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Theoretical Topics in B–Physics*

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1. Introduction; the CKM Triangle

The bottom quark should need no introduction. Other than the undiscovered top quark, it is by far the most fashionable of the six. There is good reason for this. It is bottom–quark behavior which holds out the most hope of measuring and understanding some of the most fundamental and delicate parameters of the standard model—those having to do with the origin of electroweak mixing—and thereby in all probability also the origin of quark mass. Also interwoven into this is the subject of CP violation, and its proposed interpretation in terms of electroweak mixing.

In this section we shall review the basics of electroweak mixing and how it is impacted by the study of b–quark properties. There are by now many lecture series and workshop proceedings devoted to this topic,[1] so I will not try to be comprehensive, but only hit some highlights.

The parameters of electroweak mixing are defined by the amplitudes for W decay into quark–antiquark final states.[2] There is no selection rule operative other than charge conservation and the V–A structure of the weak interaction Hamiltonian. Therefore, we essentially have

$$\frac{\mathcal{M}(W^+ \to Q\overline{q})}{\mathcal{M}(W^+ \to e^+\nu_e)} \cong V_{Qq} \equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \qquad (1.1)$$

The nine quantities V_{Qq} form a 3×3 matrix with complex entries, and it is a principal task of experiment to determine them. However from the point of view of standard–model theory, there is an additional restriction required for consistency of the electroweak gauge theory, namely that the matrix V be unitary. The reason for this will be elaborated upon later, but here we only note that the unitarity restriction reduces the eighteen real parameters in V down to nine. There are five further reductions having to do with the fact that the choice of phase given to the quark fields or wave functions are arbitrary. This looks like elimination of six more parameters, but the number is only five, because a common phase rotation of all six quark fields leaves V unaffected.

The bottom line is that in the standard model there are four independent real parameters in the matrix V to determine. It is natural to use as those parameters objects already accurately measured or with potential to be accurately measured in the future. A very natural phase choice for the elements of V is to choose the diagonal elements as real positive. This is because each is close in magnitude to unity (at least if we assume the unitarity constraint!), and in the limit of no mixing it is almost obligatory to let V approach the unit matrix. This leaves two other phase choices to make. The next-to-diagonal elements in the upper right are important ones experimentally, and we choose them to be real positive as well. V_{us} is the sine of the Cabbibo angle; well-measured and quite overdetermined through the many studies of strange particle weak decays. V_{cb} governs the dominant semileptonic decays of the B into charm final states. The lifetime and branching ratio measurements already determine its magnitude to about 20% and prospects for future improvements are good, as we will elaborate upon later in these lectures.

With the phase choices out of the way, we see that V_{ub}, in general complex, together with the magnitudes of V_{us} and V_{cb}, provide a convenient set of four independent parameters to describe the purportedly unitary matrix V (called the Kobayashi–Maskawa KM matrix, or better the CKM matrix in recognition of Cabibbo's earlier contributions to the development of the ideas of electroweak mixing.) Then the determination of the remaining parameters of the matrix using unitarity is straightforward. First get V_{ud} (which is real) by demanding the norm of the first row of the matrix be unity. Then get V_{cd} (which is complex) by demanding orthogonality of the first and second rows. (this is two real equations.) This must be done together with determination of V_{cs} via the norm condition on the second row. While in general this could entail ugly algebra, in practice things are completely straightforward because of the smallness of the off diagonal elements and the fact they decrease rapidly as one departs further from the diagonal. The same situation holds for the elements of the third row. Orthogonality with the second row determines V_{ts} to rather good accuracy. The norm condition on the third row essentially determines V_{tb} to be very near unity, and the remaining conditions, orthogonality of first and third rows, provide the most delicate and interesting relation:

$$V_{ud}V_{td}^* + V_{us}V_{ts}^* + V_{ub}V_{tb}^* = 0 \ . \tag{1.2}$$

To good approximation this is

$$V_{ub} + V_{td}^* = V_{us}V_{cb} \tag{1.3}$$

where we use

$$\begin{aligned} V_{ud} &\approx V_{tb} \approx 1 \\ V_{ts}^* &\approx -V_{cb} \ . \end{aligned} \tag{1.4}$$

This is conveniently depicted as a triangle relation in the complex plane (Fig. 1).

It appears ever more frequently in the literature, and perhaps in a decade or two

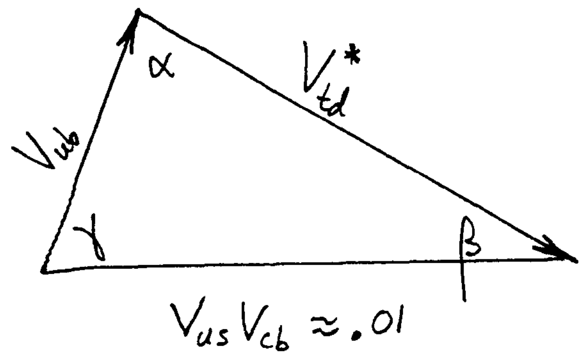

Figure 1. The unitarity triangle.

it may penetrate the hallowed pages of the Particle Data Group compilations. In any case for practical purposes we can regard the matrix (assuming its unitarity!!) to be reasonably well determined with the exception of the V_{ub} element—especially its phase—and hence also the V_{td} element as well. Thus a good representation for the matrix is

$$V \cong \begin{pmatrix} 0.97 & 0.22 & V_{ub} \\ -0.22 - 0.044\, V_{ub}^* & 0.97 & 0.044 \\ V_{td} & -0.043 - 0.22\, V_{ub}^* & 1.00 \end{pmatrix} \tag{1.5}$$

where we have availed ourselves of some recent experimental numbers. The status of the unitarity triangle will be discussed later.

However, before getting into the details of that phenomenology, it is appropriate to recall where this matrix is supposed to originate. It is in the depths of the Higgs sector of the electroweak theory, in particular the piece of the action responsible for quark mass. This is supposed to occur via Yukawa coupling of the quarks to a complex Higgs field Φ. We write this as follows:

$$\mathcal{H}_{\text{Yuk}} = \overline{q}'_L \Phi G' q'_R + \text{h.c.} \tag{1.6}$$

Here Φ is a matrix of Higgs fields which is nontrivial only in weak–isospin space; it is diagonal in the 3–dimensional "generation space". That is, we may write

$$\Phi = H + i\boldsymbol{\tau} \cdot \boldsymbol{w} \tag{1.7}$$

with four independent real components. H is the physical Higgs degree of freedom and the three w's are "eaten" by the gauge degrees of freedom according to the Higgs mechanism. The w's become the gauge–boson longitudinal degrees of freedom. The electroweak SU(2) rotations act on this matrix from the left

$$\begin{aligned} q'_L &\to U q'_L \\ q'_R &\to q'_R \end{aligned} \qquad \Phi \to U\Phi \tag{1.8}$$

so that Φ transforms as a doublet as it should, while

$$\overline{q}'_L \Phi \tag{1.9}$$

is an $SU(2)_L$ invariant combination. The coupling matrix G' is nontrivial in flavor space but does not see the $SU(2)_L$; it is an invariant. But it is an ugly invariant; it is as general as it could possibly be. It is best to write out the matrices Φ and G' in 6×6 form to really see what they look like.

The field Φ is supposed to undergo spontaneous symmetry breakdown, *i.e.* develop a classical vacuum expectation value due to its self–interactions. The expectation value is diagonal in our notation;

$$\langle \Phi \rangle = \langle H \rangle \equiv v = 246\ GeV \tag{1.10}$$

and turns the original quark–Higgs Yukawa coupling into a mass term.

$$\mathcal{H}_{\text{Yuk}} \rightarrow \bar{q}'_L \langle \Phi \rangle G' q'_R + \text{h.c.} \equiv \bar{q}'_L M' q'_R + \text{h.c.} \tag{1.11}$$

with

$$M' = \langle \Phi \rangle G' \ . \tag{1.12}$$

The message here is that the origin of mass in the standard model is to be traced to the presence of Yukawa couplings of the quarks to the Higgs degrees of freedom. The peculiar values of the quark masses are a consequence of the peculiar values of the corresponding Yukawa couplings to which they are proportional.

But where does the mixing come from? It is that the matrix G' need not be diagonal nor hermitian nor symmetric under interchange of up and down degrees of freedom. A lot of diagonalizing can be and is done on G'. One writes

$$G' = V_L G V_R^\dagger \tag{1.13}$$

with G diagonal. But one must check whether the rest of the electroweak Lagrangian commutes with this diagonalization procedure. One place where it does not is in the quark–W coupling, which depends on weak isospin:

$$\begin{aligned} \mathcal{L}_W &= g\bar{q}'_L \gamma^\mu \boldsymbol{\tau} \cdot \boldsymbol{w}_\mu q'_L \\ &\Rightarrow g\bar{q}_L \gamma^\mu V_L^\dagger \boldsymbol{\tau} \cdot \boldsymbol{w}_\mu V_L q_L \ . \end{aligned} \tag{1.14}$$

We see that the matrix V_L, which depends upon τ^3, will not commute with τ^+ or τ^-, which are the couplings of the charged W to the quarks. It is advised to write the coupling of the quarks to the W^+ explicitly in 6×6 matrix form to see how this works. The relevant matrix turns out to be

$$V_L q_L = \begin{pmatrix} V_{\rm up} & 0 \\ 0 & V_{\rm down} \end{pmatrix} \begin{pmatrix} u \\ c \\ t \\ d \\ s \\ b \end{pmatrix}_L \tag{1.15}$$

where $V_{\rm up}$ and $V_{\rm down}$ are 3×3 unitary submatrices. Then

$$\begin{aligned} V_L^\dagger \boldsymbol{\tau} \cdot \boldsymbol{w}_\mu V_L &= \begin{pmatrix} V_{\rm up}^\dagger & 0 \\ 0 & V_{\rm down}^\dagger \end{pmatrix} \begin{pmatrix} w_3/2 & \sqrt{2}\,w^+ \\ \sqrt{2}\,w^- & -w_3/2 \end{pmatrix} \begin{pmatrix} V_{\rm up} & 0 \\ 0 & V_{\rm down} \end{pmatrix} \\ &= \begin{pmatrix} w_3/2 & \sqrt{2}\,V_{\rm up}^\dagger V_{\rm down} w^+ \\ \sqrt{2}\,V_{\rm down}^\dagger V_{\rm up} w^- & -w_3/2 \end{pmatrix} \end{aligned} \tag{1.16}$$

and

$$V = V_{CKM} = V_{\rm up}^\dagger V_{\rm down} \; . \tag{1.17}$$

In contemplating the origin of V, it is clear from this point of view one must contemplate the origin of G', or equivalently the mass matrix

$$M' = \langle \Phi \rangle \; G' = G' v \; . \tag{1.18}$$

This is not so easy, because the mass matrix (which need not be hermitian) is diagonalized not only by V_L but also V_R, a matrix about which we have no experimental information. And only the CKM combination $V_{\rm up} V_{\rm down}^\dagger$ of V_L appears in the data as well. Nevertheless things can be done; most of this will however be left for the reader to work out. Some extra assumptions are typically needed to do this. A popular ansatz is that the mass matrix have "Fritzsch texture", *i.e.* it takes the form[3]

$$M'_{\mathrm{up}} = \begin{pmatrix} 0 & m_{uc} & 0 \\ m_{cu} & 0 & m_{ct} \\ 0 & m_{tc} & m_t \end{pmatrix} \qquad M'_{\mathrm{down}} = \begin{pmatrix} 0 & m_{ds} & 0 \\ m_{sd} & 0 & m_{sb} \\ 0 & m_{bs} & m_b \end{pmatrix} . \tag{1.19}$$

My own preferred attack is a little different. Because the off diagonal elements of V are small, one is tempted to assume the same for M'. Then low order perturbation theory can be used to determine the elements of M' from those of V. One writes to first order

$$\begin{aligned} V_L &= 1 + iK_L \\ V_R &= 1 + iK_R \\ M' &= M + m \end{aligned} \tag{1.20}$$

with m having only off–diagonal elements. Then, because V_L diagonalizes the hermitian matrix $M'M'^{\dagger}$,

$$i\,\left[K_L, M^2\right] \cong Mm^{\dagger} + mM \tag{1.21}$$

or

$$i\,K^L_{ij} = \frac{(m_{ij}M_j + m^*_{ji}M_i)}{(M_j^2 - M_i^2)} \tag{1.22}$$

From this one gets three useful relations.

$$
\begin{aligned}
V_{us} &= i(K_{ds}^L - K_{uc}^L) = \frac{(m_{ds}M_s + m_{sd}^* M_d)}{M_s^2 - M_d^2} - \frac{(m_{uc}M_c + m_{cu}^* M_u)}{M_c^2 - M_u^2} \\
&\cong \left(\frac{m_{ds}}{M_s} - \frac{m_{uc}}{M_c}\right) \\
V_{ub} &\cong \left(\frac{m_{db}}{M_b} - \frac{m_{ut}}{M_t}\right) \\
V_{cb} &\cong \left(\frac{m_{sb}}{M_b} - \frac{m_{ct}}{M_t}\right) .
\end{aligned}
\tag{1.23}
$$

To go further requires more assumptions. My own favorite guess is that all important off–diagonal terms of M' reside in the down–quark matrix. This leaves their numerical values all in the 10's to 100's of MeV. (If this is true for the up–quark off–diagonal mass–matrix elements, they indeed are not very important contributions to the CKM mixing.) Provided the mass matrix is anywhere near hermitian, one gets to good approximation

$$
|M_{\text{down}}| \approx \begin{pmatrix} ? & 35 & 25 \\ ? & 150 & 250 \\ ? & ? & 5000 \end{pmatrix} MeV \qquad |M_{\text{up}}| \approx \begin{pmatrix} 4 & 0 & 0 \\ 0 & 1500 & 0 \\ 0 & 0 & 140,000 \end{pmatrix} MeV .
\tag{1.24}
$$

Notice that in any case the only information on the mass matrix is on the elements above the diagonal; the remaining elements are sensitive to V_R.

