# Probing $W-\gamma$ couplings using $\gamma \gamma \rightarrow W^{+} W^{-*}$ 

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#### Abstract

We examine the process $\gamma \gamma \rightarrow W^{+} W^{-}$in the context of a $\gamma \gamma$ collider constructed by backscattering laser light off the electron beams of a $500 \mathrm{GeV} e^{+} e^{-}$ collider. We present analytic formulas for the helicity amplitudes with general $\kappa$ and $\lambda$ anomalous couplings. We calculate the effective cross section, accounting for the photon spectrum and including polarization effects. Finally, we assess the sensitivity of this experiment, and compare to those of other experiments running at a comparable energy.


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## 1. Introduction

Study of the $W$ and $Z$ bosons in general, and their electromagnetic and self interactions in particular, are some of the most important tasks of the next generation of linear $e^{+} e^{-}$colliders of $400 \mathrm{GeV}-1 \mathrm{TeV}$ energy.

The conventional process of choice to study $W$ interactions in $c^{+} c^{-}$colliders is $e^{+} e^{-} \rightarrow W^{+} W^{-}$. Due to unitarity cancellations in the standard model, the proccss is particularly sensitive to deviations from the standard model. This process has been studied extensively. ${ }^{[1]}$ However, it is complicated to extract theoretical constraints from the process because both $W \gamma$ and $W Z$ couplings are involved. An alternative process available in traditional $e^{+} e^{-}$colliders is $e \gamma \rightarrow W \nu$, where the photon originates either from bremsstrahlung or beamstrahlung. ${ }^{[2]}$ Unlike $e^{+} e^{-} \rightarrow W^{+} W^{-}$, only $W \gamma$ couplings contribute. This allows one to focus on the $W \gamma$ anomalous couplings and derive strict bounds. In this way, $e^{+} e^{-} \rightarrow W^{+} W^{-}$ and $e \gamma \rightarrow W \nu$ give complimentary information on the $W \gamma$ interactions.

An additional process involving $W \gamma$ interactions is $\gamma \gamma \rightarrow W^{+} W^{-}$. In an ordinary $e^{+} e^{-}$collider of a few hundred GeV energy, the effective $\gamma \gamma$ luminosity from radiated photons is too soft to produce a reasonable sample of $W^{+} W^{-}$events. Akerlof and Ginzburg et al. have proposed a means of constructing a dedicated $\gamma \gamma$ collider in this energy range: Begin with an $e^{-} e^{-}$linear collider, and convert the high-energy electrons to photons by backscattering laser beams from the electron beams near the interaction point. ${ }^{[3,4,5]}$ Such a collider offers essentially the same luminosity as the original $e^{+} e^{-}$collider, with only a $20-30 \%$ reduction of center-of-mass energy. We study the physics potential of the process $\gamma \gamma \rightarrow W^{+} W^{-}$in a $\gamma \gamma$ collider in which each photon beam is set up by colliding a 0.7 eV laser with a 250 GeV electron beam. The effective center-of-mass energy in such a collider is
about 350 GeV .
The reaction $\gamma \gamma \rightarrow W^{+} W^{-}$is one of the dominant processes in the 300 GeV - 1 TeV energy range. As all the vertices in the tree level diagrams of this process involve $W \gamma$ couplings, it is quite sensitive to any anomalous interactions. Thus, it should be straightforward to make a detailed analysis of this reaction. This process has been studied before ${ }^{[6]}$ but never with the full set of $W \gamma$ couplings we allow here.

When analyzing $W \gamma$ interactions in a model independent way, it is customary to parametrize the $W W \gamma$ vertex by "anomalous couplings". Only two, conventionally called $\kappa$ and $\lambda$, are allowed in a $C$ and $P$ conserving theory and are considered here. A third parameter is allowed in a theory that violates $C$ and $P$ separately, but conserves $C P$. Standard model radiative corrections contribute to this term, but we do not consider it here.

Both $e^{+} e^{-} \rightarrow W^{+} W^{-}$and $e \gamma \rightarrow W \nu$ involve only 3 -boson vertices. The standard model also predicts a specific tree-level $W W \gamma \gamma$ vertex. That vertex contributes to $\gamma \gamma \rightarrow W^{+} W^{-}$, making it a particularly important tool in measuring $W$ electromagnetic interactions. In principle, one could use the most general 4boson vertex. However, in this paper we use the simplest form of this vertex which satisfies the electromagnetic Ward identity. It should be noted that this simple consistency requirement forces the 4 -boson vertex to depend on $\lambda]^{[7]}$

The paper proceeds as follows. In the next section we discuss the theoretical setting for our work, and describe the Feynman rules we use in our calculation. In section III we introduce the experimental setting - the general properties of the backscattering mechanism and the particular set of parameters used here. Sec. IV presents the analytic formulas for the various helicity amplitudes, and points out
some of their non-trivial features. Sec. V examines the various observable quantities and assesses their measurability and sensitivity to the anomalous couplings. Sec. VI evaluates the discovery potential and compares it to that of other high-energy experiments. We present our conclusions in section VII.

## 2. Theoretical Background

The three-vector $W W \gamma$ vertex has been analyzed fully. ${ }^{[7,8,1]}$ Here we use the most general $C$ and $P$ conserving form of this vertex. The vertex with momenta labeled as in fig. 1 is conventionally written as

$$
\begin{align*}
& i \Gamma_{\mu \nu \tau}=i e\left[g_{\tau \nu}(p-\bar{p})_{\mu}+g_{\tau \mu}(\bar{p}+\kappa q)_{\nu}-g_{\mu \nu}(p+\kappa q)_{\tau}\right. \\
& +\frac{\lambda}{m_{W}^{2}}\left\{g_{\tau \nu}\left((\bar{p} \cdot q) p_{\mu}-(p \cdot q) \bar{p}_{\mu}\right)+g_{\tau \mu}\left((p \cdot \bar{p}) q_{\nu}-(p \cdot q) \bar{p}_{\nu}\right)\right.  \tag{2.1}\\
& \left.\left.\quad+g_{\mu \nu}\left((\bar{p} \cdot q) p_{\tau}-(p \cdot \bar{p}) q_{\tau}\right)-\bar{p}_{\mu} p_{\tau} q_{\nu}+p_{\mu} q_{\tau} \bar{p}_{\nu}\right\}\right]
\end{align*}
$$



Figure 1. The general $W W \gamma$ vertex.

