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FROM BIT-STRINGS (part way) TO QUATERNIONS*

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ABSTRACT

We present work in progress on constructing rotations and boosts from bit strings, and a mapping of bit-strings onto integer quaternion coordinates.

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1. INTRODUCTION

When, with my customary temerity, I announced the title "From Bit-Strings to Quaternions" for my contribution to ANPA 12, I thought that another year's work would get me past the high water marks I had set at ANPA 9,10, and 11. I was able to wash up over some of the previous material in my presentation at ANPA 12, but failed to reach my announced goal. Much of the background was set out in a technical note distributed before the conference[1]. Having taken my usual drubbing at the conference, I tried to get the material back into shape by the end of December with unsatisfactory results. I more or less froze the text then, but kept working on it into January, and sent out a technical note under the title "From Bit Strings to Quaternions or **From here to Eternity?**" to reflect what little further progress I had made on what was threatening to become a task of Sisyphus[2]. What became clear during that effort, though probably not sufficiently reflected in the words distributed, was that I would never achieve my goal along the route I was pursuing. One basic reason for my failure is that I keep forgetting that in a *discrete and finite* coordinate system, there is no way to build in *translational invariance.* Consequently there is no way to arrive at 3-vector or 4-vector addition in a discrete setting along the lines I was pursuing. That question had been raised for me a year ago by Stan Gudder. I admitted then I had not met his problem. I still have not. However, the approach sketched in the second chapter *might* allow one to discuss "translational invariance" as a cyclic invariance restricted to one or two of the longest wavelengths in the problem, reflecting the phase invariance of quantum mechanics. We would also have to be able to extend the space (up to some pre-assigned bound) by bit-string concatenation. We will see whether or not this works another time. Fortunately most of the practical physics requires only discrete versions of Lorentz transformations (both discrete velocity "boosts" and non-commuting finite angular rotations), and requires little more than the tools I have already developed.

I am currently actively pursuing an axiomatization of finite particle number (initially single particle) relativistic quantum mechanics in collaboration with Pat Suppes. The next chapter, entitled "WORK IN PROGRESS: discrete quantum mechanics" gives some insights I have come across along the way. My intent will be to get a simple axiomatization relying on discreteness but not on bit-strings and end up with the discrete versions of the Schroedinger and Dirac free particle equations discussed at ANPA WEST 6. Once this is accomplished, I can then — as in DP[3]— spell out what significant details of particle physics I hope to model before going on to our new theory. The third chapter explains how far I got toward constructing quaternions from bit-strings. As usual, I conclude with an overall table of results achieved (in my opinion) by *discrete physics* as of this writing.

2. WORK IN PROGRESS: discrete quantum mechanics

2.1 THE SYSTEM TO BE MODELED

The strategy now proposed is to start with an orthogonal system in 3+1 dimensions with integer coordinates. When we go from the model to physical interpretation, the spacial interval between coordinates will be h/mc and the temporal interval h/mc^2. We are interested in the interval between an event at $(x_1^0, x_2^0, x_3^0; t_0)$ and a second event at $(x_1, x_2, x_3; t)$. Since, for the moment, we are interested in time-like separated events, we require that the velocity so specified be a rational fraction between -1 and $+1$ whose components are given by

$$\beta_i := \frac{x_i - x_i^0}{t - t_0} \tag{2.1}$$

The restriction to rational fractions allows us to specify a *period* which defines points at events separated by this velocity can occur, called T, and $3(T-2)$ possible integers n_i in the range $0 < n_i < T$ that *quantize* the velocity components using the definition

$$n_i := \frac{T}{2}(1 + \beta_i) \tag{2.2}$$

Then we can specify the relation between the events by

$$t = t_0 + NT; \; x_i = x_i^0 + \beta_i NT \tag{2.3}$$

Neglecting the fraction t_0/NT, it is clear that N is simply the number of periods separating the two events. We emphasize that in our approach the characteristic cyclic ("wave") character of quantum mechanics stems directly from the digitization and the definition of velocity.

Two facts of the utmost importance follow from the adoption of this starting point. The first is that the ratio of the momentum components to the energy can be defined *independent* of the number of periods (provided it is greater or equal to one) simply by taking

$$p_i c = \beta_i E \tag{2.4}$$

This definition specifies a unique "origin" in momentum space, because if $p_i = 0$ for all three components, $E = mc^2$. The second fact is that angular momentum,

$$\ell_i = x_j p_k - x_k p_j \tag{2.5}$$

like the coordinate components themselves, does depend on the (currently unobservable) parameters x_i^0. We note that they will ultimately supply an unobservable phase factor in the wave function — which was the starting point of my S-Matrix gloss on Tom Phipps derivation of quantum mechanics back in 1972. They also show how the connection between lack of commutation between either the angular momentum components or position and momentum will entail the other result as a simple algebraic consequence, as already noted in another connection in FDP[4].

Two problems remain. One is the fact that we cannot use the Pythagorean sum of squares to define the square of the radial distance, the square of the energy, or the square of the angular momentum without leaving the integer domain. We suspect that the simplest way to proceed will be find a way to derive the relationships

$$L^2 = \ell(\ell+1); \ -\ell \leq \ell_z \leq +\ell; \ L_\mp L_\pm = (\ell \mp \ell_z)(\ell + 1 \pm \ell_z) \tag{2.6}$$

from which the quantum mechanical version of the Pythagorean relation follows, namely

$$\frac{1}{2}[L_- L_+ + L_+ L_-] + \ell_z^2 = L^2 \tag{2.7}$$

I think this can be done by using the *Zitterbewegung* minimal motion on the grid to construct minimal rotations. Then the definition

$$L_\pm = L_x \pm i L_y \tag{2.8}$$

will be a convenient way to bring in the algebraic meaning of the imaginary in what is to begin with a clear geometrical picture. With our quantized velocities, a similar trick will quantize our Lorentz transformations, as we discuss below.

Again, because the Pythagorean theorem doesn't work, we will in general have a radial length r and a velocity β referring directly to the particle motion and not to the components, which must also carry the same period:

$$r = r_0 + \beta N T \tag{2.9}$$

The definition of event in the bit-string theory, if it is to provide momentum conservation, requires the periods of two interacting particles to be commensurable and in inverse ratio to their masses. For consistency the origin of coordinates must be separated from the first event an integral number of periods corresponding to some mass m_0 which engaged in an event at the origin and in the event that produced the particle m at (x_0, y_0, z_0, t_0). Consequently

$$\beta_i^0 = \frac{x_i^0}{N_0 T_0};\ \beta_0 = \frac{r_0}{N_0 T_0};\ m_0 T_0 = mT \tag{2.10}$$

We emphasize that in discrete physics 3-momentum conservation arises from the definition of event and the consequent requirement of the commensurability of periods, *not* from translational invariance. The appropriate form of "translational invariance" will be reflected in the arbitrariness of the integer N_0. Provided we stay within the event horizon and the most massive elementary particle has a mass sufficiently small compared to the mass of the matter within the event horizon, there appears to be no difficulty in constructing an effective *upward* "translational invariance" within a bit-string theory by appropriate concatenations (see section on bit-strings below). The next step is to look at boosts and rotations.