There is an amusing corollary which follows from this plus a couple more assumptions. Suppose that the mass matrix is hermitian and that all off diagonal elements are pure imaginary. (This is an old suggestion of Stech.[4]) Then it turns out (the demonstration is left to the reader) that the unitarity triangle is to very good approximation a right triangle, with $\gamma = 90°$. (To get this result, one must go beyond first–order perturbation theory in the size of the off–diagonal elements.) This result does not deserve to be taken very seriously. But what is vital is to get a better handle on the origin of the peculiar properties of the mass matrix. It deserves everyone's best efforts.

Also, one must not forget that the assumed unitarity of the CKM matrix is just that—an assumption. It is easy to find models where that is not true. Perhaps the most natural way of doing that is to introduce extra down quarks which are electroweak singlets (this happens naturally in GUT theories such as E(6)) but which mix with the usual quarks.[5] Nir and Silverman[6] have given a very nice analysis of the simplest situation, where only one extra down quark mixes significantly with the other three. Evidently there will be a 4×4 mixing matrix which is unitary, although the 3×3 submatrix will not be. The unitarity condition becomes

$$V_{ud}V_{td}^* + V_{us}V_{ts}^* + V_{ub}V_{tb}^* + V_{uD}V_{tD}^* = 0 \tag{1.25}$$

which leads to a unitarity quadrangle (Fig. 2)

$$V_{td}^* + V_{ub} \cong V_{us}V_{cb} - V_{uD}V_{tD}^* \ . \tag{1.26}$$

The extra segment is constrained in a variety of ways. And one might a priori not expect it to be especially large. But the effects on CP violation measurements in the B system can nevertheless be big. It is a little premature to discuss them here, but this point must be kept in mind as we go along.

The sides and angles of the unitarity triangle have a direct experimental meaning. It is best to normalize the base to V_{us} by dividing all sides by V_{cb}. Then the other two sides are more closely related to experimental observations. Evidently

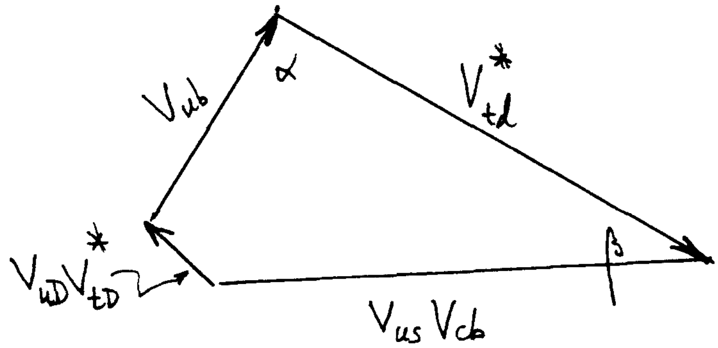

Figure 2. A unitarity quadrangle.

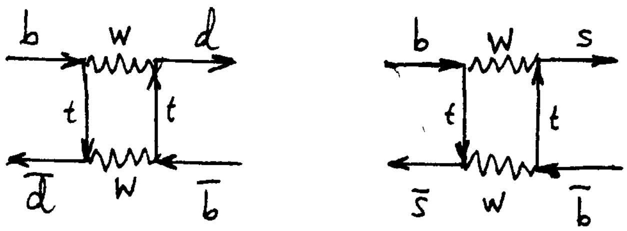

Figure 3. Mechanism of $B-\overline{B}$ mixing.

V_{ub}/V_{cb} is measured by the ratio of charmless to charmed semileptonic decays.[7] The other side ideally is measured via comparison of $B_d-\overline{B}_d$ mixing with that of the B_s. This is clear from the diagrams shown in Fig. 3, assuming they are the dominant contributors to the mixing. The formula for the mixing probability goes like

$$\frac{(\Delta M)_{B_d}}{(\Delta M)_{B_s}}=\left|\frac{V_{td}}{V_{ts}}\right|^2\frac{[\text{QCD matrix element}]_d}{[\text{QCD matrix element}]_s}\approx\left|\frac{V_{td}}{V_{ts}}\right|^2 \tag{1.27}$$

and so the ratio of the mixing probabilities leads to a lot of cancellation of theoretically uncertain factors. At present, one relies on best estimates of the separate factors to get at the value. The result is shown in Fig. 4 for an assumed top quark mass of 160 GeV. There is much ado about the best fits which I do not choose here to discuss.[8] My own view is that there is plenty of uncertainty in how the triangle will look, most of it theoretical.

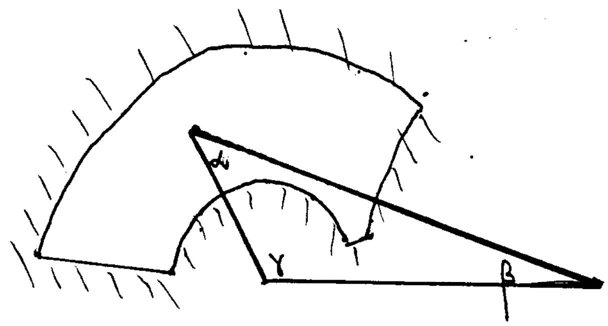

Figure 4. Allowed region for vertex of unitarity triangle for m_t = 160 GeV. From Ref. 8.

Direct observation of the angles of the triangle requires CP–violation experiments to be performed.[9] The angle beta, for example, is what is measured in the premier CP violation experiment $B_d \to \psi + K_s$ One compares the time–dependent decay of a tagged B_d with that of a tagged $\overline{B}_d$. Standard mixing theory (ignoring quite justifiably[10] lifetime mixing) gives for the state which at initial time is pure B_d

$$|B_d(t)\rangle = \left[|B_d\rangle \cos\frac{\Delta mt}{2} - i\lambda \left|\overline{B}_d\right\rangle \sin\frac{\Delta mt}{2} \right] e^{-\Gamma t/2} \tag{1.28}$$

where λ is of modulus unity, with its phase being the phase of the mass mixing term.

$$i\frac{\partial}{\partial t}|B_d\rangle = -\frac{i\Gamma}{2}|B_d\rangle + \left\langle \overline{B}_d \left| \frac{\Delta M}{2} \right| B_d \right\rangle \left|\overline{B}_d\right\rangle \ . \tag{1.29}$$

Note that with our phase convention this has the phase of the square of V_{td}. The

B_d decay amplitude into the ψ-K_s CP eigenstate is then

$$\mathcal{M}(B_d \to \psi K_s) = \mathcal{M}_0 \left[V_{cb} \cos \frac{\Delta m t}{2} \pm i\lambda\, V_{cb}^* \sin \frac{\Delta m t}{2} \right] e^{-\Gamma t/2} \tag{1.30}$$

where we have taken out the CKM element from the decay amplitude.Thus doing the same thing for the antiparticle gives

$$\begin{aligned} \frac{dN}{dt} &= N_0 \left| \cos \frac{\Delta m t}{2} \pm i \frac{V_{td}}{V_{td}^*} \cdot \frac{V_{cb}^*}{V_{cb}} \sin \frac{\Delta m t}{2} \right|^2 e^{-\Gamma t} \\ &= N_0 \left[1 \pm \sin\phi \sin \Delta m t \right] e^{-\Gamma t} \\ \frac{d\overline{N}}{dt} &= \overline{N}_0 \left| \cos \frac{\Delta m t}{2} \pm i \frac{V_{td}^*}{V_{td}} \cdot \frac{V_{cb}}{V_{cb}^*} \sin \frac{\Delta m t}{2} \right|^2 e^{-\Gamma t} \\ &= \overline{N}_0 \left[1 \pm \sin\phi \sin \Delta m t \right] e^{-\Gamma t} \end{aligned} \quad . \tag{1.31}$$

Only V_{td} has significant phase content in our convention, so that the effect depends upon

$$\phi = 2 \mathrm{Arg}\, V_{td} = 2\beta \ . \tag{1.32}$$

In a very similar way it can be seen that the angle α at the top of the unitarity triangle is measured by the decay $B_d \to \pi + \pi$. Here the factor V_{cb} in the decay amplitude is replaced by V_{ub}, and its additional phase changes the observable phase to twice α.

$$\sin\phi = \sin(2 \mathrm{Arg}\, V_{td} - 2 \mathrm{Arg}\, V_{ub}) = \sin(2\beta + 2\gamma) = -\sin 2\alpha \ . \tag{1.33}$$

The phase γ is the hardest to get at. The decay $B_s \to \rho K_s$ is a candidate; here the only relevant phase is contained in the factor V_{ub} in the decay amplitude.

In all these examples, we have used $B-\overline{B}$ mixing, together with decay into a CP eigenstate, as the technique for seeing the CP violation. There are other possible attacks as well, in particular particle–antiparticle branching–ratio differences.

$$\Gamma(B \rightarrow F) \neq \Gamma(\overline{B} \rightarrow \overline{F}) . \tag{1.34}$$

These typically utilize the existence of "Penguin" diagrams. Unfortunately there will be no time in these lectures to discuss Penguin processes.

2. Semileptonic Decays

As we indicated in the previous section, the quest for observation of CP violation in b–decay processes is the central reason for the great experimental–and theoretical–interest in the subject nowadays. But there is a long way to go before getting there, and much should be measured and understood on the way. CP violation studies in the b–system, if possible at all, should turn out to be an experimental program and not just an isolated discovery experiment. There are a variety of modes competitive in sensitivity which probe different features of the unitarity triangle (or quadrilateral). It therefore is especially important to have as good a grip on the overall phenomenology of b–decays as possible. A large, well understood data base is essential in optimizing the yield of information possible to obtain on CP–violating processes. And already we see important parameters of the CKM matrix limited by theoretical systematic errors. So there will be some emphasis in these lectures on the underlying phenomenology. The natural starting point is semileptonic decays.

Semileptonic b–decay processes are expected to be especially clean theoretically. The reason is the same as for kaon decays or charm particle decays, although there is some basis for hope that the heavy b–quark mass may allow certain nonleptonic decays to be comparable in cleanliness to semileptonic decays. The theory of semileptonic–decay phenomena is especially active nowadays, thanks to the contributions of Isgur and Wise (Wisgur).[11] They have shown that in the formal limit of infinite b– and c–quark mass, the phenomenology is greatly simplified. To me this holds out the promise of a relatively model–independent approach to these processes. While the predictions of the limiting case may not be highly accurate, there is most likely a well–defined set of first–order corrections; the model dependence is then hopefully relegated to these corrections.

So I will base the discussion here on the Wisgur limiting case; it at least has the advantage of clarity and simplicity. The basic ideas are very simple: what is surprising is that they lead to such strong consequences. They are

1. As the b–quark mass becomes very large, a B–meson becomes a cannonball. It is very hard to change its velocity; a very large momentum transfer is needed and only perturbative mechanisms (hard gluons or electroweak transitions) can do that.

2. QCD exists in the limit; nothing terrible seems to occur. It is like setting the mass of a nucleus to infinity in QED atomic physics.[12]

3. The spin degree of freedom of the b–quark decouples from the dynamics in the limit because the color hyperfine interaction scales inversely with the heavy quark mass.

4. Therefore there are new symmetries in the spectrum of states of the hadrons containing a b–quark; all members of a hyperfine multiplet have the same mass in the limit. In particular the pseudoscalar B becomes degenerate with the vector B^* in the limit. (actually they are believed to be split by 50 MeV.)

5. In the limit, the flavor label of the heavy quark, *e.g.* b vs. c, becomes irrelevant; hence a new flavor symmetry emerges as well.

The simplifications to the phenomenology occur for two reasons. The first is that in the limit the semileptonic matrix elements can depend only on the velocities of B and D (or D^*), not separately on momenta and mass. The second simplification comes from the Wisgur hyperfine symmetry. Spin rotation of the b–quark is a symmetry operation; using it one can relate matrix elements of B to D^* to those of B to D.

