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A HAMILTONIAN FORMULATION OF QED_{2+1} ON THE LIGHT CONE*

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ABSTRACT

It is shown that an improper regularization in the light-cone quantization of field theories can introduce a violation of rotational invariance as well as spurious divergences. Several methods are developed to avoid or cure these problems, as required for a consistent renormalization procedure of gauge theories. Based on these methods, the light-cone Hamiltonian for QED_{2+1} is constructed. Extension to gauge theories in $3+1$ physical dimensions is also described.

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1. Introduction

One of the main advantages of the light-cone quantization in field theory is its manifest invariance under a maximally large subgroup of the Lorentz group[1] which contains even certain boost transformations. The corresponding generators of these "simple" transformations are nondynamical operators, i.e. they do not involve any interaction terms. Such nondynamical symmetries can be preserved under a wide class of approximations,2 such as e.g. cutoffs in the number of particles. This feature greatly simplifies the task of constructing the Hamiltonian formulation of a relativistic field theory.

The price to pay for having simple generators of boost transformations is the occurrence of complicated and dynamical generators for certain rotations which implies that angular momentum is not manifestly conserved in the light-cone quantization. We will show that this results in a divergent structure of even super renormalizable theories.

Rotational invariance, is not a natural symmetry in the light-cone quantization procedure since it mixes longitudinal and transverse degrees of freedom. In particular an improper treatment of the short distance singularities due to regularization will result in a violation of rotational invariance. In fact most approximations or regularizations (if infinities are present) will spoil rotational invariance, for rotations which mix the $\underline{x} = (x^-, x^1, x^2)$ and x^+ direction.[3] In this paper we will concentrate on this aspect.

We will discuss several complementary approaches to this problem. The first, using Pauli-Villars (P-V) regularization, softens the short distance singularities and thus avoids the cause of the problem, since it regularizes symmetrically in longitudinal and transverse coordinates. The second approach starts from the

naive light-cone quantization. Any violations of rotational invariance, e.g. due to an improper treatment of the short distance singularities, are then cancelled by adding explicitly rotational noninvariant terms to the light-cone Hamiltonian.

The resulting regularization and renormalization program has a ***priori*** nothing to do with the usual renormalizations of mass and charge. As a matter of fact, while infinite mass and charge renormalization are often not necessary in less than 3-l-l dimensions, the problems which are discussed here appear in any number of dimensions (except in $1+1$, where there are no spatial rotational).

In order to emphasize this point we will mostly work in $2+1$ dimensions. This will help separate light-cone specific divergences and renormalizations from the usual ones. An extension of the techniques developed here to $3+1$ dimensions will be described at the end of this paper.

2. Pauli-Villars regularization of the light-cone quantized Yukawa model

As a simple example, which exhibits many of the light-cone related problems, we first consider the light-cone quantized Yukawa model,

$$\mathcal{L} = \overline{\psi}(i\not{\partial} - m)\psi - \phi(\Box + \lambda^2)\phi + \gamma\overline{\psi}\psi\phi \ , \tag{2.1}$$

in $2+1$ dimensions. It is easy to study the violation of rotational invariance in this model since it is-in contrast to e.g. gauge theories in the light-cone gauge—described by a fully covariant Lagrangian, i.e. even off-shell Green's functions should exhibit covariance. In particular, one should be able to express the fermion

self energy in the form

$$\Sigma(p^\mu) = (\not{p} - m)\, f_1(p^2) + f_2(p^2)\,. \tag{2.2}$$

However, naive light-cone perturbation theory yields[4] at one loop

$$\mathrm{tr}\,(\Sigma\gamma^+) = c\,p^+ \int_0^1 dx \int_{-\infty}^{\infty} dk_\perp \frac{1-x}{x(1-x)p^2 - m^2x - \lambda^2(1-x) - (k_\perp - xp_\perp)^2} - \text{“}\lambda \to \Lambda\text{”} \tag{2.3}$$

$$\mathrm{tr}\,(\Sigma\gamma^-) = \frac{c}{p^+} \int_0^1 dx \int_{-\infty}^{\infty} dk_\perp \frac{\frac{m^2 + (p_\perp - k_\perp)^2}{1-x}}{x(1-x)p^2 - m^2x - \lambda^2(1-x) - (k_\perp - xp_\perp)^2} - \text{“}\lambda \to \Lambda\text{”} \tag{2.4}$$

where $c = \gamma^2/\pi$. Adding

$$0 = \frac{1}{1-x}\,\frac{x(1-x)p^2 - m^2x - \lambda^2(1-x) - (k_\perp - xp_\perp)^2}{x(1-x)p^2 - m^2x - \lambda^2(1-x) - (k_\perp - xp_\perp)^2} - \text{“}\lambda \to \Lambda\text{”} \tag{2.5}$$

to the integrand in Eq. (2.4) one finds

$$\begin{aligned}
\mathrm{tr}\,(\Sigma\gamma^-) &= \frac{c}{p^+} \int_0^1 dx \int_{-\infty}^{\infty} dk_\perp \frac{xp^2 + m^2 - \lambda^2 + (1-x)p_\perp^2}{x(1-x)p^2 - m^2x - \lambda^2(1-x) - (k_\perp - xp_\perp)^2} \\
&\quad - \text{“}\lambda \to \Lambda\text{”} \\
&= \frac{c}{p^+} \int_0^1 dx \int_{-\infty}^{\infty} dk_\perp \frac{(1-x)(p^2 + p_\perp^2) - \frac{d}{dx}\left[x(1-x)p^2 - m^2x - \lambda^2(1-x)\right]}{x(1-x)p^2 - m^2x - \lambda^2(1-x) - (k_\perp - xp_\perp)^2} \\
&\quad - \text{“}\lambda \to \Lambda\text{”} \\
&= \frac{p^-}{p^+}\,\mathrm{tr}\,(\Sigma\gamma^+) - \frac{c\pi}{2p^+}\left[\left(\sqrt{m^2} - \sqrt{\lambda^2}\right) - \text{“}X \to \Lambda\text{”}\right].
\end{aligned} \tag{2.6}$$

Obviously two conditions, namely $\int d\lambda^2 \rho(\lambda^2)$ and $\int \lambda_\perp^D \rho(\lambda^2) = \int d\lambda^2 \sqrt{\lambda^2} \rho(\lambda^2) = 0$, are necessary to cancel the non-covariant term which implies the need for ar least two P.V. particles. This is rather unpleasant and perhaps unexpected, since-in a manifestly covariant approach-the fermion self energy in Yukawa_{2+1} is finite by power counting. As we have seen here, in light-cone quantization Σ is linearly divergent and extra degrees of freedom have to be introduced to make it finite and covariant. As far as perturbation theory is concerned, for theories with a manifestly covariant Lagrangian, the violation of rotational invariance is in principle no problem. In any Greens function one calculates only the "good components", like tr $\Sigma\gamma^+$, and uses general relations, like Eq. (2.2), to construct the "bad components". In the above example one could recover rotational invariance by defining

$$\text{tr}\, \Sigma\gamma^- = \frac{p^-}{p^+} \text{tr}\, \Sigma\gamma^+ \,. \tag{2.7}$$

However, this does not work in gauge theories in the light-cone gauge (or any non-covariant gauge), since there $\Sigma(p^\mu)$ does not have such a simple structure as in Eq. (2.2). Furthermore, in the Hamiltonian formalism, one does not calculate $\Sigma(p^\mu)$ but on mass shell matrix elements thereof. Thus in general it will be technically more difficult to develop an algorithm for extracting the noncovariant piece. Nevertheless the noncovariant terms still have observable effects which allow one to extract them. We will discuss this point later in the context of QED_{2+1}.