2.2 BOOSTS: THE "COUNTER PARADIGM" REVISITED

In our discussion of the "Counter Paradigm" in DP, pp 90-91, we noted that *"..., we will have to provide more and more precise definitions of these criteria* [relating 3- and 4- vertices at certain TICKs to the space-time volumes of laboratory counters] *as the analysis develops."*

Physicists are accustomed to "looseness of fit" between the mathematics (*representational framework*, R), the connection to quantitative laboratory measurement (*rules of correspondence, procedural framework*, P), and the objectives of the process (*epistemological framework*, E), whatever names they use for these three essential ingredients in the modeling of the *practice* of physics. In my view only many recursions through RPE in any order can be expected to yield satisfactory results. This looseness generates considerable criticism from some members of ANPA wherever I start. As a physicist, I have been more comfortable starting with E, roughing out the mathematics R enough to make a first stab at connecting to laboratory practice (*including* the way algebraic formulae and monte-carlo programs are used to compare theoretical predictions with digital laboratory results, i.e. "counter data") P, and then recursing to E to get an estimate of where we are; I can then ask what it might be profitable to scrap before going on. This has landed me in mathematical difficulties, sometimes over my head.

My initial thoughts about how to connect the "counter paradigm"[5] to Stein's "random walk" model[6–8] were *very* naive. Recently I have been trying to come to grips with some of the difficult aspects, — *after* Karmanov[9] had suggested that we might be able to go directly from a "Stein-like" model to a discrete version of the 1+1 Dirac Equation *without* going to an infinitesimal step-length "limit".

The naive counter paradigm amounts to saying that when we have two sequential counter firings a distance L apart with time separation T attributed to a single particle of mass m, we can associate the invariant interval $c^2\tau^2 = c^2T^2 - L^2$ between these events with a labeled bit-string. The label, according to rules that I am still developing, specifies the mass. If the string $\mathbf{a}(S;m)$ is of length S and Hamming measure $a(S;m) = |\mathbf{a}(S;m)|$, we take the time to be $T = Sh/mc^2$ and the distance to be $L = [2a(S;m) - S](h/mc)$ it follows that the velocity $V := L/T = \beta c = [\frac{2\mathbf{a}(S;m)}{S} - 1]c$. In practice we cannot measure the dimensions of a counter to an integral number of Compton wavelengths h/mc; the time resolution of the counters is always much coarser than h/mc^2., These practical constraints define an ensemble of strings and not a single string. Further, simply specifying the string length and the Hamming measure also defines only an ensemble of strings which may be generated in various ways. Part of the problem my lectors have with my exposition is that my language has often led them to identify a particle with a single labeled string rather than with these context-dependent ensembles. I am so used to employing this type of short-hand in going from model to experimental context and back that I tend to forget how often I need to remind others (and occasionally even myself) how inextricably this empirical context is connected to the model itself, however "mathematical" the representation of the model may look.

Once my model is spelled out this way, it is easy to think of the ensemble of strings as a "random walk", or relativistic *Zitterbewegung*, in which the particle takes a step either along or against a line connecting the two counters, each step executed at the velocity of light, defining a causal trajectory in 1+1 space-time generated by the construction of any particular string as a Bernoulli sequence. This is where the trouble starts. Such a model, in the large number case, would approximate a relativistic diffusion equation and *not* the Schroedinger equation. One can use it to derive the Lorentz transformations, as Stein did initially, by treating the step-length as the uncertainty in position; a rigorous derivation along these lines is given by McGoveran in FDP. But this is still a long cry from quantum mechanics.

Stein attacked this problem in his most recent published paper[10]. He distinguished quantum events from classical coincidences in such a way that the quantum process corresponds to a *single step*, and in this way was able to prove that in his model a Gaussian distribution exhibits the characteristic "wave packet spreading" of (non-relativistic) quantum mechanics. We convinced John Bell that Stein had in this way constructed the solutions of the Schroedinger equation for Gaussian wave packets, and I am willing to argue that he did indeed derive the 1+1 free particle (non-relativistic) Schroedinger equation in this way. Feynman and Hibbs[11] get the relativistic Schroedinger and Dirac equations out of a similar model, by taking the counter-intuitive leap of treating the step-length as *imaginary*! We will discuss why that works from our point of view on another occasion[12]. Adequate treatment requires much more care than the naive model we have sketched in this section. A preliminary treatment[13] claimed that I had derived the Feynman imaginary step length prescription, but this claim should be treated with caution.

In my first approach to the Lorentz transformations (DP pp 91-93) for the interval connecting coordinates $(0,0)$ to (z,t) in the forward light cone (in units of h/mc for z and h/mc^2 for t) I used $z = 2a(S) - S$, $t = S$, and asked for a transformation from $(z;t)$ to $(z';t')$ which keeps $\tau^2 = t^2 - z^2 = 4a(S)(S - a(S))$

invariant. This can be generated by

$$(t' + z') = \rho(t + z) = \rho\ 2a(S);\quad (t' - z') = \rho^{-1}(t - z) = \rho^{-1}2(S - a(S)) \quad (2.11)$$

or in terms of Hamming measure and string length by

$$a'(S') = \rho\ a(S);\quad S' = \gamma_\rho S;\ \gamma_\rho = \frac{1}{2}[\rho + \rho^{-1}] \quad (2.12)$$

One difficulty with this route to the Lorentz transformations, which Karmanov realized (Ref. 9) but I did not, is that if we are to retain connection to bit-string operations $\gamma_\rho S$ must be integral. This clashes with our definition of length, time and velocity based on h/mc. This early approach (unfortunately now enshrined in DP) is abandoned in this paper.

A second difficulty with this approach is that the string length is changed, while it would be natural for us to keep the string length fixed, and generate transformations under this restriction. This observation, together with the way quantized angular momentum works in ordinary quantum mechanics, provided me with the clue to the solution.

Note that the classical connection between the Lorentz transformation parameters β and γ, i.e. $\gamma^2(1 - \beta^2) = 1$, defines a circle of unit radius: $\beta^2 + \frac{1}{\gamma^2} = 1$. But we wish to exclude the light cone by the smallest experimental resolution which we can measure in the system at hand; we take this to be $\Delta\beta = 1/S$. We can now replace the classical definition by the quantum definition $\beta^2 + \gamma^2 = 1 - \frac{1}{S}$. Then the integer relations which we need are:

$$\beta := \frac{n_\beta}{S};\ \frac{1}{\gamma} := \frac{n_\gamma}{S};\ n_\beta^2 + n_\gamma^2 = S(S - 1) \quad (2.13)$$

As in the angular momentum case, we can keep $S(S - 1)$ [rather than $\ell(\ell + 1)$] invariant when we transform either β *or* γ and compute the square of the other from a difference of squares. In general n_β, n_γ and S cannot all three be integral;

this has been known since the time of Pythagoras. In a bit-string theory, S must be integral, so we have a choice. If we use velocity resolution as above, it must be n_β; the alternative of requiring n_γ integral amounts to specifying the smallest mass we will consider. If we want to transform from a system in which the velocity is $\beta = n_\beta/S$ to a system in which the velocity is $\beta' = n_{\beta'}/S$, the transformation velocity $\beta_L = n_L/S$ is readily computed from

$$n_{\beta'} = \frac{n_\beta + n_L}{S^2 + n_\beta n_L} \tag{2.14}$$

The usual form of the Lorentz transformation follows immediately, as can be readily verified.

