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Diquark Stars*

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ABSTRACT

We study neutron stars with cores consisting of a mixture of constituent mass quarks and diquarks. Diquarks are colored, two quark bound states which have been conjectured to exist in the density range above deconfinement. We compute an approximate equation of state for such a mixture. At relatively low densities this has the form of a polytrope with adiabatic index $\Gamma = 2$. We find that the maximum mass star with a quark-diquark core surrounded by a low density envelope of nucleons has mass $1.79 M_{\odot}$, radius 11.4 km and central density $1.8 \times 10^{15} g/cm^3$.

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INTRODUCTION

It is believed that the density of matter in the core of a neutron star exceeds that of nuclear matter[1], $\rho_{nuc} \simeq 2.8 \times 10^{14} g/cm^3$. Moreover, in most models of neutron stars a large part of the mass will be in this high density core region. Consequently, many properties of neutron stars depend strongly on the properties of matter at such high densities.

At densities above ρ_{nuc}, individual nucleons overlap substantially and one expects that the system should properly be described in terms of quarks and gluons. Indeed, at very high density the strong interactions are screened[2] and the quarks can be described by a weakly interacting fermi gas. Most models of neutron stars, however, use equations of state for interacting nucleons to describe the high density core. This will be a good description, if quarks remain spatially localized and correlated in color singlet states resembling nucleons. Estimates have been made of the density at which the transition between nucleon matter and quark matter takes place[3]. The transition is found to happen at many times nuclear density; above what is estimated for maximum central densities in most models. Thus, it has been thought that quark matter probably does not play a role in neutron stars.

Recently it was suggested that there might be an intermediate stage in the transition between ordinary nuclear matter and quark matter[4] in which nucleons have disassociated but a majority of quarks remain localized and correlated in spin singlet pairs, known as diquarks. Such pairing seems likely because the attractive spin-spin energy in this channel[5] is sizable on the scale of quark energies in the density region above ρ_{nuc} (a few hundred MeV). The spin-spin interaction is responsible for the splitting of about 300MeV between the spin 1/2 nucleon and the spin 3/2 Δ. Two of the quarks in a nucleon are in a spin singlet, color $\bar{3}$ combination, with the third quark carrying the overall spin. From the spin interaction there is no energetic advantage to having this third quark grouped with the others. The energy of the system would be lowered further if it found another unpaired quark and formed a spin zero diquark with it. Such a rearrangement is clearly impossible in the confined hadron phase, but in the deconfined regime at higher density it should be possible.

It seems plausible that the diquark correlated state describes part of the density range appropriate to the high density cores of neutron stars. In this paper we compute an approximate equation of state for the diquark state and use this equation of state to build neutron stars. The equation of state we find is comparable to the stiffer equations of state for nuclear matter which appear in the literature. The speed of sound in the quark-diquark mixture is more than an order of magnitude greater than the speed of sound for a gas of noninteracting nucleons. For a model neutron star composed of a charge zero mixture of diquarks and constituent mass quarks surrounded by an envelope of low density neutrons we find that the maximum mass stable star has mass $1.8 M_{\odot}$, radius $11.4 km$, and central density $1.79 \times 10^{15} g/cm^3$. We note that pairing in the diquark channel has been studied previously in the very high density regime where QCD perturbation theory is valid[6]. The binding forces in this regime are then perturbatively weak, and though pairing is found to happen through a BCS mechanism, this only involves quarks near the Fermi surface and the effect on the equation of state is small. Our considerations here are for the regime just above deconfinement where the binding forces are still expected to be fairly strong. The pairing can then involve a large fraction of the quarks and significantly alter the equation of state.

