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Simulation of a Storage Ring Free Electron Laser with a Mapping Algorithm for Distribution Functions*

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ABSTRACT

A model for the simulation of the time dependent behavior and the analysis of the equilibrium of the coupled system of storage ring and Free Electron Laser (FEL) is presented. The analysis comprises both amplifier and oscillator FEL. Bunch lengthening and energy widening due to wake forces are taken into account in a self-consistent way. The method is based on a mapping algorithm for means and correlations of the electron distribution function, pioneered by K. Hirata. The evolution of the laser field in the oscillator FEL is described by supermodes. The model is used to simulate an FEL in a small 500 MeV storage ring with 100 m circumference. Typical values for the output power, spatial, and spectral characteristics of the emitted radiation are presented.

INTRODUCTION

In this paper we present a method to investigate the interaction of a storage ring and a small signal-small gain FEL. We address the time-dependent transient behavior and the equilibrium. Special attention is paid to the temporal and spectral characteristics of the laser output in the oscillator configuration.

In the model we will consider the longitudinal phase space of the storage ring, only. Damping and quantum excitation due to synchrotron radiation lead to Gaussian bunches, which are perturbed by two processes. First, discontinuities in the vacuum chamber lead to wake fields by which the leading particles in a bunch perturb the trailing particles. Second, in an amplifier FEL the energy exchange between the transversely undulating electrons and a copropagating laser wave depends crucially on the relative phases and predominantly acts as a noise source for the electron beam. Only the non-linear character of the FEL process leads to an overall energy loss of the electrons and amplification of the laser wave. In an oscillator the situation is complicated due to the presence of a multitude of resonator modes. Furthermore, the FEL process leads to mode coupling and the formation of light pulses, called supermodes, typically having a length of a few percent of the electron bunch length. The supermodes consist of thousands of coupled modes which then act back on the electron beam and perturb it locally where the light pulse is situated, usually slightly ahead of the peak of the electron distribution.

THE STORAGE RING

Since both the wake field and the FEL depend on collective properties of the electron distribution, a statistical approach is in order. To avoid the need of huge computers we adopt K. Hirata's approach [1] and construct mappings for the means $X_i = \langle x_i \rangle$, and correlations $\sigma_{ij} = \langle (x_i - X_i)(x_j - X_j) \rangle$, of a Gaussian distribution function in the variables $x_1 = \omega_0 \nu_s z/\alpha c$ and $x_2 = (E - E_0)/E_0$. Here z and E are the longitudinal position and energy of an electron, and ω_0, ν_s, α are the revolution frequency, the synchrotron tune, and the momentum compaction factor, respectively. E_0 is the design energy. In these variables the map for oscillations reduces to a rotation matrix and for synchrotron radiation effects to $x_1' = x_1$ and $x_2' = \xi x_2 + \sqrt{1-\xi^2}\sigma_0 \hat{P}$ where $\xi = e^{-T_0/\tau_E}$ is the damping decrement, T_0/τ_E is the ratio between revolution time and longitudinal damping time, σ_0 is the natural energy spread, and $\hat{P}$ is Gaussian white noise. Here primed quantities denote values after the map. For a wake defined by the superposition of resonator impedances $Z(\omega) = \sum_n R_n/(1 + iQ_n(\omega/\omega_n - \omega_n/\omega))$, the map for an individual particle is given by $x_1' = x_1$ and $x_2' = x_2 - f(x_1)$, with $f(x_1)$ defined by [3]

$$
\begin{aligned}
f(x_1) = f_0 \sum_n \frac{A_n}{4} \exp\left[-\frac{(x_1 - X_1)^2}{2\sigma_{11}}\right] \\
\times \left[i\frac{C_n\sqrt{\sigma_{11}}}{\sqrt{2}} - i\frac{x_1 - X_1}{\sqrt{2}\sqrt{\sigma_{11}}}\right] + c.c. \ ,
\end{aligned}
\tag{1}
$$

and $A_n = \tilde{A}_n / \sum_n \tilde{A}_n$, $\tilde{A}_n = \omega_n R_n (1 + i/\sqrt{4Q_n^2 - 1})/Q_n$, $C_n = \left[(\alpha\omega_n)/(2\omega_0\nu_s Q_n)\right](1 - i\sqrt{4Q_n^2 - 1})$. f_0 is a normalization constant, proportional to the charge per bunch, and $w(z)$ is the complex error function. R_n, Q_n and ω_n are the shunt impedance, the quality factor, and the frequency of the n-th resonator mode.