2.3 BIT-STRINGS

We specify a *bit-string*

$$\mathbf{a}(S) = (e_1^a e_2^a ... e_s^a ... e_S^a) \tag{2.15}$$

by its S ordered elements

$$e_s^a \in 0,1; \quad s \in 1,2,....S; \quad 0,1,...,S \in \ \textit{ordinal integers} \tag{2.16}$$

and its norm by

$$|\mathbf{a}(S)| = \Sigma_{s=1}^S e_s^a = a(S) \tag{2.17}$$

This is the usual Hamming measure for bit-strings. Define the *null string* by $\mathbf{0}(S)$, $e_s^0 = 0$ for all s and the *anti-null* string by $\mathbf{1}(S)$, $e_s^1 = 1$ for all s.

Define *discrimination* ($\oplus$) by

$$e_s^{a\oplus b} := (e_s^a - e_s^b)^2; \ \mathbf{a} \oplus \mathbf{b} := (...e_s^{a\oplus b}...e_S^{a\oplus b}) = \mathbf{b} \oplus \mathbf{a} \tag{2.18}$$

from which it follows that

$$\mathbf{a} \oplus \mathbf{a} = \mathbf{0}; \ \mathbf{a} \oplus \mathbf{0} = \mathbf{a} \tag{2.19}$$

Note that our definition differs from the usual symmetric difference, $+_2$, XOR, *OREX, ... in that the "0" 's and "1" 's in the string are not simply arbitrary dichotomous symbols, "bits" in the computer sense, existence symbols in the logical sense, ...; they are* **ordinal integers**. *This allows us to sum them to get the Hamming measure. This also emphasizes the fact that we have assumed that ordinary integer arithmetic up to some finite ordinal named in advance is* **understood**. *We do not construct it, nor do we treat our bit-strings as binary numbers.*

Define $\bar{\mathbf{a}}(S)$ by

$$\bar{\mathbf{a}} := \mathbf{a} \oplus \mathbf{1};\ \textit{hence}\ \mathbf{a} \oplus \bar{\mathbf{a}} \oplus \mathbf{1} = \mathbf{0} \tag{2.20}$$

Distinct strings which are *discriminately independent*, or *d.i.*, are those which when combined by discrimination in all possible non-repetitive ways do not produce the null string. Discriminately and anti-discriminately independent strings, or *d.i.a.d.* strings are *d.i.* strings which also do not produce the anti-null string.

Since discrimination is only defined for bit-strings of the same length S, we can often omit reference to the string length, as we have done above. However, when the norm *and* the anti-null string are involved we need to know the string length. In particular

$$|\mathbf{1}(S)| = S;\ |\bar{\mathbf{a}}(S)| = S - a(S) \tag{2.21}$$

For two strings $\mathbf{a}(S_a), \mathbf{b}(S_b)$ we define *concatenation* ($\|$) by

$$e_k^{a\|b} := e_s^a,\ s \in 1, 2, ..., S_a;\ e_k^{a\|b} = e_j^b,\ j \in 1, 2, ..., S_b, k = S_a + j$$

$$\mathbf{a}(S_a)\|\mathbf{b}(S_b) = (....e_i^a...e_{S_a}^a)\|(....e_j^b...e_{S_b}^b) \tag{2.22}$$

$$= (......, e_k^{a\|b}.....e_{S_a+S_b}^{a\|b}) \tag{2.23}$$

Hence

$$a(S_a) + b(S_b) := |\mathbf{a}(S_a)\|\mathbf{b}(S_b)| = |\mathbf{b}(S_b)\|\mathbf{a}(S_a)| \tag{2.24}$$

but note that in general $\mathbf{a}\|\mathbf{b} \neq \mathbf{b}\|\mathbf{a}$.

2.4 ROTATIONS

In ordinary one particle relativistic quantum mechanics, the space-time reference framework is assumed understood as the normal classical continuum of special relativity. Here we cannot afford that luxury. Instead we start with two bit-strings, a reference string $\mathbf{R}(S)$ and the string of interest $\mathbf{a}(S)$ which are subject to the constraints

$$S > R := R(S) > a := a(S) > 0 \tag{2.25}$$

and select a third integer *or* half-integer parameter μ_a which lies in the range

$$-\frac{a}{2} \leq \mu_a \leq +\frac{a}{2} \tag{2.26}$$

Note that the integer and half-integer cases for μ_a are mutually *exclusive.* This parameter can be related to the two strings by adopting a standard representation for them:

$$\mathbf{R}(a,\mu_a) = \mathbf{1}(\frac{a}{2} - \mu_a - a)\|\mathbf{0}(\frac{a}{2} + \mu_a)\|\mathbf{1}(R - \frac{a}{2} + \mu_a)\|\mathbf{0}(n_0)$$

$$\mathbf{a}(R;a,\mu_a) = \mathbf{1}(\frac{a}{2} - \mu_a - a)\|\mathbf{1}(\frac{a}{2} + \mu_a)\|\mathbf{0}(R - \frac{a}{2} + \mu_a)\|\mathbf{0}(n_0) \tag{2.27}$$

$$\mathbf{R}(a,\mu_a) \oplus \mathbf{a}(R;a,\mu_a) = \mathbf{0}(\frac{a}{2} - \mu_a - a)\|\mathbf{1}(\frac{a}{2} + \mu_a)\|\mathbf{1}(R - \frac{a}{2} + \mu_a)\|\mathbf{0}(n_0)$$

Where

$$n_0 = S - [R + \frac{a}{2} + \mu_a] > 0$$

determines the string length. Note that this positive integer parameter is *arbitrary* so long as the other conditions are met. Further, so long as the *same* permutation of the positions $s \in 1,2,3,...,S$ is applied to all three strings, the properties of interest for this minimal structure are unchanged. It is the existence of these $S!$ permutations that lead to a different count for our probabilities than one would obtain by thinking of the strings as Bernoulli sequences[14]. We expect to see in due course that this arbitrariness in the string length can replace the accepted arbitrariness of the phase parameter in the quantum mechanical wave function. We expect to show that the dependence on string length in our theory will be negligible for large enough string lengths in the physical situations currently accessible technologically.

....

Define

$$[A_-A_+](a,\mu_a) := (\frac{a}{2}+\mu_a)(\frac{a}{2}+1-\mu_a) = \frac{a}{2}(\frac{a}{2}+1)+\mu_a-\mu_a^2$$

$$[A_+A_-](a,\mu_a) := (\frac{a}{2}-\mu_a)(\frac{a}{2}-1+\mu_a) = \frac{a}{2}(\frac{a}{2}+1)-\mu_a-\mu_a^2 \tag{2.28}$$

Hence

$$A_\perp^2 := \frac{1}{2}[A_-A_+ + A_+A_-](a,\mu_a) = \frac{a}{2}(\frac{a}{2}+1)-\mu_a^2 \tag{2.29}$$

Much could be accomplished by working out the implications of this definition, and giving it a geometrical interpretation in terms of a 3+1 orthogonal mesh with integer spacing.