EQUATION OF STATE

Donoghue and Sateesh [4] gave an approximate method for computing the properties of a gas of interacting diquarks. Diquarks are antisymmetric in spin and color and spatially symmetric under interchange of the quarks. They must therefore be antisymmetric in flavor in order to satisfy the exclusion rule, and so contain one up quark and one down quark. Diquarks can be described by an effective field theory for a color triplet scalar field ϕ^{α} with lagrangian

$$L_{eff} = \frac{1}{2}\left(\partial_{\mu}\phi^{\dagger}\partial^{\mu}\phi - m_{ud}^2\phi^{\dagger}\phi\right) - \lambda\left(\phi^{\dagger}\phi\right)^2 . \tag{1}$$

From the $\Delta - N$ splitting the mass of a diquark is etimated to be $m_{ud} \simeq 575\text{MeV}$. The coupling constant λ was estimated from the quark interactions using a variant of the P-matrix formalism of Jaffe and Low[7]. They found a value $\lambda = 27.8$ which we shall use in our numerical work. Note that for $\lambda = 0$ the diquarks would condense into the zero momentum state and form a pressureless gas, which is unphysical for the case at hand.

In this approximation the classical energy of a collection of diquarks is given by [4]

$$E = \int d^3k f(k) \left(k^2 + m_{ud}^2\right)^{\frac{1}{2}} + \frac{\lambda}{2V} \left(\int d^3k \frac{f(k)}{\left(k^2 + m_{ud}^2\right)^{\frac{1}{2}}} \right)^2 , \tag{2}$$

where $f(k)$ is the distribution of diquarks in momentum space. The pressure defined by $P = -\frac{\partial E}{\partial V}$, is then

$$\begin{aligned} P =& \frac{1}{3V} \int d^3k f(k) \frac{k^2}{\left(k^2 + m_{ud}^2\right)^{\frac{1}{2}}} + \frac{\lambda}{2V^2} \left(\int d^3k \frac{f(k)}{\left(k^2 + m_{ud}^2\right)^{\frac{1}{2}}} \right)^2 \\ & - \frac{\lambda}{3V^2} \left(\int d^3k \frac{f(k)}{\left(k^2 + m_{ud}^2\right)^{\frac{1}{2}}} \right) \left(\int d^3k \frac{k^2 f(k)}{\left(k^2 + m_{ud}^2\right)^{\frac{3}{2}}} \right) . \end{aligned} \tag{3}$$

The distribution $f(k)$ is approximated by a gaussian

$$f(k) = \frac{N}{\left(2\pi\sigma^2\right)^{\frac{3}{2}}} e^{-\frac{k^2}{2\sigma^2}} , \tag{4}$$

where N is the total number of diquarks. The width of the gaussian σ is taken to be the value which minimizes the energy (2).

For a general value of the overall diquark density, $n = \frac{N}{V}$, the integrals for the energy and pressure (2) and (3), and the minimization with respect to the width of the distribution must be carried out numerically. In the limits of high and low density, however, this can be done analytically. In these two limits the width σ is given by

$$\sigma = \begin{cases} 0, & \hat{n} \to 0; \\ \left(\frac{\lambda n}{\sqrt{2\pi} m_{ud}^3}\right)^{\frac{1}{3}}, & \hat{n} \to \infty, \end{cases} \tag{5}$$

where $\hat{n} = n/m_{ud}^3$. Then the energy and pressure are

$$\begin{aligned} \rho &= m_{ud} n, \qquad P = \frac{\lambda}{2m_{ud}^2} n^2, \qquad \hat{n} \to 0 \\ \rho &= a\lambda^{\frac{1}{3}} n^{\frac{4}{3}}, \qquad P = \frac{1}{3} a\lambda^{\frac{1}{3}} n^{\frac{4}{3}}, \qquad \hat{n} \to \infty, \end{aligned} \tag{6}$$

where a is a numerical factor, $a \simeq 1.76$. Eliminating the number density from these expressions we obtain the equation of state in these two limits

$$\begin{aligned} P &= \frac{\lambda}{2m_{ud}^4} \rho^2, \qquad \hat{n} \to 0 \\ P &= \frac{1}{3} \rho, \qquad \hat{n} \to \infty \end{aligned} \tag{7}$$

The numerical results for the diquark equation of state in the intermediate density range are given by curve (a) in Fig. 1.