The parameters $\kappa$ and $\lambda$ are related to the magnetic dipole moment $\mu$ and the electric quadrupole moment $Q$ of the $W$ boson by

$$
\begin{equation*}
\mu=(1+\kappa+\lambda) \frac{e}{2 m_{w}}, \quad Q=\frac{2 e}{m_{w}^{2}}(\lambda-\kappa) . \tag{2.2}
\end{equation*}
$$

In the standard model $\kappa=1$ and $\lambda=0$ at the tree level.

Unlike the $W W \gamma$ vertex, the $W W \gamma \gamma$ vertex has not been systematically analyzed. Here we do not use its most general form. Rather we use the simplest form which is still consistent with eq. (2.1) in the sense of maintaining electromagnetic gauge invariance ${ }^{[7]}$ This general form with momenta labeled as in fig. 2 is given by

$$
\begin{align*}
i \Gamma_{\mu \nu \tau \sigma} & =i c^{2}\left[2 g_{\mu \nu} g_{\sigma \tau}-g_{\mu \tau} g_{\nu \sigma}-g_{\mu \sigma} g_{\nu \tau}\right. \\
& +\frac{\lambda}{m_{W}^{2}}\left\{-g_{\mu \tau} g_{\nu \sigma}((p \cdot \bar{q})+(\bar{p} \cdot q))-g_{\mu \sigma} g_{\nu \tau}((p \cdot q)+(\bar{p} \cdot \bar{q}))\right. \\
& +g_{\mu \nu} g_{\tau \sigma}(p+\bar{p})^{2}-2 g_{\mu \nu} p_{\tau} \bar{p}_{\sigma}-2 g_{\sigma \tau} q_{\mu} \bar{q}_{\nu}  \tag{2.3}\\
& +g_{\mu \tau}\left(2 p_{\nu} \bar{p}_{\sigma}+2 \bar{p}_{\nu} q_{\sigma}-p_{\nu} q_{\sigma}\right)+g_{\mu \sigma}\left(2 p_{\tau} \bar{p}_{\nu}+2 p_{\nu} q_{\tau}-\bar{p}_{\nu} q_{\tau}\right) \\
& \left.\left.+g_{\nu \tau}\left(2 p_{\mu} \bar{p}_{\sigma}+2 \bar{p}_{\mu} \bar{q}_{\sigma}-p_{\mu} \bar{q}_{\sigma}\right)+g_{\nu \sigma}\left(2 p_{\tau} \bar{p}_{\mu}+2 p_{\mu} \bar{q}_{\tau}-\bar{p}_{\mu} \bar{q}_{\tau}\right)\right\}\right]
\end{align*}
$$

In the process $\gamma \gamma \rightarrow W^{+} W^{-}$, one can take the four external momenta to be on shell. The $W W \gamma \gamma$ vertex then takes the form


Figure 2. The general $W W \gamma \gamma$ vertex.

$$
\begin{align*}
\Gamma_{\mu \nu \tau \sigma} & =e^{2}\left[2 g_{\mu \nu} g_{\sigma \tau}-g_{\mu \tau} g_{\nu \sigma}-g_{\mu \sigma} g_{\nu \tau}\right. \\
& +\frac{\lambda}{m_{W}^{2}}\left\{-g_{\mu \tau} g_{\nu \sigma}\left(u-m_{W}^{2}\right)-g_{\mu \sigma} g_{\nu \tau}\left(t-m_{W}^{2}\right)+g_{\mu \nu} g_{\tau \sigma} s\right. \\
& -2 g_{\mu \nu} p_{\tau} \bar{p}_{\sigma}-2 g_{\sigma \tau} q_{\mu} \bar{q}_{\nu}+g_{\mu \tau}\left(2 p_{\nu} \bar{p}_{\sigma}+2 \bar{p}_{\nu} q_{\sigma}-p_{\nu} q_{\sigma}\right)  \tag{2.4}\\
& +g_{\mu \sigma}\left(2 p_{\tau} \bar{p}_{\nu}+2 p_{\nu} q_{\tau}-\bar{p}_{\nu} q_{\tau}\right)+g_{\nu \tau}\left(2 \ddot{p}_{\mu} \bar{p}_{\sigma}+2 \bar{p}_{\mu} \bar{q}_{\sigma}-p_{\mu} \bar{q}_{\sigma}\right) \\
& \left.\left.+g_{\nu \sigma}\left(2 p_{\tau} \bar{p}_{\mu}+2 p_{\mu} \bar{q}_{\tau}-\bar{p}_{\mu} \bar{q}_{\tau}\right)\right\}\right]
\end{align*}
$$

where $s=(q+\bar{q})^{2}, t=(p-q)^{2}$ and $u=(p-\bar{q})^{2}$.

## 3. Experimental Setting

The photon spectrum in a traditional $500 \mathrm{GeV} e^{+} e^{-}$collider, even with the effects of beamstrahlung, is too soft to produce experimentally useful $W$ pair production via $\gamma \gamma \rightarrow W^{+} W^{-}$. In addition, the background from $e^{+} e^{-} \rightarrow W^{+} W^{-}$is overwhelming. Therefore, we only consider measurements of $\gamma \gamma \rightarrow W^{+} W^{-}$in a dedicated photon-photon collider. Ginzburg et al. ${ }^{[4]}$ have suggested a scheme for
converting a single-pass $e^{+} e^{-}$collider into an $e \gamma$ or $\gamma \gamma$ collider. The conversion of high-energy electrons to photons is done by backward Compton scattering of high intensity laser light off the electron beams. This mechanism entails losing very little luminosity; it reduces the center-of-mass energy by $20-30 \%$.

