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ANTI–PAULI BLOCKING IN QCD_{1+1}

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ABSTRACT

We investigate the flavor asymmetry of the $q\bar{q}$ sea in a (uu)-proton for QCD_{1+1} with $SU(2)$-color. The excess of $\bar{u}$ over $\bar{d}$ which apparently seems to be in conflict with the exclusion principle is explained by taking the color degrees of freedom into account. By computing color factors for the relevant perturbation theory diagrams we show that the non–abelian sea quarks behave effectively similar to abelian bosons. For the magnitude of the flavor asymmetry of the sea we predict scaling proportional to the ratio of the quark Compton wavelength to the proton radius. We show that the sign of the asymmetry in QCD_{1+1} is different from the sign measured in $3+1$ dimensions.

From an analysis of the violation of the Gottfried sum rule[1)] it is known that the proton contains more $\bar{d}$ than $\bar{u}$ quarks (see e.g. Ref. 2). This property of antiquark distributions is conjectured to be a consequence of the Pauli principle.[3)] The argument is based on the observation that there are more u than d quarks in the valence configuration of the proton. Thus the exclusion principle apparently forbids more states with additional u quarks (from $u\bar{u}$ pair creation) than with additional d quarks. From this simple reasoning one might conclude that the dominance of $\bar{d}$ over $\bar{u}$ in the proton is an obvious consequence of the Pauli principle. Similar conclusions can be reached by looking at the pion cloud of a nucleon (see e.g. Ref.

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4). The basic argument proceeds as follows: The main contributions to sea quarks in a nucleon arise from πN intermediate states. For the proton the only possible πN states are $\pi^0 p$ and $\pi^+ n$. Since the π^0 contains the same ammount of $\bar{u}$ and $\bar{d}$, whereas the π^+ contains only $\bar{d}$ (at least in a valence quark picture) one expects more $\bar{d}$ than $\bar{u}$ contributions from the pion cloud. For more details we refer to Ref. 4 and references therein.

The purpose of this talk is to show that the excess of $\bar{d}$ over $\bar{u}$ in the proton is not as natural as it seems to be if one follows these arguments. We will provide an example where the above reasoning seems to apply, though the actual effect will go into the opposite direction, i.e. more $\bar{u}$ than $\bar{d}$ will be present in our $1+1$ dimensional toy model. To avoid any misunderstanding we should emphasize that we do not claim that the above arguments are wrong in QCD_{3+1}. In fact they reproduce the experimentally observed trend. However, we think that the validity of these arguments should be checked more carefully to see why they are successful in QCD_{3+1} but fail in QCD_{1+1}.

The model we are considering is QCD in 1+1 dimensions. For simplicity we work with $SU(2)$ color[#1] and two degenerate flavors. The valence configuration of our "proton" consists of (uu) in a color antisymmetric state. Certainly the above reasoning would predict an excess of $\bar{d}$ over $\bar{u}$ in the (uu)-proton. We investigated this system by using the method of Discrete Light-Cone Quantization (DLCQ).[5,6] In this approach the hadronic wave function is expanded in terms of Fock states at equal light-cone time. For fixed quantum numbers the expectation value of the light-cone Hamiltonian is minimized by varying the wave functions in the various Fock components. The resulting eigenvalue equation is solved by discretization in momentum space and subsequent matrix diagonalization. In the numerical work we restricted ourselves to two classes of Fock states, the valence configuration and the Fock component with one additional sea quark pair. For the values of the coupling constant considered ($g^2 C_F/\pi \leq m_q^2$) this is justified since already one $u\bar{u}$ or $d\bar{d}$ pair appears only rarely. If taken into account, higher Fock components are in general negligible for ground state hadrons in QCD_{1+1}.[6]

#1 As far as the flavor asymmetry of the sea in QCD_{1+1} is concerned there is nothing special about $N_C = 2$. In fact one obtains similar results for $N_C = 3$. However, $N_C = 2$ is much easier to deal with numerically as well as conceptually since fewer degrees of freedom have to be considered.

