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Hidden Local Symmetry and Induced Topological Terms*

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ABSTRACT

Extending the previous derivation of an effective gauge theory from a generalized Nambu–Jano–Lasinio model, we show how the topological θ–term, $\theta\widetilde{F}F$, can be generated dynamically in a similar fashion. These results provide a vivid demonstration of hidden local symmetry at work. Possible Phenomenological implications of this approach are also discussed.

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The dynamical chiral symmetry breaking (DχSB) phenomenon has been extensively studied in the past three decades. Although it is inherently non–perturbative, and calculational tools are limited, yet lots of interesting results have been obtained. It was first shown by Nambu and Jona–Lasinio (NJL)[1] in a 4–fermion model in 4–dimensions that if the coupling constant squared exceeds a certain critical value, then a mass for the fermion can be generated dynamically in the Hartree–Fock approximation. The chiral symmetry in the model is dynamically broken when the fermion mass acquires a non–zero value, and the result of this DχSB is the formation of a pseudo–scalar bound state of the fermion pairs. Later, Eguchi *et al.*[2] and others[3,4] went a step further and demonstrated that a purely fermionic NJL model which bears only global chiral symmetry can generate not only the scalar bosons but also the composite gauge bosons, which acquire their kinetic terms via quantum corrections as a consequence of DχSB, *i.e.* when certain gap equations are satisfied. In modern language, such a symmetry transmutation phenomenon can be understood as hidden local symmetry (HLS).[5] Hidden local symmetry states that any non–linear sigma model based on the manifold G/H is gauge–equivalent to a linear model with $G_{\text{global}} \otimes H_{\text{local}}$ symmetries. H_{local} is a HLS which possesses composite gauge bosons as its corresponding gauge fields. Although it is not sure at present if 4–dimensional non–linear sigma models may also provide the kinetic term of the gauge boson of HLS, Kugo *et al.*[6] have demonstrated with specific CP^{N-1} models that the appearance of massless composite gauge bosons can in fact by–pass the Weinberg–Witten[7] no–go theorem. Ebert and Reinhardt[8] have also constructed a massive Yang-Mills theory via bosonization of a generalized NJL model in 4–dimensions. Thus we have good indications that the phenomenon of the generation of hidden composite gauge bosons with a kinetic term is a gen-

eral feature in a wide class of non–linear sigma models in 4–dimensions. On the phenomenological side, by identifying the dynamically generated gauge boson as the vector meson ρ, the authors in Ref. 9 obtained very successful results in the low–energy hadronic physics. So, it is worthwhile to see if HLS can be used to generate all the possible effective gauge invariant operators that are allowed by the symmetries of the theory beyond the leading order, or, in the language of Eguchi[2] and others,[3,4] beyond the divergent diagrams. Actually this can be tested by looking at higher dimension operators that involves the effective topological terms $\widetilde{F}F = 4\epsilon_{\mu\nu\alpha\beta}(\partial^\mu G^\nu \partial^\alpha G^\beta + 2i\partial^\mu G^\nu G^\alpha G^\beta)$ which bears a non–trivial relative coefficient between terms in it. In this paper, we demonstrate that terms involving $\widetilde{F}F$ can indeed be correctly generated by an explicit calculation. We will also discuss some possible phenomenological implications of this approach.

To see the essential features of HLS as a consequence of DχSB in a generalized NJL model with $U(N)_L \otimes U(N)_R$ chiral symmetry we consider the Lagrangian

$$\mathcal{L} = \overline{\psi} i \not{\partial} \psi + g[(\overline{\psi}\lambda_a\psi)^2 - (\overline{\psi}\gamma_5\lambda_a\psi)^2] - g'[(\overline{\psi}\gamma_\mu\lambda_a\psi)^2 + (\overline{\psi}\gamma_5\gamma_\mu\lambda_a\psi)^2] \ , \qquad (1)$$

where $\lambda_a (a = 0, \ldots, N^2 - 1)$ is an $U(N)$ matrix. This model was studied by Chakrabarti and Hu[3], and Kikkawa.[4] If we introduce the auxiliary fields

$$s = -2g\lambda_a\overline{\psi}\lambda_a\psi \ , \qquad (2a)$$

$$p = -2gi\lambda_a\overline{\psi}\gamma_5\lambda_a\psi \ , \qquad (2b)$$

$$V_\mu = 2g'\lambda_a\overline{\psi}\gamma_\mu\lambda_a\psi \ , \qquad (2c)$$

$$A_\mu = 2g'\lambda_a\overline{\psi}\gamma_5\gamma_\mu\lambda_a\psi \ , \qquad (2d)$$