3. BIT-STRING COORDINATES

3.1 Greider's Quaternions

Our objective in this chapter is to map bit-strings onto quaternion coordinates which are integral, or rational. Our strategy is to construct the ingredients used by Greider[15] in his systematic development of the scalars, 4-vectors, bivectors, trivectors and pseudoscalars needed in relativistic quantum field theory. We choose his approach because he has demonstrated that ambiguities in formulating the free field conservation laws using the tensor notation are uniquely resolved within his formalism; further, his approach can readily be extended to general relativity. He starts from the basic bivector product

$$\mathbf{e}_\mu \mathbf{e}_\nu + \mathbf{e}_\nu \mathbf{e}_\mu = 0; \mu \neq \nu; \;\; \mu, \nu \in 0, 1, 2, 3 \tag{3.1}$$

and the scalar products

$$\mathbf{e_1}^2 = \mathbf{e_2}^2 = \mathbf{e_3}^2 = -\mathbf{e_0}^2 = +1 \tag{3.2}$$

He defines a 4-vector $\mathbf{v}$ by its projection onto this basis, i.e.

$$\mathbf{v} := v^0 \mathbf{e_0} + v^1 \mathbf{e_1} + v^2 \mathbf{e_2} + v^3 \mathbf{e_3} = (v^1, v^2, v^3; v^0) = (\vec{v}; v^0) \tag{3.3}$$

from which the Lorentz-invariant (*space-like* positive) 4-vector product

$$\sigma^2 := \vec{a} \cdot \vec{b} - a^0 b^0 \tag{3.4}$$

follows immediately. Note Greider's arbitrary choice of a space-like metric for σ^2 rather than the *time-like* positive metric

$$\tau^2 := -\sigma^2 = a^0 b^0 - \vec{a} \cdot \vec{b} \tag{3.5}$$

which I prefer. We note here that Phipps[16] points out that "time-dilation" and "mass-increase" for time-like intervals connected to a single particle have ample empirical confirmation, but that "length contraction" has no corresponding *direct* empirical evidence to support it. Clearly evidence against the "Lorentz contraction" of rigid rods would prevent us from using the facile $\sigma^2 = -\tau^2$ assumption we employ in this paper.

Greider remarks that "The four basis vectors $\mathbf{e}_\mu$ are part of the 16 linearly independent elements that form the (Dirac) C_4 algebra, and the v^μ are scalar coefficients. The other 12 elements of C_4 are obtained by multiplication of the $\mathbf{e}_\mu$;..." Note that the *finite* length of the unit basis vectors can be thought of as a first step toward quantization.

In the past I have sometimes simplified my notation $\mathbf{a}(S)$ for a bit-string of length S by dropping the dependence on S. Since this could create confusion with Greider's notation for a 4-vector, I will try not to do so in what follows. In order to distinguish Greider's *space-like* metric (Eq.'s 3.2 - 3.4) from the standard notation for *on-shell* 4-vectors p in momentum space[17] we write

$$p := (p^0, p^1, p^2, p^3) = (E, \vec{p}); \; p^2 = E^2 - \vec{p} \cdot \vec{p} = m^2$$

Although our strategy for mapping bit-strings onto quaternions works in a formal sense, we do not in this way succeed in achieving translational invariance or vector addition. As we roughed out in the last chapter, bit-string operations should suffice to describe finite and discrete rotations and boosts using strings of fixed length. Further, bit-string concatenation allows us to define multiplication of a single bit-string by positive and negative integers and their combinations, including the scalar "0". This makes our coordinate desciption meaningful, provided we can supply a *macroscopic* ("laboratory") definition of the *directions* of the vector basis strings. I believe this will suffice for the physics modeling I have done and intend to do. I suspect that my failure to construct the full vector addition in our theory has deep roots, but these cannot be explored in this paper.

3.2 AMSON INVARIANCE

My tactical motivation for mapping bit-strings onto quaternions is explained at the start of Sec. 3.1; basically, Greider's approach to the free field equations provides us with a familiar point of departure, which we can qualify as we go along. My earlier philosophical motivation for mapping bit-strings onto quaternions came primarily from Amson[18] invariance. This started long ago when I found it useful to obtain "anti-particles" by discriminating with the anti-null string.

It is often emphasized in discussions of bit-strings that so long as the two symbols used in the ordered string are distinct, the choice of what symbols to use is *arbitrary.* Hence there is a basic symmetry in the representational starting point of a theory modeled using bit-strings. John Amson emphasized this fact by raising the basic question of where these two symbols come from in the first place. His answer was the "Bi-Orobourous", which is supposed to make them self-contained.

If we define "discrimination" by

$$a \oplus a := 0 = b \oplus b; \;\; a \oplus b := 1 = b \oplus a$$

where a and b are the two arbitrary, *distinct* symbols already mentioned, it is clear that the two additional symbols "0" and "1" are *also* arbitrary. Using them to replace a and b in a system whose notation is still fluid can be dangerous. If I understand John Amson correctly, keeping one pair fixed and interchanging the other pair changes one system into its "dual" system. Then, if I am still on track, this basic symmetry can be collapsed by taking either the a and b or the 0 and 1 as the completed hierarchy in one representation and its dual representation as the initial arbitrary, distinct symbols and starting all over again.

Once we have collapsed the notation by replacing a and b by 0 and 1, we obtain the usual XOR of computer practice in which the symmetry between the two symbols is *broken*, in that the "0" in the definition refers to the symbols being "the same" and the "1" to their being "different". I suspect that this asymmetry is related to Parker-Rhodes' starting point in *Agnosia*[19] and *The Inevitable Universe*[20], where he distinguishes between the ontological statement "something exists" and the information-theoretic statement, "this ontological statement conveys no information".

I make my definition of discrimination still more concrete by defining bit-strings as strings of dichotomous symbols *ordered* by the ordinal integers. I take the normal arithmetic properties of the integers — both with regard to addition and to multiplication — as "given" up to some integer *fixed in advance.* By identifying the dichotomous symbols in the strings — the "0" and "1" — as *ordinal* integers, I make what I claim to be a consistent step, *provided* I define *discrimination* by

$$b_s^{a\oplus b} := (b_s^a - b_s^b)^2;\ b_s^x \in 0,1;\ s \in 1,2,3,....,S$$

rather than using the "symmetric difference" definition given above, or some binary equivalent. This possibility was, like many other things, one that Clive Kilmister and I ran into together when working in his office at King's nearly a decade ago. I reiterate here my contention that I see no need for deriving the integers from a more primitive starting point so long as my aim is to model the *practice* of physics in such a way as to *construct* a consistent finite and discrete relativistic quantum mechanics. The philosophical point I wish to make about either my approach, or John Amson's, or Fredrick Parker-Rhodes's, or (so far as I can see) Clive Kilmister's and Ted Bastin's, is that there is a tension between the *broken symmetry* that is an inevitable part of the hierarchy construction as usually presented and the initial indistinguishable duality. I find this contrast fruitful rather than paradoxical.

