta^*_- = 0.5$ mm in a distance $s^*_- \simeq 2.5$ mm.

For an underdense plasma interacting with a positron beam, the plasma electrons are drawn toward the beam axis by the focusing potential provided by the positron beam. This results in a complex motion of these electrons that simultaneously oscillate across and co-move with the beam. In each cycle of oscillation the plasma electron spends a fractional amount of time inside the core of the positron beam, resulting in an effective concentration of negative charges that partially neutralize the positron beam's space-charge force, and the self-focusing is thus induced. Since the focusing force is nonlinear, the net effect is not simple to describe analytically. We shall instead adopt the theoretical model developed in Ref. 2. The description of the positron beam focusing in this approach is only approximate, and further theoretical efforts are needed to better elucidate the process. It is shown that the aberrations due to a thin-plasma lens can be described by an *aberration power* P that transforms the Twiss parameters as $\alpha = \alpha_0/P, \beta = \beta_0/P, \epsilon = \epsilon_0 P$, and $\alpha^* = (\alpha_0 + \beta_0/f)/P$, where the aberration power is defined by

$$P \equiv \left[1 + \left(\frac{\beta_0}{f}\delta\right)^2\right]^{1/2} \quad . \tag{7}$$

Here f is the focal length and δ is the effective total divergence-increase due to the r and z variations in K. By grouping all the position dependent effects under P, this treatment recovers an effective focusing strength that is independent of (r, z). From the simulations in Ref. 4, it is estimated that $\delta \simeq 0.28$ for a mildly underdense lens for positron focusing. We see from Eq. (7) that the aberration is more severe when the focusing is stronger, since in that case the focal length is shorter.

Strictly speaking, this treatment of the degradation is not self-consistent in the case of thick lenses, as the dilution in phase space should really be convoluted with the focusing strength, which in turn influences the optics further downstream. Moreover, the effective focusing strength $\bar{K}$ for the positron beam can in principle be larger than that in the corresponding electron beam case. This is because in the latter case the maximum focusing attainable is by total rarefaction of the plasma electrons *inside* the beam volume, whereas in the former case more supplies of plasma electrons can be drawn in from *outside* the beam. For a rough estimate of the effect, however, we shall assume that this model indeed applies to thick lenses, and that $\bar{K} = K$. Then we first need to determine the equivalent focal length of the lens. The final spot size is shown[3] to be

$$\frac{\sigma_{\pm}^{*\,2}}{\sigma_0^{*\,2}} \equiv \frac{\beta_{\pm}^{*}\epsilon_{\pm}}{\beta_0^{*}\epsilon_0} = \frac{P^2}{P^2 + (\alpha_0 + \beta_0/f)^2} \quad . \tag{8}$$

We may deduce the focal length from the ideal aberration-free condition by setting $P = 1$ in Eq. (8) ($\alpha_0 = 0, \beta_0 = \beta_0^*$), and combining it with Eqs. (5) and (6). We find

$$f = \frac{s_-^*}{\sqrt{(\pi/2)^2 - (s_-^*/\beta_0^*)^2}} \quad . \tag{9}$$

Once the focal length is determined from the ideal lens parameters, we insert it back into Eq. (8) to calculate the aberration-prone positron final spot size. Similarly, the s^* for the positron beam can be derived to be

$$s_+^* = \frac{\pi}{2}\beta_0^* \left[\frac{P}{P^2 + (\beta_0^*/f)^2}\right]^{1/2} \quad . \tag{10}$$

Having the optics for the electron and the positron beams described, next we look into the physics of beam-beam interaction inside a plasma. The disruption effect due to the mutual pinching between the colliding e^+e^- beams in vacuum has been studied in details in the past.[10,11] In the situation where the e^+e^- beams collide inside a plasma, the mechanism is in principle different. When the two colliding beams overlap, the total beam current is increased; thus we expect an increase of the "return current" induced in the plasma. In principle, the return current acts to reduce the self-focusing effect and the mutual beam-beam pinching. However, one important underlying assumption which makes the concept of plasma self-focusing possible is that the initial beam transverse size is much smaller than the plasma wavelength[1]: $\sigma_{r0} \ll \lambda_p(\lambda_p = \sqrt{\pi/r_e n_p})$. In this regime, the plasma return current runs mostly outside the beam, and does not effectively reduce the self-focusing force. At the final focus where beams collide, the spot sizes are supposed to be even smaller, thus the effect of return current during collision should be diminishingly small. On the other hand, in the same beam over-lapping region the net space charge is reduced. Therefore, we expect a decrease of the space-charge perturbation in the plasma. This helps to reduce the plasma influence on the disruption effect. Most importantly, around the collision point, the plasma is extremely underdense: $n_p/n_b^* = \eta\beta^*/\beta_0^* \ll 1$. So even if there is a residual plasma effect on a colliding beam, it should be negligibly small compared to the force exerted by its oncoming beam. We thus conclude that the beam-beam disruption effect in an underdense plasma is effectively the same as that in the vacuum.