One should emphasize that the term which violates the rotational invariance depends only on the external p^+ but not on $p_\perp$ or p^2. Furthermore a simple calculation shows that tr $(\Sigma\gamma^+)$ and tr $(\Sigma\gamma^\perp)$ do not contain such extra terms.

This implies that we can write (if we do not regularize)

$$\sum\nolimits^{LC}(p^\mu) = \sum\nolimits^{cov}(p^\mu) + \text{const.}\,\frac{\gamma^+}{p^+}\,. \tag{2.8}$$

This is a general result which also holds for higher loops[5] -provided all noncovariant terms have been removed for subloops -and for other field theories like e.g. QED in light cone gauge. This has various practical consequences. First one might be able to remove this term by adding a counterterm to the Hamiltonian (i.e. by changing the mass of the fermion in the kinetic energy term). Secondly this allows one to develop simple subtraction procedures in perturbative calculations to get rid of such terms.[6]

A last point which we are going to make in the context of Yukawa_{2+1} concerns the "over regularization" of the theory. As we mentioned already there are no P-V particles necessary in covariant perturbation theory whereas we needed two of them for a more one loop treatment. At higher loops the situation becomes a little better, namely one P-V particle is sufficient (provided subloops are rendered covariant) but it is also necessary in general as the example in Appendix B shows. For renormalization theories where P-V regularization poses no extra problems, like QED_{3+1}, this means that there is no more regularization necessary than one would need. However, in nonabelian gauge theories P-V regularization violates gauge invariance and we would have to restore it by further counterterms. We also emphazise, and this can also be read off from the example in Appendix B, that dimensional regularization does not take care of the noncovariant terms. The reason for this is that dimensional regularization in the transverse coordinate does not regularize the longitudinal coordinates.

3. Hamiltonian formulation for QED_{2+1} in the light cone gauge (Pauli–Villars regularization)

We start our considerations from the QED-Lagrangian in two space and one time dimensions with gauge fixing term $(n_\mu A^\mu = A^+)$

$$\mathcal{L} = \mathcal{L}_{\text{Ferm}} + j_\mu A^\mu - \frac{1}{4} F_{\mu\nu}F^{\mu\nu} - \lim_{\xi\to\infty} \frac{\xi}{2} (n_\mu A^\mu)^2 \ . \tag{3.1}$$

For the purpose of P-V regularization (as well as if one wants to introduce an IR-regulator) it is necessary to specify how to introduce a mass for the A-field. One might be tempted to add just a term like $\frac{\Lambda^2}{2} A_\mu A^\mu$ to Eq. (3.1). However, since $A_\mu A^\mu = A^+A^- - A_\perp^2 = -A_\perp^2$ (note: $A^+ = 0$) this means that only the I-degrees of freedom become massive whereas the longitudinal degrees of freedom remain massless. In terms of the photon propagator this means

$$D_\Lambda^{\mu\nu} = -\lim_{\xi\to\infty} \left[(k^2 - \Lambda^2)g^{\mu\nu} + \xi n^\mu n^\nu - k^\mu k^\nu\right]^{-1} = -\frac{g^{\mu\nu} - \frac{k^\mu n^\nu + k^\nu n^\mu}{kn} + \frac{\Lambda^2 n^\mu n^\nu}{(nk)^2}}{k^2 - \Lambda^2 + i\epsilon} \tag{3.2;}$$

i.e. even at the tree level, the photon propagator does not vanish for $\Lambda^2 \to \infty$ and the "instantaneous"contribution

$$\lim_{\Lambda\to\infty} D_\Lambda^{\mu\nu} = \frac{n^\mu n^\nu}{(nk)^2} \tag{3.3}$$

remains. What one has to do, in addition to adding an $\frac{\Lambda^2}{2} A_\mu A^\mu$ term to $\mathcal{L}$, is to introduce a dynamical longitudinal degree of freedom: a scalar field ϕ of mass A which couples with strength $\frac{e\Lambda}{k^+}$ to the current j^+, i.e.

$$\delta\mathcal{L}_{\text{long}} = -\phi(\Box + \Lambda^2)\phi + ie\Lambda\phi \frac{1}{n^\mu\partial_\mu} n^\mu j_\mu \ . \tag{3.4}$$

The effect of this scalar field can be absorbed into the photon propagator, yielding

$$\widetilde{D}^{\mu\nu}_{\Lambda}(\text{eff}) = D^{\mu\nu}_{\Lambda} + D^{\mu\nu}_{\Lambda}(\text{longitudinal}) = -\frac{g^{\mu\nu} - \frac{n^\mu k^\nu + n^\nu k^\mu}{nk}}{k^2 - \Lambda^2} \,. \tag{3.5}$$

Since for on-shell Greens functions the $n^\mu k^\nu$ terms do not contribute', all S-matrix elements should exhibit rotational invariance--even for finite Λ^2!

Having specified how to treat the A-field we can now proceed to construct the Hamiltonian. As a matter of convenience we choose to represent the Hamiltonian using discrete light-cone quantization (DLCQ). [8,9] Except for the longitudinal field this has been done already by A. Tang [10] for QED_{3+1} so that we do not have to go into the details. For one flavor of fermion (b^+ = fermion, d^+ = antifermion) and one massive photon (a^+ = transverse photon, c^+ = longitudinal photon) one finds in 2 + 1 dimensions

$$H = H_0 + V_{\text{flip}} + V_{\text{no flip}} + V_{\text{inst phot}} + V_{\text{long}} + V_{\text{inst ferm}} + V_{NO} \tag{3.6}$$

where

$$\begin{aligned} H_0 = &\sum_{\underline{p}} \frac{1}{p} \left[\lambda^2 + \left(\frac{p_\perp \pi}{L_\perp} \right)^2 \right] \left[a^+_{\underline{p}} a_{\underline{p}} + c^+_{\underline{p}} c_{\underline{p}} \right] \\ &+ \sum_{s,\underline{n}} \frac{1}{n} \left[m^2 + \left(\frac{n_\perp \pi}{L_\perp} \right)^2 \right] \left[b^+_{s,\underline{n}} b_{s,\underline{n}} + d^+_{s,\underline{n}} d_{s,\underline{n}} \right] , \end{aligned} \tag{3.7}$$