$\mathcal{L}$ can be rewritten as

$$\mathcal{L} = \overline{\psi}\, \not{D}\psi - \frac{1}{4g}\,(s^2 + p^2) + \frac{1}{4g'}\,(V_\mu^2 + A_\mu^2)\ , \tag{3}$$

where

$$\not{D} = i\ \not{\partial} - m - U\ , \tag{4}$$

and

$$U = s - m + i\gamma_5 p + \gamma_\mu V^\mu + \gamma_5\gamma_\mu A^\mu\ . \tag{5}$$

The generating functional is

$$Z = \frac{1}{N}\int [d\psi\, d\overline{\psi}\, ds dp\, dV_\mu dA_\mu]\, e^{i\int d^4x[\mathcal{L}+\overline{\eta}\psi+\overline{\psi}\eta]}\ , \tag{6}$$

η and $\overline{\eta}$ being sources for the fermion fields. Integrating out the fermion degrees of freedom, we obtain

$$Z = \frac{1}{N'}\int [ds\, dp\, dV_\mu dA_\mu]\, e^{i\int d^4x\mathcal{L}_{\rm eff}+\overline{\eta}\not{D}^{-1}\eta]}\ , \tag{7}$$

where

$$\mathcal{L}_{\rm eff} = -\frac{1}{4g}\,(s^2 + p^2) + \frac{1}{4g'}\,(V_\mu^2 + A_\mu^2) - i{\rm Tr}\,\ell n\left(1 - \frac{1}{i\ \not{\partial} - m}U\right)\ . \tag{8}$$

The last term in $\mathcal{L}_{\rm eff}$ can be expanded in powers of U:

$$-i\,{\rm Tr}\,\ell n\left(1 - \frac{1}{i\ \not{\partial} - m}U\right) = \sum_{n=1}^{\infty}\frac{i}{n}\,{\rm Tr}\left(\frac{1}{i\ \not{\partial} - m}U\right)^n \tag{9}$$

which can be represented by a series of Feynman diagrams as shown in Fig. 1. As the cutoff $\Lambda \to \infty$, the set of diagrams can be divided into two classes: divergent

and convergent. Including only the divergent terms, the authors in Refs. 3 and 4 derived a non–Abelian gauge theory in which both the Higgs and gauge bosons arise as collective excitation of the fermion pairs.

As HLS should guarantee the generation of all gauge invariant terms, it would be worthwhile to see if other interesting higher dimension terms also appear correctly. Since in a previous publication[10] we derived the Chern–Simon term in 3–dimensions, it would be interesting to see if the non–trivial topological θ–term, $\theta\widetilde{F}F$, in 4–dimensions can also be induced in this way. For this purpose, we shall focus our attention on the $U(1)$ component of p and the $SU(N)$ components of V_μ:

$$\theta = -2gi\overline{\psi}\gamma_5\psi \ , \tag{10a}$$

$$G_\mu = 2g'\lambda_i\overline{\psi}\gamma_\mu\lambda_i\psi \ , \qquad i = 1, \ldots, N^2 - 1 \ . \tag{10b}$$

To derive the θ–term we have to go beyond the leading order, viz, the divergent terms, and include the finite terms. After a long and tedious calculation, it is a remarkable demonstration of HLS that all the correct numerical factors are in place to reproduce the exactly desired gauge invariant form. The effective Lagrangian comprising only the θ and G_μ fields relevant for our present purpose is:

$$\mathcal{L}'_{\text{eff}} = \mathcal{L}^{\theta G}_{\text{eff}} + \mathcal{L}_T \ , \tag{11}$$

$$\mathcal{L}^{\theta G}_{\text{eff}} = \frac{c_1}{3}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{3}{8c_1}\left(\frac{1}{g'} - c_0 - m^2c_1\right)G_\mu G^\mu \right.$$
$$\left. + \frac{3}{2}\cdot\frac{1}{2}\,\partial_\mu\theta\partial^\mu\theta + 3\cdot\frac{1}{2}m^2\theta^2 - \frac{1}{2}\theta^4\right] , \tag{12}$$

$$\mathcal{L}_T = \frac{1}{16\pi^2 m}\,\theta\widetilde{F}F\ , \tag{13}$$

$$F_{\mu\nu} = \partial_\mu G_\nu - \partial_\nu G_\mu + i[G_\mu, G_\nu]\ , \tag{14a}$$

$$\widetilde{F}F = 4\epsilon_{\mu\nu\alpha\beta}\left(\partial^\mu G^\nu \partial^\alpha G^\beta + 2i\partial^\mu G^\nu G^\alpha G^\beta\right)\ , \tag{14b}$$

and

$$c_1 = -4i\int^{\Lambda}\frac{d^4k}{(2\pi)^4}\,\frac{1}{(k^2-m^2)^2} = \frac{1}{4\pi^2}\left[\ell n\left(1+\frac{\Lambda^2}{m^2}\right) - \frac{\Lambda^2}{\Lambda^2+m^2}\right]\ , \tag{15a}$$

$$c_0 = 8i\int^{\Lambda}\frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2-m^2} = \frac{1}{2\pi^2}\left[\Lambda^2 - m^2\ell n\left(1+\frac{\Lambda^2}{m^2}\right)\right]\ . \tag{15b}$$