Let us begin with the $B \to D + e + \nu$ decay. It is just like Ke_3 decay as far as kinematics is concerned. Normal conventions put the matrix element of the weak current (pure vector; no axial contribution) in the following general form:

$$\langle D\,|V_\mu|\,B\rangle = \frac{1}{\sqrt{4E_D E_B}}\ \left[F^+(q^2)(P_B + P_D)_\mu + F^-(q^2)q_\mu\right]\ . \tag{2.1}$$

There are two form factors as shown, but only F^+ contributes because

$$q_\mu J^\mu_{\text{lepton}} = (P_B - P_D)_\mu J^\mu_{\text{lepton}} = 0\ . \tag{2.2}$$

We now compare this with the expression resulting from the infinite–mass limit requirement.

$$\langle D\,|V_\mu|\,B\rangle = \sqrt{\frac{M_B M_D}{4E_B E_D}}\,F(\widehat{t})\left(\frac{P_B}{M_B} + \frac{P_D}{M_D}\right)_\mu \tag{2.3}$$

$$V_\mu = \bar{c}\gamma_\mu b$$

There are two changes. One is rather superficial; the traditional $1/\sqrt{2E}$ normalization factors for the wave functions are replaced by $\sqrt{M/2E}$. This form depends only on the Lorentz γ of the heavy meson, as appropriate for the limit. A more vital change is the appearance of only one form factor. This occurs for a combination of two reasons. The first is that only the combination $P_\mu/M = v_\mu$, the invariant velocity, can appear in the matrix element. The other is that matrix element of the vector current between B and D at a given velocity transfer has to equal the matrix element of the vector current

$$\begin{aligned} \left\langle B' \left| \widetilde{V}_\mu \right| B \right\rangle &= \frac{1}{\sqrt{4E'_B E_B}} F\left(\frac{q^2}{4M_B^2}\right) \ (P'_B + P_B)_\mu \\ \widetilde{V}_\mu &= \bar{b}\gamma_\mu b \end{aligned} \tag{2.4}$$

between B' and B for the same velocity transfer. This happens because the spectator system of light valence quark and its accompanying cloud of gluons and $q\bar{q}$ pairs cannot distinguish between a b and c quark source; the flavor label carries no dynamical information in the infinite mass limit. But the elastic B matrix element is characterized by only one form factor.

Notice the remarkable feature that a form factor for a process involving a timelike momentum transfer is related to one with spacelike momentum transfer. The synthesis occurs because what matters is velocity transfer. The invariant velocity transfer is

$$\widehat{t} = \frac{1}{4}(v' - v)^2 = \frac{1}{4}\left(\frac{P'}{M'} - \frac{P}{M}\right)^2 \tag{2.5}$$

and for the weak transition of interest this is related to q^2 as follows:

$$\widehat{t} = \frac{q^2 - (M_B - M_D)^2}{4M_B M_D} \ . \tag{2.6}$$

When $\widehat{t}$ vanishes, the (timelike) momentum transfer to the dilepton is maximized; the mass of the dilepton is just the B–D mass difference. In this limit, the D or D^*

remains at rest (in the B rest frame) and there is no recoil motion of the spectator system.

This procedure does not reduce the number of independent vector and axial $B \to D^*$ matrix elements, of which there are four. However these all get related to the above form factor using the Wisgur symmetry. The direct way of getting this result is to relate B–to–D^* matrix elements to B–to–D matrix elements by applying a spin rotation to the D^* in its rest frame. The easy way is to use

$$S_z = \int d^3x \, c^\dagger(x) \sigma_z c(x) \tag{2.7}$$

which is a symmetry operator, *i.e.* commutes with the Hamiltonian, in the infinite mass limit. One easily finds (up to annoying phase conventions)

$$S_z \left| D \right\rangle = \frac{1}{2} \left| D^* \right\rangle_{\text{long}} \qquad S_z \left| D^* \right\rangle_{\text{long}} = \frac{1}{2} \left| D \right\rangle \ . \tag{2.8}$$

Thus

$$\left\langle D^*_{\text{long}} \left| J_\mu \right| B \right\rangle = 2 \left\langle D \left| S_z J_\mu \right| B \right\rangle = 2 \left\langle D \left| [S_z, J_\mu] \right| B \right\rangle = \left\langle D \left| J'_\mu \right| B \right\rangle \tag{2.9}$$

and the commutator can be evaluated explicitly for any choice of current, yielding a matrix element of some other current operator between B and D. Upon doing this repeatedly, one finds that all the form factors indeed can be determined in terms of the single (normalized) form factor introduced above.

We shall not go through that line of argument in any detail, but instead write down the answer in a compact form which allows further generalization.[13]

$$\left\langle D \text{ or } D^* \left| J_\mu \right| B \right\rangle = \sqrt{\frac{M_B M_D}{4 E_B E_D}} \, \mathrm{Tr} \, \overline{\mathcal{D}} \mathcal{J}_\mu \mathcal{B} \rho \ . \tag{2.10}$$

In this formula, each of the factors is a 4×4 Dirac matrix. The matrix $\mathcal{B}$ is the wave function of the initial B:

$$\mathcal{B} = \left(\frac{\not{p}_B + M_B}{2M_B}\right)\gamma_5 = \Lambda(v_B)\gamma_5 \ . \tag{2.11}$$

Were the initial state a B^*, γ_5 would be replaced by $\gamma \cdot \epsilon$, with epsilon the polarization vector of the B^*. The matrix $\overline{D}$ is defined similarly

$$\overline{\mathcal{D}} = \gamma_0 \mathcal{D}^\dagger \gamma_0 \ . \tag{2.12}$$

The matrix for the current is γ_μ for vector, etc. The remaining matrix ρ represents the physics of the spectator system, namely the amplitude that the light–quark spectator system for the B is carried away by the D or D^* without additional hadron emission. It is dependent upon the B and D, D^* four–velocities (and γ matrices), and some routine Dirac algebra shows that it can be reduced to a multiple of the unit matrix and factored out of the trace. It is just the form factor, dependent upon the invariant velocity transfer, introduced above. The reader is urged to work out the results for the $B \to D, D^*$ matrix elements from the trace formula to see how easy it is—and to verify the $B \to D$ example we already derived in detail.

More general matrix elements can likewise be written down immediately using the trace formalism:

$$\langle D \text{ or } D^*; k_1 \ldots k_n \,|J_\mu|\, B\rangle = \sqrt{\frac{M_B M_D}{4E_B E_D}}\, \mathrm{Tr}\, \overline{\mathcal{D}} \mathcal{J}_\mu \mathcal{B} \rho(v, v'; k_1 \ldots k_n) \ . \tag{2.13}$$

Only the object ρ changes; it in general depends on all the variables defining the extra particles in the final state; it is the spectator system which is responsible for their emission, because the heavy quark dynamics—in the infinite mass limit we use here—is trivial. In the general case ρ will be a nontrivial 4×4 matrix, and carries with it the nature of the correlation of the D, D^* final state variables with the remaining ones.

The formalism for charmless final states is similar. For the general process, one simply writes

$$\langle k_1 \dots k_n \,|J_\mu|\, B\rangle = \sqrt{\frac{M_B}{2E_B}}\, \mathrm{Tr}\, \mathcal{J}_\mu \mathcal{B} \phi(v; k_1 \dots k_n) \ . \tag{2.14}$$

An important application, noted by Wise,[14] is that the same formula applies for the processes

$$B \longrightarrow \{k_1 \dots k_n\} + \ell + \nu \tag{2.15}$$

and

$$D \longrightarrow \{k_1 \dots k_n\} + \ell + \nu \ . \tag{2.16}$$

Therefore the measurement of semileptonic (Cabibbo–forbidden) D decays into charmless, nonstrange final states gives one information, in this limit, on semileptonic B decays into the same final state. The information is only partial, because the invariant mass of the final hadron system must in the former case be small (in practice not much more than a GeV) while in the latter case it can be quite a bit larger.

A simple example of a charmless final state is nothing at all, namely the pure leptonic decays. Here one writes

$$\begin{aligned}\langle 0\,|J^\mu|\, B\rangle &= \sqrt{\frac{M_B}{2E_B}}\, \mathrm{Tr}\, \mathcal{J}^\mu \mathcal{B} \phi = \sqrt{\frac{M_B}{2E_B}}\, \mathrm{Tr}\, \gamma_5 \gamma^\mu \left(\frac{\not{P} + M_b}{2M_B}\right) \gamma_5 \phi \\ &= \sqrt{\frac{2M_B}{E_B}}\, \frac{P_B^\mu}{M_B}\, \phi\end{aligned} \tag{2.17}$$

where ϕ is again proportional to the unit Dirac matrix, in fact just a number. The usual way of writing these decay amplitudes is in terms of the decay constant

$$\langle 0 \,| J^{\mu} |\, B \rangle = \frac{1}{\sqrt{2E_B}} F_B \, P_B^{\mu} \; . \tag{2.18}$$

The relation between them is

$$F_B = \frac{2\phi}{\sqrt{M_B}} \tag{2.19}$$

so that the scaling–law is $(\text{mass})^{-1/2}$. This is a well–known piece of folklore for the lattice QCD community, amongst others.[15] There is some skepticism on whether this asymptotic behavior is "precocious"; lattice calculations (not to mention experiments) are the best hope for an answer.

Another important application may eventually be to the $b \to s$ flavor changing neutral current ("Penguin") processes, where the matrix elements can be related to the dominant semileptonic D–decay amplitudes.[16] Yet another application is for baryonic semileptonic decays.[17] For the principal $\Lambda_b \to \Lambda_c$ matrix elements, the formalism is very simple, because these states have no hyperfine partners to mix with. Therefore the matrix elements are simply

$$\langle k_1 \ldots k_n, \Lambda_c \,| J_{\mu} |\, \Lambda_b \rangle = \sqrt{\frac{M'M}{4E'E}}\, \overline{u}(P')\gamma_{\mu}(1-\gamma_5)u(P)\, F(v',v;k_1 \ldots k_n) \; . \tag{2.20}$$

The spectator system has the quantum numbers of a spinless diquark, so there is no correlation between it and the heavy quark system other than the dependence on initial and final velocities. The spin correlation properties of the Λ_b and Λ_c should be just like the structureless heavy quarks within them, independent of the remaining light–hadron final state accompanying it!

For the elastic transition when no additional particles are emitted, the form factor depends again only on the invariant velocity transfer introduced before, and at $\hat{t} = 0$ the form factor must be normalized to unity, because the spectator diquark is unaffected by the transition.

In all these cases, the consequences of the Wisgur limit can be written down for the most general matrix elements. Therefore it is also possible to consider the consequences for inclusive quantities, *i.e.* squared matrix elements summed over a set of final states. As an example here we consider the baryonic semileptonic decays, because they are simplest, and also because to my knowledge the results for this case have not yet been written down elsewhere. We saw above that the matrix elements factorize into a kinematic piece involving spinor products, multiplied by a form factor depending upon the spinless spectators and the emitted pions, etc. The decay width then has the structure

$$d\Gamma(\Lambda_b \to \Lambda_c + k_1 \dots k_n) = d\Gamma_0 \cdot \left|F(v, v'; k_1 \dots k_n)\right|^2 \prod_{i=1}^{n} \frac{d^3 k_i}{2\omega_i (2\pi)^3} \tag{2.21}$$

where

$$\begin{aligned} d\Gamma_0 = \frac{G_F^2}{2} |V_{cb}|^2 \cdot \frac{M}{2E} \left|\overline{u}(p')\gamma_\mu(1-\gamma_5)u(p)\overline{\ell}\gamma^\mu(1-\gamma_5)\nu\right|^2 \\ \times \frac{d^3\ell}{2\ell_0} \frac{d^3\nu}{2\nu_0} \frac{d^3p' M'}{2E'(2\pi)^5} \delta^4(p' - p - \ell - \nu - \Sigma k_i) \ . \end{aligned} \tag{2.22}$$

Now sum over all hadronic final states of given final mass W and over all dilepton states of fixed mass q

$$\begin{aligned} W &= p' + \Sigma k_i \\ q &= \ell + \nu \\ W^2 &\equiv (M' + \epsilon)^2 \ . \end{aligned} \tag{2.23}$$

In the infinite mass limit, the k_i within the momentum conserving delta–function can be safely neglected, and a straightforward calculation then gives the differential width as

$$\frac{d\Gamma}{dq^2 d\epsilon} = 2W \frac{d\Gamma}{dq^2 dW^2} = \frac{d\Gamma_0(p,q)}{dq^2} w(\epsilon, \hat{t}) \tag{2.24}$$

where $d\Gamma_0$ is the expression for the differential width in the free quark limit (it eventually goes as M_b^5), and where the structure function $w(\hat{t}, \epsilon)$ is defined as

$$w(v, v'; \epsilon) \equiv \sum_n \int \prod_{i=1}^{n} \frac{d^3 k_i}{2\omega_i (2\pi)^3} \left| F(v, v'; k_1 \ldots k_n) \right|^2 \delta \left(\sum_{j=1}^{n} k_j \cdot v' - \epsilon \right) \ . \tag{2.25}$$

For a fixed velocity transfer $\hat{t}$ we must expect that the important values of W will involve only a finite amount of excitation, *i.e.* a finite value of ϵ. The physics is that the spectator diquark, originally at rest, must respond to its heavy–quark "nucleus", suddenly moving away at a finite (possibly relativistic, but still finite) velocity or γ. The response will include emission of hadrons, but few if any with gamma larger than that of the receding heavy quark. A qualitative estimate of the energy of the extra emitted hadrons in the Λ_b rest frame (or for that matter in the Λ_c rest frame) is

$$\epsilon \sim \langle m_{\text{diquark}} \rangle \cdot \gamma = \langle m_{\text{diquark}} \rangle \, v \cdot v' = \langle m_{\text{diquark}} \rangle \, (1 - 2\hat{t}) \ . \tag{2.26}$$

Notice that the physics of this structure function is different from the usual deep–inelastic structure–function physics. It is the response of the entire spectator to the acceleration of its heavy–quark source, not its response to the acceleration of one of its constituents.