For low densities the diquark energy density is just the mass density. Since the diquark mass is less than twice the constituent quark mass ($m_q \simeq 360$MeV), at low density as many quarks as possible will be in diquark clusters. As the density is increased, interactions raise the diquark chemical potential to a point where it is energetically favorable for some of the quarks to remain unpaired. We should then consider a mixture of quarks and diquarks. We will neglect the interactions of the quarks, keeping only the diquark-diquark interactions, which are necessary to avoid a bose condensate. Starting with fixed overall densities of up and down quarks, we minimize the energy of the mixture to find how many of each species of quark will be paired. For an isoscalar mixture (equal number of up and down quarks) we find that all quarks will be paired below $\rho \simeq 2.7 \times 10^{14}\ g/cm^3 \simeq \rho_{nuc}$. The fraction of quarks paired drops to 1/2 at $\rho \simeq 2.3 \times 10^{15}\ g/cm^3$. For a charge zero mixture coming from pure neutron matter (the number of down quarks equals twice the number of up quarks) all the up quarks will be paired until $\rho = 8.5 \times 10^{14} g/cm^3$ and at ten times nuclear density the fraction in pairs reaches 2/3. We see that the fraction of diquarks remains significant for quite large densities. The equation of state for the isoscalar mixture is given by curve (b) in Fig. 1. For comparison curve (c) is the equation of state for an isoscalar mixture of constituent mass quarks. The equation of state for the charge zero mixture is given by curve (a) in Fig. 2. Curve (b) in Fig. 2 gives the equation of state for a charge zero mixture of quarks.

Diquark Stars

The Einstein equation for a spherically symmetric, static spacetime, with perfect fluid stress energy, reduces to the well known Oppenheimer-Volkoff[8] equations,

$$\begin{aligned} \frac{dm}{dr} &= 4\pi r^2 \rho, \\ \frac{dP}{dr} &= -\frac{\rho m}{r^2}\left(1+\frac{P}{\rho}\right)\left(1+\frac{4\pi P r^3}{m}\right)\left(1-\frac{2m}{r}\right)^{-1}. \end{aligned} \tag{8}$$

This must be supplemented by an equation of state. Then there is a one-parameter family of solutions, parameterized by the central density ρ_c. In general relativity, there is a maximum mass which a static star can have. Here we find the mass and radius as a function of ρ_c and, in particular, the maximum mass.

It is instructive to first look at diquark stars in the Newtonian limit, in which case the energy density is equal to the mass density and only the first term in the pressure equation is significant. This is a good approximation if the pressure is small compared to the energy density and the gravitational field is weak. For diquarks, this first requirement implies $\rho \ll 1.8 \times 10^{15} g/cm^3$. The low density diquark equation of state has the form of a polytrope, $P = K\rho^\Gamma$, with $\Gamma = 2$ and $K = \frac{\lambda}{2m_{ud}^4}$. For comparison, an ideal Fermi gas at $T = 0$ has $\Gamma = 5/3$ in the non-relativistic limit. For $\Gamma = 2$, the Newtonian equations can be reduced to a set of linear equations and solved exactly. The solution is

$$\rho = \rho_c \frac{sin\,(\pi r/R)}{\pi r/R}, \tag{9}$$

where the radius R of the star is given by

$$R = \sqrt{\frac{\pi\lambda}{4}} \frac{m_{pl}}{m_{ud}} \simeq 34.0 km. \tag{10}$$

Note that the radius is independent of the central density. This fixed value for the radius comes out in the ball park of the radii of compact stars, rather than say the size of the universe, because the diquark mass is the same order of magnitude as the nucleon mass. The total mass of the star is given by

$$M = \left(2\pi^2\lambda\right)^{\frac{3}{2}} \frac{m_{pl}^3}{m_{ud}^6} \rho_c \simeq 5.9 \times \left(\frac{\rho_c}{\rho_{nuc}}\right) M_\odot,$$

rising linearly with the central density. Also, $2GM/R = \frac{\lambda}{27} 0.58(\rho_c/\rho_{nuc})$ for the diquark stars, so we expect that general relativity cannot be neglected for $\rho_c/\rho_{nuc} > 1.7$.