Note that the single particle wake map is a localized map in the sense that the longitudinal position of the particle does not change. In Refs. [2,3] it is shown that from localized maps of the type $x_1' = x_1$, $x_2' = x_2 + g(x_1, x_2)$, maps for the statistical quantities X_i, σ_{ij} can be constructed which are given by

$$
\begin{aligned}
X_1' &= X_1, \qquad & X_2' &= X_2 + R_1 \ , \\
\sigma_{11}' &= \sigma_{11}, \qquad & \sigma_{12}' &= (1 + R_{22})\sigma_{12} + R_{21}\sigma_{11} \ , \\
\sigma_{22}' &= (1 + 2R_{22})\sigma_{22} + 2R_{21}\sigma_{12} + R_3 \ ,
\end{aligned}
\tag{2}
$$

where the functions R depend on X_i and σ_{ij}. For arbitrary $g(x_1, x_2)$, and in particular for the wake map, they are calculated in Ref. [3].

The concatenation of the maps for wake, synchrotron radiation effects, and synchrotron oscillation defines the one-turn-map for the simple model storage ring. The period-1 fixed point of this map defines the equilibrium values for X_i and σ_{ij}, and can be determined using the "fast fixed point finding algorithm" described in Ref. [3].

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Influence of the Beam Quality on FEL Performance

The effect of the beam quality on the performance of an FEL oscillator is suitably described by the theory of supermodes [4], which leads to an eigenvalue equation for the longitudinal electric field profile of the light pulse. This eigenvalue equation can be solved with an ansatz

$$E(\tilde{z}) \propto e^{\lambda_n \tau} H_n(a(\tilde{z}+b)) e^{-c(\tilde{z}+d)^2}, \tag{3}$$

where a, b, c, d are functions of the electron beam parameters [3,4,5]. $H_n(x)$ are Hermite polynomials and λ_n is the gain coefficient whose real part determines the growth rate of the $n-$th supermode. τ is the scaled time and $\tilde{z} = z/\sigma_z$. The length and displacement of the light pulse can be calculated from Eq. (3), and using the Fourier transform, the bandwidth can be obtained. In the simulation code [3] only the fundamental supermode is taken into account.

The gain of the amplifier FEL can be obtained from λ_n by setting $n = 0$ and the electron bunch length to ∞.

The Effect of the FEL on the Electron Beam

The FEL affects the electron beam in two ways. First, on a macroscopic scale it introduces noise, because one of two particles separated by half a laser wave length looses energy while the other gains energy. Second, due to the non-linear character of the FEL interaction the electron bunch as a whole looses or gains energy. Since the electrons do not move longitudinally within the bunch during their traversal the map for the FEL can also be written as $x_1' = x_1$, $x_2' = x_2 + \delta\varepsilon$. Here we have to note that $\delta\varepsilon$ does not only depend on the macroscopic quantities x_1 and x_2, but also on the microscopic relative phases of the electrons' undulation and the laser wave. In order to perform the averages we first have to "microaverage" on the scale of the laser wavelength. We will denote this process by a bar over the averaged quantity. Second, we "macroaverage" over the macroscopic dimensions of the bunch, and denote this process by acute brackets. As an example R_1 and R_3 for the FEL are given by $R_1 = \langle\overline{\delta\varepsilon}\rangle$ and $R_3 = \langle\overline{\delta\varepsilon^2}\rangle$.

For an oscillator FEL $\overline{\delta\varepsilon}$ and $\overline{\delta\varepsilon^2}$ can be expressed as [3]

$$\begin{aligned} \overline{\delta\varepsilon^2} &= \tilde{N}_0 W_0 S_1(x_1 - X_1)\left(\frac{\sin(2\pi N\varepsilon)}{2\pi N\varepsilon}\right)^2 , \\ \overline{\delta\varepsilon} &= \frac{1}{2}\frac{\partial}{\partial\varepsilon}\langle\overline{\delta\varepsilon^2}\rangle , \end{aligned} \tag{4}$$

where $S_1(x_1) = \exp[-\Gamma_1 x_1^2 + \Gamma_2 x_1 + \Gamma_3]$ and the Γ_i are expressable in terms of the a, b, c, d that determine the spatial profile of the supermodes. W_0 is the peak energy density of the fundamental supermode and $\tilde{N}_0$ is a normalization constant. $\varepsilon = x_2 - x_r$ is the difference between the resonance energy of the FEL x_r and the energy of the electrons. Using Eq. (4) and with the help of a few tricks reported in Ref. [3] all functions R_i in Eq. (2) can be calculated in closed form in terms of complex error functions [3].

The coefficients R_i for the map for the amplifier FEL is obtained by setting $\Gamma_i = 0$ [3] and then using the expressions for the oscillator FEL.