$$\begin{aligned} V_{\text{flip}} = \frac{em}{2\sqrt{\pi L_\perp}} \sum_{\underline{p}\,\underline{m}\,\underline{n}} \frac{a_{\underline{p}}}{\sqrt{p}} \Bigg\{ &\left(b^+_{\uparrow \underline{m}} b_{\downarrow \underline{n}} - b^+_{\downarrow \underline{m}} b_{\uparrow \underline{n}} \right) \left(\frac{1}{m} + \frac{1}{n} \right) \delta^{(2)}_{\underline{n}+\underline{p},\underline{m}} \\ &+ \left(d^+_{\uparrow \underline{m}} d_{\downarrow \underline{n}} - d^+_{\downarrow \underline{m}} d_{\uparrow \underline{n}} \right) \left(\frac{1}{m} - \frac{1}{n} \right) \delta^{(2)}_{\underline{n}+\underline{p},\underline{m}} \\ &+ \left(b^+_{\uparrow \underline{m}} d^+_{\uparrow \underline{n}} - b^+_{\downarrow \underline{m}} d^+_{\downarrow \underline{n}} \right) \left(\frac{1}{m} - \frac{1}{n} \right) \delta^{(2)}_{\underline{n}+\underline{m},\underline{p}} \Bigg\} + h.c. \,, \end{aligned} \tag{3.8}$$

$$V_{\text{no flip}} = \frac{e}{2L_\perp}\sqrt{\frac{\pi}{L_\perp}} \sum_{s,\underline{p},\underline{mn}} \frac{a_{\underline{p}}}{\sqrt{p}} \left(2\frac{p_\perp}{p} - \frac{n_\perp}{n} - \frac{m_\perp}{m}\right)$$

$$\cdot \left\{ b^+_{s\underline{m}} b_{s\underline{n}} \delta^{(2)}_{\underline{n}+\underline{p},\underline{m}} - d^+_{s,\underline{m}} d_{s\underline{n}} \delta^{(2)}_{\underline{n}+\underline{p},\underline{m}} + b^+_{s\underline{m}} d^+_{-s\underline{n}} \delta^{(2)}_{\underline{n}+\underline{m},\underline{p}} \right\} + \text{h.c.} \,, \tag{3.9}$$

$$V_{\text{inst phot}} = \frac{e^2}{\pi L_\perp} \sum_{s,t,\underline{k},\underline{\ell},\underline{m},\underline{n}} \left\{ [k-m|n-\ell] \left[b^+_{s\underline{k}} d^+_{-t\underline{\ell}} b_{s\underline{m}} d_{-t\underline{n}} \right.\right.$$

$$\left. - \frac{1}{2} b^+_{s\underline{k}} b^+_{t\underline{\ell}} b_{s\underline{m}} b_{t\underline{n}} - \frac{1}{2} d^+_{s\underline{k}} d^+_{t\underline{\ell}} d_{s\underline{m}} d_{t\underline{n}} \right]$$

$$+ [k-m|\ell + n] \cdot \left[d^+_{s\underline{k}} d_{t\underline{\ell}} d_{s,\underline{m}} b_{-t\underline{n}} + b^+_{s\underline{k}} b_{t\underline{\ell}} b_{s\underline{m}} d_{-t\underline{n}} + h.c. \right.$$

$$\left.\left. - b^+_{s\underline{k}} d^+_{-s\underline{\ell}} b_{t\underline{m}} d_{-t\underline{n}} \cdot [k+\ell|m+n] \right\} \right. , \tag{3.10}$$

$$V_{\text{long}} = \frac{e\lambda}{\sqrt{\pi L_\perp}} \sum_{s,\underline{k},\underline{\ell},\underline{m}} \frac{1}{\ell^{3/2}} c_{\underline{\ell}} \cdot \left\{ \right.$$

$$\left. \left(b^+_{s\underline{k}} b_{s,\underline{m}} - d^+_{s\underline{k}} d_{s,\underline{m}} \right) \delta^{(2)}_{\underline{k},\underline{m}+\underline{\ell}} + b^+_{s\underline{k}} d^+_{-s\underline{m}} \delta^{(2)}_{\underline{\ell},\underline{k}+\underline{m}} \right\} + h.c. \,, \tag{3.11}$$

$$V_{\text{inst ferm}} = \frac{e^2}{4\pi L_\perp} \sum_{s,p,q,\underline{m},\underline{n}} \frac{1}{\sqrt{pq}} \left\{ \right.$$

$$a^+_{\underline{p}} a_{\underline{q}} \left[b^+_{s\underline{m}} b_{s\underline{n}} + d^+_{s\underline{m}} d_{s\underline{n}} \right. \cdot [\{p+m|q+n\} - \{p-n|q-m\}]$$

$$+ a^+_{\underline{p}} a^+_{\underline{q}} b_{s\underline{m}} d_{-s\underline{n}} \cdot \{p-m|-q+n\} + h.c. \tag{3.12}$$

$$+ a^+_{\underline{p}} a_{\underline{q}} b_{s\underline{m}} d_{-s\underline{n}} \cdot [\{p-m|q+n\} - \{p-n|q+m\}] + h.c.$$

$$\left. + a_p a_q [b^+_{sm} b_{s\underline{n}} + d^+_{s\underline{m}} d_{s\underline{n}}] \cdot \{p+n|-q+m\} + h.c. \right\}$$

Here

$$\begin{gathered} p, q = 2,4,6,\ldots \\ k, l, m, n = 1,3,5,\ldots \\ p_\perp, q_\perp, k_\perp, l_\perp, m_\perp, n_\perp = 0, \pm 1, \pm 2, \ldots \\ s, t = \uparrow, \downarrow \end{gathered} \tag{3.13}$$

$$\begin{aligned} \{m|n\} &= \delta^{(2)}_{\underline{m},\underline{n}} \cdot \frac{1}{m} \\ [m|n] &= \delta^{(2)}_{\underline{m},\underline{n}} \cdot \frac{1}{m^2} \,. \end{aligned} \tag{3.14}$$

V_{NO} represents the normal ordering terms which are part of the $\mathcal{O}(e^2)$ contributions to the self energies. Since they arise from instantaneous interactions they are independent of particle masses and thus vanish in P-V regularization.[11]

We leave the explicit construction of the P-V regularized Hamiltonian to the Appendix. For perturbative calculations we will weight the contributions from the various electrons and photons (physical and P-V) with coefficients c_i^2 and c_j^2 which are later determined such that all unwanted terms vanish. E.g. the $\mathcal{O}(e^2)$ contributions to the self energy of a transverse photon with momentum $\underline{p}$ are $(\hat{p}_\perp = p_\perp \frac{\pi}{L_\perp})$:

$$\begin{aligned} \delta E^{\text{trans}}_{\underline{p}} = \frac{e^2}{4\pi L_\perp} \frac{1}{p} \sum_i c_i^2 \sum_n \\ \times \frac{m_i^2 \left(\frac{1}{n} + \frac{1}{p-n}\right)^2 + \left(\frac{2\hat{p}_\perp}{p} - \frac{\hat{n}_\perp}{n} - \frac{\hat{p}_\perp - \hat{n}_\perp}{p-n}\right)^2}{\frac{\lambda^2 + \hat{p}_\perp^2}{p} - \frac{m_i^2 + \hat{n}_\perp^2}{n} - \frac{m_i^2 + (\hat{p}_\perp - \hat{n}_\perp)^2}{p-n}} \end{aligned} \tag{3.15}$$

