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Simulation of accelerating structures with large staggered tuning*

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Abstract

One idea for controlling multibunch beam breakup in future linear colliders is to use accelerating structures that have a large (~10%) staggered tuning, that is, a large cell-to-cell spread in frequencies of the transverse dipole modes. Simulations of such structures using the programs TBCI and LINACBBU are compared, and issues relevant to the design and optimization of such structures, such as choice of cell frequency distribution and role of cell-to-cell coupling are discussed.

I. INTRODUCTION

A variation in the cell dimensions in each accelerating section of the SLAC linac was originally introduced to make the acceleration gradient nearly constant over the length of the structure. This variation also turned out to be invaluable for reducing the severity of cumulative beam break-up, because of the resulting cell-to-cell spread (a few percent) in the frequencies of the transverse dipole modes ("staggered tuning") [1]. It has been suggested that an even larger staggered-tuning (~10% or so) could help control multibunch beam break-up in future linear colliders [2]. Such staggered tuning might be useful in conjunction with damped acceleration structures [3], especially if there should turn out to be practical limitations in going to extremely low transverse mode Q's in the damped structures. Our simulations indicate that staggered tuning with an appropriately chosen frequency distribution, combined with moderate ($Q \lesssim 100$) de-Q'ing of the transverse modes, can reduce the multibunch beam break-up to very low levels in future collider designs. We shall also compare the results of simulations with and without cell-to-cell coupling, and investigate the effects of errors in the design frequency distribution, in order to estimate the reliability of the simulations of our proposed solution.

II. SIMULATION METHODS

Two different programs were used to simulate the transverse wake field and its effect on a train of bunches, in structures with large staggered tuning; each method has its own limitations and advantages.

The program TBCI [4] calculates the wake field, taking into account the coupling between cells. A disadvantage is that it is difficult for the program to accurately represent the small differences between cells in the stagger-tuned structures, due to limitations of the meshing in the program. Another disadvantage is that the program takes much CPU time, and thus is impractical for examining more than a few cases.

The other program used was LINACBBU [5]. This program does not actually calculate wake field modes for a given structure, but rather takes a set of wake field modes for a single cell as input. These modes can then be frequency-split in any desired fashion, to represent the staggered tuning. Since we are interested in the average wake, we simply regard all components as distributed over all cells. Coupling between cells is ignored, so for a stagger-tuned structure, the wake is less accurately represented at longer distances. However, if one can show that the wake field obtained using LINACBBU is accurate enough over the range needed, then LINACBBU is an efficient tool for examining various stagger-tuned distributions and calculating the resulting beam break-up.

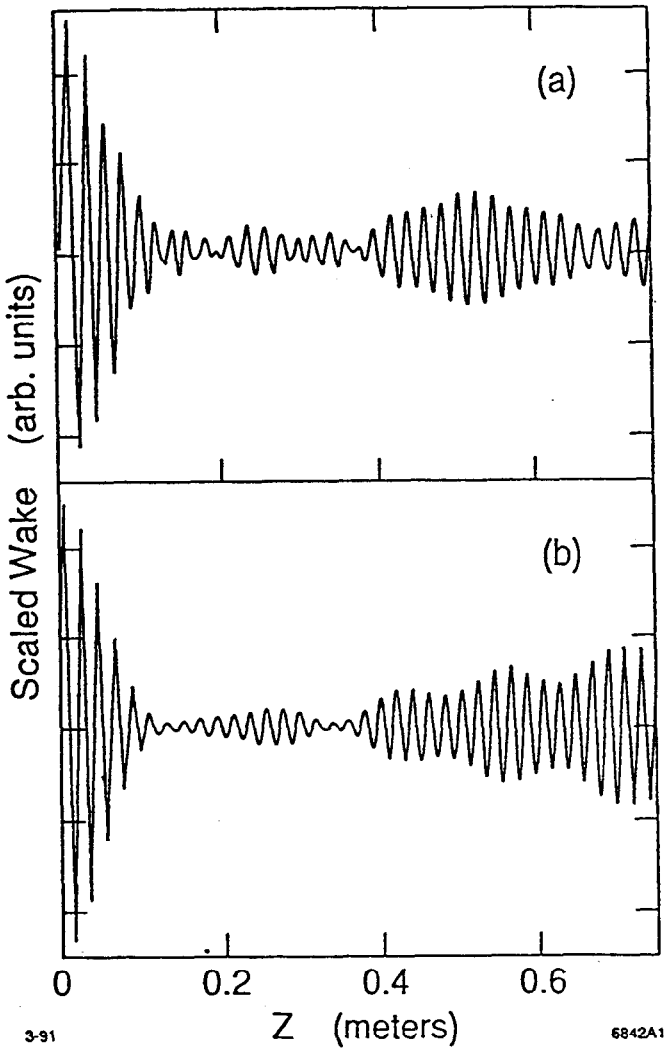

Figure 1. Plots of the fundamental transverse mode wake field, for a 37% full-width, $\pm 2.5\sigma$ truncated-Gaussian, stagger-tuned structure, (a) calculated by TBCI, and (b) as represented without coupling in LINACBBU. [The small (1 cm) horizontal offset in the TBCI plot is due to the fact that $z = 0$ is at the -4σ point in the bunch used to calculate the wake, with $\sigma = 2.5$ mm.]