The formula above has the structure of the spectator model of heavy flavor decays. And the spectator model would be recovered were there a sum rule for the structure function w.

$$\int d\epsilon\, w(\widehat{t}, \epsilon) \stackrel{?}{=} 1 \ . \tag{2.27}$$

Then the decay width, differential in $\widehat{t}$, *i.e.* differential in the final–state dilepton mass q, would be identical to that calculated in the free quark model. This has always been regarded as an inevitable consequence of the heavy–quark limit. If no constraints are put on the final state, the physics of the decay is controlled solely by what happens at the quark level.

The sum rule can be shown to be true. This will not be done in detail here. A way of getting it is to start with an equal–time current commutation relation known to be true

$$[J_0^\dagger(\vec{x},0),\, J_0(0)] = \widetilde{J}_0(0)\, \delta^3(\vec{x}) \tag{2.28}$$

with

$$J_\mu(x) = \bar{c}(x)\, \gamma_\mu b(x) \qquad \widetilde{J}_\mu = \bar{b}\gamma_\mu b - \bar{c}\gamma_\mu c \tag{2.29}$$

and putting it between Λ_b states. Contributions of so–called z–graphs need to be included, but when the dust settles the sum rule written down above indeed emerges. Extraction of the elastic contribution gives the result

$$1 = |F(\widehat{t})|^2 + \int\limits_{\substack{0 \\ \text{inelastic} \\ \text{channels}}}^{\infty} d\epsilon\, w(\epsilon, \widehat{t}\,) \ . \tag{2.30}$$

At zero velocity transfer this reduces to $1 = 1$, because we already know (or should know) that the elastic Λ_b to Λ_c form factor is normalized to unity there. The first derivative of the sum rule evidently relates the slope of the form factor to the sum of inelastic contributions to the width. This is analogous to the Cabibbo–Radicati sum rule for ordinary current–algebra sum rules.[18]

Similar results exist for the mesonic transitions.[13] The factorization structure only emerges after summation over D and D^* in the final states (but no average over B and B^* in the initial state is necessary.) The contribution of the elastic channels to the sum is a little different and reads

$$1 = (1-\hat{t})|F(\hat{t})|^2 + \int_{\text{inelastic}} d\epsilon\, w(\epsilon, \hat{t}) \; . \tag{2.31}$$

Again there is a Cabibbo–Radicati sum rule:

$$F'(\hat{t})|_{\hat{t}=0} = \frac{1}{2}\left[1 + \int_{\text{inelastic}} \frac{d\epsilon}{|\hat{t}|}\, w(\epsilon, \hat{t})\right] \; . \tag{2.32}$$

It evidently demands that the radius of the form factor exceed 1/2. (From the analyticity of F, we expect a radius of order unity.)

It is perhaps useful here to record the separate contributions to the differential width from the D, longitudinal D^*, and transverse D^*, since experiments can eventually sort these out via angular correlation measurements. Use of the trace formula yields the results

$$\begin{aligned}
B \to D: \quad & W = \left(\frac{\gamma_m + 1}{\gamma_m - 1}\right) \frac{(\gamma^2-1)}{4} \left|F(\hat{t})\right|^2 \delta(\epsilon) \\
B \to D^*_L; \quad & W_L = \frac{(1+\gamma)^2}{4} \left|F(\hat{t})\right|^2 \delta(\epsilon) \\
B \to D^*_T: \quad & W_T = \left(\frac{\gamma_m - \gamma}{\gamma_m - 1}\right) \frac{\gamma(\gamma+1)}{2} \left|F(\hat{t})\right|^2 \delta(\epsilon) \\
B \to D^*_{\text{total}}: \quad & W_L + 2W_T \; .
\end{aligned} \tag{2.33}$$

Here we have used the Lorentz boost γ instead of $\hat{t}$ in the formulae. The relevant relations are

$$\begin{aligned}
\gamma &= 1 - 2\hat{t} \\
\gamma_m &= \frac{M_B^2 + M_D^2}{2M_B M_D} \\
&= \text{value of } \gamma \text{ at endpoint of spectrum} \\
&\quad\ (q^2 = 0;\ \text{zero dilepton mass}).
\end{aligned} \tag{2.34}$$

Most of these contributions have endpoint zeroes; the exception is that for the longitudinal D^*. Normalizing the others to it and plotting the results gives a simple pattern. Comparison with model calculations (Fig. 5) yields good agreement, with the most important discrepancy occurring in the ratio of D to longitudinal D^* at the maximum velocity transfer, *i.e.* when $q^2 = 0$.

As yet there is not enough data on this kind of thing in the B system to provide constraints on the theory. But there are analyses for $D \to K, K^*\ell\nu$ transitions, some quite recent.[19,20] It is of interest to see whether these ideas apply at all, despite having to assume that the strange quark is heavy. (Maybe heavier is good enough.). Here there seems to be trouble in the comparison with those model calculations which go along lines parallel to the infinite–mass limit approach.

The Fermilab photoproduction experiment E691 has recently completed an analysis of the angular correlation structure in this process.[20] They express the weak matrix element as follows

$$\begin{aligned}
\sqrt{4EE'}\,\langle K^*(p',\epsilon\,|J_\mu|\,D(p)\rangle &= (M+m')A_1(q^2)\epsilon_\mu \\
&- \frac{2A_2(q^2)}{(M+m')}(\epsilon\cdot p)p_\mu - \frac{2iV(q^2)}{(M+m')}\epsilon_{\mu\nu\rho\sigma}\epsilon^\nu p^\rho (p')^\sigma
\end{aligned} \tag{2.35}$$

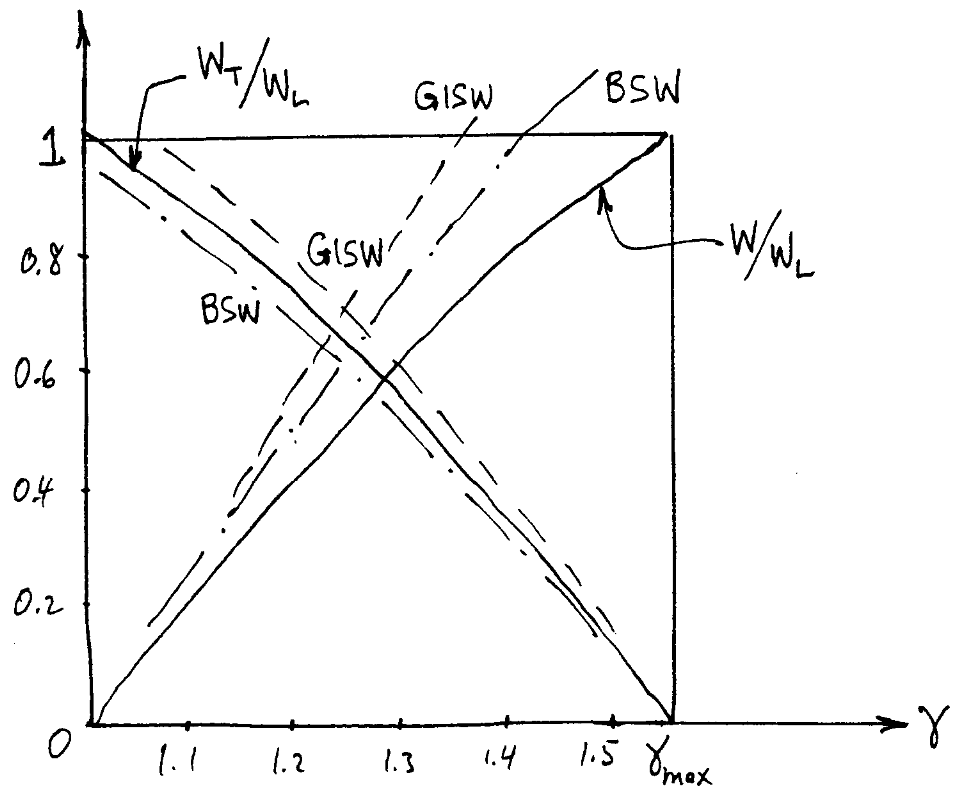

Figure 5. Predicted ratios of semileptonic partial widths. The solid line is from Wisgur; BSW and GISW are Refs. 21 and 22 respectively.

and find for the three form factors the values

$$
\begin{aligned}
A_1(0) &= 0.46 \pm 0.06 \pm 0.03 \\
A_2(0) &= 0.0 \pm 0.2 \pm 0.1 \\
V(0) &= 0.9 \pm 0.3 \pm 0.1 \ .
\end{aligned} \tag{2.36}
$$

The infinite–mass limit results are easily worked out and give for these the expressions

Table I	E691	Models IS	BW	GS	KS
$A_1(0) \to A_1(t_{\max})$	$0.46 \to 0.54$	$0.8 \to 1.0$	$0.9 \to 1.1$	$0.8 \to 0.9$	$1.0 \to 1.2$
$A_2(0) \to A_2(t_{\max})$	0.0	$0.8 \to 1.0$	$1.2 \to 1.4$	$0.6 \to 0.7$	$1.0 \to 1.2$
$V(0) \to V(t_{\max})$	$0.9 \to 1.2$	$1.1 \to 1.4$	$1.3 \to 1.7$	$1.5 \to 1.9$	$1.0 \to 1.3$
		Isgur[23] Scora	Bauer[24] Wirbel	Gilman[25] Singleton	Koerner[26] Schuler

$$\sqrt{\frac{4EE'}{MM'}}\langle K^* | J_\mu | D\rangle = \mathrm{Tr}\, \not{\epsilon} \left(\frac{\not{p}' + m'}{2m'}\right) \gamma_\mu (1-\gamma_5) \left(\frac{\not{p} + M}{2M}\right) \gamma_5 F(\hat{t})$$

$$A_1 = \frac{1}{\sqrt{M_D M_{K^*}}} \frac{(M_D m_K^* + P_D \cdot P_{K^*})}{(M_D + M_K^*)} \approx 1.2 \tag{2.37}$$

$$V = A_2 = \frac{(M_D + M_K^*)}{2\sqrt{M_D M_K^*}} \approx 1.1 \ .$$

In addition, the $B \to D$ semileptonic transition form factor F_+ also has been measured by the same group, with the result[19]

$$F_+(0) = 0.8 \pm 0.05 \pm 0.06 \ . \tag{2.38}$$

This is not too far from the expectation in the infinite–mass limit:

$$F_+(q^2_{\max}) = \frac{(m_K + m_D)}{2\sqrt{m_K m_D}} = 1.2 \ . \tag{2.39}$$

In any case we see trouble. The ratio of A_2 to V, expected to be unity in the Wisgur limit, is apparently considerably smaller. This also seems to be the case for the other axial form factor A_1, whose ratio to F_+ also seems to be considerably too small. In Table 1, as presented by the E691 collaboration, one sees that the explicit model calculations also suffer from the same disease as the infinite–mass limit approach.

It is a little hard to assess the robustness of the E691 result, since it depends upon a difficult likelihood analysis involving a function of several variables. But there does appear to be a serious problem here, and I cannot see an easy fix.

Will the infinite–mass, Wisgur limit turn out to be of use? I find it a very promising development. If the corrections can be systematized, then the model dependence of the predictions is relegated to that of the correction terms. The value of the method will end up being dependent on the size of those corrections and how well they can be kept in theoretical control. There is probably a considerable amount of work to be done before the value of the Wisgur method can really be assessed.

What are the nature of the corrections? One class is basically kinematic; reduced mass corrections and kinematic 1/M corrections, e.g. coming from small components of Dirac wave functions.[27] Other 1/M corrections are associated with the chromomagnetic interactions. Another class of corrections are associated with hard gluon emission, real or virtual, from the heavy quark system. These must be velocity–changing, so the running coupling associated with these processes will be evaluated at a heavy quark mass scale; hence be small. Some of these corrections have been worked out, in particular ratios of renormalization factors of B and D states, which differ because of the different masses.[28] This gives rise to the endemic factors

$$\left[\frac{\alpha_s(M_B^2)}{\alpha_s(M_D^2)}\right]^d \tag{2.40}$$

where the exponent is of order 1/4.

Finally there may be important effects associated with anomalous thresholds.[29] The elastic form factor of a D^*, in the Wisgur limit, should be identical to the elastic form factor of a D. But the D^* can be viewed as a loosely bound system of pion and D with a very large radius, proportional to the square root of the binding energy. In the infinite–mass limit the mass difference must be small compared to a pion mass, which isn't at all the case. While the correction may be big, one may hope that it can be accurately taken into account, because the anomalous threshold contribution can be precisely defined and calculated. But the work has to be done. A good place to start may be for the $D \to K^*$ problem.