In order to obtain a finite results in the continuum limit we have to require $\sum_i c_i^2 = 0$. This allows us to simplify the numerator by using the replacement

$\left(\widehat{n}_\perp - \widehat{p}_\perp \frac{n}{p}\right)^2 \to -m_i^2 + \frac{\lambda^2}{p}\left(\frac{1}{n} + \frac{1}{p-n}\right)^{-1}$, *i.e.*

$$\delta E_{\underline{p}}^{\text{trans}} = \frac{e^2}{4\pi L_\perp}\frac{1}{p}\sum_i c_i^2 \sum_n \times \frac{4m_i^2 + \lambda^2 \frac{(p-2n)^2}{p^2}}{n(p-n)\left\{\frac{\lambda^2}{p} - \left(\frac{1}{n}+\frac{1}{p-n}\right)\left[\left(\widehat{n}_\perp - \widehat{p}_\perp\frac{n}{p}\right)^2 + m_i^2\right]\right\}}$$

$$= \frac{e^2}{4\pi L_\perp}\frac{1}{p}\sum_i c_i^2 \times \frac{4m_i^2 + \lambda^2\left[1 - 8\frac{n}{p}\left(1-\frac{n}{p}\right)\right]}{n(p-n)\left\{\frac{\lambda^2}{p} - \left(\frac{1}{n}+\frac{1}{p-n}\right)\left(\widehat{n}_\perp - \widehat{p}_\perp\frac{n}{p}\right)\left[\left(\widehat{n}_\perp - \widehat{p}_\perp\frac{n}{p}\right)^2 + m_i^2\right]\right\}}$$

$$+ \delta E_p^{\text{long}}, \tag{3.16}$$

where we have already separated the self energy of a longitudinal photon

$$\delta E_{\underline{p}}^{\text{long}} = \frac{e^2}{4\pi L_\perp}\frac{1}{p}\sum_i c_i^2 \sum_n \frac{4\lambda^2/p^2}{\frac{\lambda^2}{p} - \left(\frac{1}{n}+\frac{1}{p-n}\right)\left[\left(\widehat{n}_\perp - \widehat{p}_\perp\frac{n}{p}\right)^2 + m_i^2\right]}. \tag{3.17}$$

In the continuum limit the self energies of longitudinal and transverse photons must be equal-otherwise rotational invariance is broken. To analyze this condition further we transform this term into an integral

$$\delta E^{\text{trans}} - \delta E^{\text{long}} \to \frac{e^2}{4\pi^2}\frac{1}{p}\sum_i c_i^2 \int_0^1 dx \int_{-\infty}^{\infty} dk_\perp \frac{4m_i^2 + \lambda^2[1 - 8x(1-x)]}{\lambda^2 x(1-x) - m_i^2 - k_\perp^2}$$

$$= -\frac{e^2}{\pi}\frac{1}{p}\sum_i c_i^2\sqrt{m_i^2} \tag{3.18}$$

and our second P-V condition has to be $\sum_i c_i^2\sqrt{m_i^2} = 0$.

We have performed similar calculations for the on-shell self energy of an electron. Since this is a gauge invariant quantity we can require that our calculation in light-cone gauge and light-cone quantization reproduces the covariant result obtained in Feynman gauge and 2 + 1 dimensional symmetrical integration. An alternative approach-which will be elaborated in more detail in the next section-is to calculate the one-loop corrections to the Compton cross section and compare with well known results. Both methods lead to the same condition, namely

$$\sum_j c_j^2 = 0 \qquad \sum_j c_j^2 \sqrt{\lambda_j^2} = 0 \ . \tag{3.19}$$

For practical calculations it is useful to reduce the number of P-V conditions. To achieve this one can add a counterterm to the Hamiltonian which cancels those terms which are multiplied by $c_i^2\sqrt{m_i^2}$ and $c_j^2\sqrt{\lambda_j^2}$ in the self energies of photons and electrons respectively. At one loop this reduces-by construction-the number of P-V conditions required. However, and this is a highly nontrivial result, numerical calculations of the self energies as well as the example in Appendix B show that this is also true for higher loops, ***i.e.*** the second P-V particle is only necessary at one-loop. Once we avoid it by adding a suitable one-loop counterterm there is only one P-V particle needed at two loops and most probably (we have not checked this numerically) also for higher loops.

There might be various reasons for this special behavior at one loop. First of all there are ambiguities in how to treat normal ordering divergences which are of $\mathcal{O}(e^2)$ and contribute only to the one-loop self energies. Secondly, power counting in light-cone coordinates is different from the usual covariant power counting.[12] One has to count separately powers in $k_\perp$ and $1/k^+$ in order to properly estimate the degree of divergence. Here it turns out that the strongest divergence (e.g. a

quadratic $k_\perp$ divergence in 3 + 1 divergences) occurs only at the one loop level. The situation here is similar to scalar QED in equal-time quantization.

4. Renormalization using noncovariant count ert erms

QED_{2+1} is super renormalizable and only two graphs are superficially divergent in Feynman perturbation theory (the one and two loop vacuum polarization are finite if gauge invariant regularization is used.) However the presence of terms which break rotational invariance has forced us to introduce four P-V particles (two photons and two electrons), i.e. the Fock space content of the theory has increased considerably. Even after calculating the one-loop counterterms by hand one has to deal with one P-V photon and one P-V electron, i.e. the number of degrees of freedom still increases by a factor of four compared to the unregularized theory.

Furthermore practical calculations require in general some approximations which in general lead to further violations of rotational invariance.[13] In this work we deal only with those violations of rotational invariance which are induced by an improper treatment of the high energy degrees of freedom (large $k_\perp$, small x) if no P-V regularization, or anything equivalent, is applied. (The methods, which we are going to develop for the latter problem, should however also be applicable for approximation-induced effects.)