The overall enhancement on luminosity in our scheme can be estimated as

$$H_D = \frac{H_{D1} H_{D2}}{H_{D0}} \quad , \tag{11}$$

where H_{D1} is the "geometric" enhancement due to the reduction of the beam sizes from the plasma lens, and H_{D0} and H_{D2} are the disruption enhancement due to beam-beam interaction with and without the plasma lens, respectively. Since the plasma-focused e^+e^- beams are different in sizes, the "geometric" enhancement (excluding depth of focus and disruption effects) in luminosity is

$$H_{D1} = \frac{2\sigma_{r0}^{*2}}{\sigma_-^{*2} + \sigma_+^{*2}} \quad . \tag{12}$$

To illustrate the effectiveness of this thick lens scheme, we demonstrate three numerical examples, based on beam parameters that are inspired by the current SLC running conditions. The β_0^* from the newly installed superconducting final quadrupole is expected to be 5 mm. We assume the normalized emittance is $\epsilon_n = 4 \times 10^{-3}$ cm. This would give an initial beam size of $\sigma_{r0}^* = 1.4$ μm. The bunch populations are $N = 3, 4, 5 \times 10^{10}$. The nominal bunch length is $\sigma_z = 1$ mm. This bunch length is indeed close to the operating value along the linac. But the eventual bunch length at the collision point in SLC can be made much smaller than this value.[12] It is known that by accelerating the beam at an RF phase which is off from the crest, a coherent energy spread (as a function of the particle longitudinal position inside the bunch) is induced in the beam. If the phase is properly chosen, then the subsequent beam transport through the SLC arc would help to compress the the core of the beam before it eventually arrives at the final focus. The minimum attainable SLC bunch length is then governed essentially by the incoherent energy fluctuation in the beam, which in the parameters that we consider is roughly 0.12 mm.[13] We shall thus assume that a bunch length of $\sigma_z = 0.2$ mm is attainable in SLC.

The shortness of the bunch is essential to the success of applying our scheme to SLC for three major reasons. First, given the constraints on other beam parameters, which is usually harder to be improved, one should minimize the bunch length in order to raise the beam collective field beyond the tunneling ionization threshold. Second, with the densest possible incoming beam, the matching plasma density can be raised to provide the strongest possible focusing and smallest possible β^*. Finally, having the β^* sufficiently reduced, one also likes to have the bunch length sufficiently short, such that the further luminosity enhancement from beam-beam disruption is not damped due to the depth of focus effect[12] when $A \equiv \sigma_z/\beta^* \gtrsim 1$.

With the above choice of beam parameters, we fix the plasma density by letting $\eta = 1/2$. The corresponding plasma densities are $n_p = 2.4, 3.2, 4.0 \times 10^{18}$ cm^{-3}, respectively. Other relevant physical parameters can be derived, and are listed in Table 1. The disruption parameter in the absence of plasma, $D_0 = r_e \sigma_z N/\gamma\sigma_{r0}^{*2}$, ranges from 0.10 to 0.16 in the three cases. The corresponding H_{D0} is essentially of the order unity. For H_{D2}, the physics is more complex, as it involves beams with unequal sizes and divergences. Computer simulations using the code ABEL[14] have been performed under the above argument of negligible plasma effects, to estimate both H_{D0} and H_{D2}. These results are also shown in Table 1. We see that the resultant luminosity can, in principle, be raised by a factor $\sim$ 12 to 23. Assuming a collision repetition rate of $f_p = 120$/sec, the luminosity will be enhanced to $\sim$ 6 to 30×10^{30} cm^{-2}.

So far we have studied the luminosity enhancement in an ideal situation. It is important to study sensitivity of the scheme to various nonperfect conditions. The issue can be generally categorized in three types: fluctuation of various beam parameters, fluctuation of plasma density and thickness, and the transverse jitters between the two colliding beams.

In the underdense regime, the focusing strength and the optics are determined by the plasma density. Thus the system is relatively insensitive to the fluctuation of beam parameters, as long as the underdense condition is sufficiently satisfied. On the other hand, the fluctuation on the plasma density causes the variation of

K, which in turn changes β^* and s^*. To ensure that the beams collide around the waist of the β-function, we demand that $\delta s^* \lesssim \beta^*$. Combining Eqs.(5) and (6), we obtain the constraint

$$\frac{1}{2}\frac{s^*}{\beta^*}\frac{\delta n_p}{n_p} \lesssim 1 \quad . \tag{13}$$

For our parameters, this corresponds to $\delta n_p/n_p \lesssim 30\%$, which is not at all stringent.

