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REPORT FOR WORKING GROUP ON COHERENT SYNCHROTRON RADIATION*

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ABSTRACT

The working group on Coherent Synchrotron Radiation met jointly with the working group on Impedances and Wake Fields. Since coherent radiation is strongly affected by the shielding due to the vacuum chamber,[1] the two subjects have much in common. In fact the theory of coherent radiation might be described as the theory of impedances and wake fields with curved particle trajectories. Parts of the theory have been pursued now and then over many years, whereas experiments are a relatively new development. For experiments see Ref. 2 , and references therein. I will review separately our discussions on experiments and theory.

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Experiments

(1) The experiment of Nakazato *et al.* at Tohoku University,[2] the first to claim definitive evidence for coherent synchrotron radiation, was discussed. J. Bisognano raised the question of how to be certain that the observed radiation was really a curvature effect. In this experiment there is a sharp transition in the transverse dimension of the vacuum chamber at the entrance to the bending magnet. The incoming beam tube is about 10 cm across. It connects to a large tank in the region of the magnet, about 30 cm wide and 1 m long. Could it be that the sharp transition results in excitation of modes in the tank, irrespective of any curvature effect, with the particles in the bunch radiating coherently into those modes? As in the case of true coherent synchrotron radiation, the intensity of this "induced r.f." would vary as N^2, where N is the number of particles in the bunch. This issue was already addressed in Ref. 2 , which mentioned a theoretical estimate of the induced r.f. and also an attempt to intercept r.f. by a thin aluminum window at a point upstream from the point of emission of observed light. In the working group, T. Nakazato affirmed his belief that the effect is not important, pointing out that a displacement of the beam was found to have a strong effect on the intensity at the fixed detector; i.e., the radiation had pronounced directionality. Also, strong polarization of the radiation was observed, as would be expected of synchrotron radiation but not of induced cavity radiation.

Remark: The frequency distribution of coherent synchrotron radiation depends very sensitively on both the charge distribution in the bunch and the characteristics of the shielding. This is especially noticeable near the frequency threshold for appreciable coupling impedance. Thus, a theoretical curve such as the dotted curve in Fig. 2a of Ref. 2 should be regarded as quite model-dependent.

(2) S. Okuda reported on a new experiment in progress at Osaka University. The experiment makes use of a very high intensity 38 meV beam from the ISIR linac, with $N = 2 - 3 \cdot 10^{11}$ and a clean bunch profile of Gaussian appearance, with length $\sigma \approx 9$ mm. The experimenters find an enhancement in intensity of about 10^{11} in comparison with computed incoherent synchrotron radiation at the same wavelength (around $2-3$ mm). They plan to measure the frequency spectrum, and to vary the bunch length by means of a bunch compressor.

(3) We learned of a proposed experiment at the Cornell University injector linac, by Eric Blum *et al.*. Unlike the Tohoku experiment, there would be no abrupt change in vacuum chamber dimensions at the entrance to the bending region. Another proposed experiment,[3] by A. Hofmann, L. Rivkin, and B. Zotter at LEP, was described by Rivkin in the plenary session. It appears to me that the parameters quoted for this experiment make the observation of coherent radiation a doubtful matter. My estimates, based on either of two models of the shielding,[1] indicate that coherent radiation in the LEP experiment should be totally suppressed by the shielding, unless the bunch spectrum has a much higher proportion of high frequency components than a Gaussian would have. The authors of Ref.3 base their proposal on a formula derived from an early paper of Schiff[4]. They and Hofmann[5] make the puzzling statement that Schiff's model consists of *two* infinite, parallel, conducting plates, with the beam circulating in a plane midway between the plates. Schiff states that his model has only *one* plate, and that the model gives an *upper bound* on the coherent power, not necessarily a close estimate for the actual power. I am not sure of the pedigree of Schiff's formula, since he gives no derivation, but it clearly has no resemblance to the well-known formula for two parallel plates.[1] In particular, it does not display the sharply defined frequency threshold for appreciable coupling impedance that has been confirmed by several investigators in several models that are more realistic than the single plate model (parallel plates, concentric cylinders, pillbox, torus). In the LEP experiment, the threshold is so high in frequency that it lies far outside the bunch spectrum (assumed to be roughly Gaussian). See the numerical considerations in item **(5)** below.

(4) F. Caspers raised the possibility of studying curvature effects by means of bench measurements. One could set up a curved strip line in the proximity of a corresponding curved metal surface. According to Caspers, it is well known that radiation from such a configuration can be observed. One could excite the line with a fixed frequency, or with a pulse, and look at the angular distribution of intensity.