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A comparison between the wake field generated by TBCI and the wake field obtained from LINACBBU for the same set of mode frequencies is shown in Fig. 1, for a structure designed to operate at 11.4 Ghz accelerating frequency. Only the fundamental transverse wake mode (frequency $\approx$ 14.4 GHz) will be considered throughout this paper, as it is strongly dominant for these structures. The structure in this example is assumed to consist of 48 detuned cells, with the mode frequency distribution approximately Gaussian in density, truncated at $\pm 2.5\sigma$, where $\sigma = 1.062$ GHz. The density in this example is actually a "stair-step" approximation to Gaussian, due to limitations in representing a more smoothly varying distribution in TBCI.

The initial roll-off of the wake field due to the approximately Gaussian distribution agrees well between TBCI and the simpled, uncoupled representation used in LINACBBU. Recent measurements at the ANL advanced accelerator test facility are also in good agreement with LINACBBU simulations [6].

For larger distances z, the wake as represented by LINACBBU is expected to deviate from reality, since coupling has been ignored. Our strategy is to obtain a strong initial reduction of the wake via the staggered tuning distribution, before one bunch spacing, which is about 42 cm in the present SLAC design for a future linear collider. Beyond this distance, the effects of coupling, as well as effects due to random errors in the chosen frequency distribution, make the calculation of the details of the wake somewhat uncertain. However, we rely upon the de-Q'ing of the transverse modes to control this residual wake at longer distances.

III. FREQUENCY DISTRIBUTIONS

Consider a single wake mode of wavenumber $k_0 = \omega_0/c$, which when unsplit gives a wake function

$$W(z) = W_0 \sin(k_0 z) \exp(-k_0 z/2Q_0) \quad . \qquad (1)$$

To represent staggered-tuning in LINACBBU, we assume this mode is split into N_{spr} components of equal weight. Given a full-spread Δk_{tot}, the components run from $(k_0 - \frac{\Delta k_{tot}}{2})$ to $(k_0 + \frac{\Delta k_{tot}}{2})$. For the linear distribution, there is a uniform spacing $\Delta k_{tot}/(N_{spr}-1)$ between adjacent components. For the truncated-Gaussian distribution, with given σ, the density of frequency components near wavenumber k is proportional to $\exp[-(k-k_0)^2/2\sigma^2]$. This means that the spacing between adjacent components is given by

$$\mathrm{erf}\left(\frac{k_i - k_0}{\sqrt{2}\,\sigma}\right) = \mathrm{erf}\left(\frac{k_{i-1} - k_0}{\sqrt{2}\,\sigma}\right) + A \quad , \qquad (2)$$

where $A \equiv 2\,\mathrm{erf}(\frac{\Delta k_{tot}}{2\sqrt{2}\,\sigma})/(N_{spr}-1)$ and $\mathrm{erf}(x) \equiv \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2}\, du$.

We shall mainly focus on the truncated-Gaussian case, since it can give a very strong initial roll-off of the wake, with less partial-recoherence of the wake within the length of a bunch train ($\sim$ 4 meters) than in the linear case.