3. Nonleptonic Decays

The theory of nonleptonic decays of kaons and even charmed mesons has been fraught with uncertainty. This does not create much cause for encouragement that things will be manageable for bottom decays. However I am guardedly optimistic that under certain circumstances some nonleptonic B decays may be comparable in cleanliness to semileptonic decays. This statement is subject to plenty of criticism. But in this lecture I will try to explain what I mean.

The starting point for describing nonleptonic B–decays is the naive, unadorned current–current Lagrangian

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} \left[V_{cb}^* \bar{b}\gamma^\mu(1-\gamma_5)c + V_{ub}^* \bar{b}\gamma^\mu(1-\gamma_5)u\right] \cdot \left[V_{ud}\bar{u}\gamma_\mu(1-\gamma_5)d + V_{us}\bar{u}\gamma_\mu(1-\gamma_5)s\right] . \tag{3.1}$$

In the following we concentrate on only the first, dominant term. There are two immediate issues to address. One is how to take matrix elements of this interaction between hadron final states. The other is how the virtual hard gluons of QCD influence the form of this interaction. With respect to the first issue, one hypothesis is that of "factorization", namely the most important contribution comes from final state configurations such that the system on one side of the exchanged W does not talk to that on the other (Fig. 6). This hypothesis in general looks quite arbitrary. But there may be circumstances where it is justified. For example in the decay

$$B_s \to D_s + \overline{D}_s \tag{3.2}$$

there may be enough relative momentum of the subsystems and small enough interquark interactions to make the final-state effects small.

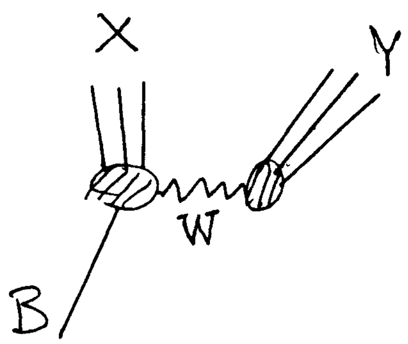

Figure 6. Factorized decay amplitude.

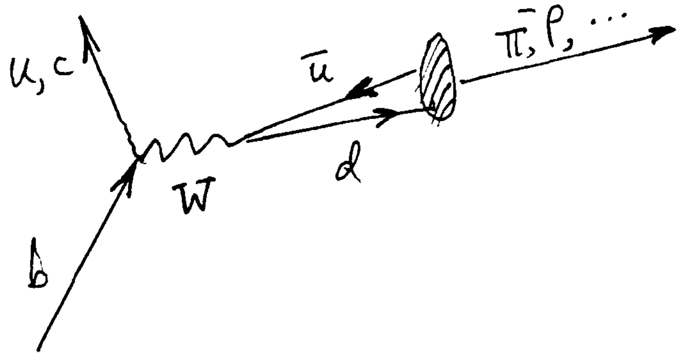

Figure 7. Factorized decay amplitude for low mass emitted meson.

Another class of processes I especially like is shown in Fig. 7. The $u\bar{d}$ system emitted from the virtual W is presumed to be of low mass, which eventually materializes into a pion or rho. It begins its life as a pointlike color singlet; furthermore it moves off with a quite large Lorentz γ, of order 5 or so. Therefore its evolution from a small, perturbative color dipole is time–dilated. By the time it grows into a large, strongly interacting hadron–like entity, it is probably several fermis away from its point of origin—and from the spectator system of the parent b quark. Therefore it is too late for the final state interaction to occur. I am told by experts that this is a well–known piece of folklore. But I don't know anywhere where the argument is laid out in detail and made respectable.[30]

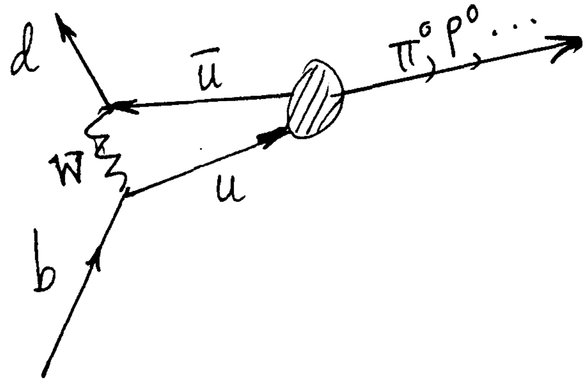

Figure 8. Factorized decay amplitude for "neutral current" contribution.

If the argument is right, it seems that it also should work for low mass neutral pairs. For example, in charmless decays (Fig. 8) one could pair up the $u\bar{u}$ system. This requires rewriting the original weak Lagrangian in charge–retention form by making a Fierz transformation. For the record the rules for these Fierz rearrangements are as follows:

$$(\bar{b}_L\gamma_\mu c_L)\,(\overline{u}_L\gamma^\mu d_L)$$

$$= \frac{1}{3}\,(\bar{b}_L\gamma_\mu d_L)\,(\overline{u}_L\gamma^\mu c_L) + 2\sum_{A=1}^{8}(\bar{b}\gamma_\mu t^A d_L)\,(\overline{u}_L\gamma^\mu t^A c_L)$$

$$\sum_{A=1}^{8}(\bar{b}_l\gamma_\mu t^A c_L)\,(\overline{u}_L\gamma^\mu t^A d_L) \tag{3.3}$$

$$= \frac{4}{9}\,(\bar{b}_L\gamma_\mu d_L)\,(\overline{u}_L\gamma^\mu c_L) - \frac{1}{3}\sum_{A=1}^{8}(\bar{b}_c\gamma^\mu t^A d_L)\,(\overline{u}_L\gamma_\mu t^A c_L)\ .$$

(Here the t^A are one–half the Gell–Mann 3×3 color matrices λ^A.) One would be tempted, therefore, to drop the color octet piece (at least with regard to calculating the decay of interest). The strength of the remainder piece is diluted in amplitude by a factor three because of this color–singlet projection.

If factorization works, the problem of nonleptonic decays is "reduced" to that of the semileptonic decays. I would not necessarily expect it to work for general, generic, multibody final states. But many of the most interesting channels are the low multiplicity states for which the above argument applies. I think it is extremely important that a careful experimental program be devoted to a critical study of how well factorization works. We will return to this question later, after the complications of hard–gluon radiative corrections are included.

The discussions of perturbative–QCD corrections to nonleptonic decays go back to the pioneering work of Gaillard and (Ben) Lee,[31] and of Altarelli and Maiani[32] more than fifteen years ago. The context was nonleptonic K decays, and the calculation was a leading–logarithm, renormalization–group analysis. This has served as the basic framework for the discussion of the b–decay corrections as well.

But because the b–mass scale is fairly large, not much is lost by looking at only the lowest order corrections. This is what will be done here, with a guide at the end as to how the first order analysis relates to what one finds in the books.

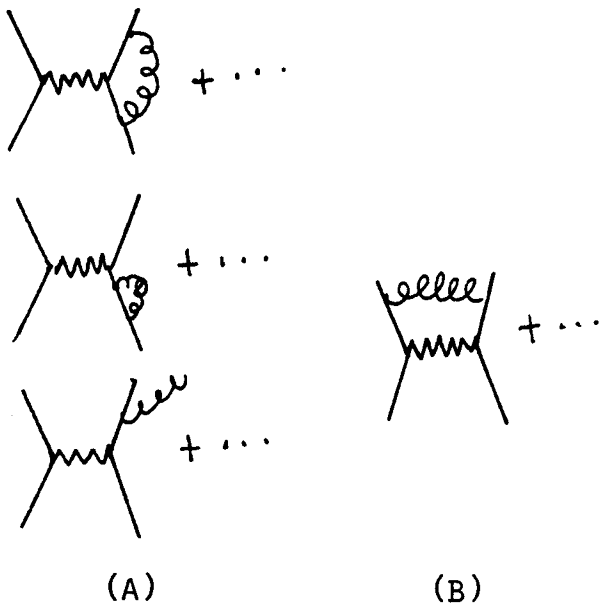

Figure 9. QCD Radiative Corrections to nonleptonic B decay.

One starts from the assertion that the effective interaction at the scale of the W mass suffers no large ultraviolet renormalization effects. To see how reasonable this is, consider the Feynman diagrams in Fig. 9. If this is to be regarded as a parton–model process, say resonant quark–antiquark scattering at the W mass, the assertion is not at all true. (This is not what Gaillard and Lee assume; they put the external quarks all highly virtual, with spacelike mass of order the W mass). But no matter what, there is no ultraviolet divergence in the "factorizable' diagrams of Fig. 9(a), because self–energy and vertex divergences cancel just as in QED. The remaining diagrams of Fig. 9(b) converge and have no large logarithms. But logarithms will be generated as the energy scale for the process goes down, because the W propagator effectively contracts to a point and the remaining amplitude is a vertex part cut off at the W mass.

What about the factorizable pieces in Fig. 9(a)? If the external lines are treated as partons, *i.e.* more or less on-mass-shell, their QCD radiative corrections will be much like those in electron-positron annihilation. The total decay width into hadron states with $u\overline{d}$ quantum numbers will suffer only a minor radiative correction

$$\frac{\Gamma(W^+ \to u\overline{d} + \text{gluons, etc.})}{\Gamma(W^+ \to e^+\nu_e)} = |V_{ud}|^2 \left(1 + \frac{\alpha_s}{\pi} + \ldots\right) . \tag{3.4}$$

But if the final state is restricted to only collinear u and $\overline{d}$ jets and no extra gluon jets there will be a big form factor effect. Thus, experience with e^+e^- radiative effects, along with the fact that the presence of factorizable radiative corrections does not affect the factorization hypothesis, encourages us to omit from further consideration the factorizable pieces and only look at the remainder. It is clear that in the remaining terms of Fig. 9(b) the exchange must include a unit of charge and an octet of color. Since fermion masses are neglected, helicity is conserved. Up to an overall coefficient this determines the basic form of the correction to be

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} V_{cb} \cdot f \cdot \frac{\alpha_s}{\pi} \ell n \frac{m_W^2}{\mu^2} \cdot [\overline{b}\gamma_\mu(1-\gamma_5)t^A c]\,[\overline{u}\gamma^\mu(1-\gamma_5)t^A d] \tag{3.5}$$

where μ is the low mass scale of interest. The Feynman diagram calculation gives the value of the coefficient to be

$$f = -\frac{3}{2} \, . \tag{3.6}$$

(There is still a question, not to be considered further here, of how this piece behaves for "parton–model" external lines, and whether infrared effects occur, such as for the factorizable piece.)

We conclude that for the original W–exchange channel, there is no first–order correction to the factorization approximation to consider. But of course, Fierz rearrangement of this correction will give a color–singlet neutral–exchange contribution to add to what one gets from the Fierz rearrangement of the leading term:

$$\text{“}\mathcal{M}_{NC}\text{”} = \frac{G_F}{\sqrt{2}} V_{cb} \left[\bar{b}\gamma_\mu(1-\gamma_5)d\right] \left[\bar{u}\gamma^\mu(1-\gamma_5)c\right] \left\{\frac{1}{3}\cdot\frac{4}{9} f \cdot \frac{\alpha_s}{\pi} \ell n \frac{m_W^2}{\mu^2}\right\} . \quad (3.7)$$

Note that the radiative correction leads to a significant suppression of the leading order contribution (cf. Eq. (3.3)).

It is now time to make contact with the formalism found in the literature.[33] In order to sum the leading logarithms, a different combination of interaction terms is introduced. Before radiative corrections one writes for the leading term alone

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} V_{cb} \begin{cases} \frac{1}{2}\left[\bar{b}\gamma_\mu(1-\gamma_5)c\cdot\bar{u}\gamma^\mu(1-\gamma_5)d + \bar{b}\gamma_\mu(1-\gamma_5)d\cdot\bar{u}\gamma^\mu(1-\gamma_5)c\right] \\ +\frac{1}{2}\left[\bar{b}\gamma_\mu(1-\gamma_5)c\cdot\bar{u}\gamma^\mu(1-\gamma_5)d - \bar{b}\gamma_\mu(1-\gamma_5)d\cdot\bar{u}\gamma^\mu(1-\gamma_5)c\right] . \end{cases} \quad (3.8)$$

This is done because these are the combinations that get multiplicatively renormalized when the higher order effects (which for the b–physics applications are not very big) are included. As already mentioned, the solution of renormalization–group equations under these circumstances always gives a factor

$$c_\pm = \left(\frac{\alpha_s(\mu^2)}{\alpha_s(m_W^2)}\right)^{-d_\pm} . \quad (3.9)$$

Therefore the first line of the equation above gets multiplied by c_+, while the second line gets multiplied by c_-.