Using the light-cone power counting rules one shows that light- cone QED in $3+1$ and $2+1$ dimensions is renormalizable .[11] This implies that the violations of rotational invariance (which in our case are induced by an improper handling of arbitrarily high energies) can be compensated by a redefinition of terms in the Hamiltonian. In general such a renormalization procedures can be quite lengthy since, at

least in principle, the e^- masses which appear in the kinetic energy and in the vertex, the various e^- charges and the various photon masses can all require different renormalizations, i.e. instead of three renormalization constants (m, λ, e) we would have to deal with nine $(m_{\text{kin}}, m_{\text{vertex}}, e_{\text{flip}}, e_{\text{no flip}}, e_{\text{inst phot}}, e_{\text{inst ferm}}, \lambda_{\text{long}}, \lambda_{\text{trans}}, \lambda_{\text{vertex}})$. However, practical calculations[14] have shown that violations of rotational invariance in LC gauge occur only in two point functions and there only in a very specific form,[15] namely

$$\Sigma = \Sigma^{P.V.} + \frac{\gamma^+}{p^+} c_1 \qquad \prod^{\mu\nu} = \prod^{\mu\nu}_{P.V.} + \delta^{\mu\perp}\delta^{\nu\perp} c_2 \ , \tag{4.1}$$

for electron and photon self energies respectively. i.e.the deviations from the P-V regularized results-which lead to rotational invariant observables — can be parametrized by only two additional constants c_1, c_2. The burden of fitting nine renormalization constants has thus been reduced to fitting five. In practice one adds two extra counterterms

$$\begin{aligned} \delta H^{(1)} &= \sum_{s,\underline{n}} \frac{b^+_{s\underline{n}} b_{s\underline{n}} + d^+_{s\underline{n}} d_{s\underline{n}}}{n} \, \delta m^2_{\text{kin}} \\ \delta H^{(2)} &= \sum_{\underline{p}} \frac{a^+_{\underline{p}} a_{\underline{p}}}{p} \, \delta \lambda^2_{\text{trans}} \end{aligned} \tag{4.2}$$

to the Hamiltonian and adjusts δm^2_{kin} and $\delta \lambda^2_{\text{trans}}$ such that rotational invariance is restored (this point will be discussed below). The next step, which is not necessary in QED_{2+1}, would then be the usual mass and charge renormalization.[16]

The constants δm^2_{kin} and $\delta \lambda^2_{\text{trans}}$ are determined as follows. Fixing $\delta \lambda^2_{\text{trans}}$ is rather easy: one diagonalizes the Hamiltonian (within some approximations like e.g. cutoff in Fock space) for a given $\delta \lambda^2_{\text{trans}}$ and compares the physical masses

(eigenvalues of the Hamiltonian) of longitudinal and transverse photons. $\delta\lambda^2_{\mathrm{trans}}$ is then tuned until these eigenvalues coincide.

For $\delta m^2_{\mathrm{kin}}$ two methods are suggested. The first method is based on the fact that instantaneous e^- exchange becomes singular for small p^+ transfer (e.g. in Compton back scattering). This is of course an unphysical singularity which has to be cancelled by noninstantaneous e^- exchange. At tree level it is crucial for the cancellation that the kinetic mass of an electron (m in ***Ho(3.7))*** equals the vertex mass (m in $V_{\mathrm{flip}}(3.8)$). **At** one loop the interaction will renormalize m_{kin} and m_{vertex} differently and one can easily convince oneself that the cancellation will be spoiled unless one renormalizes m_{kin} differently from m_{vertex}. This defines already the renormalization procedure, namely tuning m^2_{kin} until finiteness of the Compton back scattering amplitude for zero p^+ transfer is achieved.

The second method uses the degeneracy of the positronium spectrum due to rotational invariance. A glance at the Hamiltonian , Eq. (3.6) shows that, for zero perpendicular momenta, an annihilation of an e^+e^- pair into a transverse photon is possible if and only if both have a parallel spin but not for the S = 1, S_z = 0 state. Another annihilation process is possible via longitudinal or instantaneous photons but only from the S = 1, S_z = 0 state. In the first case the vertex mass appears whereas in the second it does not. For degeneracy of the S_z = 0, $\pm$ 1 states it is important that both interactions have the same strength. Again this is achieved at tree level by choosing $m_{\mathrm{kin}} = m_{\mathrm{vertex}}$ but if loops are taken into account the condition changes. Degeneracy of the S_z = 0, ± 1 states in the ground state of positronium can thus be used as a renormalization condition.

The first method seems to be superior from a practical point of view, since it requires to look at the $e^-\gamma$ system only and not at $e^-e^+\gamma$ states as for the

second method. However, from a practical point of view we are interested in the positronium spectrum, i.e. we diagonalize the Hamiltonian. The second methods thus requires only little effort to implement-namely, diagonalizing $\boldsymbol{H}$ for two spin configurations and repeating this a few times (to fit δm_k^2 iteratively). Furthermore, and this will also be of practical importance, the renormalization constants will thus be evaluated automatically to the same loop order and with the same approximations as the actual positronium calculations are done.

5. Extension to 3 + 1 Dimensions

For those theories considered in this work (Yukawa and QED) an extension to 3 + 1 dimensions is straightforward. The only difference will be that more coefficients have to be renormalized and that there will be in general an infinite renormalization.

In practice the following steps have to be performed. If one wants to render all loops covariant, i.e. even the one loop graphs, using P-V there will be three P-V conditions for photons and electrons, namely[17]

$$\begin{aligned} \int d\lambda^2 \rho(\lambda^2) &= 0 \\ \int d\lambda^2 \lambda^2 \rho(\lambda^2) &= 0 \\ \int d\lambda^2 \lambda^2 \log \lambda^2 \rho(\lambda^2) &= 0 \end{aligned} \tag{5.1}$$

which is awkward from a numerical point of view. Thus one should only use the improved version of the P-V approach, where the one-loop counterterms are constructed "by hand" and only one P-V condition has to be imposed for higher loops. The number of degrees of freedom will thus be the same as in a covariant

approach (e.g. euclidean integration) with P-V regularization. The method of noncovariant counterterms might also be very useful. For example, if one uses a kinetic energy cutoff further violations of rotational invariance are induced. The algorithm described in Section 4 would automatically remedy this without further effort.

The extension to nonabelian gauge theories is not as straightforward. All methods discussed in this work violate local gauge invariance at least in intermediate steps. For QED this is not a problem since, e.g. the P-V regularization preserves the Ward identities. In QCD this is not the case and one has to add further gauge breaking counterterms which restore gauge invariance.[18]

6. Summary and Conclusion

Naive light-cone quantization without careful regularization violates rotational invariance. In theories with a covariant Lagrangian we have demonstrated this by investigating the covariant structure of self energies. In the case of a non-covariant Lagrangian (QED in the light-cone gauge) the Lorentz transformation properties of Green's functions are nontrivial and therefore possible violations of Lorentz invariance are not obvious.

However, these effects must show up in the calculation of physical processes. To study them it is convenient to select those processes which are sensitive to violation of its covariant structure as well as technically rather easy to deal with. In QED the degeneracy of the triplet positronium state with parallel and antiparallel spin as well as Compton back scattering are such processes.

The violation of rotational invariance is not limited to one loop, althought one might expect this since normal-ordering ambiguities arise only in one-loop self

energies. In fact, unless regularized properly, the normal ordering contributions lead to violation of rotational symmetry. However, those terms are not the only source of violations of this kind as our explicit two loop calculations show. The induced divergences are less severe there, though.