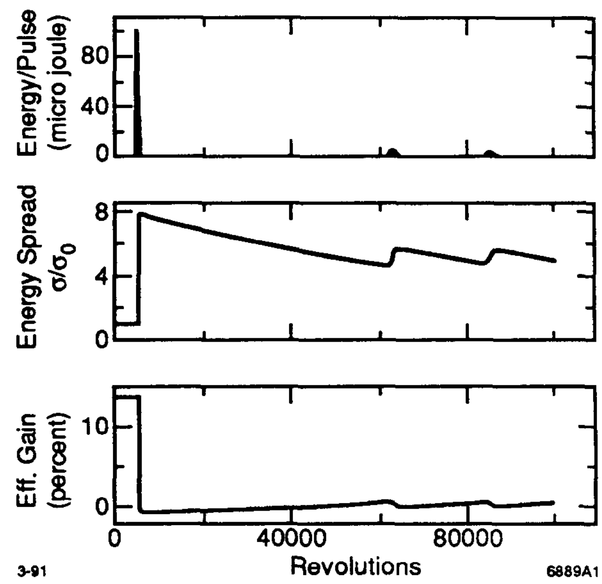

Figure 1. The extracted energy per micropulse in μJ (a), the normalized energy spread $\sqrt{\sigma_{22}}/\sigma_0$ and the effective gain $g_0 \operatorname{Re}[\lambda_0] - \gamma_T$ for a simulation over 100 000 revolutions. After the FEL is switched on after 5000 revolutions the first superpulse appears which in turn degrades the energy spread such that the laser turns itself off.

Simulation Results

The presented maps are implemented in a FORTRAN code that runs interactively on IBM PCs. The storage ring-FEL system is modelled by the following succession of maps: *Wake* $\mapsto$ *FEL* $\mapsto$ *Radiation* $\mapsto$ *Oscillation.* Since only 10 quantities $(X_i, \sigma_{ij}, a, b, c, d, W_0)$ are mapped, the simulation is very fast. On an 8 MHz PC-XT with math coprocessor 20,000 revolutions take about one hour.

Fig. 1 a,b,c show a plot of the energy per microbunch (a), the normalized energy spread (b), and the total effective gain (c) for a small storage ring over 100, 000 revolutions. The FEL is switched on after 5,000 revolutions, and due to the small energy spread and short bunch length, the gain is large and the output laser power rises. This explains the first large peak in Fig. 1a. Then the large laser peak power increases the energy spread (b) which in turn degrades the gain (c) down to the level of the mirror losses of 1 %. This switches the laser pulse off (a) for about 50,000 revolutions and the energy spread damps down (b) which in turn lets the gain increase until the next laser pulse appears. This sequence repeats with increasingly smaller amplitude towards an equilibrium configuration.

The coupled system exhibits three interesting dynamical regimes: the first large superpulse, the oscillations, and the equilibrium which is reached after a long time. The equilibrium was investigated in Refs. 6,7. Using the fixed point option in the simulation code it was verified that the laser output power in equilibrium is proportional to the emitted synchrotron radiation power. In our model the energy stored in about 1 ps-long micro pulses is on the order of 0.2 μJ.

The oscillation that starts to appear in Fig. 1 can be accounted for by a simple model presented in Ref. [8]. The oscillations indicate that the system has a resonance in the 100 Hz range which makes it susceptible to external perturbations.

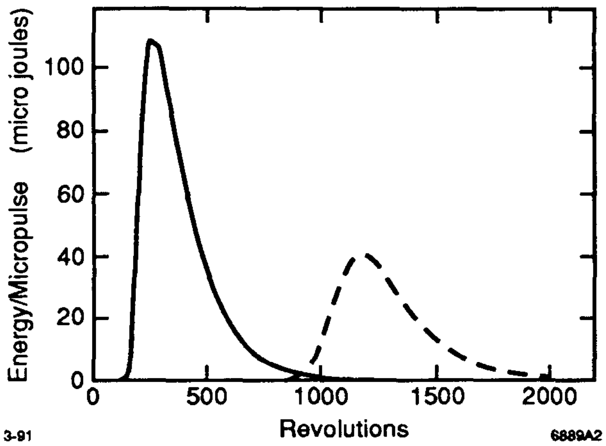

Figure 2. Comparison of the energy per micro bunch of the super pulse over the first 2 000 turns for a configuration without wake (solid) and with a wake (dashed) that produces a threefold initial energy spread.

The first large superpulse with total stored energy of about 30 mJ consists of 300 micropulses, each of which is approximately 1 ps long and stores around 50 to 100 μJ energy. Note that the energy in a micropulse of the large superpulse is three orders of magnitude larger than in equilibrium, which makes Q-switching mechanisms attractive [8].

Fig. 2 shows the effect of a wake that produces a threefold initial energy spread on the energy per micro pulse for the large super pulse over the first 2000 revolutions after the laser is turned on. The larger pulse is identical to the one in Fig. 1a and the second smaller pulse is the equivalent pulse with wake. Clearly the increased energy spread reduces the gain and the laser pulse takes longer to build up and its peak assumes a smaller value.

Conclusion

The presented mapping algorithm for distribution functions for the first time allows the realistic investigation of the time dependence of the coupled system of FEL and storage ring. It unifies the simpler models [6,7,8] previously used to explain; e.g., the oscillations or the equilibrium.

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