Writing for the running coupling constant

$$\frac{1}{\alpha_s(m_W^2)} \cong \frac{1}{\alpha_s(\mu^2)} + \frac{b}{\pi}\,\ell n\,\frac{m_W^2}{\mu^2} \tag{3.10}$$

with

$$b = \frac{33 - 2n_f}{12} \approx 2.1 \tag{3.11}$$

and expanding the renormalization factors out to first order gives

$$c_\pm = \left[1 + \frac{b}{\pi}\,\alpha_s(\mu^2)\,\ell n\,\frac{m_W^2}{\mu^2}\right]^{-d_\pm} \approx 1 - (bd_\pm)\,\frac{\alpha_s}{\pi}\,\ell n\,\frac{m_W^2}{\mu^2}\,. \tag{3.12}$$

In our case the fact that the first order correction is pure color and charge exchange, along with the Fierz identities above, allows the radiative correction to be written solely in terms of color-singlet–exchange operators:

$$\begin{aligned}
\mathcal{L} &= \frac{G_F}{\sqrt{2}}\,V_{cb}\Big\{\bar{b}\gamma_\mu(1-\gamma_5)c\cdot\bar{u}\gamma^\mu(1-\gamma_5)d \\
&\quad + f\cdot\frac{\alpha_s}{\pi}\,\ell n\,\frac{m_W^2}{\mu^2}\sum_A \bar{b}\gamma_\mu t^A(1-\gamma_5)c\cdot\bar{u}\gamma^\mu t^A(1-\gamma_5)d\Big\} \\
&= \frac{G_F}{\sqrt{2}}\,V_{cb}\Big\{\bar{b}\gamma_\mu(1-\gamma_5)c\cdot\bar{u}\gamma^\mu(1-\gamma_5)d\left(1-\frac{f}{6}\cdot\frac{\alpha_s}{\pi}\,\ell n\,\frac{m_W^2}{\mu^2}\right) \\
&\quad + \frac{f}{2}\cdot\frac{\alpha_s}{\pi}\,\ell n\,\frac{m_W^2}{\mu^2}\cdot\bar{b}\gamma_\mu(1-\gamma_5)d\cdot\bar{u}\gamma_\mu(1-\gamma_5)c\Big\}
\end{aligned} \tag{3.13}$$

and hence

$$\begin{aligned}
c_1 &\equiv \frac{1}{2}(c_+ + c_-) \cong 1 - \frac{f}{6}\cdot\frac{\alpha_s}{\pi}\,\ell n\,\frac{m_W^2}{\mu^2} \\
c_2 &\equiv \frac{1}{2}(c_+ - c_-) \cong \frac{f}{2}\cdot\frac{\alpha_s}{\pi}\,\ell n\,\frac{m_W^2}{\mu^2}\,.
\end{aligned} \tag{3.14}$$

Use of the definitions of the anomalous dimensions in Eq. (3.9) allows the determinations

$$bd_+ = -\frac{f}{3} = +\frac{1}{2} \qquad bd_- = +\frac{2f}{3} = -1 \; . \tag{3.15}$$

The choice of renormalization scale μ is naturally taken at the mass of the b quark. Thus one gets for the numerical value of the expansion parameter

$$\frac{\alpha_s}{\pi}\, \ell n \,\frac{m_W^2}{m_b^2} \approx 0.3 \; . \tag{3.16}$$

The effective Lagrangian is conventionally quoted as follows

$$\begin{aligned} \mathcal{L}_{\text{eff}} = \frac{G_F}{\sqrt{2}}\, V_{cb} \Big\{ & c_1 \bar{b}\gamma_\mu(1-\gamma_5)c \cdot \bar{u}\gamma^\mu(1-\gamma_5)d \\ & + c_2 \bar{b}\gamma_\mu(1-\gamma_5)d \cdot \bar{c}\gamma^\mu(1-\gamma_5)u \Big\} \; . \end{aligned} \tag{3.17}$$

By our estimates

$$c_1 \approx 1.07 \qquad c_2 \approx -0.23 \; . \tag{3.18}$$

The more official numbers are

$$c_1 = 1.13 \qquad c_2 = -0.29 \; . \tag{3.19}$$

Once the effective lagrangian has been written down, either in first order or with the higher orders included, there still is a question of double counting as whether the Fierz–rearranged pieces should be taken into account phenomenologically. Bauer, Stech and Wirbel,[34] in perhaps the most comprehensive study of nonleptonic decays done so far, include a parameter ζ in their analyses

$$a_1 = c_1 + \zeta c_2 \qquad a_2 = c_2 + \zeta c_1 \; . \tag{3.20}$$

A vanishing ζ corresponds to omitting the Fierz rearrangement and a ζ of $1/3$ corresponds to keeping it all. In their analyses of charm decays a vanishing ζ

seems to be preferred, although it seems to me the value 1/3 has more logical consistency. But the case for factorization made above is very weak for the charm decays.

A compendium of BSW predictions and data are given in the accompanying tables. The data has been provided by the Argus[35] and CLEO[36] collaborations this year. In general their calculations (which do depend upon their model of semileptonic decays!) work quite well. In particular the Argus group fits their branching ratios to the model predictions and obtains as best values (assuming a_2 is negative)

$$a_1 = 1.03 \pm 0.09 \qquad a_2 = -0.20 \pm 0.03 \qquad (\chi^2 = 6.5/10)\ . \tag{3.21}$$

While again a small ζ seems to be preferred, the success of this fit is grounds for encouragement that factorization works. But this is not a substitute for the direct, model independent experimental tests of factorization.[37] Within the present data set, there are already some fairly direct tests. We can classify the data as belonging to three categories. The first is D or D^* plus pions, the second is ψ plus K plus possible pions, and the third is D or D^* plus D_s.

Observation of the tables invites several checks. The $\rho/\pi/a_1$ ratios are without uncertainty because the coupling of charged weak current to those states are determined in tau–lepton decays (cf. Fig. 10). The Argus group quotes, for example

$$\frac{\Gamma(\overline{B} \to D^+\rho^-)}{\Gamma(\overline{B} \to D^+\pi^-)} = 3.2 \pm 1.2 \tag{3.22}$$

to be compared with the BSW estimate of Eq. (2.6). Also

$$\frac{\Gamma(B \to D^*\rho)}{\Gamma(B \to D^*\pi)} = 2.5 \pm 1.2 \tag{3.23}$$

B^- decay modes

B decay	signal events	branching ratio
$B^- \to D^0\pi^-$	12 ± 5	$(0.20 \pm 0.08 \pm 0.06)\%$
$B^- \to D^0\rho^-$	19 ± 6	$(1.3 \pm 0.4 \pm 0.4)\%$
$B^- \to D^{*0}\pi^-$	9 ± 3	$(0.40 \pm 0.14 \pm 0.12)\%$
$B^- \to D^{*0}\rho^-$	7 ± 4	$(1.0 \pm 0.6 \pm 0.4)\%$
$B^- \to D^{*+}\pi^-\pi^-$	11 ± 6	$(0.26 \pm 0.14 \pm 0.07)\%$
$B^- \to {D^{(*)}}^0_J\rho^-$	6 ± 3	see text
$B^- \to D^{*+}\pi^-\pi^-\pi^0$	26 ± 10	$(1.8 \pm 0.7 \pm 0.5)\%$
$B^- \to {D^{(*)}}^0_J\pi^-$	5 ± 3	see text
$B^- \to D^{*+}\pi^-\pi^-\pi^-\pi^+$	< 9	$< 1.0\%$ at 90% C.L.
$B^- \to J/\psi K^-$	6	$(0.07 \pm 0.03 \pm 0.01)\%$
$B^- \to \psi' K^-$	5	$(0.18 \pm 0.08 \pm 0.04)\%$
$B^- \to J/\psi K^{*-}$	2	$(0.16 \pm 0.11 \pm 0.03)\%$
$B^- \to \psi' K^{*-}$	< 3.9	$< 0.49\%$ at 90% C.L.
$B^- \to J/\psi K^-\pi^+\pi^-$	< 8	$< 0.16\%$ at 90% C.L.
$B^- \to \psi' K^-\pi^+\pi^-$	3	$(0.19 \pm 0.11 \pm 0.04)\%$

$\overline{B}^0$ decay modes

B decay	signal events	branching ratio
$\overline{B}^0 \to D^-\pi^-$	22 ± 5	$(0.48 \pm 0.11 \pm 0.11)\%$
$\overline{B}^0 \to D^-\rho^-$	9 ± 5	$(0.9 \pm 0.5 \pm 0.3)\%$
$\overline{B}^0 \to D^{*-}\pi^-$	12 ± 4	$(0.28 \pm 0.09 \pm 0.06)\%$
$\overline{B}^0 \to D^{*+}\rho^-\pi^0$	51 ± 10	$(1.8 \pm 0.4 \pm 0.5)\%$
$\overline{B}^0 \to D^{*+}\rho^-$	19 ± 9	$(0.7 \pm 0.3 \pm 0.3)\%$
$\overline{B}^0 \to D^{*+}\pi^-\pi^-\pi^+$	26 ± 7	$(1.2 \pm 0.3 \pm 0.4)\%$
$\overline{B}^0 \to D^{*+}\pi^-\pi^-\pi^+\pi^0$	28 ± 10	$(4.1 \pm 1.5 \pm 1.6)\%$
$\overline{B}^0 \to J/\psi K^0_S$	2	$(0.04 \pm 0.03 \pm 0.01)\%$
$\overline{B}^0 \to \psi' K^0_S$	< 2.3	$< 0.14\%$ at 90% C.L.
$\overline{B}^0 \to J/\psi \overline{K}^{*0}$	6	$(0.11 = 0.05 \pm 0.02)\%$
$\overline{B}^0 \to \psi' \overline{K}^{*0}$	< 3.9	$< 0.23\%$ at 90% C.L.
$\overline{B}^0 \to \psi' K^-\pi^+$	< 2.3	$< 0.10\%$ at 90% C.L.

Table 2: ARGUS[35] data on B decays.

B Branching Ratios (%)

Mode	CLEO 1987	CLEO 1985†	ARGUS	Bauer, et al. Model[2]
$B^- \to D^0\pi^-$	$0.44 \pm 0.07 \pm 0.07$	$0.54 \pm 0.17 \pm 0.11$	$0.20 \pm 0.08 \pm 0.06$	$0.48(a_1 + 0.75a_2)^2$
$B^- \to D^{*+}\pi^-\pi^-$	< 0.4	$0.23 \pm 0.15 \pm 0.07$	$0.26 \pm 0.14 \pm 0.07$	
$B^- \to \psi K^-$	$0.08 \pm 0.02 \pm 0.02$	$0.10 \pm 0.07 \pm 0.2$	$0.07 \pm 0.03 \pm 0.01$	$1.01\ a_2^2$
$B^- \to \psi K^{*-}$	$0.13 \pm 0.09 \pm 0.03$		$0.16 \pm 0.11 \pm 0.03$	$4.33\ a_2^2$
$B^- \to \psi K^-\pi^+\pi^-$	$0.12 \pm 0.06 \pm 0.03$		< 0.16	
$B^- \to \psi' K^-$	< 0.05		$0.18 \pm 0.08 \pm 0.04$	$0.28\ a_2^2$
$B^- \to \psi' K^{*-}$	< 0.35		< 0.49	$1.91\ a_2^2$
$B^- \to D^0 D_s^-$	$1.8 \pm 0.8 \pm 0.8$			$0.73\ a_1^2$
$\overline{B}^0 \to D^+\pi^-$	$0.25 \pm 0.06 \pm 0.04$	$0.51 \pm 0.27 \pm 0.14$	$0.48 \pm 0.11 \pm 0.11$	$0.48a_1^2$
$\overline{B}^0 \to D^{*+}\pi^-$	$0.36 \pm 0.09 \pm 0.07$	$0.27 \pm 0.13 \pm 0.08$	$0.28 \pm 0.09 \pm 0.06$	$0.37a_1^2$
$\overline{B}^0 \to D^{*+}\rho^-$	$1.9 \pm 0.9 \pm 1.3$		$0.7 \pm 0.3 \pm 0.3$	$1.18a_1^2$
$\overline{B}^0 \to D^{*+}\ a_1^-$	$2.6 \pm 0.5 \pm 0.6$			$1.63a_1^2$
$\overline{B}^0 \to D^0\rho^0$	< 0.1			$0.07a_2^2$
$\overline{B}^0 \to \psi\overline{K}^0$	$0.06 \pm 0.03 \pm 0.02$		$0.08 \pm 0.06 \pm 0.02$	$1.02a_2^2$
$\overline{B}^0 \to \psi\overline{K}^{*0}$	$0.11 \pm 0.05 \pm 0.03$	$0.35 \pm 0.16 \pm 0.03$	$0.11 \pm 0.05 \pm 0.02$	$4.36a_2^2$
$\overline{B}^0 \to \psi K^-\pi^+$	$0.10 \pm 0.04 \pm 0.03$		< 0.10	
$\overline{B}^0 \to \psi'\overline{K}^0$	< 0.15		< 0.28	$0.28a_2^2$
$\overline{B}^0 \to \psi'\overline{K}^{*0}$	$0.14 \pm 0.08 \pm 0.04$		< 0.23	$1.91a_2^2$
$\overline{B}^0 \to D^+D_s^-$	$0.75 \pm 0.21 \pm 0.32$			$0.67a_1^2$
$\overline{B}^0 \to D^{*+}D_s^-$	$1.5 \pm 0.9 \pm 0.7$			$0.30a_1^2$

† The previous CLEO results have been renormalized for equal charged and neutral B production on the $\Upsilon(4S)$.