In describing our machine parameters, we use the dimensionless variables ${ }^{[9]}$

$$
\begin{equation*}
x=\frac{4 E \omega_{0}}{m_{e}^{2}}, \quad y=\frac{\omega}{E} \tag{3.1}
\end{equation*}
$$

where $E$ is the electron beam energy, $\omega_{0}$ is the energy of a laser photon and $\omega$ is the energy of a scattered photon. The parameter $x$ is just $\left(s / m_{e}^{2}\right)$ for the Compton scattering process. The maximum energy of a scattered photon is given by

$$
\begin{equation*}
y \leq y_{m}=\frac{x}{x+1} \tag{3.2}
\end{equation*}
$$

Due to the onset of $e^{+} e^{-}$pair-production between backscattered and laser photons, conversion efficiency drops considerably for $x>2+2 \sqrt{2} \approx 4.82$. $^{[4,10]} \mathrm{We}$ assume $x=2+2 \sqrt{2}$, which, given 250 GeV electrons, corresponds to laser energy of about 0.7 eV .

The photon spectrum depends sensitively on $\lambda_{e} P_{c}$ where $\lambda_{e}$ is the mean electron helicity and $P_{c}$ is the mean laser photon helicity. Larger negative values of $\lambda_{e} P_{c}$ give a harder, more monochromatic photon spectrum, resulting in a larger total cross section of $\gamma \gamma \rightarrow W^{+} W^{-}$. See ref. 10 for a thorough discussion of the experimental consequences of electron beam polarization. The sensitivity to anomalous couplings, however, does not increase significantly with a harder photon spectrum, while measuring the actual $\lambda_{e}$ introduces new systematic errors. Therefore, we assume that the electron beam is unpolarized. On the other hand, the
laser can be easily polarized almost completely, and this polarization can serve as an important experimental tool. We assume that $\left|P_{c}\right|=1$. The photon spectrum $\sigma_{c}^{\prime}(y)=d \sigma_{c}(y) / d y$ is given by ${ }^{[4]}$

$$
\begin{equation*}
\sigma_{c}^{\prime}(y)=\frac{2(1+x)^{2}\left(2 x^{2}-4 x y-4 x^{2} y+4 y^{2}+4 x y^{2}+3 x^{2} y^{2}-x^{2} y^{3}\right)}{(1-y)^{2}\left(x\left(16+32 x+18 x^{2}+x^{3}\right)-2\left(8+20 x+15 x^{2}+2 x^{3}-x^{4}\right) \log (1+x)\right)} \tag{3.3}
\end{equation*}
$$

Fig. 3 shows the photon spectrum used here.


Figure 3. Photon spectrum $\sigma_{c}^{\prime}(y)$ for $x=2+2 \sqrt{2}$. Solid curve shows total spectrum. Dashed curve shows spectrum of photons with helicity $-P_{c}$.

Let $z$ be the ratio of the $\gamma \gamma$ collision energy to that of the of the original $e^{-} e^{-}$system. The spectral $\gamma \gamma$ luminosity $d \mathcal{L}_{\gamma \gamma} / d z$ is given by folding together the spectra of both beams:

$$
\begin{equation*}
\frac{d \mathcal{L}_{\gamma \gamma}}{d z}=2 z \int_{z^{2} / y_{m}}^{y_{m}} \frac{\sigma_{c}^{\prime}(y) \sigma_{c}^{\prime}\left(z^{2} / y\right)}{y} d y \tag{3.4}
\end{equation*}
$$

Fig. 4 shows the effective $\gamma \gamma$ luminosity used in our calculations.


Figure 4. The $\gamma \gamma$ effective luminosity $d \mathcal{L} / d z$ as a function of $z$.

Using a polarized laser beam results in the scattered photons being polarized. If the laser beam polarization does not have a linear component, coherence between left- and right-handed photons is lost upon integration over the azimuthal angle of the Compton scattering process. Since the photons are emittcd within 0.1 mrad of the electron direction, the angle is unobservable, and the integration is done automatically. The average helicity $\xi_{2}$ of the photon beam is then given by ${ }^{[4]}$

$$
\begin{equation*}
\xi_{2}=-\frac{x(x-2 y-x y)\left(2-2 y+y^{2}\right)}{2 x^{2}-4 x y-4 x^{2} y+4 y^{2}+4 x y^{2}+3 x^{2} y^{2}-x^{2} y^{3}} \tag{3.5}
\end{equation*}
$$

The dashed line in fig. 3 show the spectrum of photons with helicity $-P_{c}$.

## 4. Helicity Amplitudes

The three Feynman diagrams contributing to $\gamma \gamma \rightarrow W^{+} W^{-}$are shown in fig. 5. The matrix element (in unitary gauge) for this process is given by

$$
\begin{align*}
\mathcal{M}_{\lambda_{1}, \lambda_{2}, \lambda_{3}, \lambda_{4}}= & \epsilon_{\lambda_{1}}^{\mu} \epsilon_{\lambda_{2}}^{\nu} \epsilon_{\lambda_{3}}^{* \tau} \epsilon_{\lambda_{1}}^{* \sigma} \mathcal{M}_{\mu \nu \tau \sigma} \\
i \mathcal{M}_{\mu \nu \tau \sigma}= & \left(i \Gamma_{\mu \tau \rho_{1}}\right) \frac{-i\left(g^{\rho_{1} \rho_{2}}-\left(p_{1}-p_{3}\right)^{\rho_{1}}\left(p_{1}-p_{3}\right)^{\rho_{2}} / m_{W}^{2}\right)}{t-m_{W}^{2}}\left(i \Gamma_{\nu \rho_{2} \sigma}\right) \\
& +\left(i \Gamma_{\mu \rho_{1} \sigma}\right) \frac{-i\left(g^{\rho_{1} \rho_{2}}-\left(p_{1}-p_{4}\right)^{\rho_{1}}\left(p_{1}-p_{4}\right)^{\rho_{2}} / m_{W}^{2}\right)}{u-m_{W}^{2}}\left(i \Gamma_{\nu \tau \rho_{2}}\right)  \tag{4.1}\\
& +i \Gamma_{\mu \nu \tau \sigma} .
\end{align*}
$$