The numerical results for various quark masses ($m = m_u = m_d$) are shown in Table 1.

Table 1

Total admixture of $\bar{u}$ and $\bar{d}$ quarks and flavor asymmetry of the sea for various quark masses.

$(m^2\pi/g^2C_F)$	1.	10.	100.	1000.
$\bar{u} = \int_0^1 dx\, f_{\bar{u}}(x)$	$0.18 \cdot 10^{-2}$	$0.17 \cdot 10^{-3}$	$0.45 \cdot 10^{-5}$	$0.75 \cdot 10^{-7}$
$\bar{d} = \int_0^1 dx\, f_{\bar{d}}(x)$	$0.91 \cdot 10^{-3}$	$0.11 \cdot 10^{-3}$	$0.35 \cdot 10^{-5}$	$0.62 \cdot 10^{-7}$
$(\bar{u} - \bar{d})/\frac{1}{2}(\bar{u} + \bar{d})$	0.66	0.42	0.29	0.096

The results for the flavor asymmetry of the sea are quite surprising. First, and most important, its sign is positive. In the $SU(2)_C$ (uu)-proton, it is more likely to find $u\bar{u}$ sea quarks than it is to find $d\bar{d}$ sea quarks. Although the total number of $\bar{u}$ and $\bar{d}$ quarks is small for the quark masses considered, the relative flavor asymmetry of the sea reaches 2/3 for strong coupling $g^2C_F/m^2\pi = O(1)$.

Furthermore, one can observe that the effect of the antisymmetrization vanishes for weak coupling ($g^2C_F/m^2\pi \to 0$). This result can be easily understood. The momentum spectrum of sea quarks is quite wide—the typical scale being one quark mass — whereas the spectrum of the valence quarks is narrow (a typical scale is the inverse hadron radius). Only for such sea quarks whose momenta are present in the valence quark wavefunction is antisymmetrization important. Thus the flavor asymmetry should scale like the fraction of sea quarks with momentum of the order of the inverse proton radius; i.e. like the ratio of the quark Compton wavelength to the proton radius. For large quark masses we have verified the power law $(\bar{u} - \bar{d})/(\bar{u} + \bar{d}) \propto g^{2/3}m^{-2/3}$ implied by such a scaling behavior.

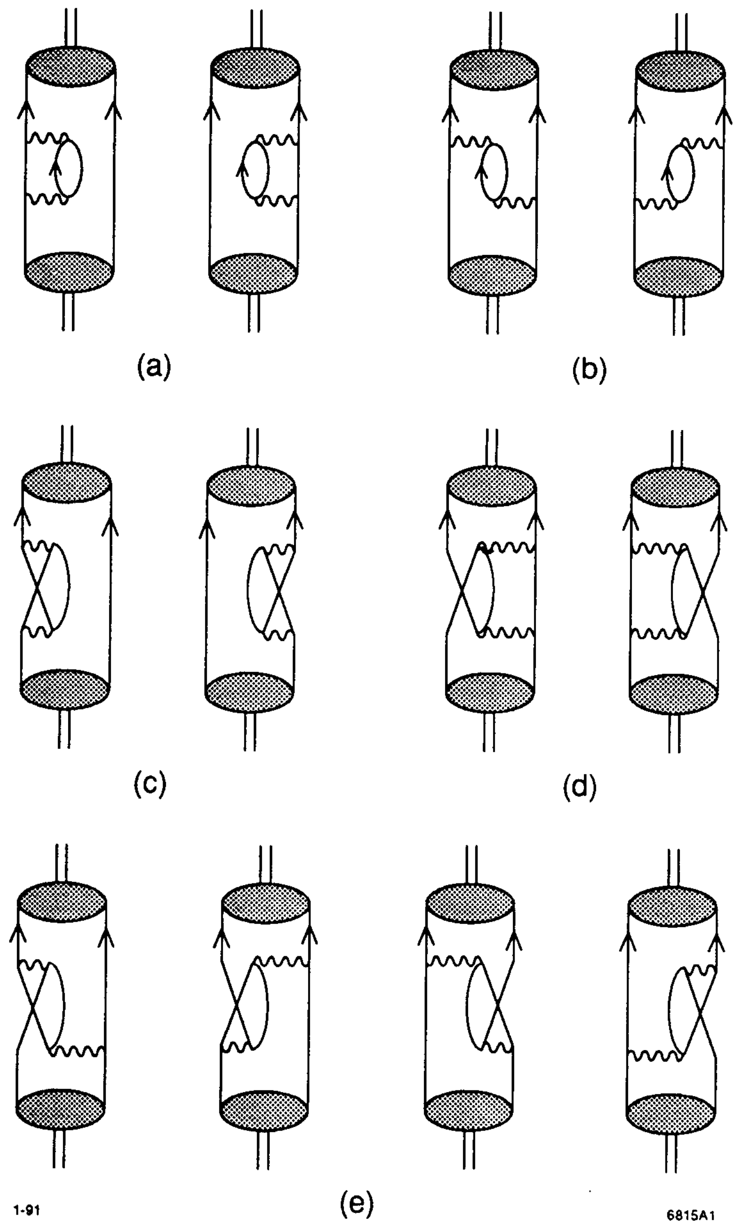
Let us now return to the more interesting question of the sign of the flavor asymmetry. In order to develop some intuitive understanding we concentrate on weak coupling ($m^2\pi/g^2C_F \geq 10$) where sea quarks can be incorporated perturbatively. More precisely one can neglect all interactions in states containing sea quarks and use second order perturbation theory starting from the valence configuration[#2] while producing less than 5% errors for the asymmetries.[7)]