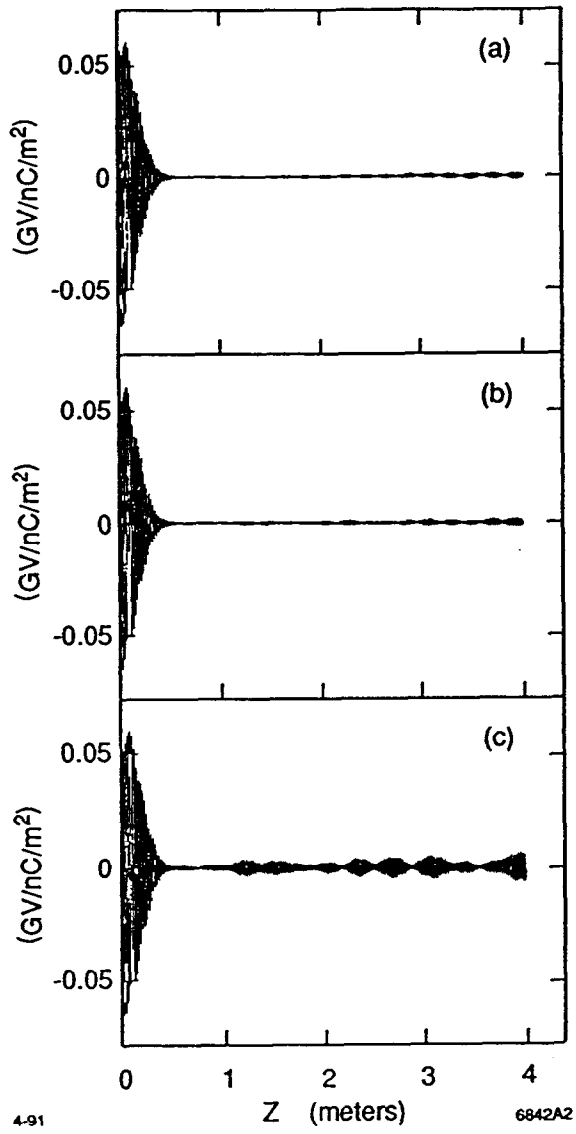

Figure 2. Wake functions with varying size of random errors superimposed on the split mode frequencies, for a truncated-Gaussian distribution, with $\Delta k_{tot}/k_0 = 5\sigma/k_0 = 10\%$, and $N_{spr} = 151$. We show cases of (a) no errors, (b) random errors with $\sigma_E/k_0 = 10^{-4}$, (c) random errors with $\sigma_E/k_0 = 10^{-3}$. No mode damping is included ($Q = \infty$).

IV. EFFECT OF ERRORS

We shall also include the effect of errors in the underlying error-free distributions discussed in the preceding section. We shall assume that a random error is added to each component of a split mode, and that the errors have a Gaussian probability distribution. Thus,

$$P(\delta k)\, d(\delta k) = \frac{1}{\sqrt{2\pi}\sigma_E} \exp[-(\delta k)^2/2\sigma_E^2]\, d(\delta k) \qquad (3)$$

is the probability of an error in wavenumber of size δk in the interval $d(\delta k)$.

For a given longitudinal separation z, the phases of the various split frequency components will remain close to those of the underlying error-free distribution provided that $\sigma_E z << 1$, while the phases are essentially randomized by the errors if $\sigma_E z >> \pi$. In the intermediate region

$1 \lesssim \sigma_E z \lesssim \pi$, the distribution is significantly modified by the errors but not completely randomized. This behavior can be seen in the wake functions $W(z)$ in Fig. 2, which wakes including errors of various sizes, in a stagger-tuned frequency distribution for a single mode with $f_0 =$ 14.4 GHz and $Q = \infty$. The staggered-tuning distribution is truncated-Gaussian, with $\Delta k_{tot}/k_0 = 5\sigma/k_0 = 10\%$, and $N_{spr} = 151$. Fig. 2(a) shows the wake function with no errors. Fig. 2(b) shows a typical wake function with fractional errors $\sigma_E/k_0 = 10^{-4}$, and Fig. 2(c) shows one with $\sigma_E/k_0 = 10^{-3}$. From mechanical tolerances, a reasonable estimate of the fractional error is $\sigma_E/k_0 \approx 5 \times 10^{-4}$.

V. EXAMPLES

We assume the bunch spacing $l_b = 0.42$ m; note that the corresponding number of e-foldings between bunches is $k_0 l_b/2Q \approx 64/Q$. The charge per bunch is $2 \times 10^{10}\, e$, and there are 10 bunches in a train, so the total length of the bunch train is about 3.8 m. We consider some examples with $Q = 100$, thus, if there were no staggered tuning, the influence of the wake would remain significant out to a few bunch spacings. We take $N_{spr} = 151$, which is comparable to the typical number of cells in a structure. Results for three examples of truncated-Gaussian staggered-tuning distributions are shown in Fig. 3; random errors with $\sigma_E/k_0 = 5 \times 10^{-4}$ were included in all cases. In Fig. 3(a), $\sigma/k_0 = 2.5\%$ and $\Delta k_{tot}/k_0 = 10\%$, in Fig. 3(b), $\sigma/k_0 = 3\%$ and $\Delta k_{tot}/k_0 = 10\%$, and in Fig. 3(c), $\sigma/k_0 = 3\%$ and $\Delta k_{tot}/k_0 = 15\%$.