Figure 5. Feynman diagrams for $\gamma \gamma \rightarrow W^{+} W^{-}$.
$\lambda_{1}, \lambda_{2}, \lambda_{3}$ and $\lambda_{4}$ are the helicities of the two photons, the $W^{-}$, and the $W^{+}$, respectively. $\lambda_{1}$ and $\lambda_{2}$ take the values -1 and $1 . \lambda_{3}$ and $\lambda_{4}$ take the values $-1,0$ and 1. The total cross section (averaged over incoming polarizations) is then

$$
\begin{align*}
\sigma & =\frac{1}{4} \int d \Omega d z \mathcal{L}_{\gamma \gamma}(z) \frac{(2 \pi)^{4}}{2 s z^{2}} \sum_{\lambda_{1} \lambda_{2} \lambda_{3} \lambda_{4}}\left|\mathcal{M}_{\lambda_{1}, \lambda_{2}, \lambda_{3}, \lambda_{4}}\right|^{2} \frac{\sqrt{1-4 m_{W} / s z^{2}}}{8(2 \pi)^{6}} \\
& =\frac{1}{128 \pi s} \int_{2 m_{W} / \sqrt{s}}^{y_{m}} d z \frac{\sqrt{1-r}}{z^{2}} \mathcal{L}_{\gamma \gamma}(z) \int_{-1}^{1} d(\cos \theta) \sum_{\lambda_{1} \lambda_{2} \lambda_{3} \lambda_{4}}\left|\mathcal{M}_{\lambda_{1}, \lambda_{2}, \lambda_{3}, \lambda_{4}}\right|^{2} \\
& =\frac{3 R}{32} \int_{2 m_{W} / \sqrt{s}}^{y_{m}} d z \frac{\sqrt{1-r}}{z^{2}} \mathcal{L}_{\gamma \gamma}(z) \int_{-1}^{1} \frac{d(\cos \theta)}{\left(1-\cos ^{2} \theta+r \cos ^{2} \theta\right)^{2}} \sum_{\lambda_{1} \lambda_{2} \lambda_{3} \lambda_{4}}\left|\tilde{\mathcal{M}}_{\lambda_{1}, \lambda_{2}, \lambda_{3}, \lambda_{4}}\right|^{2}, \tag{4.2}
\end{align*}
$$

where

$$
\begin{equation*}
r=\frac{4 m_{W}^{2}}{s z^{2}}, \quad \mathcal{M}_{\lambda_{1}, \lambda_{2}, \lambda_{3}, \lambda_{4}}=\frac{4 \alpha \pi \tilde{\mathcal{M}}_{\lambda_{1}, \lambda_{2}, \lambda_{3}, \lambda_{4}}}{1-\cos ^{2} \theta+r \cos ^{2} \theta} \tag{4.3}
\end{equation*}
$$

and

$$
\begin{equation*}
R=\frac{4 \pi \alpha^{2}}{3 s}=\frac{86.8 \mathrm{fb}}{\left(E_{\mathrm{CM}}(\mathrm{TeV})\right)^{2}} \tag{4.4}
\end{equation*}
$$

The helicity amplitudes are given by

$$
\begin{aligned}
\tilde{\mathcal{M}}_{++00}= & \frac{1}{2 r}\left[4 r^{2}+8 \Delta \kappa\left(\sin ^{2} \theta+r \cos ^{2} \theta\right)+8 \lambda r \sin ^{2} \theta\right. \\
& +\Delta \kappa^{2}\left(4 \sin ^{2} \theta+r\left(3 \cos ^{2} \theta-1\right)\right)+2 \Delta \kappa \lambda r\left(3 \cos ^{2} \theta-1\right) \\
& \left.+\lambda^{2}\left(1+\cos ^{2} \theta\right)\left(2 \sin ^{2} \theta+r\left(2 \cos ^{2} \theta-1\right)\right)\right] \\
\tilde{\mathcal{M}}_{++0+}= & \frac{-\cos \theta \sin \theta}{2 \sqrt{2 r}}\left[4 \Delta \kappa(1-\beta-r)+4 \lambda(1-\beta+r)+\Delta \kappa^{2}(2-2 \beta-r)\right. \\
& \left.+2 \Delta \kappa \lambda(2+r)-\lambda^{2}\left(2(1+\beta)(1 / r-1) \sin ^{2} \theta+r\right)\right] \\
\tilde{\mathcal{M}}_{+++-}= & \frac{\sin ^{2} \theta}{4 r}\left[8 \lambda(2-r)+\Delta \kappa^{2} r+2 \Delta \kappa \lambda(4-r)-\lambda^{2}\left(2 \sin ^{2} \theta+\left(2 \cos ^{2} \theta-1\right) r\right)\right]
\end{aligned}
$$

$$
\begin{align*}
& \tilde{\mathcal{M}}_{+-00}=\frac{\sin ^{2} \theta}{2 r}\left[4 r(1+r)+8 \Delta \kappa r+\Delta \kappa^{2}(2+r)+2 \Delta \kappa \lambda(2-r)\right. \\
& \left.+\lambda^{2}\left(2 \cos ^{2} \theta+r\left(1-2 \cos ^{2} \theta\right)\right)\right] \\
& \tilde{\mathcal{M}}_{+-++}=\frac{\sin ^{2} \theta}{4 r}\left[8 r^{2}+8 \Delta \kappa r+16 \lambda(1-r)+3 \Delta \kappa^{2} r+2 \Delta \kappa \lambda(4-3 r)\right. \\
& \left.-\lambda^{2}\left(2 \sin ^{2} \theta-\left(3-2 \cos ^{2} \theta\right) r\right)\right] \\
& \tilde{\mathcal{M}}_{++++}=\frac{-1}{4 r}\left[8 r(2-2 \beta-r)+8 \Delta \kappa r\left(2-\beta-\beta \cos ^{2} \theta\right)\right. \\
& +8 \lambda\left(2(1+\beta) \sin ^{2} \theta-r\left(1-\beta-3 \cos ^{2} \theta-\beta \cos ^{2} \theta\right)\right) \\
& +\Delta \kappa^{2}\left(2(1+\beta) \sin ^{2} \theta+r\left(3+3 \cos ^{2} \theta-2 \beta \cos ^{2} \theta\right)\right) \\
& +2 \Delta \kappa \lambda\left(2(1+\beta) \sin ^{2} \theta-r\left(3-5 \cos ^{2} \theta-2 \beta \cos ^{2} \theta\right)\right) \\
& +\lambda^{2}\left(4(1+\beta) \sin ^{2} \theta\left(1-3 \cos ^{2} \theta\right) / r\right. \\
& -2\left(6+3 \beta-13 \cos ^{2} \theta-9 \beta \cos ^{2} \theta+3 \cos ^{4} \theta+2 \beta \cos ^{4} \theta\right) \\
& \left.\left.+r\left(3-11 \cos ^{2} \theta-2 \beta \cos ^{2} \theta+2 \cos ^{4} \theta\right)\right)\right] \\
& \tilde{\mathcal{M}}_{-+0+}=\frac{-(1+\cos \theta) \sin \theta}{2 \sqrt{2 r}}\left[8 r+4 \Delta \kappa(1+r)+4 \lambda(1-r)+\Delta \kappa^{2}(3-\cos \theta+r \cos \theta)\right. \\
& +\Delta \kappa \lambda(4(1-\cos \theta) / r-2(1-3 \cos \theta)-2 r \cos \theta) \\
& \left.+\lambda^{2}\left(2(1-\cos \theta)^{2} / r-(1-\cos \theta)(1-2 \cos \theta)+r \cos ^{2} \theta\right)\right] \\
& \tilde{\mathcal{M}}_{-++-}=\frac{-(1-\cos \theta)^{2}}{4 r}\left[8 r+8 \Delta \kappa r+\Delta \kappa^{2}(2(1+\cos \theta)+(1-2 \cos \theta) r)\right. \\
& +2 \Delta \kappa \lambda(2(1+\cos \theta)-(1+2 \cos \theta) r)+\lambda^{2}(4(1+\cos \theta)(3+\cos \theta) / r \\
& \left.\left.-2\left(6+11 \cos \theta+3 \cos ^{2} \theta\right)+\left(1+6 \cos \theta+2 \cos ^{2} \theta\right) r\right)\right] \tag{4.5}
\end{align*}
$$