The diagrams contributing to sea quark production within these approximations are listed in Fig. 1.

Figure 1

Second order diagrams contributing sea quarks to the (uu)-proton.

a,b) bubble graphs; c,d,e) exchange graphs.



#2 Which is obtained by diagonalizing the light-cone Hamiltonian in the two quark subspace of the Fock space.

The bubble graphs (Figs. 1a,b) are the only ones contributing to $d\bar{d}$ production whereas all of them contribute to $u\bar{u}$ production. Thus, in order to understand the difference between $d\bar{d}$ and $u\bar{u}$ production it suffices to consider the exchange graphs (Figs. 1c-e). In QED the exchange graphs differ form the bubble graphs only by a sign from Fermi statistics[#3] and thus exchange graphs cancel part of the contribution from the bubble graphs. This cancellation applies only to quarks with the same flavor as the valence quarks and we expect $u\bar{u}$ to be suppressed compared to $d\bar{d}$ (we confirmed this numerically). The basic reason is the fact that the group theory factors are the same for direct and exchange graphs. More precisely the group theory factors in QED are given by the product of charges at the photon-fermion vertices, i.e. they are given by[#4]

$$\begin{aligned} G^{a}_{QED} &= 1, \quad G^{b}_{QED} = -1 \\ G^{c/d}_{QED} &= 1, \quad G^{e}_{QED} = -1 \end{aligned}$$

(the letters a-e refer to the graphs in Figs.1a-e). The signs simply arise from attaching the photons to positively and negatively charged fermions: e.g. the first graph in Fig.1a and the first graph in Fig.1b differ by connecting the lower photon to different valence fermions. Since we had chosen opposite charges for the valence fermions this results in opposite signs for the group theory factors of these two graphs. Note that graphs which differ only by exchanging two fermion lines (e.g. exchanging two fermion lines in Fig.1a leads to Fig.1 c or d) have the same group theory factor. In QED these statements are almost trivial since group theory factors can be evaluated by multiplying some numbers.

In QCD the situation is quite different since the group theory factors are not just products of numbers but rather sums of traces of products of matrices. Thus by simply "looking" at the graph it is not so easy to guess the group theory factor and calculating them often yields surprising results. The corresponding group theory

#3 Here we are interested only in a qualitative understanding of the flavor asymmetries. We thus neglect the orbital matrix elements which are the same in QED and QCD within the order of perturbation theory considered in this work. Of course the numerical results in Table 1 contain the orbital part.