We have discussed from two basic methods to restore rotational invariance, the Pauli Villars method and the method of noncovariant counterterms. Both methods seem to require a large number of additional degrees of freedom or counterterms. However, because of the specific structure of rotational invariance violation in light cone quantization-the worst problems are restricted to one loop and only certain components of two point functions (the γ^+ component of the fermion self energy and the $\perp\perp$ – components of the vacuum polarization) are affected. This allows us to optimize these methods considerably. We give analytic expressions for one loop counterterms. As a result the P.V. approach then requires only one ghost per particle to offset the violations of rotational symmetry at higher loops.

The method of noncovariant counterterms requires only two additional counterterms (compared to a manifest covariant approach) , namely a mass term for transverse photons and an additional correction to the fermion mass term which appears at spin flip photon-electron vertices. To fix the additional constants one has to specify the renormalization conditions. This can be achieved by considering the degenerate ground state of positronium as well as the degeneracy of the longitudinal and transverse photons.

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APPENDIX A

The Pauli-Villars regularized Hamiltonian for QED_{2+1}

As discussed in the section about Pauli-Villars regularization, one Pauli-Villars condition

$$\int dm^2 \rho_e(m^2) = 0 \tag{A.1}$$

$$\int d\lambda^2 \rho_\gamma(\lambda^2) = 0 \,. \tag{A.2}$$

for electrons and photons respectively is sufficient to guarantee covariant regularization in all calculations beyond one loop- provided all one loop subgraphs have been rendered covariant (e.g. by constructing the necessary one-loop counterterms). One can easily convince oneself that the sum rules **(A.l)** and (A.2) can be achieved by introducing one additional electron and photon field respectively which are quantized with the wrong metric. One way to do so in practice is to introduce an extra factor of $\sqrt{-1}$ for all heavy photon vertices and another factor of $\sqrt{-1}$ for all heavy electron pair creation and annihilation vertices. In addition the heavy electron has to be quantized as a boson.

In practice this implies

$$H^{\mathrm{PV}} = H_0 + V_{\mathrm{flip}} + V_{\mathrm{no\ flip}} + V_{\mathrm{long}} + V_{\mathrm{inst\ ferm}} + V_{1\ \mathrm{loop}} \tag{A.3}$$

where

$$H_0 = \sum_{\underline{p}} \frac{1}{p} \left[\left(\frac{p_\perp \pi}{L_\perp} \right)^2 + \lambda^2 \right] \cdot \left[a^+_{\underline{p}} a_{\underline{p}} + c^+_{\underline{p}} c_{\underline{p}} \right]$$

$$+ \sum_{s\underline{n}} \frac{1}{n} \left[\left(\frac{n_\perp \pi}{L_\perp} \right)^2 + m_e^2 \right] \left[b^+_{s,\underline{n}} b_{s,\underline{n}} + d^+_{s,\underline{n}} d_{s,\underline{n}} \right]$$

$$+ \sum_{\underline{p}} \frac{1}{p} \left[\left(\frac{p_\perp \pi}{L_\perp} \right)^2 + \Lambda^2 \right] \left[A^+_{\underline{p}} A_{\underline{p}} + C^+_{\underline{p}} C_{\underline{p}} \right] \qquad \text{(A.4)}$$

$$+ \sum_{s\underline{n}} \frac{1}{n} \left[\left(\frac{n_\perp \pi}{L_\perp} \right)^2 + M^2 \right] \left[B^+_{s,\underline{n}} B_{s,\underline{n}} + D^+_{sm\underline{n}} D_{s,\underline{n}} \right]$$

$$V_{\text{flip}} = \frac{e}{2\sqrt{\pi L_\perp}} \sum_{\underline{pmn}} \frac{(a_{\underline{p}} + iA_{\underline{p}})}{\sqrt{p}} \left\{ \right.$$

$$\times \left[m_e (b^+_{\uparrow \underline{m}} b_{\downarrow \underline{n}} - b^+_{\downarrow \underline{m}} b_{\uparrow \underline{n}}) + M (B^+_{\uparrow \underline{m}} B_{\downarrow \underline{n}} - B^+_{\downarrow \underline{m}} B_{\uparrow \underline{n}}) \right] \left(\frac{1}{m} - \frac{1}{n} \right) \delta^{(2)}_{n+p,m}$$

$$- \left[m_e (d^+_{\uparrow \underline{m}} d_{\downarrow \underline{n}} - d^+_{\downarrow \underline{m}} d_{\uparrow \underline{n}}) + M (D^+_{\uparrow \underline{m}} D_{\downarrow \underline{n}} - D^+_{\downarrow \underline{m}} D_{\uparrow \underline{n}}) \right] \left(\frac{1}{m} - \frac{1}{n} \right) \delta^{(2)}_{n+p,m}$$

$$+ \left[m_e (b^+_{\uparrow \underline{m}} d^+_{\uparrow \underline{n}} - b^+_{\downarrow \underline{m}} d^+_{\downarrow \underline{n}}) + iM (B^+_{\uparrow \underline{m}} D^+_{\uparrow \underline{n}} - B^+_{\downarrow \underline{m}} D^+_{\downarrow \underline{n}}) \right] \left(\frac{1}{m} + \frac{1}{n} \right) \delta^{(2)}_{n+m,p} \Bigg\}$$

+ "h.c." (A.5)

$$V_{\text{no flip}} = e \sqrt{\frac{\pi}{L_\perp}} \frac{1}{2L_\perp} \sum_{s\underline{pmn}} \frac{a_{\underline{p}} + iA_{\underline{p}}}{\sqrt{p}} \left(\frac{2p_\perp}{p} - \frac{n_\perp}{n} - \frac{m_\perp}{m} \right)$$

$$\times \Big\{ (b^+_{s\underline{m}} b_{s\underline{n}} + B^+_{s\underline{m}} B_{s\underline{n}}) \, \delta^{(2)}_{n+p,m} - (d^+_{s\underline{m}} d_{s\underline{n}} + D^+_{s\underline{m}} D_{s\underline{n}}) \, \delta^{(2)}_{n+p\,m} \qquad \text{(A.6)}$$

$$+ (b^+_{s\underline{m}} d^+_{-s\underline{n}} + iB^+_{s\underline{m}} D^+_{-s\underline{n}}) \, \delta^{(2)}_{p,n+m} \Big\} + \text{"h.c."}$$

$$V_{\text{long}} = \frac{e}{\sqrt{\pi L_\perp}} \sum_{s\underline{k}\underline{p}\underline{m}} \frac{1}{p^{3/2}} (\lambda c_{\underline{p}} + i\Lambda C_{\underline{p}} \Big\{$$

$$\times [b^+_{s\underline{k}} b_{s\underline{m}} + B^+_{s\underline{k}} B_{s\underline{m}}] \delta^{(2)}_{m+p,k} - [d^+_{s\underline{k}} d_{s\underline{m}} + D^+_{s\underline{k}} D_{s\underline{m}}] \delta^{(2)}_{m+p,k} \qquad \text{(A.7)}$$

$$+ [b^+_{s\underline{k}} d^+_{-s\underline{m}} + iB^+_{s\underline{k}} D^+_{-s\underline{m}}] \delta^{(2)}_{k+m,p} \Big\}$$