where $\beta=\sqrt{1-r}$. The other amplitudes are related to those by the following relations:

$$
\begin{align*}
& \mathcal{M}_{\lambda_{1}, \lambda_{2}, \lambda_{3}, \lambda_{4}}=\mathcal{M}_{-\lambda_{1},-\lambda_{2},-\lambda_{3},-\lambda_{4}} \\
& \mathcal{M}_{\lambda_{1}, \lambda_{2}, \lambda_{3}, \lambda_{4}}=\mathcal{M}_{\lambda_{2}, \lambda_{1}, \lambda_{3}, \lambda_{4}} \quad(\cos \theta \rightarrow-\cos \theta) \\
& \mathcal{M}_{\lambda_{1}, \lambda_{2}, \lambda_{3}, \lambda_{4}}=\mathcal{M}_{\lambda_{1}, \lambda_{2}, \lambda_{4}, \lambda_{3}} \quad(\cos \theta \rightarrow-\cos \theta)  \tag{4.6}\\
& \mathcal{M}_{\lambda_{1}, \lambda_{2}, \lambda_{3}, \lambda_{4}}=\mathcal{M}_{\lambda_{1}, \lambda_{2},-\lambda_{4},-\lambda_{3}} \quad(\beta \rightarrow-\beta)
\end{align*}
$$

The various helicities for $\gamma \gamma \rightarrow W^{+} W^{-}$were calculated with the aid of a set of Mathematica ${ }^{[11]}$ packages (see Appendix A.) Figure 6 shows the differential cross section for producing $W$ 's of various helicity combinations. Figure 7 shows the dependence on $\kappa$ and $\lambda$ of the differential cross section at $\cos \theta=0$. These figures are the idealized cross-sections, assuming monochromatic beams of polarized photons.

There are several nontrivial points worth noting:

1. The cross section for producing two opposite-helicity $W$ 's from an initial state with total spin component along beam axis $J_{z}=0((+++-)$ term $)$ is exactly zero in the standard model.
2. The cross section for producing one longitudinal and one transverse $W$ from a $J_{z}=0$ initial state $((++0+)$ term $)$ is exactly zero in the standard model.
3. The differential cross section at $\cos \theta=0$ for producing one longitudinal and one transverse $W$ from a $J_{z}=0$ state $((++0+)$ term) is exactly zero for all values of $\kappa$ and $\lambda^{[12]}$.
4. The cross section for producing two longitudinal $W$ 's from a $J_{z}=0$ photon combination is suppressed by a factor of $m_{W}^{2} / s$ in the standard model. The


Figure 6. Differential cross sections for producing $W$ pairs of specific polarization with center of mass energy of 350 GeV as a function of $\cos \theta$. The solid lines are standard model couplings ( $\kappa=1, \lambda=0$.) The dashed curve was calculated using $\kappa=1.1, \lambda=0$ while the dotted curve is for $\kappa=1, \lambda=0.1$. Here and henceforth, cross sections are given in units of $R$.
same factor is well known to appear in the production of charged scalars $\left(\gamma \gamma \rightarrow \pi^{+} \pi^{-}.\right)$
5. The cross section for producing two right- (left-) handed $W$ bosons from two


Figure 7. Differential cross sections with center of mass energy of 350 GeV at $\cos \theta=0$.
Solid curves show dependence on $\Delta \kappa=\kappa-1$. Dashed curves show dependence on $\lambda$.
left- (right-) handed photons is suppressed by a factor of $\left(m_{w}^{2} / s\right)^{2}$.
Information on the polarization of a $W$ boson is obtained by looking at the angular distribution of its decay products. For clarity, let us discuss the decay $W^{-} \rightarrow l^{-} \bar{\nu}$. Let $\chi$ be the angle between the lepton momentum and the $W$ direction of motion, as measured in the $W$ center-of-mass frame. The $\chi$ distribution is then
given by

$$
\begin{equation*}
\frac{d \sigma}{d \cos \chi}=\sum_{\lambda} \sigma_{\lambda} P_{\lambda}(\chi) \tag{4.7}
\end{equation*}
$$

where

$$
\begin{equation*}
\sigma_{\lambda}=\sum_{\lambda^{\prime}} \sigma\left(\gamma \gamma \rightarrow W_{\lambda^{\prime}} W_{\lambda}\right) \tag{4.8}
\end{equation*}
$$

and

$$
\begin{align*}
P_{ \pm 1}(\chi) & =\frac{3(1 \mp \cos \chi)^{2}}{8}  \tag{4.9}\\
P_{0}(\chi) & =\frac{3\left(1-\cos ^{2} \chi\right)}{4}
\end{align*}
$$