#4 Since QED is confining in $1+1$ dimensions we assume here that the two valence fermions have opposite charges.

factors for the graphs in Fig.1 are

$$G^a_{QCD} = 2/3, \quad G^b_{QCD} = -2/3$$
$$G^{c/d}_{QCD} = -3/16, \quad G^e_{QCD} = 3/16.$$

Besides the smaller absolute value (which is like a $1/N_C$ suppression) the exchange graphs have a negative color factor compared to the direct graphs (and also compared to the charge factors in QED) which compensates for the above mentioned sign from Fermi statistics. Thus the non-abelian sea quarks, which are of course fermions, behave similarly as abelian bosons would do. What happens here is the following: when a valence quark emits a $q\bar{q}$ pair via one gluon exchange, the color group factors at the vertices make it more likely to find the produced quark in a color odd state than to find it in a color even state with the valence quark. If the sea quark has the same flavor as the valence quark the total wave function must be odd under exchange of sea and valence quark; i.e. the spatial part must be even in the color odd case (the more likely one) and vice versa.[#5] Therefore, after "integrating out" the color degrees of freedom, the sea quarks are more likely to behave like bosons with respect to the valence quarks. If the spatial part has bosonic symmetry there is constructive interference between direct and exchange graphs, i.e. there is enhancement compared to distinguishable quarks (the case when sea and valence quarks have different flavor).

Besides the (uu)-proton for $SU(2)_C$ we investigated $u\bar{u}$ and $d\bar{d}$ sea quarks in $u\bar{u}$ mesons for general N_C. Furthermore we estimated the effect for the $SU_C(3)$ proton. In these examples the sign of the asymmetry of the sea is always the same although its absolute value decreases like $1/N_C$.

One might be tempted to generalize these results to QCD$_{3+1}$. This would however contradict the experimental results.[2)] These clearly show an excess of $\bar{d}$ over $\bar{u}$ in the proton. We will not try to resolve this problem here but simply point out several important differences between this work and QCD$_{3+1}$. Due to the running of the QCD coupling the sign of $\frac{d}{dq^2}(\bar{u} - \bar{d})$ is reversed; although the color factors are the same in 1+1 and 3+1 dimensions, radiative corrections tend

#5 Remember there is no spin in $1+1$ dimensions. In QCD_{3+1} one has to take spin degrees of freedom into account which complicates the discussion. We don't know how this changes the results.

to favor $\bar{d}$ over $\bar{u}$ in the real proton[7] and perturbation theory in QCD_{3+1} predicts the correct sign for the asymmetry. Most importantly however the predicted result is much smaller than the experimentally observed asymmetry[8] though there is some uncertainty in the extrapolation of the data to small x.[9] Thus it is not clear at the moment whether the observed excess of $\bar{d}$ over $\bar{u}$ in the real proton is a nonperturbative effect. Furthermore, it would be very interesting to see which effect or which degrees of freedom are primarily responsible for the sign reversal in the asymmetry if we go from $1+1$ to $3+1$ dimensions and to get some intuitive understanding about the underlying mechanism. As we have already pointed out, the most obvious difference between $1+1$ and $3+1$ dimensions is the spin, which does not exist in $1+1$ dimensions. One might think that inclusion of spin degrees of freedom might reverse the effect studied here. In principle this is possible but we do not know what really makes the difference between real experiments and our $1+1$ dimensional results. However, we would like to emphasize that if the spin degrees of freedom account for the basic difference between QCD_{1+1} and QCD_{3+1}, then the naive argument based on the Pauli principle must be wrong for QCD_{3+1} also since spin plays no role whatsoever in the argument.