Next compare the diquark stars to stars made of nonrelativistic fermions. The equation of state for fermions in this limit is given by

$$P = \frac{1}{5}\left(\frac{6\pi^2}{g}\right)^{2/3} m^{-8/3}\rho^{5/3}, \tag{11}$$

where ρ is the total density, and g is the degeneracy factor. From the known solution to the Newtonian equations for $\Gamma = \frac{5}{3}$ equation of state of state[9] one finds

$$R \simeq 3.6\left(\frac{5K}{8\pi G}\right)^{\frac{1}{2}} \rho_c^{-\frac{1}{6}}, \qquad M = 34.0\left(\frac{5K}{8\pi G}\right)^{\frac{3}{2}} \rho_c^{\frac{1}{2}}. \tag{12}$$

For a star made of an equal number of protons and neutrons ($g = 4$ and $m =$

940MeV) we then have

$$R_N = 14\left(\frac{\rho_c}{\rho_{nuc}}\right)^{-\frac{1}{6}} km, \qquad M_N = .29\left(\frac{\rho_c}{\rho_{nuc}}\right)^{\frac{1}{2}} M_\odot. \tag{13}$$

Now, fix the number of baryons, and compare a diquark star to a star made of nucleons. We see that

$$M_N = 1.1 M_{ud},$$

$$\frac{R_{ud}}{R_N} = 3.6\left(\frac{\lambda}{27}\right)^{1/2}\left(\frac{M_N}{M_\odot}\right)^{1/3} = 2.4\left(\frac{\lambda}{27}\right)^{1/2}\left(\frac{\rho_{c,N}}{\rho_{nuc}}\right)^{1/6}, \tag{14}$$

$$\left(\frac{\rho_{c,ud}}{\rho_{nuc}}\right) = .046\left(\frac{\rho_{c,N}}{\rho_{nuc}}\right)^{1/2}\left(\frac{27}{\lambda}\right)^{3/2}$$

The nucleon star is about 10% heavier. For central densities above ρ_{nuc} the diquark star is considerably larger and the central density of the diquark star is much lower. As the coupling λ decreases, the radius and central density of the diquark star become closer to the values for the nucleons. These qualitative features also occur in the general relativistic solutions.

It is also of interest to compare the diquark star to one composed of an ideal gas of constituent mass up and down quarks. The solutions again have the form (12), but with $g = 12$ and $m_q = 360$MeV. Hence for isoscalar, Newtonian quark stars,

$$R_q = 34\left(\frac{\rho_c}{\rho_{nuc}}\right)^{-1/6} km, \qquad M_q = 4.4\left(\frac{\rho_c}{\rho_{nuc}}\right)^{1/2} M_\odot \tag{15}$$

Comparing a quark star to a diquark star with the same baryon number implies

$$M_q = 1.25 M_{ud},$$

$$\frac{R_{ud}}{R_q} = .61\left(\frac{\lambda}{27}\right)^{1/2}\left(\frac{M_q}{M_\odot}\right)^{1/3} = 1.0\left(\frac{\lambda}{27}\right)^{1/2}\left(\frac{\rho_{c,q}}{\rho_{nuc}}\right)^{1/6}, \tag{16}$$

$$\frac{\rho_{c,ud}}{\rho_{nuc}} = .60 \left(\frac{\lambda}{27}\right)^{3/2} \left(\frac{\rho_{c,q}}{\rho_{nuc}}\right)^{1/2} .$$

The diquark star is less massive than the quark star by about 20%, is approximately the same size and has a lower central density.