Another possibility would be to put a wire inside a toroidal chamber, so as to simulate a beam in a storage ring. The questions to be answered by such measurements are not yet clear. The interpretation of experiments in which a conductor replaces a beam could be materially different from what we are accustomed to in the usual configurations without curvature (straight pipe with cavities, etc.). The dispersion relation for resonances of a smooth toroidal chamber is completely different from that of cavity resonances, and that may have an impact in the interpretation of wire measurements, just as it has an impact in the study of coherent instabilities of a beam in such a chamber. It may be possible to study the problem along the lines followed by Gluckstern and Li for a wire in a straight tube.[6]

(5) The simplest useful model of shielding consists of two infinite, parallel, perfectly conducting plates. The beam follows a circular orbit in a plane midway between the plates, which are separated by a distance h. According to Faltens and Laslett,[7] the maximum value of Re $Z(n, n\omega_o)/n$ is

$$\max_n \left[\frac{\text{Re } Z(n, n\omega_o)}{n}\right] \approx 300\frac{g}{R} \text{ ohms} \quad , \tag{1}$$

where $g = h/2$ and R is the radius of the orbit. Also, Re $Z(n, n\omega_o)/n$ is negligibly small for n less than a threshold given roughly by

$$n = \pi(R/h)^{3/2} \quad . \tag{2}$$

The results (1) and (2) were obtained by numerical evaluation of a somewhat complicated formula for the impedance that is stated in terms of high-order Bessel functions.[1] During the workshop, I noticed that the formula could be simplified so as to make these results obvious. Using appropriate asymptotic forms of the Bessel functions, one finds that the following formula holds to a good approximation:

$$\frac{\text{Re } Z(n, n\omega_o)}{n} = 2Z_o\left[\frac{\pi R}{hn}\right]^2 \exp\left[-\frac{2}{3n^2}\left(\frac{\pi R}{h}\right)^3\right] \quad . \tag{3}$$

Here only the dominant term (axial mode number $p = 1$) has been included, and the vertical size of the beam is much less than h. The maximum value of the expression (3) is

$$\frac{720}{e}\frac{g}{R} \approx 265\frac{g}{R} \quad , \tag{4}$$

in agreement with (1) . The maximum occurs at $n = \pi 2^{1/2}(R/h)^{3/2}$, and (2) is a good representation of the threshold.

The formula (3) makes it much easier to calculate the radiated power and the wake field, following the formulas given in Ref. 1. To find the radiated power by Eq.(2.11) of Ref. 1, we must evaluate $|\lambda_n|^2\mathrm{Re}(n, n\omega_o)$ near its maximum, where λ_n is the Fourier coefficient of the longitudinal charge distribution. For a Gaussian bunch of length σ this quantity is proportional to

$$\frac{1}{n}\exp\big[-(\frac{n\sigma}{R})^2 - \frac{2}{3n^2}(\frac{\pi R}{h})^3\big] \quad . \tag{5}$$

Since the factor $1/n$ has little effect on locating the maximum, we look for the maximum of the exponential factor, and find that it lies at the point n such that the exponent of the bunch spectral density $|\lambda_n|^2$ is

$$-(\frac{n\sigma}{R})^2 = -(\frac{2}{3})^{1/2}\frac{\sigma}{R}(\frac{\pi R}{h})^{3/2} \quad . \tag{6}$$

In the most favorable case for the LEP experiment (90° lattice, observation in mini-wiggler) we have $h = 6\mathrm{cm}$, $\sigma = 1.8\mathrm{mm}$, $R = 250\mathrm{m}$, and (6) has the value -8.8, so that the bunch spectral density is down by a factor $1.5 \cdot 10^{-4}$ from its maximum value. Since the spectral density has to be fairly close to its maximum to get appreciable radiation, this does not look favorable for the LEP experiment. By contrast, in the Tohoku experiment (with $\sigma = 2.2\mathrm{mm}$, $h = 30\mathrm{cm}$, and $R = 2.44\mathrm{m}$) the spectral density is at 9/10 of its maximum. The toroidal model will predict even less coherent radiation at LEP, since the threshold is at a somewhat higher frequency.

(6) S. Heifets pointed out that the usual concept of impedance does not always apply when the trajectory bends through an angle $\alpha < 2\pi$. This is true if one defines impedance by imposing the synchronism constraint $\omega = n\omega_o$ on the general impedance $Z(n,\omega)$, which is the Fourier transform of the Green function $G(\theta - \theta', t - t')$. By keeping n and ω as independent variables, one can treat an arc of a circle. Responding to Heifets' remark, I found the following formula for the energy change during traversal of an arc of angle α:

$$\Delta U = -(q\alpha)^2\frac{1}{2\pi}\sum_{n=-\infty}^{\infty}|\lambda_n|^2\int_{-\infty}^{\infty} d\omega S^2(\frac{\alpha}{2}(\frac{\omega}{\omega_o} - n))\mathrm{Re}Z(n,\omega) \quad , \tag{7}$$

where q is the total charge, λ_n is the Fourier coefficient of the longitudinal charge distribution as defined in Ref. 1, and

$$S(x) = \frac{\sin x}{x} \quad . \tag{8}$$

Here it is assumed that the charge is created at the beginning of the arc and destroyed at the end. A more elaborate calculation has been set up, in which charge conservation is accounted for by allowing the charge to come in and go out on straight trajectories extending to infinity. Although the integrals for the straight paths have not yet been evaluated, it appears at first sight that they have a minor effect.