One has a choice here. The asymmetric structure clearly has a great deal to do with the hierarchical ordering of the scale constants. I claim to have gone a considerable ways toward using this structure to interpret the elementary particle quantum numbers, coupling constants and mass ratios. However, conventional elementary particle physics cannot be formulated without ending up with a theory in which $\mathcal{CPT}$ is necessarily unbroken, even though $\mathcal{C}$,$\mathcal{P}$, $\mathcal{CP}$ (and hence presumably $\mathcal{T}$) are broken *both* empirically *and* in the standard model of quarks and leptons. This was my motivation for invoking "Amson Invariance" a long time ago as the symmetry in our theory which allows us to model this empirical situation. Early on I used discrimination with the anti-null string to distinguish "particles" from "anti-particles". In the current paper I show that my definition of coordinates provides *all* these discrete symmetries. I am working out the details of how this relates, quantitatively, to the way the coupling constants break these symmetries in a manner consistent with experiment.

One important aspect of the theory as I am formulating it is that one has the choice between either breaking $\mathcal{CPT}$ or requiring it. This already gives our approach a critical advantage over conventional theories. A colleague of mine (Helen Quinn) asserts "All relativistic quantum field that anyone has written down are Lagrangian field theories." Further, a standard textbook by Itzykson and Zuber[21] states that "In any quantum field theory derived from a Lagrangian, the PCT theorem holds"; they provide a proof and references to the literature. Max Dresden informs me that the theorem applies only to *local* Lagrangian theories, and that non-local theories have more freedom. Non-local theories would, necessarily, introduce a dimensional parameter for which there is no current experimental motivation. In contrast, the hierarchy construction necessarily breaks $\mathcal{CPT}$ symmetry in any application along the lines I have pioneered; the breaking parameter is part of the theory. In elementary particle physics it is one part in $2^{127} + 136$. In our cosmology it is one part in 256^4, which is a good estimate of the empirical breaking parameter: the number of baryons per photon.

Hamming measure (number of 1's in a string) necessarily breaks "Amson invariance". This fact motivates dropping Hamming measure in favor of a symmetric definition by the way we introduce metric coordinates (see below). In terms of McGoveran's definition of *attribute distance*, what we do is to use some string with an equal number of 0's and 1's as our *reference ensemble.* (Hamming measure uses the null string as the reference ensemble.) This restricts us to using basis strings of even length. We find this to be a good move, because it gives us a simple way to discuss $\mathcal{CPT}$ invariance (see below).

3.3 Orthogonal Bit-String Basis Vectors

In order to give meaning to a vector basis for vectors with integer coefficients constructed from bit-strings we start with a set of D d.i.a.d. *vector basis strings* of the same length S which we call $\mathbf{B}_\alpha(S)$; $\alpha \in 1, 2, ..., D$. Note that, in contrast with the d.i. basis strings used in the construction of the four levels of the *combinatorial hierarchy* — which by definition exclude the null string — we exclude the anti-null string as well. This suffices for 3-vectors, but for 4-vectors we adjoin the anti-null string explicitly as one of the basis vectors:

$$\mathbf{B}_0(S) := \mathbf{1}(S) \tag{3.6}$$

Once we have selected a d.i.a.d. basis, our next step in defining bit-string coordinates is to construct a meaning for addition of the basis bit-strings and for multiplying them by a positive integer.

$$\mathbf{B}_\alpha(S) + \mathbf{B}_\alpha(S) := 2\mathbf{B}_\alpha(S) := \mathbf{B}_\alpha(2S) := \mathbf{B}_\alpha(S)\|\mathbf{B}_\alpha(S) \tag{3.7}$$

and hence by recursion

$$(n+1)\mathbf{B}_\alpha(S) := \mathbf{B}_\alpha(S)\|\mathbf{B}_\alpha(nS) = \mathbf{B}_\alpha(nS)\|\mathbf{B}_\alpha(S) = \mathbf{B}_\alpha(S)(n+1) \tag{3.8}$$

Note that

$$|n\mathbf{B}_\alpha(S)| = |\mathbf{B}_\alpha(nS)| = n|\mathbf{B}(S)| = n\ B_\alpha(S) = B_\alpha(nS) = B_\alpha(S)\ n \tag{3.9}$$

Consequently we have indeed succeeded in defining the multiplication of a basis string by a positive integer.

In order to extend this definition to negative integers and multiplication by zero, We define *addition*, +, and *subtraction*,–, of a basis string as follows

$$0 := 0\ \mathbf{B}_\alpha(S) := \mathbf{B}_\alpha(S) + \bar{\mathbf{B}}_\alpha(S) \tag{3.10}$$

Hence, since "–"is to have the usual meaning as the inverse of "+",

$$\mathbf{B}_\alpha(S) = -\bar{\mathbf{B}}_\alpha(S) := -\mathbf{B}_\alpha(-S) \tag{3.11}$$

and by recursive definition similar to Eq. 3.9

$$(m \pm n)\mathbf{B}_\alpha(S) = m\mathbf{B}_\alpha(S) \pm n\mathbf{B}_\alpha(S) = \mathbf{B}_\alpha(mS)\|\mathbf{B}_\alpha(\pm nS)$$

$$= \mathbf{B}_\alpha(\mp mS)\|\mathbf{B}_\alpha(-nS) = -\mathbf{B}_\alpha(-S)(m \pm n),\ etc. \tag{3.12}$$

We have already restricted ourselves to a d.i.a.d. basis because of our desire to preserve Amson invariance; this motivation *also* requires us to restrict our vector basis strings to strings of even length. For strings of even length (i.e. $\frac{S}{2} \in$ *positive integer*), we call our *vector basis strings* $\mathbf{E}_\mu(S), \mu \in 0, 1, 2, 3...$ and require that

$$\mathbf{E}_0(S) := \mathbf{1}(S);\ |\mathbf{E}_i(S)| = \frac{S}{2},\ i \in 1, 2, 3... \tag{3.13}$$

Then we can *define* the *components* a^μ of any string of length S by

$$a^\mu := |\bar{\mathbf{a}}(S) \oplus \mathbf{E}_\mu(S)| - S/2 \tag{3.14}$$

from which it follows that

$$(E_\mu)^\mu = \frac{S}{2};\ (E_0)^i = 0 = (E_i)^0,\ i \in 1, 2, 3... \tag{3.15}$$

Thus, any "spacial" vector basis string $\mathbf{E}_i(S)$ can be said to be *orthogonal* to the "temporal" vector basis string $\mathbf{E}_0(S)$. In order to have orthogonal coordinates in a $D + 1$ space, we must obviously require that

$$(E_i)^j = \delta_{ij}\frac{S}{2},\ i, j \in 1, 2, ..., D \tag{3.16}$$

We discuss below how this requirement can be met. Once we have established an orthogonal basis of dimensionality D using strings of length S, we can extend the system to include a larger number of coordinates by the "length multiplication" described above. This is simply an (upward) scale change because once this is applied to all the vector basis strings, it is easy to show that

$$(n\mathbf{a}(S))^{\mu} = (\mathbf{a}(nS))^{\mu} = na^{\mu} \tag{3.17}$$

Our mapping of basis vector strings onto basis vectors can now be written as

$$\frac{2}{S}\mathbf{E}_{\mu}(S) \rightarrow \mathbf{e}_{\mu} \tag{3.18}$$

and for repetitive vector basis strings

$$\frac{2}{nS}\mathbf{E}_{\mu}(nS) \rightarrow \mathbf{e}_{\mu} \tag{3.19}$$

3.4 How Many Dimensions?

McGoveran (FDP, Theorem 13) has shown that any discrete space of D "homogeneous and isotropic" dimensions synchronized by a universal ordering operator can have no more than *three* indefinitely continuable dimensions; three separate out and the others "compactify" after a surprisingly small number of constructive operations. The proof starts from the assumption that we have D independent generators of sequences of two dicotomous symbols. The sequences share a common ordinal integer n which is "0" when the sequences start ("initial synchronization") and which counts the number of symbols which have been added sequentially to each sequence; the basic assumption is that whatever method we use to generate the sequences cannot allow any subset of the $d = 1, 2, ..., D$ generators to be distinguished from any of the rest other than by this arbitrary numbering.