Of course, it is incorrect to take the matter to be diquarks when the density falls below nuclear; we must match onto a low density equation of state for nucleons at some density. This matching density is arbitrarily chosen at this point. We do not know the density at which deconfinement might allow the diquark state to be realized. We chose to do the matching when the baryon number in the quark-diquark soup falls to one baryon per sphere of radius 1fm, $n_b = 2.3 \times 10^{38} cm^{-3}$. The corresponding energy density for the charge zero quark-diquark soup is $\rho_{ud} = 4.4 \times 10^{14} g/cm^3$ and the corresponding pressure is $P_{ud} = 3.9 \times 10^{34} dynes/cm^2$. Pressure is continuous in the star, so we match onto a low density equation of state at this pressure. Hence there will be a discontinuity in the energy density at the boundary. For example, if we match onto the equation of state for free nucleons, the energy density in nucleons at the matching pressure is quite large $\rho_N = 9.4 \times 10^{14} g/cm^3$. Such a large discontinuity seems unphysical. However, at these high densities interactions are certainly important, and we should be matching onto an equation of state for interacting nucleons. After all, if the density in nucleons is much above nuclear, the picture seems inconsistent. A variety of equations of state appear in the literature and we can at least get an idea of the direction of the effect. Consulting, e.g., a graph of a number of these equations[10], we see that interacting equations of state have lower energy densities than free neutrons in the range of the transition pressure. The energy density of the quark-diquark mixture is comparable to that of the stiffer equations of state displayed. For simplicity we chose to match onto the equation of state for free nucleons. This seems reasonable in order to get an idea of the effect of adding a quark-diquark core. It also allows us to vary the ratio of charge to baryon number of the surrounding material easily.

Now let us study general relativistic stars, in which case the densities and pressures may be large and the gravitational fields may be strong. Therefore we use the numerically determined equation of state for the diquark-quark soup. Perhaps the most interesting feature of general relativistic solutions is that there exist maximum masses for stable equilibria. It is the masses and rotation rates of compact stars that can be observed (or more fairly, one can hope to infer the mass from other observations). So one way to "tell" if stars are diquarks or nucleons, is if the mass-radius relation for stable stars is different for the two equations of state, and a star is observed which can be explained by one but not the other.

The "most physical" model we will look at is a star having a charge zero quark-diquark core surrounded by an envelope of neutrons. It is instructive to compare these results with a number of other cases in order to illustrate the separate effects of adding quarks in the core and the envelope of low density nucleons to the basic diquark core.

We integrated (8) numerically using a trapezoidal algorithm. This method gave quite good results for the solutions to the Newtonian equations with polytrope equations of state, which can be compared to known results. Solutions to the full OV equations agreed with the Newtonian solutions in the appropriate limits.

Our results are summarized in Table 1. M_{core} and R_{core} are respectively the mass and radius of the region above the matching pressure. With the exception of the second line, these results are for the maximum mass stars. First consider the simplest case, when the whole star is diquarks. The maximum mass of a stable diquark star is $2.56 M_{\odot}$, which occurs at a central density $\rho_c = 1.4 \times 10^{15} gm/cm^3$ and has a radius of $12.8 km$. As anticipated from the Newtonian results, the diquark stars are larger, and hence less dense, than stars made of noninteracting nucleons. We also see that the maximum mass for a stable diquark star is at the high end of neutron star masses and occurs at a fairly low central density. This is a general feature of neutron star models with stiff equations of state for the core material.

The next case is a star made of the isoscalar diquark-quark soup, but no crust.

Compared to the pure diquark star, the maximum mass is somewhat larger in this case and occurs at a much lower central density. Hence the soup interpolates between pure diquarks and pure (massive) quarks, which is also listed for comparison. Of course, the equation of state for the mixture interpolates, and we can understand the stability properties of the mixture as follows. The star is stable to radial perturbations if the adiabatic index $\Gamma_1 = \frac{\partial ln p}{\partial ln \rho}$ satisfies[1] $\int dvp(\Gamma_1 - \frac{4}{3}k|E|/M) \geq 0$ where E is the Newtonian gravitational self-energy and k is a constant (which depends on the equation of state) of order unity. If the star is the sum of two species the condition for stability becomes

$$\int dv\{p_a(\Gamma_a - \frac{4}{3}) + \ (\Gamma_b - \frac{4}{3}) \ - p\frac{k|E|}{M}\} \geq 0 \tag{17}$$

This assumes $\rho_b \simeq c\rho$ and $\rho_a \simeq (1-c)\rho$, where c is a constant. If this is not a good approximation then Γ_i is replaced by $\Gamma_i \frac{\rho}{\rho_i}\frac{\partial \rho_i}{\partial \rho}$ in (17). Therefore the stability properties of the mixture interpolates. Suppose $\Gamma_a \rightarrow 4/3$ at a lower density than Γ_b. Then "b" stabilizes the star, and there are stable equilibria at higher densities than if only "a" were present. On the other hand, the presence of "a" means that the net force for collapse is stronger than if only "b" were present, tending to destabilize the mixture.