To ensure gauge invariance in Eq. (11), one imposes the compositeness condition

$$\frac{1}{g'} - c_0 - m^2 c_1 = 0\ . \tag{16}$$

The existence of this gap equation will justify the formation of composite gauge field G. The important point in this exercise is that there exists no explicit dependence on the original 4–fermion coupling g and g' in the low energy effective Lagrangian $\mathcal{L}_{\text{eff}}^{\theta G}$ which is also a general feature of second order phase transitions when one goes from a symmetric phase to a broken phase. Once we input a value for c_1, there is no free parameter in $\mathcal{L}_{\text{eff}}^{\theta G}$ and therefore $\mathcal{L}_{\text{eff}}^{\theta G}$ indeed bears predictive power at low energies. One should also note that $\mathcal{L}_T$ is suppressed by m^{-1} which is of order of Λ^{-1} as it should be.

Some phenomenological implications can also be drawn in this exercise.

1. In order to have weakly interacting composite gauge bosons at low energy (*e.g.* a composite model for the standard electroweak theory) one has to fine tune the model to generate a hierarchy of mass scales. This can easily be seen by renormalizing $\mathcal{L}'_{\rm eff}$ through field redefinitions: $\widehat{G}_\mu = \sqrt{c_1/3}\, G_\mu$ and $\widehat{\theta} = \sqrt{c_1/2}\,\theta$ which result in an induced gauge coupling $g_{\rm ind} = \sqrt{3/c_1} \approx (2\sqrt{3}\pi/\ell n(\Lambda/m))$. $g_{\rm ind}$ is small only when we fine tune the theory in such a way that the cutoff Λ is much greater than the dynamical mass m.

2. When on applies this HLS approach to hadron physics, one will naturally obtain a strongly interacting hadronic physics because the confining scale of QCD is of order of the constituent quark masses. Moreover, if the gluons of QCD are composite at all, then they must arise from theories without a scale hierarchy.

3. $\mathcal{L}'_{\rm eff}$ can have further spontaneous symmetry breakings when the renormalized $\widehat{\theta}$ field gains a vacuum expectation value, $\left\langle \widehat{\theta} \right\rangle = (m/2)\sqrt{3c_1}$. Therefore, if QCD really arises from some fundamental theories, then the effective strong CP violating parameter $\theta_{\rm QCD}$ will be large naturally when one requires that QCD interact strongly. This fact will certainly create serious problems for building composite theories for QCD.

4. When one applies $\mathcal{L}_T$ to describe hadron physics by identifying θ as the pion and the composite gauge boson as the ρ–meson, then the G–parity violating $\pi\rho\rho$ coupling is naturally suppressed as seen in Eq. (13). Whether the effective $\pi\rho\rho$ coupling derived in this way is consistent with the vector dominance predictions will be discussed in a forthcoming publication.

FIGURE CAPTIONS

1) Feynman diagrams that correspond to the expansion in Eq. (9).

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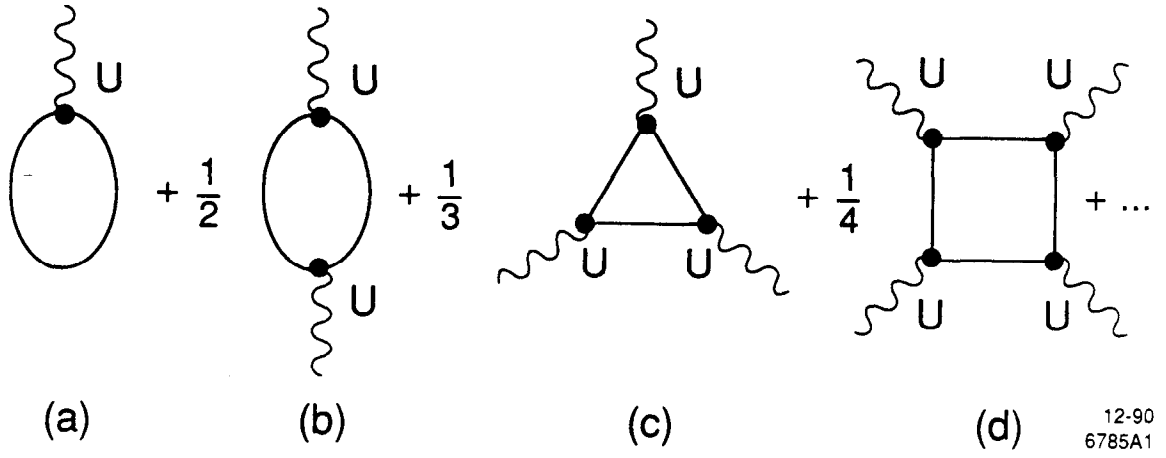

Fig. 1