For example, we could run the generators until we had produced sequences all D of which are discriminately independent at a length which we could call N_L, the *label length*. Then which we call $d = 1, 2, 3, ..., D$ is an arbitrary replacement for these generated sequences; this is the way part of *PROGRAM UNIVERSE I* operates (cf DP). In Parker-Rhodes terminology, these D sequences are *indistinguishables* with cardinality "D" and ordinality "1". In our context, this is what we mean by "homogeneous and isotropic dimensions". This allows us to invoke a result proved by Feller[22] for D independently generated Bernoulli sequences (i.e. arbitrary sequences of the symbols 0, 1). Feller proved that the probability that after n synchronized trials all will have accumulated the same number of "1" 's is less than $n^{-\frac{1}{2}(D-1)}$. [The exact expression for this probability is $\frac{1}{2^{nD}}\Sigma_{k=1}^{n}(\frac{n!}{k!(n-k)!})^{D}$.] Consequently the probability of this criterion being met vanishes like $n^{-\frac{3}{2}}$ for $D = 4$, and increasingly rapidly for higher numbers of independently generated sequences. McGoveran met various objections to this interpretation in Ref. 1, Appendix II. For completeness, I quote the relevant passage here.

"Now regarding the difficulty of giving finite combinatorial meaning to Feller's Theorem vis–a–vis statistically unlikely circumstances. While I cannot avoid the statistical character of the proof, I can remove the problem of combinatorial interpretation. This problem arises because of the way Feller invokes convergence and difference theorems and therefore limit theorems. The asymptotic continuation of the combinatorial terms of the series seems to be essential. However, one need not resort to this method to see the validity of the theorem.

"In particular, suppose that a $3 + n$ space has been generated up to some finite extent. Because of the probabilities involved, the most dense constructible 1–dimensional d–subspace will have a denser sequence of metric points than every constructible 2–dimensional d–subspace, and the most dense 2–dimensional d–subspace denser than every 3–dimensional d–subspace. However, this situation reverses at 4–dimensions so that the most dense $4 + n$–dimensional d–subspaces are now ordered as more dense than every 5+n-dimensional d-subspace (where n is an element of 0, 1, 2, ...)! This means that every $4 + n$–dimensional d–subspace is separable into a number of isotropic and homogenous 1, 2, and 3–dimensional d–subspaces, but NOT into isotropic and homogenous 1, 2, 3 and 4–dimensional d–subspaces.

"Again, there might be some (and indeed perhaps a large number) of "exceptional" generators of homogeneous and isotropic m–dimensional d–subspaces with $n > 3$*. The algorithm for this generator would be deterministic. However, it is my claim that no such deterministic algorithm can be correct for other reasons as explained regarding "arbitrariness" and the very definition of ordering operator in Foundations: the complexity of the algorithm for an ordering operator is such that it cannot be given a full interpretation within the generated system.*

"For PU, the generators of our d–space, therefore, are of such complexity that the "next" metric mark cannot be represented in terms of all those generated so far. This precludes the possibility that the generation of the space is deterministic in the way required: namely that we can predict deterministically from the d–space generated so far and the distribution of metric marks where/when the next metric mark will be generated. Every c–dimensional d–space with $n > 3$ *is not algorithmicly extensible within the system. It is therefore subject only to statistical characterization. I realize this is not a formal argument and hope to make it formal in my next major effort: Foundations II.*

"Not long ago I questioned Pierre's reference to "McGoveran's Theorem" regarding there being only three conserved unique quantum numbers (which I take to mean that only three quantum units or parameters are possible for global descriptions and what you mean by Pierre's conservation theorem). I subsequently convinced myself that it was OK, with the fourth number being only a locally usable number. If this fourth number is color, we have "color confinement" and "asymptotic freedom". Conservation is not the issue here. (Indeed I insist that nothing ever gets "conserved" but that similar structures are recursively generated so that a "conserved property" is found to have the same "value" over some causal trajectory—see ANPA 11 paper.)

"The argument is simple. PU generates strings with arbitrary quantum numbers (QN's hereinafter) selected from all those allowed. We can imagine a generation which orders the sets of strings with QN's of each type: a set of strings

ordered by spin QN, another by angular momentum, etc. We now synchronize the generators so that a d–space is constructed with a diagonal of n strings, one with each of these QN's and therefore n–dimensions. Feller's Theorem now applies.

"I agree that synchronization is the bridge between combinatorics and geometry — at least that is why and how I have used it."

This theorem has a powerful corrolary in our bit-string coordinate context. Eq. 3.13 identifies the vector basis string $\mathbf{E}_0(S)$ with the *unique* anti-null string. If we identify this with the time direction, then all strings which have the same time coordinate $t = a^0(t)$ have the same Hamming measure

$$|\mathbf{a}(S;t)| = t + \frac{S}{2} \tag{3.20}$$

But strings with the same Hamming measure satisfy the condition required by McGoveran's Theorem in Feller's context (i.e. all have the same number of "1" 's). Consequently, any simultaneous (i.e. same "t") points in our finite and discrete space, when constructed from independently generated, but synchronized, sequences of dichotomic variables of the same length S projected onto a coordinate system with D spacial dimensions have a rapidly diminishing probability of satisfying this "distant simultaneity" criterion for large $t + S/2$ and $D > 3$. The critical $D = 3$ case does allow what we call here *DISTANT simultaneity* to be defined for large (but finite) $t + S/2$.

It is important to realize that this DISTANT simultaneity is *non-local* in the usual quantum mechanical sense when we make the interpretation $a^0 = t, \vec{a} = (a^1 = x, a^2 = y, a^3 = z)$. We intend to prove that the basic "three-vertices" correspond to normal relativistic velocity addition in spite of this non-locality of events. When we make the interpretation $a^0 = E, \vec{a} = \vec{p} = (a^1 = p_x, a^2 = p_y, a^3 = p_z)$, and impose the 4-event criterion that 4 strings discriminate to the null string, this is equivalent to 3-momentum conservation in appropriate contexts, in particular when we require the commensurability of the periods and masses of interacting particles.