Next consider the effect of replacing the low density region of the star with an envelope of nucleons. We would like to understand why the maximum mass decreases. To isolate this effect, compare a star made just out of diquarks, to one with a diquark core and a nucleon envelope. The gravitational self-energy term which appears in condition (17) has three contributions, the the self-energy of the core, the self-energy of the shell, and an interaction piece. Look at the case for $\rho_c = 1.4 \times 10^{15} g/cm^3$; this is the highest central density possible for a stable diquark star. For a fixed diquark core, the nucleon envelope is considerably thinner than the diquark envelope. If the same mass were packed into the two envelopes, the self-energy of the nucleons would be much higher, which destabilizes the star. The actual nucleon envelope has a smaller mass, which helps stability, and so the star with nucleons is less massive. Now increase the central density to $\rho_c = 1.7 \times 10^{15} g/cm^3$, the value at the maximum mass for a stable diquark plus nucleon star. This central density is higher than that for a stable diquark star, and the self-energy of the diquark core is quite high. The total self-energy is kept sufficiently low by putting even less mass in the envelope. One can check these remarks by computing $|E|/M$, with $E_{core} \simeq -\frac{1}{2}G\frac{M_c^2}{R_c}$, $E_{int} \simeq -\frac{1}{2}G\frac{M_e M_c}{M_c}(1+\frac{R_c}{R_T})$ and $E_{env} \simeq -\frac{1}{8}G\frac{M_e^2}{R_c}(1+3\frac{R_c}{R_T})$. One then sees that E/M is approximately the same for the three cases just discussed; the core contribution increases and the contribution from the successively less massive envelopes decreases.

Finally we come to our "most physical" case of a charge zero quark-diquark soup surrounded by an envelope of neutrons. The maximum mass in this case is $1.79 M_\odot$ at central density $1.8 \times 10^{15} g/cm^3$ with radius $11.4 km$. Here again we see that adding the envelope lowers the maximum mass, but raises the maximum central density. Figs. 3and 4give plots of the mass vs. central density and mass vs. radius relations for the charge zero soup with (curve a) and without (curve b) the neutron envelope.

CONCLUSIONS

We have studied the thermodynamics of a mixture of interacting diquarks and a fermi sea of noninteracting, constituent mass, up and down quarks, which may be a useful model of matter at densities above nuclear. At low densities, the mixture is dominated by diquarks, which have a stiff polytrope equation of state, with adiabatic index two. At high densities, the equation of state of the diquarks approaches that for a fermi gas of relativistic particles. (However these densities are probably too high to be of interest for the cores of neutron stars.) The global properties of neutron stars having a diquark-quark core, surrounded by an envelope of nucleons, are similiar to those of stars constructed from stiff equations of state for interacting nucleons. It would be of interest to see what effect the inclusion of diquarks has on modelling supernovae bounces. It would also be interesting to see how strange matter calculations change when the diquark pairing interaction is included.

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TABLE CAPTIONS

1: Some results for stars calculated using different equations of state which are discussed in the paper.

FIGURE CAPTIONS

1) The equations of state are displayed for an isoscalar mixture of quarks (curve c), a pure diquark gas (curve a) and an isoscalar soup of diquarks and quarks (curve b).