The ratio between left- and right-handed $W$ bosons is related to the $\chi$ forwardbackward asymmetry $\chi_{\text {FB }}$ by

$$
\begin{equation*}
\chi_{\mathrm{FB}}=\frac{\sigma(\cos \chi>0)-\sigma(\cos \chi<0)}{\sigma(\cos \chi>0)+\sigma(\cos \chi<0)}=\frac{3}{4} \frac{\sigma_{-1}-\sigma_{1}}{\sigma_{\mathrm{TOT}}} \tag{4.10}
\end{equation*}
$$

## 5. Observables

In this section we study the various observables and assess their dependence on $\kappa$ and $\lambda$. All the calculations in this chapter include the effects of the beam energy and helicity distributions from equations (3.3) and (3.5).

The most straightforward observable is the total cross section $\sigma_{\text {тот }}$. In other processes such as $e \gamma \rightarrow W \nu$ and $e^{+} e^{-} \rightarrow W^{+} W^{-}$it is found to be quite sensitive to anomalous couplings. To reduce problems associated with particles escaping detection by going near the beam pipe, we cut the angular integration at $|\cos \theta|=$ 0.8. Measuring the total cross section is prone to systematic errors. In this paper we assume a systematic error of $5 \%$ in total cross section measurements. The
total number of events, given an integrated luminosity of $10 \mathrm{fb}^{-1}$, is about $2 \times 10^{5}$. Statistical errors are much less than $1 \%$, and are neglected here. Even with one tenth that luminosity, our results would not change significantly. Figure 8 shows the total cross section for $\gamma \gamma \rightarrow W^{+} W^{-}$as a function of $\kappa$ for several values of $\lambda$, while fig. 9 shows its dependence on $\lambda$ for several values of $\kappa$.


Figure 8. $\sigma_{\text {тот }}$ for $\gamma \gamma \rightarrow W^{+} W^{-}$as a function of $\kappa$ for $\lambda=-0.1,0,0.1$.


Figure 9. $\sigma_{\text {тот }}$ for $\gamma \gamma \rightarrow W^{+} W^{-}$as a function of $\lambda$ for $\kappa=0.9,1,1.1$.

The process $\gamma \gamma \rightarrow W^{+} W^{-}$is symmetric with respect to interchanging the two initial photons, so there cannot be any forward-backward asymmetry. The angular distribution of the $W$ does, however, carry important information. We quantify this information by looking at the ratio

$$
\begin{equation*}
I O=\frac{\sigma(|\cos \theta|<0.4)}{\sigma(|\cos \theta|<0.8)} \tag{5.1}
\end{equation*}
$$

Systematic errors associated with luminosity measurement cancel. We assume here that $I O$ can be measured to 0.03 . Figures 10 and 11 show $I O$ 's dependence on $\kappa$ and $\lambda$.


Figure 10. IO as a function of $\kappa$ for $\lambda=$ $-0.1,0,0.1$.


Figure 11. $I O$ as a function of $\lambda$ for $\kappa=$ $0.9,1,1.1$.

As in $e^{+} e^{-} \rightarrow W^{+} W^{-}$, the angle $\chi$ can be measured for one of the $W^{\prime}$ s in events in which one $W$ decays hadronically while the other decays leptonically. About $44 \%$ of all events are of that nature. By measuring the $\chi$ distribution one can easily derive the ratio $L / T$ of longitudinally to transversely polarized $W$ production. Here we assume that the $L / T$ ratio can be measured to within 0.03 . Figures 12 and 13 show $L / T$ 's dependence on $\kappa$ and $\lambda$.

Polarizing the laser beam causes the photon spectrum to be polarized. The most energetic photons are always polarized with opposite helicity with respect to the laser photons (see fig. 3.) Setting the two laser sources to have the same polarization makes the most energetic events have $\left|J_{z}\right|=2$. We call this setting ' $J_{z}=2$ dominated'. Similarly, setting the two laser sources to have opposite polarization makes the most energetic events have $J_{z}=0$. This setting is referred to as ' $J_{z}=0$ dominated'. The ratio

$$
\begin{equation*}
(0 / 2)=\frac{\sigma\left(J_{z}=0 \text { dominated }\right)}{\sigma\left(J_{z}=2 \text { dominated }\right)} \tag{5.2}
\end{equation*}
$$



Figure 12. $L / T$ ratio as a function of $\kappa$ for $\lambda=-0.1,0,0.1$.


Figure 13. $L / T$ ratio as a function of $\lambda$ for $\kappa=0.9,1,1.1$.
is another, independent, measurable quantity. Lasers can be easily and accurately polarized, and therefore virtually all systematic errors are canceled in measuring this ratio. We assume it can be measured to 0.01 . Let us stress that this measurement does not require the electron beam to bë polarized. Figures 14 and 15 show the dependence of the ratio $(0 / 2)$ on $\kappa$ and $\lambda$.


Figure 14. (0/2) ratio as a function of $\kappa$ for $\lambda=-0.1,0,0.1$.


Figure 15. (0/2) ratio as a function of $\lambda$ for $\kappa=0.9,1,1.1$.

Finally, with the photon beams polarized, one can measure not only the $|\cos \chi|$ distribution which carries the information on the $L / T$ ratio, but also the $\chi$ forwardbackward asymmetry $\chi_{\text {FB }}$ which carries information on the ratio between positively and negatively polarized $W$ bosons (see eqn. (4.10).) Here we assume that the $\chi_{\mathrm{FB}}$ can be measured to 0.03 . Figures 16 and 17 show the dependence of the $\chi_{\text {FB }}$ on $\kappa$ and $\lambda$.


Figure 16. $\chi_{\text {Fb }}$ as a function of $\kappa$ for $\lambda=$ $-0.1,0,0.1$.


Figure 17. $\chi_{\text {fb }}$ as a function of $\lambda$ for $\kappa=$ $0.9,1,1.1$.