(7) It is often assumed that the energy radiated from an arc of angle α is approximately $\alpha/2\pi$ times that radiated from a full circle. The accuracy of this assumption can be checked through Eq. (7). For this purpose, one can get an approximation for ReZ analogous to Eq. (3), but allowing n and ω to be independent. Approximating $S^2(x)$ by a triangle for $|x| < \pi$ and by 0 elsewhere, one then finds

$$\Delta U \approx -q^2 \alpha \omega_o \sum_n |\lambda_n|^2 \mathrm{Re}Z(n, n\omega_o) \quad , \tag{9}$$

provided that

$$\alpha \gg (h/R)^{1/2} \quad . \tag{10}$$

This is indeed $\alpha/2\pi$ times the result for a full circle given in Ref. 1. Heifets noted that $(h/R)^{1/2}$ is roughly $n^{-1/3}$ at the mode n where the impedance is maximum, and that in any mode n the angular spread in the radiation about the plane of the orbit is also around $n^{-1/3}$ (in the usual theory for radiation from a point charge, without shielding). Thus, the condition (10), which was invoked to justify certain expansions in the derivation of (9), can be stated as the requirement that the angle α be much larger than the angular spread of (unshielded) radiation about the orbital plane (at the preferred frequency corresponding to maximum impedance with shielding).

(8) There was some discussion aimed at finding a simple explanation of the threshold condition Eq. (2). Notice that the value of (2) is typically much higher than the familiar waveguide cutoff, which lies near $n = R/h$. The lowest synchronous resonance in a smooth toroidal chamber (rectangular cross section, width w, height h) lies at a value of n somewhat greater than $n_o = \pi R^{3/2}/hw^{1/2}$. Thus the effective threshold in the toroidal model is at $n = n_t > n_o$, since the impedance is negligible below the lowest resonance. In an *impromptu* remark, R. Gluckstern offered a way to understand this threshold by imagining what might happen when a straight rectangular wave guide is bent into a circle of average radius R. In a straight guide of width w and height h, the phase velocity $v_\phi = \omega/k$ is determined by

$$(\frac{\pi m}{w})^2 + (\frac{\pi p}{h})^2 + k^2 - (\frac{\omega}{c})^2 = 0 \quad , \tag{11}$$

where the integers m and p are not both zero (for TE modes) or both not zero (for TM modes). If n is considerably larger than the pipe cutoff value, we can expand v_ϕ to lowest order in powers of n^{-2}, where $n = \omega R/c$, to obtain

$$v_\phi = c\Big[1 + \frac{1}{2}(\frac{R}{n})^2\big[(\frac{\pi m}{w})^2 + (\frac{\pi p}{h})^2\big]\Big]^{1/2} \quad . \tag{12}$$

Now suppose that the guide is bent to form a torus with outer (inner) radius $R \pm w/2$. The velocity of a point on a wave front will vary linearly with r. Suppose that the wave is in phase with the particle of velocity c at $r = R$; then its phase velocity at the outer wall is $c(1 + w/2R)$. It is reasonable to identify this velocity with the phase velocity (12) for the straight pipe; that is, to assume that bending *decreases* the phase velocity, except at the outer wall. For $m = 0$ and $p = 1$ this gives exactly the value n_o stated above. A closer look shows that this is not a complete story, since the wave guide modes and torus modes are not in proper correspondence. As the discussion of Ref. 8 shows, each torus mode is a superposition of TE and TM wave guide modes; therefore the $m = 0, p = 1$ case, a pure waveguide TE mode, cannot correspond completely to the lowest torus resonance.

S. Heifets and A. Mikhailichenko also expressed some ideas about the intuitive basis of the threshold and the maximum value of ReZ/n. The reader may consult their written account, prepared after the workshop.[9]

In my view, a clear and reliable physical picture is likely to come only from further thought about the exact theory, which is gradually becoming simpler and clearer. It is possible to derive the exact results for the parallel-plate model in just a few lines, and the approximation (3) in a few more, as I will show in a later paper.

(9) F. Caspers pointed out that there is much theoretical work on curved waveguides in the microwave literature. Although there is no account of beam current in such work, the methods used to solve the homogeneous Maxwell equations still could be of use in our subject. Caspers called attention to the book of Lewin *et al.*[10] that describes techniques for handling waveguides of general cross-sectional form, treating curvature by a perturbative method. After the workshop, I learned that H. Hahn and S. Tepikian[11] have applied a perturbative method to treat the toroidal chamber at low frequencies.

(10) Unfortunately, we did not have time to discuss the possible role of curvature effects in coherent instabilities in storage rings. As Bisognano emphasized, it is easy to believe in coherent synchrotron radiation, but relatively difficult to see whether it plays a role in bunch stability. It is not enough simply to have values of the coupling impedances, since the usual threshold criteria for instability may not hold in this novel dynamical situation. Particles following trajectories with different radii of curvature synchronize with different resonant modes of the structure. Furthermore, the toroidal chamber has a peculiar dispersion curve $\omega = \omega(n)$ that is almost parallel to the synchronism line $\omega = \omega_o n$. Therefore the tiny change in bending radius between one side of the bunch and the other can bring in a fairly wide band of resonances. This effect should be carefully accounted for in a stability study based on the Vlasov equation. One hopes that there will be results on this problem at the next conference on collective effects in short bunches.

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