Although relativistic 3-momentum is conserved, there is no guarantee that a 3-vertex is "on-shell" in the sense that $E^2 - p^2 = m^2$ for all the "particles". This fact is the starting point of our finite particle number relativistic scattering theory based on relativistic Faddeev-Yakubovsky equations with exactly unitary (flux-conserving) solutions. The asymmetry between the representational properties of "position" and "momentum" already implied by the "counter paradigm" is the reason why an S-matrix type of approach is natural for us. It is also important to realize that our distant simultaneity is *independent* of any concept of *causal continuity* of the type usually associated with special relativity, unless or until we specify in more detail how the strings are generated. That *program universe*-type generators lead to acausal, supraluminal connectivity without allowing supraluminal signaling has been argued elsewhere[23,24] .

3.5 Rational Quaternions

Our mapping of bit-strings onto an orthogonal coordinate system with spacial dimension D works only for even string length and some set of strings which satisfy Eq. 3.16. Further, if $S/2$ is odd, the indistinguishability condition for $D > 1$ implied by Eq. 3.16 cannot be met because two bit-strings with odd Hamming measure discriminate to a bit-string with even Hamming measure. Consequently the simplest basis system we can use for $D > 1$ must have strings which are multiples of some basis of length four. There are $(S!)/(\frac{S}{2}!)^2 = 6$ candidates for the vector basis strings with $S = 4$, $(n = 1)$, but three of these can be obtained from the other three by discrimination with the anti-null string, and correspond to *finite* and *discrete* rotations or reflections of the basis. One allowed basis in three plus one dimensions which I have been studying for some time is

$$\mathbf{E}_0(nS) := n(1111) = \mathbf{1}(nS)$$

$$\mathbf{E}_1(nS) := n(1010);\ \mathbf{E}_2(nS) := n(1001);\ \mathbf{E}_3(nS) := n(1100) \tag{3.21}$$

Since we saw in Sec. 2.5 that we need at most 3+1 dimensions, we will confine ourselves to this system from now on.

In order to conform to his notation, Clive Kilmister suggests that we use instead

$$\mathbf{K}_0(nS) := n(1111) = \mathbf{1}(nS)$$

$$\mathbf{K}_1(nS) := n(1001);\ \mathbf{K}_2(nS) := n(0101);\ \mathbf{K}_3(nS) := n(0011) \qquad (K3.21)$$

Here I have called his suggestion $\mathbf{K}_\mu(nS)$, and used my scalar multiplication notation. The advantage is that we can then write

$$\mathbf{K}_i(nS) = n(i4);\ \mathbf{K}_0(nS) = n(1234)$$

This move looks good; it does not change anything below, so far as I can see.

Another interesting property of these integer quaternions noted by Kilmister is that if we compute Hamilton's "quaternion norm", i.e. $[a_0^2 + a_1^2 + a_2^2 + a_3^2]^{\frac{1}{2}}$ then "for bit strings of Hamming norm n, the square of the quaternion norm must lie between 4 and $4[1 + n(n-1)]$. You can see this by doing a little algebra which will prove that the square of the quaternion norm is exactly

$$4 + 4\Sigma_{i=1}^{4}(B_i^2 - B_i)$$

where

$$B_1 = b_1 + b_5 + ...$$

$$B_2 = b_2 + b_6 + ...$$

and so on. The proof of the inequality then comes easily by looking for the maximum of the constraint under the constraint $\Sigma b_i = n$." I am sure that this restriction on the "Euclidean" norm in 4-space will have considerable significance once we work out the constraints due to rotation and Lorentz boost invariance in our discrete space.

Discrimination of 1, 2, 3 or 4 basis vectors with the anti-null string correspond to well known *discrete* symmetry operations in 3+1 space-time. We list these:

$\mathbf{E}_0(nS)' = \boldsymbol{\mathcal{T}}\mathbf{E}_0(nS) := \mathbf{1}(nS) \oplus \mathbf{E}_0(nS)$ corresponds to TIME inversion.

$\mathbf{E}_i(nS)' = \boldsymbol{\mathcal{M}}\mathbf{E}_i(nS) := \mathbf{1}(nS) \oplus \mathbf{E}_i(nS)$ corresponds to MIRROR REFLECTION across the jk plane.

$\mathbf{E}_{i,j}(nS)' = \boldsymbol{\mathcal{R}}\mathbf{E}_{i,j}(nS) := \mathbf{1}(nS) \oplus \mathbf{E}_i(nS), \mathbf{E}_j(nS)$ corresponds to ROTATION through 180^o around the k axis in either sense.

$\mathbf{E}_{i,j,k}(nS)' = \boldsymbol{\mathcal{P}}\mathbf{E}_{i,j,k}(nS) := \mathbf{1}(nS) \oplus \mathbf{E}_1(nS), \mathbf{E}_2(nS), \mathbf{E}_3(nS)$ corresponds to SPACE inversion — the PARITY operation.

We emphasize the fact that our construction leads immediately to the *discrete* space-time symmetries $\boldsymbol{\mathcal{P}}, \boldsymbol{\mathcal{T}}$ including the degenerate rotation and reflection options. Once we have discussed particulate *quantum numbers*, it will be easy to extend our discussion to $\boldsymbol{\mathcal{C}}$ and the role $\boldsymbol{\mathcal{CPT}}$ invariance plays in our discrete theory.

To go from here to rational quaternions is immediate. Simply define

$$\mathbf{e}_\mu{}^2 = 1 - \frac{4}{nS}(\mathbf{E}_\mu(nS))^0 \tag{3.22}$$

which insures that our basis vectors satisfy Eq. 3.2. We can now follow Greider by adopting the constraint given by Eq. 3.1; then use the components a^μ given by Eq. 3.14 to define a 4-vector given by Eq. 3.3. If the integers we start with do not provide a fine enough mesh to describe the phenomena we are modeling, we can *rescale* as explained above; if we wish to replace integer coordinates by rational coordinates with a smallest aliquot part $1/N_x$ named in advance we can divide all components by this factor. This measure can be fixed in particle physics in the *context* of anticipated experimental resolution. If we wish to use a time-like rather than a space-like metric, all we need do is change the sign of Eq. 3.22,

$$(\mathbf{e}_\mu{}^2)' = \frac{4}{nS}(\mathbf{E}_\mu(nS))^0 - 1 \tag{3.23}$$

So far as *coordinate description* goes, this completes our mapping of bit-strings onto quaternions.

Having gone this far, a temptation for both physicists and continuum mathematicians is to view this mapping of the bit-string spacial coordinates as an *embedding* in R_3, and of the quaternion coordinates as an *embedding* in the space-time of special relativity. Then *coordinate transformations* could be carried through in a conventional way. But this would cut the umbilical cord connecting the mapping to bit-strings. This can easily be seen by trying to go backward after a coordinate transformation and ask what this corresponds to in terms of bit-strings, which was one of Karmanov's criticisms of the original derivation of the Lorentz boosts (Ref. 9). So we have to do more work on coordinate transformations in order to discover which can be expressed in terms of bit-string operations and which cannot. This is well worth the effort, since the bit-string generated "space" is much sparser in "points" than pedestrian "discretizations of the continuum" might lead one to expect. This fact could provide us with a start toward understanding in a new way why our theory gives us the limiting velocity of relativity and the non-commutativity of quantum mechanics *without* producing at the same time "self-energies" which go to infinity in physically interesting situations, and like horrors.