Table 3: CLEO data[36] on B decays.

Decay Mode	Theory	Decay Mode	Theory
$\overline{B}^0 \to D^+\pi^-$	$0.48\ a_1^2$	$B^- \to D^0\pi^-$	$0.48\ (a_1 + 0.75a_2)^2$
$\overline{B}^0 \to D^+\rho^-$	$1.25\ a_1^2$	$B^- \to D^0\rho^-$	$1.25\ (a_1 + 0.34\ a_2)^2$
$\overline{B}^0 \to D^+\pi^-$	$0.37\ a_1^2$	$B^- \to D^{*0}\pi^-$	$0.37\ (a_1 + 1.04\ a_2)^2$
$\overline{B}^0 \to D^+\rho^-$	$1.18\ a_1^2$	$B^- \to D^{*0}\rho^-$	$1.18\ (a_1 + 0.79\ a_2)^2$
$\overline{B}^0 \to D^+D_S^-$	$0.67\ a_1^2$	$B^- \to D^0D_S^-$	$0.67\ a_1^2$
$\overline{B}^0 \to D^+D_S^-$	$0.73\ a_1^2$	$B^- \to D^0D_S^{*-}$	$0.73\ a_1^2$
$\overline{B}^0 \to D^+D_S^-$	$0.30\ a_1^2$	$B^- \to D^{*0}D_S^-$	$0.30\ a_1^2$
$\overline{B}^0 \to D^+D_S^-$	$2.03\ a_1^2$	$B^- \to D^{*0}D_S^{*-}$	$2.02\ a_1^2$
$\overline{B}^0 \to \pi^+\pi^-$	$0.17\ a_1^2\|V_{ub}/V_{cb}\|^2$	$B^- \to \pi^0\pi^-$	$0.08\ (a_1 + 1.00\ a_2)^2\|V_{ub}/V_{cb}\|^2$
$\overline{B}^0 \to \pi^+\rho^-$	$0.46\ a_1^2\|V_{ub}/V_{cb}\|^2$	$B^- \to \pi^0\rho^-$	$0.23\ (a_1 + 0.50\ a_2)^2\|V_{ub}/V_{cb}\|^2$
$\overline{B}^0 \to \rho^+\pi^-$	$0.11\ a_1^2\|V_{ub}/V_{cb}\|^2$	$B^- \to \rho^0\pi^-$	$0.06\ (a_1 + 2.01\ a_2)^2\|V_{ub}/V_{cb}\|^2$
$\overline{B}^0 \to \rho^+\rho^-$	$0.37\ a_1^2\|V_{ub}/V_{cb}\|^2$	$B^- \to \rho^0\rho^-$	$0.19\ (a_1 + 1.00\ a_2)^2\|V_{ub}/V_{cb}\|^2$
$\overline{B}^0 \to \pi^+D_S^-$	$0.28\ a_1^2\|V_{ub}/V_{cb}\|^2$	$B^- \to \pi^0D_S^-$	$0.13\ a_1^2\|V_{ub}/V_{cb}\|^2$
$\overline{B}^0 \to \pi^+D_S^-$	$0.40\ a_1^2\|V_{ub}/V_{cb}\|^2$	$B^- \to \pi^0D_S^{*-}$	$0.19\ a_1^2\|V_{ub}/V_{cb}\|^2$
$\overline{B}^0 \to \rho^+D_S^-$	$0.13\ a_1^2\|V_{ub}/V_{cb}\|^2$	$B^- \to \rho^0D_S^-$	$0.07\ a_1^2\|V_{ub}/V_{cb}\|^2$
$\overline{B}^0 \to \rho^+D_S^-$	$0.82\ a_1^2\|V_{ub}/V_{cb}\|^2$	$B^- \to \rho^0D_S^{*-}$	$0.41\ a_1^2\|V_{ub}/V_{cb}\|^2$
$\overline{B}^0 \to \pi^0D^0$	$0.13\ a_2^2$		
$\overline{B}^0 \to \pi^0D^0$	$0.19\ a_2^2$		
$\overline{B}^0 \to \rho^0D^0$	$0.07\ a_2^2$		
$\overline{B}^0 \to \rho^0D^0$	$0.38\ a_2^2$		
$\overline{B}^0 \to \overline{K}^0J/\psi$	$1.02\ a_2^2$	$B^- \to K^-J/\psi$	$1.01\ a_2^2$
$\overline{B}^0 \to \overline{K}^0J/\psi$	$4.36\ a_2^2$	$B^- \to K^{*-}J/\psi$	$4.33\ a_2^2$
$\overline{B}^0 \to D^+D^-$	$4.10^{-2}\ a_1^2$		
$\overline{B}^0 \to D^+D^-$	$4.10^{-2}\ a_1^2$		
$\overline{B}^0 \to D^0\overline{K}^0$	$2.10^{-2}\ a_2^2$		
$\overline{B}^0 \to D^{+0}\overline{K}^0$	$2.10^{-2}\ a_2^2$		

Table 4: Branching ratios (given in %) for two-particle decay modes of B. $|V_{cb}| = .05$ has been used for the theoretical predictions. From Wirbel[38].

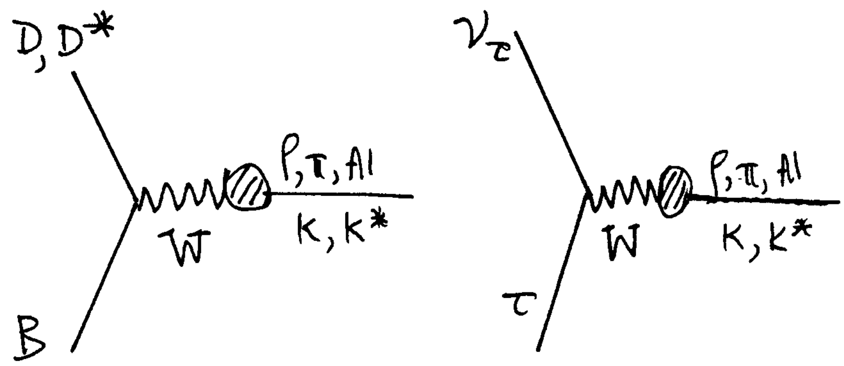

Figure 10. Factorized amplitudes in B and τ Decay.

in fine agreement with the BSW expectation of 3.0. The Cornell measurement

$$\frac{\Gamma(\overline{B}^0 \to D^{*+}\rho^-)}{\Gamma(\overline{B}^0 \to D^{*+}\pi^-)} = 5.3 \pm 2.6 \pm 2.9 \tag{3.24}$$

also is in reasonable agreement with the prediction.

The above predictions evidently depend upon the models of semileptonic form factors used. However, if one puts together the infinite–mass–limit Wisgur predictions together with the factorization hypothesis, there are no free parameters. In addition the final states obtained by replacement of D by D^*, or vice versa, are related. For example,

$$\frac{\Gamma(B \to D^*\pi^+)}{\Gamma(B \to D\pi^+)} = \frac{\sum_\epsilon |\mathrm{Tr}\, \not{\epsilon}(\not{P}_D + M_D)\not{q}(1-\gamma_5)(\not{P}_B + M_B)\gamma_5|^2}{|\mathrm{Tr}\, \gamma_5(\not{P}_D + M_D)\not{q}(1-\gamma_5)(\not{P}_B + M_B)\gamma_5|^2} = 1\ . \tag{3.25}$$

This agrees well both with the BSW estimate of 0.8 and the data.

An interesting channel is $D^0 + \pi^-$, because the neutral–current piece interferes with the charged current piece (Fig. 11) The ratio (which has considerable model

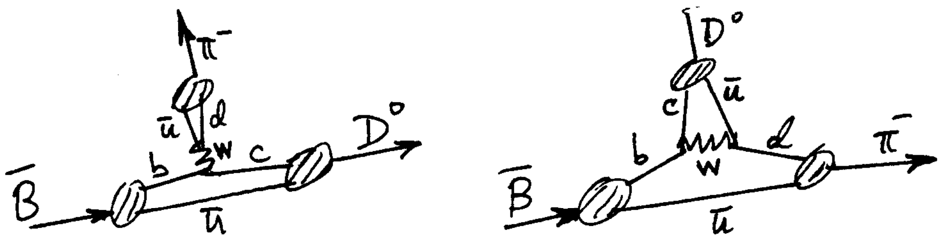

Figure 11. Mechanisms for the decay $B \to D^0\pi^-$.

dependence in it)

$$\frac{\Gamma(B^- \to D^0\pi^-)}{\Gamma(B^0 \to D^+\pi^-)} = \left(1 + 0.75\,\frac{a_2}{a_1}\right)^2 \sim (0.8)^2 \tag{3.26}$$

tests its presence, but as yet the data is inconclusive.

Note that upon assuming factorization and the Wisgur limit, the process

$$\frac{d\Gamma}{dq^2}\,(B_d \to D, D^* + X + \ell\nu)\Big|_{q^2=(\ell+\nu)^2=m_\pi^2} \tag{3.27}$$

is related to

$$B \to D, D^* + X + \pi\ . \tag{3.28}$$

This is the endpoint region of the semileptonic decays where $\hat{t}$ is largest, and form factors matter the most. The elastic D and D^* channels will be suppressed by at least a factor two by form factor effects. But the total yield, according to the sum rule, does not decrease. Therefore higher states such as $D^* + \pi$ and/or D^{**} should be considerably more prominent than they appear in the overall semileptonic branching ratios. This is clearly evidenced by the data,[35] where the branching ratio

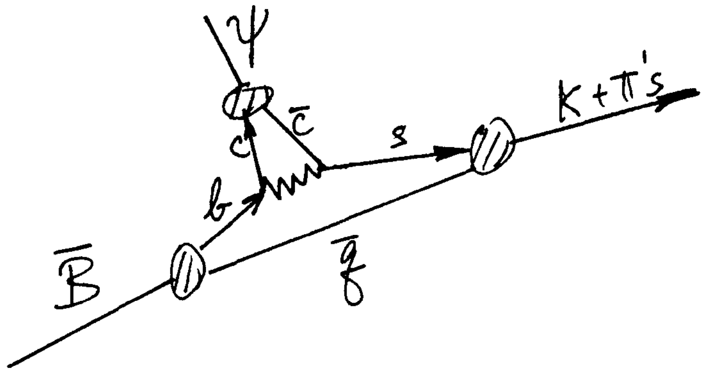

Figure 12. Mechanism for B decay to ψ final states.

for $D^{*+}\pi^-\pi^-$ is just as large as others. Argus in particular has seen fairly good evidence for D^{**} resonances in these higher mass final states as well.

A second class of decays involve psi final states. For this class of processes, neutral current factorization seems to me to be eminently reasonable, namely that the $c - \bar{c}$ onium system does not have significant final state interactions with the remainder of the system (Fig. 12). Because these processes sense only a_2, they are an excellent testing ground for the correctness of factorization and the presence of the big, destructively interfering QCD radiative correction.

$$\frac{\Gamma(B \to \psi K^*)}{\Gamma(B \to \psi K)} \lesssim 2 \pm 1 \ . \tag{3.29}$$

A good test is in ψ'-to-ψ ratios; the expectation is

$$\frac{\Gamma(B \to \psi' K^*)}{\Gamma(B \to \psi K^*)} = 0.7 \qquad \frac{\Gamma(B \to \psi' K)}{\Gamma(B \to \psi K)} = 0.28 \tag{3.30}$$

although I don't see the reason for BSW getting such a big difference in the ratios for K and K^* respectively. In any case, the data is only barely emergent:

$$\frac{\Gamma(B \to \psi' K^*)}{\Gamma(B \to \psi K^*)} \approx 1.3 \pm 0.8 \qquad \frac{\Gamma(B \to \psi' K)}{\Gamma(B \to \psi K)} < 0.6 \ . \tag{3.31}$$

Finally the channels D/D^* plus D_s again provide a combined check of factorization and Wisgur. I have not worked this one out. There are new theoretical contributions on the subject.[39] I would be surprised if the answer differs a lot from BSW, who give

$$\frac{\Gamma(\overline{B} \to D^* D_s^-)}{\Gamma(\overline{B} \to D^+ D_s^-)} \approx 0.45 \ . \tag{3.32}$$

The data are

$$\frac{\Gamma(\overline{B} \to D^{*+} D_s^-)}{\Gamma(\overline{B} \to D^+ D_s^-)} \sim 2.0 \pm 1.2 \ . \tag{3.33}$$

I conclude from all this that the BSW approach looks pretty good, but that the really quantitative, model–independent tests are still in the future. This is a very important issue, because the predictions for the very rare decay amplitudes proportional to V_{ub} are done the same way, as are those for "Penguin" processes. Both classes of decays are vital in a large variety of CP–violation measurements. So far there are many calculations and a large number of experimental limits, some of which are close to the predictions. But these will not be discussed here.