Important points to note are: (1) σ/k_0 was chosen to make the initial roll-off occur before one bunch spacing, and (2) $Q = 100$ gives sufficient damping that the wake beyond the large initial roll-off is kept very small. Increasing σ/k_0 from 2.5% to 3% in going from Fig. 3(a) to Fig. 3(b) makes the initial roll-off of the wake occur sooner, but at the expense of introducing larger "lumps" of partial-recoherence just beyond the initial large roll-off. Going to the larger spread of 15% in Fig. 3(c), while keeping the same σ/k_0 as in Fig. 3(b), reduces the size of these lumps, since the truncation of the Gaussian occurs farther out. The beam blow-up in all these cases is quite small, only a few percent.

VI. CONCLUSIONS

The combination of damping to $Q \sim 100$ and staggered tuning with a distribution approximating truncated-Gaussian, appears promising for controlling multibunch beam break-up in future linear colliders. Our simulations indicate results comparable to those obtained using damped structures of very low Q.

It is conceivable that due to coupling and other effects (e.g., localization vs distribution of the split-mode frequencies), a large fraction of the modes might conspire to recohere at some z within the length of the bunch train, for some particular frequency distributions, producing a lump in the wake function that is larger than calculated here. In this case, $Q = 100$ might not be enough to completely suppress beam break-up. Thus, if a scheme relying on both staggered tuning and damping seems desirable (for instance, if it turns out to be impractical to achieve extremely low Q's in the damped structures), then more detailed simulations of a final structure design must be pursued, along with experimental tests. One may also have the option of going to Q's lower than 100, but without needing to go to Q's as low those required to obtain similar results using mode damping alone.

We thank the other members of the SLAC NLC structures group for interesting discussions related to this work.

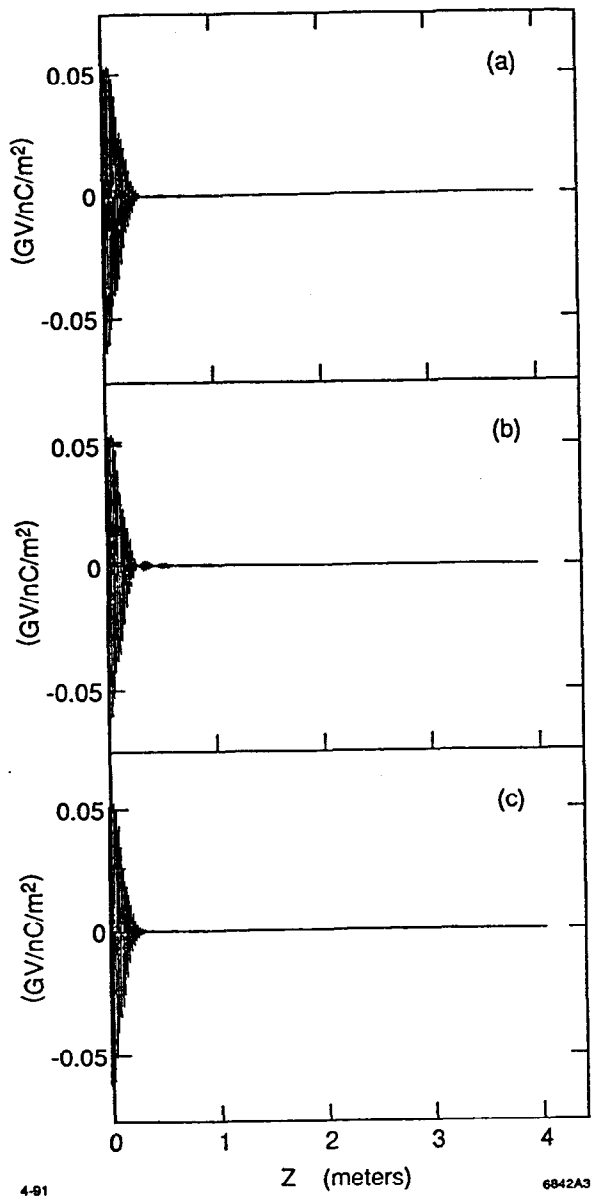

Figure 3. Examples showing combination of damping to $Q = 100$ with three truncated-Gaussian, staggered-tuning distributions. (a) $\sigma/k_0 = 2.5\%$ and $\Delta k_{tot}/k_0 = 10\%$, (b) $\sigma/k_0 = 3\%$ and $\Delta k_{tot}/k_0 = 10\%$, (c) $\sigma/k_0 = 3\%$ and $\Delta k_{tot}/k_0 = 15\%$. In all cases, $N_{spr} = 151$, and errors with $\sigma_E/k_0 = 5 \times 10^{-4}$ were included.

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