$$V_{\text{inst ferm}} = e^2 \frac{1}{4\pi L_\perp} \sum_s \sum_{pqmn} \frac{1}{\sqrt{pq}} \Big\{ (a^+_{\underline{p}} + iA^+_{\underline{p}})(a_{\underline{p}} + iA_{\underline{p}})$$

$$\times (b^+_{s\underline{m}} b_{s\underline{n}} + B^+_{s\underline{m}} B_{s\underline{n}} + d^+_{s\underline{m}} d_{s\underline{n}} + D^+_{s\underline{m}} D_{s\underline{n}}) \delta^{(2)}_{\underline{p}+\underline{m},\underline{q}+\underline{n}}$$

$$\times (\{p + m|q + n\} = \{p - n|q - m\})$$

$$- (a^+_p + iA^+_{\underline{p}})(a^+_{\underline{q}} + iA^+_{\underline{q}})(d_{-s\underline{n}} b_{s\underline{m}} + iD_{-s\underline{n}} B_{s\underline{n}})$$

$$\times \delta^{(2)}_{\underline{p}+\underline{q},\underline{m}+\underline{n}} \{p - m| - q + n\} + \text{“h.c.”}$$

$$+ (a^+_p + iA^+_{\underline{p}})(a_{\underline{q}} + iA_{\underline{q}})(d_{-s\underline{n}} b_{s\underline{m}} + iD_{-s\underline{n}} B_{s\underline{m}})$$

$$\times \delta^{(2)}_{\underline{p},\underline{q}+\underline{m}+\underline{n}} (\{p - n|q + m\} - \{p - m|q + n\}) + \text{“h.c.”}$$

$$+ (a_{\underline{p}} + iA_{\underline{p}})(a_{\underline{q}} + iA_{\underline{q}})(b_{s\underline{m}} b_{s\underline{n}} + B^+_{s\underline{m}} B_{s\underline{n}} + d^+_{s\underline{m}} d_{s\underline{n}} + B^+_{s\underline{m}} D_{s\underline{m}})$$

$$\times \delta_{\underline{m},\underline{p}+\underline{q}+\underline{n}} \{p + n| - q + m\} + \text{“h.c.”} \qquad \text{(A.8)}$$

$$V_{1\text{ loop}} = \frac{e^2}{4\pi L_\perp}\sum_p \frac{a_p^+ a_p}{p}\left\{\sum_{i,n}\frac{\lambda^2\left[8\frac{n}{p}\left(1-\frac{n}{p}\right)-1\right]-4m^2}{\frac{\lambda^2+\hat{p}_\perp^2}{p}-\frac{m^2+\hat{n}_\perp^2}{n}-\frac{m^2+(\hat{p}_\perp-\hat{n}_\perp)^2}{p-n}} - \text{“}m\to M\text{”}\right\}$$

$$+\text{“}a_p^+ a_p \to A_p^+ A_p, \lambda^2\to\Lambda^2\text{”} + \frac{e^2}{4\pi L_\perp}\sum_{n,s}\frac{[b_{sn}^+ b_{sn} + d_{sn}^+ d_{sn}]}{n}$$

$$\times\left\{\sum_P \frac{\lambda^2 - s\left[m^2+\hat{n}_\perp^2\right]\cdot\frac{p}{n}+2\hat{n}_\perp\hat{p}_\perp \quad 1}{\frac{m^2+\hat{n}_\perp^2}{n}-\frac{\lambda^2+\hat{p}_\perp}{P}-\frac{m^2+\hat{n}_\perp-\hat{p}_\perp)^2}{n-p}p(p-n)} - \text{“}\lambda^2\to\Lambda^2\text{”}\right\}$$

$$+\text{“}b^+ b\to B^+ B, d^+ d\to D^+ D, m^2\to M^2\text{”}\,. \tag{A.9}$$

The conventions are the same as in Eqs. (3.6)–(3.12). a_p, A_p, c_p, C_q, $B_{s,m}$, $D_{s,n}$ obey usual boson commutation relations, $b_{s,m}$ $d_{s,n}$ fermion anticommutation relations. “h.c.” indicates Hermitian conjugation only for field operators-not for c-numbers, i.e. iA_p + “h.c.” = $iA_p + iA_p^+$. Of course H is thus not hermitian but this should not influence unitarity below the production threshold for the heavy photons and electrons. There is no instantaneous photon exchange term since those terms cancel among the light and heavy photons.[19] The one-loop counterterms have been constructed such that they, together with one loop corrections induced by H, avoid all one-loop self energies which would be proportional to $\int dm^2\rho(m^2)\sqrt{m^2}$ or $\int d\lambda^2\rho(\lambda^2)\sqrt{\lambda^2}$ in the continuum limit. Without the one-loop counterterms more Pauli–Villars particles would be necessary to make all such terms vanish.

APPENDIX B

The two-loop self energy in $\text{Yukawa}_{D_\perp+2}$

In light-cone pertubation theory (LCPTh) the strongest divergences (quadratic in 3 + 1) occur at the one loop level. Thus one might be tempted to expect that the violations of rotational invariance occur also only at one loop. This is not true as the following simple example shows.

We consider a massless fermion coupled to a massive scalar boson via a Yukawa interaction term. As a specific example we evaluate explicitly the rainbow graph (Fig.1) contribution to the γ^+ component of the one-shell fermion self energy. If we choose vanishing $p_\perp$ for the incoming electron, i.e. $p_{\text{in}}^- = p_\perp^2/p^+ = 0$, it follows from (2.2) that this component should be zero.

In order to separate one loop and two loop effects we allow the masses of the inner (X) and the outer boson (A) in the diagram to be different from each other. This also makes it easy to regularize the inner loop "sufficiently" while leaving the outer loop unregularized for the moment. Applying LCPTh one easily finds[20] (up to the same constants)

$$\Sigma(p^\mu) = C \int d^{D_\perp}k_\perp \int_0^1 \frac{dx}{x(1-x)^2} \frac{\not{p}_1}{\frac{k_\perp^2}{1-x} + \frac{k_\perp^2+\lambda^1}{x}} I^{1LOOP}(p_1) \frac{\not{p}_1}{\frac{k_\perp^2}{1-x} + \frac{k_\perp^2+\lambda^2}{x}} \quad \text{(B.1)}$$

where

$$p_1^+ = p^+(1-x) \qquad p_1^- = -\frac{k_\perp^2 + \lambda^2}{xp^+} \quad \text{(B.2)}$$

and

$$I^{1LOOP}(p_1) = \frac{\not{p}_1}{1-x} \int d^{D_\perp}q_\perp \int_0^1 \frac{dy}{y(1-x)} \int d\Lambda^2 \frac{(1-y)\rho(\Lambda^2)}{p_1^- - \frac{q_\perp^2+\Lambda^2}{y(1-x)} - \frac{(k_\perp+q_\perp)^2}{(1-y)(1-x)}} . \quad \text{(B.3)}$$