In the meson cloud model the situation is much clearer and it seems that we have found an effect which reverses the asymmetry if we go from $1+1$ to $3+1$ dimensions. In order to show this we will investigate the consequences from another important difference between QCD_{3+1} and QCD_{1+1} in the appendix. In QCD_{1+1} the η meson is almost degenerate with the pion, i.e. there the η cannot be neglected in the meson cloud of a nucleon. We will show that inclusion of the η reverses the sign of the asymmetry in QCD_{1+1}. In $3+1$ dimensions the large η mass makes this meson much less important for radiative corrections than the π and we can safely omit the η. In other words, if we would include the η in $3+1$ dimensions this would not make much of a difference.

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APPENDIX: the contribution from the η in QCD_{1+1}

The disagreement of our findings with the naive meson cloud is so surprising that we will try to resolve the apparent contradiction. In 1+1 dimensions there is no spin (because there are no rotations), i.e. there is nothing like a nucleon but only Δ's. Thus let us modify the mesonic argument such that it applies to those baryons and consider the π admixture to a Δ^+ state which can couple to $\pi^-\Delta^{++}$, $\pi^0\Delta^+$ and $\pi^+\Delta^0$.

The (very naive, as we will see) argument here would be: The $\pi^0\Delta^+$ contributes the same ammount of $\bar{u}$ and $\bar{d}$; the $\pi^+\Delta^0$ (which contributes only $\bar{d}$) would be favored compared to $\pi^-\Delta^{++}$ (which contributes only $\bar{u}$) because of isospin Clebsch-Gordan coefficients and altogether one expects more $\bar{d}$ than $\bar{u}$. However our numerical estimates have shown that there is actually more $\bar{u}$ than $\bar{d}$ in this state, i.e. even in this simplified hadron model there seems to be a contradiction.

Let us see what went wrong with the naive $\pi\Delta$ argument. The most important mistake has been made by neglecting the η meson which is almost degenerate with the π^0 in $1+1$ dimensions.#6 Since the η has similar flavor content as the π^0 there will be important interference effects between these mesons.#7 If we assume that η and π are degenerate (which is an excellent approximation in QCD_{1+1}) we can talk about $u\bar{u}$ and $d\bar{d}$ mesons instead. In order to understand the effect of the η let us neglect antisymmetrization for the moment (which means we should get the same amount of $\bar{u}$ and $\bar{d}$ if we sum up all contributions). From quark counting it should be clear that the coupling of a Δ^+ to a $u\bar{u}$ is twice as large as to $d\bar{d}$, i.e. considering neutral mesons only we get an excess of $u\bar{u}$. Actually, if one neglects antisymmetrization this compensates exactly the excess of $d\bar{d}$ from the other $\pi\Delta$ states. If one takes antisymmetrization into account then $u\bar{u}$ mesons are even more favored. The basic reason is that the spatial wavefunction of the Δ is

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#6 The basic reason is that the axial $U(1)$ current is not anomalous in QCD_{1+1}. Intuitively one can understand this by looking at the discrete quantum numbers of these mesons. Both mesons have $(P,C)=(-,+)$ in $3+1$ dimensions and $(P,C)=(-,-)$ in $1+1$ dimensions (in $3+1$ dimensions the C-operation picks up an extra minus when acting on the odd spin wavefunction of a pseudoscalar). Thus the $1+1$ dimensional η cannot couple to a two gluon intermediate state which is sometimes mentioned as a reason for the large mass of the η. Furthermore there are no instantons in QCD_{1+1}.

#7 Naively one might believe that including the η doesn't change anything since, like the π^0, the η contains the same amount of $\bar{u}$ and $\bar{d}$. However, the operator measuring the flavor asymmetry of the sea is an isospin triplet operator, i.e. it has nonzero matrix elements between π and η states and there is interference between π and η contributions.

totally symmetric and thus the quarks behave effectively like bosons. Ultimately, this allows one to explain the numerically obtained excess of $\bar{u}$ over $\bar{d}$ in QCD_{1+1}.

Summarizing one can say that it is the interference of the η with the π which turns the apparent $\bar{d}$ dominace into a $\bar{u}$ dominance in QCD_{1+1}.

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