Unlike conventional optics, in the self-focusing plasma lens, the axis of symmetry is determined by the incoming beam axis and the initial offset is not proportionally demagnified. It is thus essential to express the tolerance on such beam jitters in terms of the initial beam size. The condition for less than one e-folding of the *kink instability* at the time when the two beam cores collide is found to be[15]

$$\exp\left\{\sqrt{\sqrt{2\pi}D_2/8}\right\}\frac{\sigma^*_{r0}}{\sigma^*_{\pm}}\frac{\delta r}{\sigma^*_{r0}} \lesssim 1 \quad . \tag{14}$$

For our parameters, we find the constraint to be $\delta r/\sigma^*_{r0} \lesssim 15\%$. This is somewhat stringent, but not incompatible with the current SLC running condition.

We have seen that the proposed scheme of beam self-induced tunneling ionization, with the subsequent self-focusing and collision in a plasma, looks quite promising. There are, however, several issues yet to be addressed. As the field is dependent on (r, z) in a bi-Gaussian beam, the tunneling ionization is generally not saturated near the head of the bunch. Thus the beam particles ahead of a certain position z_s will not be focused well by the plasma, resulting in an effective loss of beam particles. The condition for the saturation of tunneling ionization is $1 = \int_{-\infty}^{z_s} W dz/c$, where the position dependence of W in Eq.(1) is through the longitudinal variation of the field, $E(z) = E_{max}e^{-z^2/2\sigma_z^2}$. It can be shown that for $E_{max} \gtrsim E_{th}$,

$$\frac{z_s}{\sigma_z} \sim -2.72\left[1 - \frac{2}{3}\frac{\alpha^3}{\lambda_c}\frac{mc^2}{eE_{max}}\log^{-1}\left(11.5\alpha^2\frac{\sigma_z}{\lambda_c}\right)\right] \quad . \tag{15}$$

In our examples, the peak fields are 7.5 to 12.6 eV/Å, so the front-most points of saturation are $z_s \simeq -1.8\sigma_z$ to $-2.1\sigma_z$, respectively. Since the field also varies radially, the saturation near the core of the beam is not reached as rapidly. In fact, the tunneling ionization would never penetrate down to the beam axis, where the field vanishes. This results in a ring-shaped region of plasma. In this paper, the degradation off the plasma focusing and off the effective number of bunch particles was not included in the luminosity calculations, so our result is an over estimate in this regard. However, since the peak field in our examples are way above the threshold, the inner radius of the plasma "ring" is a small fraction of σ^*_{r0}, even at the start of the lens. As the beam becomes tighter during focusing, the inner radius will shrink rapidly; thus the degradation should not be too severe.

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Table 1. Plasma Lens Parameters for SLC

Beam Parameters			
N [10^{10}]	3	4	5
$\mathcal{E}$ [GeV]	45	45	45
β_0^* [mm]	5	5	5
ϵ_0 [10^{-10} m-rad]	4	4	4
σ_z [mm]	0.2	0.2	0.2
Plasma Lens Parameters			
n_p [10^{18} cm^{-3}]	2.4	3.2	4.0
$s^*(= s_-^* + s_+^*)$ [mm]	4.9	4.3	4.0
Beam Optics Parameters			
ζ [cm^{-1}]	23.6	31.5	39.4
s_-^* [mm]	2.3	2.0	1.8
β_-^* [mm]	0.42	0.32	0.25
σ_-^* [μm]	0.41	0.36	0.32
f [mm]	1.5	1.3	1.2
P	1.36	1.46	1.53
s_+^* [mm]	2.6	2.3	2.2
σ_+^* [μm]	0.54	0.50	0.47
Luminosity Enhancement			
D_0	0.10	0.13	0.16
H_{D0}	1.08	1.10	1.11
H_{D1}	8.70	10.5	12.4
H_{D2}	1.49	1.83	2.05
H_D	12.0	17.5	22.9
f_{rep} [sec^{-1}]	120	120	120
$\mathcal{L}_{00}$ [10^{30} cm^{-2}]	0.43	0.77	1.20
$\mathcal{L}_0(= H_{D0}\mathcal{L}_{00})$ [10^{30} cm^{-2}]	0.46	0.84	1.34
$\mathcal{L}_1(= H_{D1}\mathcal{L}_{00})$ [10^{30} cm^{-2}]	3.74	8.05	14.8
$\mathcal{L}(= H_{D2}\mathcal{L}_1)$ [10^{30} cm^{-2}]	5.78	14.8	30.2