Here we have already used $\int d\Lambda^2\rho(\Lambda^2) = 0$, $\int d\Lambda^2\rho(\Lambda^2)(\Lambda^2)^{D_\perp/2} = 0$ to cast I^{1Loop} into a rotationally invariant form.[21] Using (note that p_1 is an energy shell; see Eq. (B.2))

$$p_1^2 = -(1 - x)\left[\frac{k_\perp^2}{1 - x} + \frac{k_\perp^2 + \lambda^2}{x}\right] \tag{B.4}$$

one finds

$$\text{tr}\,(\Sigma\gamma^-) = C\int d^{D_\perp}k_\perp \int d^{D_\perp}q_\perp \int_0^1 \frac{dx}{x^2(1-x)^2}\int_0^1 \frac{dy}{y} \int d\Lambda^2\rho(\Lambda^2)$$

$$\times \frac{k^2 + \lambda^2}{\frac{k^2}{1-x} + \frac{k^2+\lambda^2}{x}\;\frac{k^2+\lambda^2}{x} + \frac{q^2+\Lambda^2}{y(1-x)} + \frac{(k+q)^2}{(1-y)(1-x)}} \tag{P-5}$$

$$= C\,\pi^{D_\perp}\,\frac{\Gamma(1 - D_\perp)}{1 - (D_\perp/2)}\,\frac{\pi}{\sin\pi(D_\perp/2)}\int \frac{d\Lambda^2\rho(\Lambda^2)}{(\Lambda^2)^{1-D_\perp}} \neq 0\,.$$

First and most important, the γ^+ component of Σ is nonzero and rotational invariance is thus violated since $p^- = 0$. Secondly, the result is independent of the outer boson mass λ; i.e. a Pauli–Villars regularization (with condition $\int d\lambda^2\rho(\lambda^2) = 0$) would have rendered tr $(\Sigma\gamma^-)$ zero.

This is a rather typical result for higher loop graphs and implies the following. Once one has (over-) regularized the short distance singularities so much that one can handle the one-loop singularities in a rotationally invariant way (as in P-V) then the (milder) higher loop singularities should be no problem any more if one uses the same (over-) regularized versions of the theory there.

It is however not sufficient to add only a one-loop counterterm and add no two loop counterterms at all. although one might be tempted to do so, because e.g. in 2 + 1 dimensions this would not introduce additional infinities, this violates rotational invariance by a finite term (for $D_\perp = 1$).

Figure caption

Fig.1 Rainbow diagram contribution to the two loop fermion self energy in the Yukawa model.

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2. A simple and well-known example is rotational invariance in the context of equal time quantized field theory. The angular momentum operator is a sum of single particle operators and generally approximation schemes preserve rotational invariance.
3. The situation here is very similar to boost transformations in equal time quantization which mix $\vec{x}$ and x^0 .
4. Note that we have already introduced one P-V regulator field with mass A in order to avoid the ambiguity in the treatment of normal-ordering divergences.
5. Unfortunately we were not able to prove this in general. It is however plausible, since the worst singularities in LC loop calculations occur usually for the γ^+ component where they lead to a violation of rotational invariance. We have verified the statement for all two loop diagrams.
6. M. Burkardt and A. Langnau , in preparation.
7. J.D. Bjorken and S.D.Drell, "Relativistic Quantum fields "(McGraw-Hill, New York 1966)
8. H.-C. Pauli and S. J. Brodsky, Phys. Rev. D32, 1993 (1987); 2001 (1987); T. Eller, H.-C. Pauli and S. J. Brodsky, Phys. Rev. D35, 1493 (1987); K. Hornbostel, Ph.D. Thesis, SLAC-0333 (1989).
9. It should be emphasized that we could have chosen another basis set for the expansion of the fields. In fact we are always free to replace sums by integrals and thus obtain a continuum formulation. Our choice here does not restrict generality.

10. A. Tang, Ph.D. Thesis, SLAC-PUB-351 (1990).

11. In the next chapter we will construct noncovariant counterterms which display the same structure as the normal-ordering terms. i.e., in the latter approach we can also omit V_{NO} since taking it into account just corresponds to a redefinition of parameters.

12. S. J. Brodsky, R. Roskies and R. Suaya, Phys. Rev. D8, 4574 (1973).

13. This is similar to approximations used in equal time approximation which lead to violations of boost invariance.

14. A. Langnau, Ph.D. Thesis, in preparation.

15. This is true if all noncovariant terms have been removed in subloops.

16. Note that no wave function renormalization is necessary in the Hamiltonian approach since wave functions are automatically normalized. The normalization constants cancel, due to the Ward identifies, in all physical observables.

17. C. Bouchiat, P. Fayet and N. Sourlasi Lett. Nuovo Cim. 4, 9 (1972); S.-J. Chang and T.-M. Yan, Phys. Rev. D7, 1147 (1973).

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19. Note that the original instantaneous photon exchange term (3.10) is independent of the photon mass.

20. Here we have used the Brodsky trick to combine instantaneous fermions with those fermion lines which extend only over one intermediate state by putting the corresponding fermions on energy shell in the numerator.

21. In fact I^{1Loop} can be written as $I^{1Loop} = \not{p}_1 f(p_1^2)$.

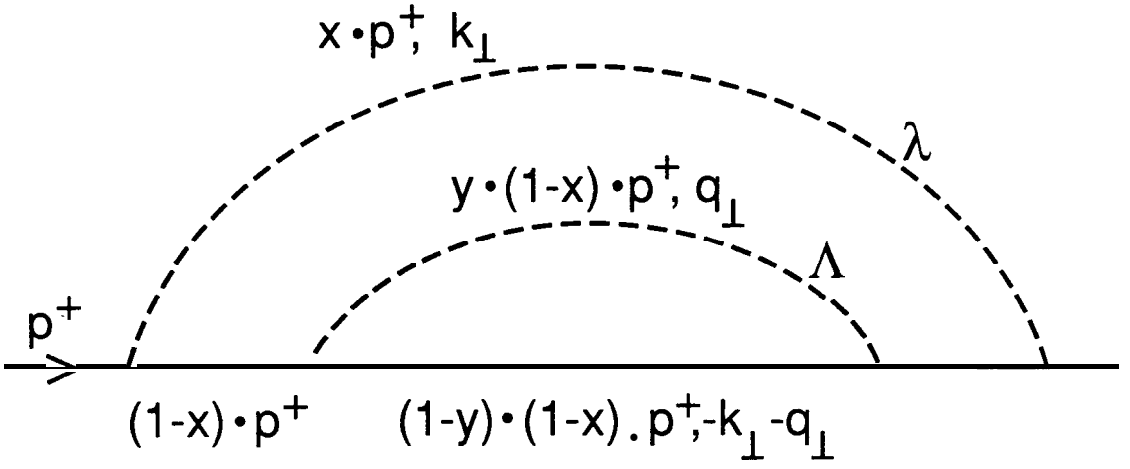

FIGURE 1