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PREDICTIONS MADE BY DISCRETE PHYSICS

April, 1991

For background see papers by H.P.Noyes and D.O.McGoveran: "An Essay on Discrete Physics", *Physics Essays*, **2**, 17-100 (1989) and SLAC-PUB-4528; "Foundations for a Discrete Physics", SLAC-PUB-4526; and "Discrete Gravity", *Physical Interpretations of Relativity Theory, II*, M.C.Duffy, ed., Imperial College, London, 1990, pp 196-201 and SLAC-PUB-5218.

Empirical Input

$c, \hbar$ and m_p as understood in the "Review of Particle Properties", Particle Data Group, *Physics Letters*, **B 239**, 12 April 1990. Numbers are quoted in the format [( )] = empirical value (error) or range.

[a][$G^2_{\pi N}$ = 13.3(3) from R.A.Arndt *et.al.*, . *Phys. Rev. Lett.*, **65**, 157 (1990). F.Sammarruca and R.Machleit (*Bull. Amer. Phys. Soc.*, **36**, No. 4 (1991)) note most modern models for the nuclear force use the strong empirical ρ coupling and therefore require $G^2_{\pi N} > 13.9$; the smaller vector-meson-dominance-model value for ρ is compatible with the Arndt value.]

COUPLING CONSTANTS

Coupling Constant	Calculated	Observed
$G^{-1}\frac{\hbar c}{m_p^2}$	$[2^{127}+136]\times[1-\frac{1}{3\cdot 7\cdot 10}] = 1.693\ 37\ldots \times 10^{38}$	$[1.69358(21)\times 10^{38}]$
$G_F m_p^2/\hbar c$	$[256^2\sqrt{2}]^{-1}\times[1-\frac{1}{3\cdot 7}] = 1.02\ 758\ldots\times 10^{-5}$	$[1.02\ 682(2)\times 10^{-5}]$
$sin^2\theta_{Weak}$	$0.25[1-\frac{1}{3\cdot 7}]^2 = 0.2267\ldots$	$[0.2259(46)]$
$\alpha^{-1}(m_e)$	$137\times[1-\frac{1}{30\times 127}]^{-1} = 137.0359\ 674\ldots$	$[137.0359\ 895(61)]$
$\alpha_s(m_\pi^2)$	$\frac{1}{7} = \frac{m_\pi}{m_N}$	[? ?]
$G^2_{\pi N}$	$[(\frac{2M_N}{m_\pi})^2 - 1]^{\frac{1}{2}} = [195]^{\frac{1}{2}} = 13.96..$	[a]$[13,3(3), > 13.9?]$

MASS RATIOS

Mass ratio	Calculated	Observed
$\left[\frac{M_{Planck}}{m_{scale}}\right]^2 := \frac{\hbar c}{Gm^2}$	$[2^{127}+136] = 1.70147\times 10^{38}$	$m_{proton}\approx m_{scale}$
m_p/m_e	$\frac{137\pi}{\frac{3}{14}\left(1+\frac{2}{7}+\frac{4}{49}\right)\frac{4}{5}} = 1836.15\ 1497\ldots$	$[1836.15\ 2701(37)]$
$m_\pi^\pm/m_e$	$275[1-\frac{2}{2\cdot 3\cdot 7\cdot 7}] = 273.12\ 92\ldots$	$[273.12\ 67(4)]$
m_{π^0}/m_e	$274[1-\frac{3}{2\cdot 3\cdot 7\cdot 2}] = 264.2\ 143\ldots$	$[264.1\ 373(6)]$
m_μ/m_e	$3\cdot 7\cdot 10 = 210$	$[206.768\ 26(13)]$

General structural results

- 3+1 asymptotic space-time
- combinatorial free particle Dirac wave functions
- supraluminal synchronization and correlation *without* supraluminal signaling
- discrete Lorentz transformations for event-based coordinates
- relativistic Bohr-Sommerfeld quantization
- non-commutativity between position and velocity
- conservation laws for Yukawa vertices and 4- events
- crossing symmetry, CPT, spin and statistics
- "Fields" replaced Wheeler-Feynman "action at a distance"

Gravitation and Cosmology

- consistent formulation of gravitational charge
- electromagnetic and gravitational unification
- the three traditional tests of general relativity
- event horizon
- zero-velocity frame for the cosmic background radiation
- mass of the visible universe: $(2^{127})^2 m_p = 4.84 \times 10^{52}\ gm$
- fireball time: $(2^{127})\hbar/m_p c^2 = 3.5$ million years
- critical density: of $\Omega_{Vis} = \rho/\rho_c = 0.01175$ $[0.005 \leq \Omega_{Vis} \leq 0.02]$
- dark matter = 12.7 times visible matter [10??]
- baryons per photon $= 1/256^4 = 2.328\ldots \times 10^{-10}$ $[2 \times 10^{-10}?]$

Unified theory of elementary particles

- quantum numbers of the standard model for quarks and leptons
with confined quarks and exactly 3 weakly coupled generations
- gravitation: $\hbar c/Gm_p^2 = [2^{127} + 136] \times [1 - \frac{1}{3\cdot 7\cdot 10}] =$
  $1.70147\ldots[1 - \frac{1}{3\cdot 7\cdot 10}] \times 10^{38} = 1.693\ 37\ldots \times 10^{38}$ $[1.693\ 58(21) \times 10^{38}]$
- weak-electromagnetic unification:
  $G_F m_p^2/\hbar c = (1 - \frac{1}{3\cdot 7})/256^2\sqrt{2} = 1.02\ 758\ldots \times 10^{-5}$ $[1.02\ 684(2) \times 10^{-5}]$;
  $sin^2\theta_{Weak} = 0.25(1 - \frac{1}{3\cdot 7})^2 = 0.2267\ldots$ $[0.2259(46)]$
  $M_W^2 = \pi\alpha/\sqrt{2}G_F sin^2\theta_W = (37.3\ Gev/c^2 sin\ \theta_W)^2;\ M_Z cos\ \theta_W = M_W$
- the hydrogen atom: $(E/\mu c^2)^2[1 + (1/137N_B)^2] = 1;\ \mu = m_e m_p/(m_p + m_e)$
- the Sommerfeld formula: $(E/\mu c^2)^2[1 + a^2/(n + \sqrt{j^2 - a^2})^2] = 1$
- the fine structure constant: $\frac{1}{\alpha} = \frac{137}{1 - \frac{1}{30\times 127}} = 137.0359\ 674\ldots$ $[137.0359\ 895(61)]$
- $m_p/m_e = \frac{137\pi}{\frac{3}{14}\left(1+\frac{2}{7}+\frac{4}{49}\right)\frac{4}{5}} = 1836.15\ 1497\ldots$ $[1836.15\ 2701(37)]$
- $m_\pi^\pm/m_e = 275[1 - \frac{2}{2\cdot 3\cdot 7\cdot 7}] = 273.1292\ldots$ $[273.12\ 67(4)]$
- $m_{\pi^0}/m_e = 274[1 - \frac{3}{2\cdot 3\cdot 7\cdot 2}] = 264.2\ 1428..$ $[264.1\ 373(6)]$
- $\alpha_s(m_\pi^2) = \frac{1}{7}$
- $(G_{\pi N}^2 m_{\pi^0})^2 = (2m_p)^2 - m_{\pi^0}^2 = (13.868..\ m_{\pi^0})^2$