## 6. Discovery Limits

In assessing the discovery potential of this experiment we assume that standard model results are actually measured. We then ask what region in the $\kappa$ - $\lambda$ plain is still allowed based on the measured results. We present the allowed region for each measurement separately, as well as the combined results of all the measurements. Figure 18 shows the allowed regions on the $\kappa-\lambda$ plane.

Next we compare the sensitivity of $\gamma \gamma \rightarrow W^{+} W^{-}$to anomalous couplings to that of other processes taking place at a comparable collider, namely $e^{+} e^{-} \rightarrow$


Figure 18. Allowed regions ( $2 \sigma$ ) in the $\kappa$ - $\lambda$ plain from various measurements of $\gamma \gamma \rightarrow W^{+} W^{-}$. The regions in the middle correspond to $68 \%$ and $90 \%$ confidence level bounds from the combination of all measurements.
$W^{+} W^{-}$and $e \gamma \rightarrow W \nu$.
The formulas for the cross sections of $e^{+} e^{-} \rightarrow W^{+} W^{-}$were taken from ref. 1 . We assume a 500 GeV center-of-mass $e^{+} e^{-}$collider. The $e^{+} e^{-} \rightarrow W^{+} W^{-}$cross section depends on both $W \gamma$ and $W Z$ couplings. Here we reduce the number of independent variables by assuming $\kappa_{Z}=1$ and $\lambda_{Z}=\lambda_{\gamma}$ as suggested from low energy experiments. ${ }^{[13]}$ The measureables we used (and the corresponding experimental errors) are: the total cross (5\%), the forward-backward asymmetry $F B$ (0.03), the in-out ratio $I O(0.03)$ and the longitudinal to transverse $W$ production ratio $L / T(0.03)$.

The expressions for the helicity amplitudes for $e \gamma \rightarrow W \nu$ are given in Appendix B. They were calculated using a new and simple technique for symbolic calculation of matrix element level spinor expressions, to be described elsewhere. The
measureables used for $\mathrm{e} \gamma \rightarrow W \nu$ (and their corresponding experimental errors) are: the total cross section (5\%), the forward-backward asymmetry $F B(0.03)$, the in-out ratio $I O(0.03)$ and the polarization ratio ( $1 / 3$ ) defined in analogue to $(0 / 2)$ (eqn. (5.2)) as the ratio between ' $\left|J_{z}\right|=1 / 2$ dominated' and ' $\left|J_{z}\right|=3 / 2$ dominated' configurations ${ }^{[14]}$ (0.01).

Figure 19 shows allowed region from the combination of all measurements $(90 \%$ confidence level) of the processes $\gamma \gamma \rightarrow W^{+} W^{-}, e^{+} e^{-} \rightarrow W^{+} W^{-}$and $e \gamma \rightarrow W \nu$. As the figure shows, both $e^{+} e^{-} \rightarrow W^{+} W^{-}$and $e \gamma \rightarrow W \nu$ give very strict, 0.03-0.05 bounds on both $\kappa$ and $\lambda . e \gamma \rightarrow W \nu$ is less sensitive, giving bounds of order 0.050.10. Reference 13 discusses possible bounds from the proposed Superconducting Super Collider (SSC). Using the process $q \bar{q} \rightarrow W \gamma$, a very strong bound (of order 0.01 ) can be imposed on $\lambda$. The same process is much less sensitive to deviations in the value of $\kappa$. Our results nicely complement the SSC bound by giving a very strict bound on possible $\kappa$ values. ${ }^{[15]}$

## 7. Conclusion

A $500 \mathrm{GeV} e^{+} e^{-}$collider is a powerful tool for measuring $W \gamma$ and $W Z$ couplings. The process $e^{+} e^{-} \rightarrow W^{+} W^{-}$is uniquely sensitive to the various couplings under examination. The two other processes we consider, e $\rightarrow W \nu$ and $\gamma \gamma \rightarrow W^{+} W^{-}$can significantly add to our knowledge of $W$ interactions. Compared to $e^{+} e^{-} \rightarrow W^{+} W^{-}$, both processes involve only $W \gamma$ vertices, allowing a seperation of $W \gamma$ and $W Z$ effects. Their large cross sections allow detailed study even with relatively small integrated luminosity. Finally, $\gamma \gamma \rightarrow W^{+} W^{-}$involves the $W W \gamma \gamma$ vertex. A careful analysis of experimental results should give us insight into its structure.


Figure 19. Allowed regions ( $90 \%$ confidence level) in the $\kappa-\lambda$ plane from various experiments.

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## APPENDIX A

The calculations described in this paper were performed in two independent ways. In the first, we used explicit expressions for the various polarization vectors. In the second, we used spinor techniques ${ }^{[16]}$ to represent the photon polarizations and to sum over $W$ polarizations. For this second approach, the phase space integration was performed numerically. In addition, our results agree with Ginzburg's analytical results. ${ }^{[6]}$

In performing the calculations we used HIP ${ }^{[17]}$ - a set of packages for performing high-energy calculations using Mathematica. ${ }^{[11]}$ Mathematica is a program for performing mathematical calculations, both symbolic and numeric, on the computer. HIP was written by the author and A. Hsieh.

HIP contains several components:

1. Covariant vector manipulation: Lorentz index contraction and dot product substitution, including automatic treatment of Mandelstam variables.
2. Full relativistic dynamics, including boosting and decaying four-vectors.
3. Dirac algebra: commuting gamma matrices and taking traces.
4. Spinor techniques for handling fermions and vector polarization.
5. Automatic phase space integration.
6. Automatic generation of C code for numeric evaluation and integration.
7. A database of standard model Feynman rules.