4. Example of a CP–Violating Process: $B_d \to \pi^+\pi^-\pi^0$

We conclude with a prototype of the kind of studies of CP–violating effects in the B system which is being pursued so actively nowadays by both experimentalists and theorists. The process I have chosen has some of the richness of complicated cases under study and the simplicity of the by now classic channels discussed in the first lecture.

The decay

$$B_d \rightarrow \pi^+ \quad \pi^- \quad \pi^0 \tag{4.1}$$

can be described by specifying the amplitudes for producing the pions at a given point of the Dalitz plot (Fig. 13).[40]

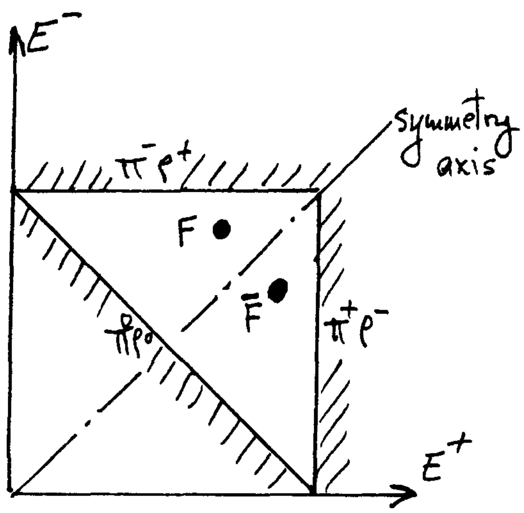

Figure 13. Dalitz plot for $B_d \rightarrow 3\pi$.

We see from the figure that there are probably three regions of importance corresponding to collinear final state configurations with any one of the three pions being the isolated one. The interior of the Dalitz triangle is very likely to be quite sparsely populated, although it is not at all out of the question that events will in fact be found there. The angular momentum of the low mass dipion, if formed from a $q-\overline{q}$ pair created in the weak transition, must be unity; hence a ρ. If this is not the case, and the dipion includes the absorption of the spectator system, then its angular momentum can be anything. Spins 0, 1, and 2, for both charged and neutral pion pairs, are all interesting.

According to the diagrams in Fig. 14 we see that the horizontal and vertical edges of the Dalitz plot will be fed by both B and $\overline{B}$ via charged–current factorization amplitudes. The diagonal edge is neutral–current, again fed by B and $\overline{B}$, but no doubt relatively small. And on the horizontal and vertical edges of the triangle, the "background" of non–p–wave dipions comes only from B or $\overline{B}$, not both. But a background can in general be expected.

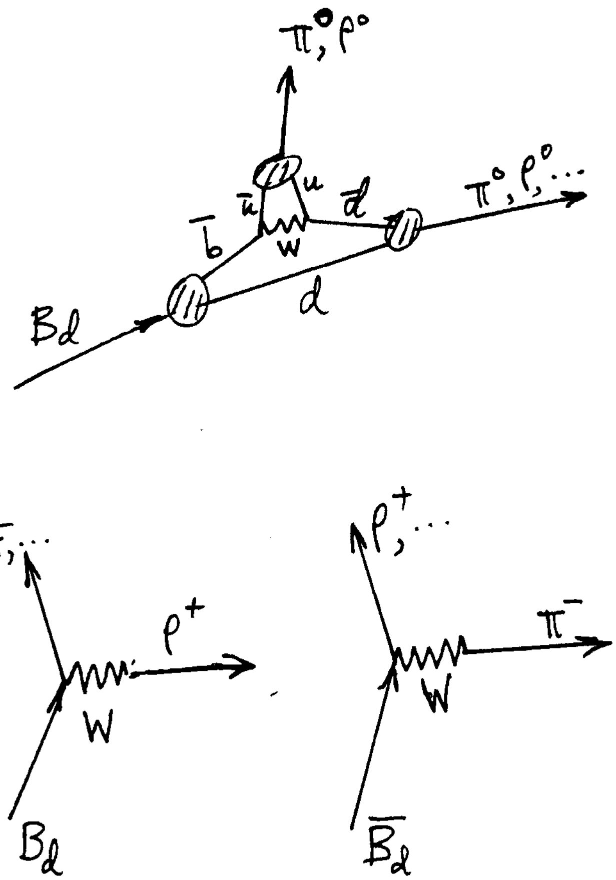

Figure 14. Decay mechanisms for $B_d \to 3\pi$.

Let us now generalize the analysis of the time–dependent interference effects expected when a B_d is produced in association with a $\overline{B}$ whose identity is known with certainty. (There are complications when the process occurs at the $\Upsilon(4S)$ and the associated particle is a neutral B itself (undergoing mixing). The quantum mechanics is beautiful but does not change the essence of the CP–violation physics we are discussing here.)[41] We need four different decay amplitudes, namely

$$\begin{aligned}
\mathcal{M}(B_d \to F) &= M e^{i(\gamma+\delta)} \\
\mathcal{M}(B_d \to \overline{F}) &= \overline{M} e^{i(\gamma+\overline{\delta})} \\
\mathcal{M}(\overline{B}_d \to F) &= \overline{M} e^{i(-\gamma+\overline{\delta})} \\
\mathcal{M}(\overline{B}_d \to \overline{F}) &= M e^{i(-\gamma+\delta)} \ .
\end{aligned} \tag{4.2}$$

Here the labels F and $\overline{F}$ denote the locations in the Dalitz plot of the final states. F is related to $\overline{F}$ by reflection about the diagonal, *i.e.* interchange of π^+ with π^-. The amplitudes M and $\overline{M}$ are defined as real, and their phases are explicitly exhibited. The CKM phase, namely the phase of V_{ub}, is denoted by γ. Now we can put these expressions into the time–dependent amplitudes we had in the first lecture

$$\begin{aligned}
|B_d(t)\rangle &= \left[|B_d\rangle \cos \frac{\Delta m t}{2} - i\lambda \left|\overline{B}_d\right\rangle \sin \frac{\Delta m t}{2} \right] e^{-\Gamma t/2} \\
\left|\overline{B}_d(t)\right\rangle &= \left[\left|\overline{B}_d\right\rangle \cos \frac{\Delta m t}{2} - i\lambda^* \left|B_d\right\rangle \sin \frac{\Delta m t}{2} \right] e^{-\Gamma t/2}
\end{aligned} \tag{4.3}$$

and obtain

$$|\mathcal{M}(B_d \to F)|^2 = \left| e^{i(\gamma+\delta)} M \cos \frac{\Delta m t}{2} - i\lambda\, e^{-i(\gamma-\delta)} \overline{M} \sin \frac{\Delta m t}{2} \right|^2 e^{-\Gamma t}, \ldots \tag{4.4}$$

Thus, in obvious notation,

$$
\begin{aligned}
\frac{1}{n}\frac{dn}{dt}(B_d(t) \to F) = \Bigg[1 &+ \left(\frac{M^2 - \overline{M}^2}{M^2 + \overline{M}^2}\right) \cos \Delta mt \\
&+ \left(\frac{2M\overline{M}}{M^2 + \overline{M}^2}\right) \sin(2\alpha - \Delta) \sin \Delta mt\Bigg] e^{-\Gamma t} \\
\frac{1}{n}\frac{dn}{dt}(B_d(t) \to \overline{F}) = \Bigg[1 &- \left(\frac{M^2 - \overline{M}^2}{M^2 + \overline{M}^2}\right) \cos \Delta mt \\
&- \left(\frac{2M\overline{M}}{M^2 + \overline{M}^2}\right) \sin(2\alpha + \Delta) \sin \Delta mt\Bigg] e^{-\Gamma t} \\
\frac{1}{n}\frac{dn}{dt}(\overline{B}_d(t) \to F) = \Bigg[1 &- \left(\frac{M^2 - \overline{M}^2}{M^2 + \overline{M}^2}\right) \cos \Delta mt \\
&- \left(\frac{2M\overline{M}}{M^2 + \overline{M}^2}\right) \sin(2\alpha - \Delta) \sin \Delta mt\Bigg] e^{-\Gamma t} \\
\frac{1}{n}\frac{dn}{dt}(\overline{B}_d(t) \to \overline{F}) = \Bigg[1 &+ \left(\frac{M^2 - \overline{M}^2}{M^2 + \overline{M}^2}\right) \cos \Delta mt \\
&+ \left(\frac{2M\overline{M}}{M^2 + \overline{M}^2}\right) \sin(2\alpha + \Delta) \sin \Delta mt\Bigg] e^{-\Gamma t}
\end{aligned}
\tag{4.5}
$$

where α is the vertex angle of the unitarity triangle and

$$
\Delta = \delta - \overline{\delta} \tag{4.6}
$$

is the strong-interaction phase difference of the two amplitudes into F and $\overline{F}$. Right away, we see that averaging over the identity of the initial $B/\overline{B}$ removes all oscillatory contributions to the decay into the final state F (or $\overline{F}$). Thus the basic asymmetry to consider is

$$
\begin{aligned}
A_F(t) &= \frac{n(B_d(t) \to F) - n(\overline{B}_d(t) \to F)}{n(B_d(t) \to F) + n(\overline{B}_d(t) \to F)} \\
&= \left(\frac{M^2 - \overline{M}^2}{M^2 + \overline{M}^2}\right) \cos \Delta m t + \left(\frac{2M\overline{M}}{M^2 + \overline{M}^2}\right) \sin(2\alpha - \Delta) sin \Delta m t \ .
\end{aligned}
\tag{4.7}
$$

This asymmetry averaged over F and $\overline{F}$ gives

$$
\frac{1}{2}\left(A_F + A_{\overline{F}}\right) = -\left(\frac{2M\overline{M}}{M^2 + \overline{M}^2}\right) (\cos 2\alpha)(\sin \Delta) \sin \Delta m t
\tag{4.8}
$$

and vanishes in the absence of relative final–state phases δ. On the other hand, the double asymmetry survives even in the absence of final–state effects associated with δ:

$$
\begin{aligned}
\frac{1}{2}\left(A_F - A_{\overline{F}}\right) = &\left(\frac{M^2 - \overline{M}^2}{M^2 + \overline{M}^2}\right) \cos \Delta m t \\
&+ \left(\frac{2M\overline{M}}{M^2 + \overline{M}^2}\right) (\sin 2\alpha)(\cos \Delta) \sin \Delta m t \ .
\end{aligned}
\tag{4.9}
$$

We see that, not surprisingly, a necessary condition to see CP violation via interference of mixing and decay is that the amplitudes M and $\overline{M}$ be not too different in magnitude, although even a ratio of a factor three in amplitudes only gives a factor 0.6 dilution in possible interference effects.

Only if backgrounds are present underneath the expected dominant $\rho\pi$ channels will δ be nonvanishing. If this is the case, the analysis is clearly more complicated. But there are also more interference effects and therefore more handles on determining the CKM phase of interest (which clearly is twice α, the same as in the simpler $\pi - \pi$ channel). For example, were $\alpha = \pi/2$, and were enough information on the strong amplitudes known, the CP violation might still be observable. How well one does depends upon how well all the contributions are understood. This in turn must come from understanding the overall Dalitz distribution. Information on this in turn comes from three–pion final states in charged B decays. If factorization is trusted (and the measurement is feasible) even the semileptonic decay into pion–pair plus dilepton contributes information.

But the main message I want to leave here is that angular correlation measurements in CP violating processes promise to be powerful handles—and perhaps interference between different well–understood strong amplitudes will provide even more handles.[42] What turns out to be useful will be greatly shaped by the nature of the data itself.

5. Concluding remarks

What comes next? Of course the next round of data will as always be very welcome. But meanwhile there are a lot of theoretical issues to deal with:

1. Corrections to the Wisgur limit need classification and estimation. Especially important to understand is the apparent large suppression of axial–current matrix elements in D decays and whether anything like that is seen in the B system.
2. There is more to do on the Wisgur limit itself. Important to me is the question of sum rules for B decays into charmless final states. Work on that is underway.[43] And QCD radiative corrections to all of the sum rules need to be understood. There probably is more to be done with Penguin processes and matrix elements of the neutral current operators.

3. Critical studies of factorization, both for neutral and charged channels, are needed. They should be as model–independent as possible.

4. On the experimental side it will be nice to see more on the nature of the $c\bar{c}s$ final states. The Wisgur developments impact on them in an interesting way.

5. In the Wisgur limit, we saw that in some cases the decay properties of heavy baryons are simpler than those of the heavy mesons. This may stimulate more attention on this important sector, both experimental and theoretical.

In these lectures much has been left out. But I hope that at the least the reader shares this author's view that B–physics is of vital importance and will be around for a long time. But to do it justice will require the building up of a large data base. Already a principal limitation to the extraction of useful results lies in the inadequacies of the theory. But there is a lot of progress, along with possible obstacles. It is clear that there is great opportunity for fruitful interplay between theory and experiment, and that there may be emerging relatively model–independent ways of dealing with semileptonic processes. And if factorization can be trusted—at least in a set of limited but well–defined cases—the progress in the semileptonic–decay theory will spill over to nonleptonic decays as well. The example of the preceding section shows full well how interconnected all these questions are. There may be a lot of apparently tedious work ahead between now and that hopeful, wonderful world of CP–violation measurements. But having done it may ultimately pay off in a big way.

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