As an illustration of some of the capabilities of HIP and Mathematica, we present the Mathematica code used to calculate the total cross section for $\gamma \gamma \rightarrow$ $W^{+} W^{-}$in the standard model.

```
(* General Preparations *)
PrepareIndex[mu, nu, sig, tau, ro1, ro2]
SetMandelstam[{p1, p2, p3, p4}, {0, 0, mw, mw}, s, t, u]
p1 = p3 + p4 - p2
u = 2mw^2-s - t
SetReal[mu, nu]
Mass[W] "= mw
(* The following objects are already defined:
    WWP - W W photon vertex
    HVP - Heavy vector propagator
    PPWW - W W photon photon vertex
    HEps - Heavy vector polarization
*)
```

```
(* t-channel *)
tch = WHP[{p3, p1-p3}, {sig, ro1, mu}] HVP[W, p1-p3, ro1, ro2] *
                                    WWP[{p2-p4, p4}, {ro2, tau, nu}]
(* u-channel *)
uch = WWP[{p3, p2-p3}, {sig, ro1, nu}] HVP[W, p2-p3, ro1, ro2] *
                        WWP[{p1-p4, p4}, {ro2, tau, mu}]
(* 4-vertex *)
v4 = PPWW[sig, tau, mu, nu]
(* matrix element *)
me = Contract[tch + uch + v4, {ro1, ro2}]/.
                                    {p2[nu]->0, p3[sig]->0, p4[tau]->0}
(* matrix element squared *)
me2 = Contract[me Conjugate[me], {mu, nu}] /.
                                    {p3[sig]->0, p4[tau]->0}
me2 = Contract[me2 AbsSquared[HEps[p3, sig] HEps[p4, tau]],
                                    {sig, tau, Conjugate[sig], Conjugate[tau]}]
me2 = Together[me2]
(* go back to representing dot products explicitly *)
Clear[p1, p2, p3, p4]
SetMass[{p1, p2, 0}, {p3, p4, mw}]
t = mw^2 - 2 DotProduct[p2, p4]
(* set up and perform phase space integration *)
ps = CrossSection[me2, p1->{0, 0, Sqrt[s]/2, Sqrt[s]/2},
        p2->{0, 0, -Sqrt[s]/2, Sqrt[s]/2}, Cylindrical[p3, p4]]/4
cs = EvaluatePhaseSpaceIntegrate[ps]
(* in terms of r = s / (4 mw^2) *)
cs = cs /. {s->1, mw->Sqrt[r/4]}
```

After some rearrangement, the final value of cs is


## APPENDIX B

In this appendix we present expressions for the various helicity amplitudes for $e \gamma \rightarrow W \nu$. Let $\mathcal{M}_{\lambda_{1}, \lambda_{2}}$ be the amplitude for fixed photon helicity $\lambda_{1}$ and $W$ helicity $\lambda_{2}, \Delta \lambda_{1}=\lambda_{1}+1 / 2, \Delta \lambda_{2}=\lambda_{2}+1 / 2$ and $J_{0}=\max \left(\left|\Delta \lambda_{1}\right|,\left|\Delta \lambda_{2}\right|\right)$. Separating some overall angular dependence in terms of the conventional $d$-functions, ${ }^{[18]}$

$$
\begin{equation*}
\mathcal{M}_{\lambda_{1}, \lambda_{2}}(\theta, \phi)=e g d_{\Delta \lambda_{1}, \Delta \lambda_{2}}^{J_{0}}(\theta) \tilde{\mathcal{M}}_{\lambda_{1}, \lambda_{2}}(\theta) \tag{B.1}
\end{equation*}
$$

simple expressions are obtained for the matrix elements $\tilde{\mathcal{M}}$. Only left-handed electrons participate in the reaction; for those one finds

$$
\begin{equation*}
\tilde{\mathcal{M}}_{\lambda_{1}, \lambda_{2}}(\theta)=-\beta\left(A_{\lambda_{1}, \lambda_{2}}+\frac{B_{\lambda_{1}, \lambda_{2}}(\theta)}{(1-\cos \theta+r(1+\cos \theta))}\right) \tag{B.2}
\end{equation*}
$$

where $r=m_{w}^{2} / s, \beta=\sqrt{1-r} . A_{\lambda_{1}, \lambda_{2}}$ and $B_{\lambda_{1}, \lambda_{2}}(\theta)$ are given in table 1.
The total cross section for $e \gamma \rightarrow W \nu$ is then given by folding in the photon energy spectrum $\sigma_{c}^{\prime}(y)$ (eq. (3.3)):

$$
\begin{align*}
\sigma & =\int d \Omega d y \sigma_{c}^{\prime}(y) \frac{(2 \pi)^{4}}{2 y s} \frac{1-m_{W}^{2} / y s}{8(2 \pi)^{6}} \sum_{\lambda_{1}, \lambda_{2}}\left|\mathcal{M}_{\lambda_{1}, \lambda_{2}}\right|^{2} \\
& =\frac{1}{32 \pi s} \int_{m_{W}^{2} / s}^{y_{m}} d y \frac{1-m_{W}^{2} / y s}{y} \sigma_{c}^{\prime}(y) \int_{-1}^{1} d(\cos \theta) \sum_{\lambda_{1}, \lambda_{2}}\left|\mathcal{M}_{\lambda_{1}, \lambda_{2}}\right|^{2} \tag{B.3}
\end{align*}
$$

Table 1. The coefficients $A_{\lambda_{1}, \lambda_{2}}$ and $B_{\lambda_{1}, \lambda_{2}}(\theta)$ of eq. (B.2) for the general $W W \gamma$ vertex (eq. (2.1)). $r=m_{W}^{2} / s$ and $\beta=\sqrt{1-r}$.

| $\left(\lambda_{1} \lambda_{2}\right)$ | $A_{\lambda_{1}, \lambda_{2}}$ | $B_{\lambda_{1}, \lambda_{2}}(\theta)$ |
| :---: | :---: | :---: |
| $(--)$ | $\sqrt{2}$ | $-(2(3-\cos \theta)-2 r(1-\cos \theta)+2 \Delta \kappa-\lambda(1-\cos \theta)) / \sqrt{2}$ |
| $(-0)$ | $-1 / \sqrt{r}$ | $(1-\cos \theta+r(1+\cos \theta)+\Delta \kappa+\lambda \cos \theta) / \sqrt{r}$ |
| $(-+)$ | 0 | $\sqrt{2 / 3} \lambda / r$ |
| $(+-)$ | 0 | $\sqrt{2 / 3}\left(2 r^{2}+\Delta \kappa r+\lambda(1-r)\right) / r$ |
| $(+0)$ | 0 | $(-4 r-\Delta \kappa(1+r)-\lambda(1+r)) / \sqrt{3 r}$ |
| $(++)$ | 0 | $-\sqrt{2}(2+\Delta \kappa)$ |

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