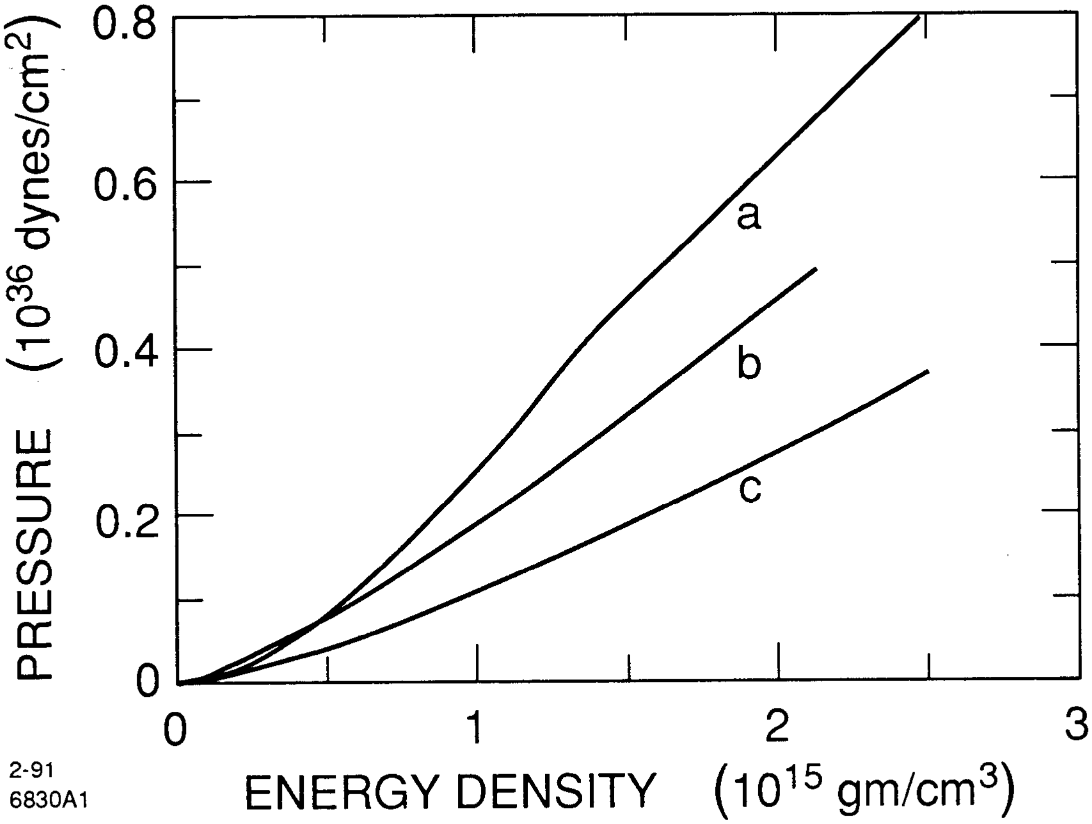
2) The equations of state are displayed for a charge zero mixture of quarks (curve b) and a charge zero mixture of diquarks and quarks.

3) The mass vs. central density relations are plotted for stars made of a charge zero mixture of diquarks and quarks with (curve a) and without (curve b) a low density neutron envelope.

4) The mass vs. radius relations are plotted for stars made of a charge zero mixture of diquarks and quarks with (curve a) and without (curve b) a low density neutron envelope.

model	$\rho_c\ (g/cm^3)$	mass ($M_\odot$)	radius (km)	M_{core} ($M_\odot$)	R_{core} (km)
diquarks	1.4×10^{15}	2.56	12.8	1.6	10.3
diquarks	1.7×10^{15}	2.55	12.5	1.7	10.2
diquarks + nucleons	1.4×10^{15}	2.35	11.8	1.6	10.3
diquarks + nucleons	1.7×10^{15}	2.38	11.5	1.7	10.2
isoscalar soup	5.2×10^{14}	2.74	22.7	~ 0	~ 0
isoscalar quarks	5.2×10^{14}	1.91	22.4	-	-
charge 0 soup	1.1×10^{15}	2.09	16.5	1.00	9.1
charge 0 soup + neutrons	1.8×10^{15}	1.79	11.4	1.2	9.2

Table 1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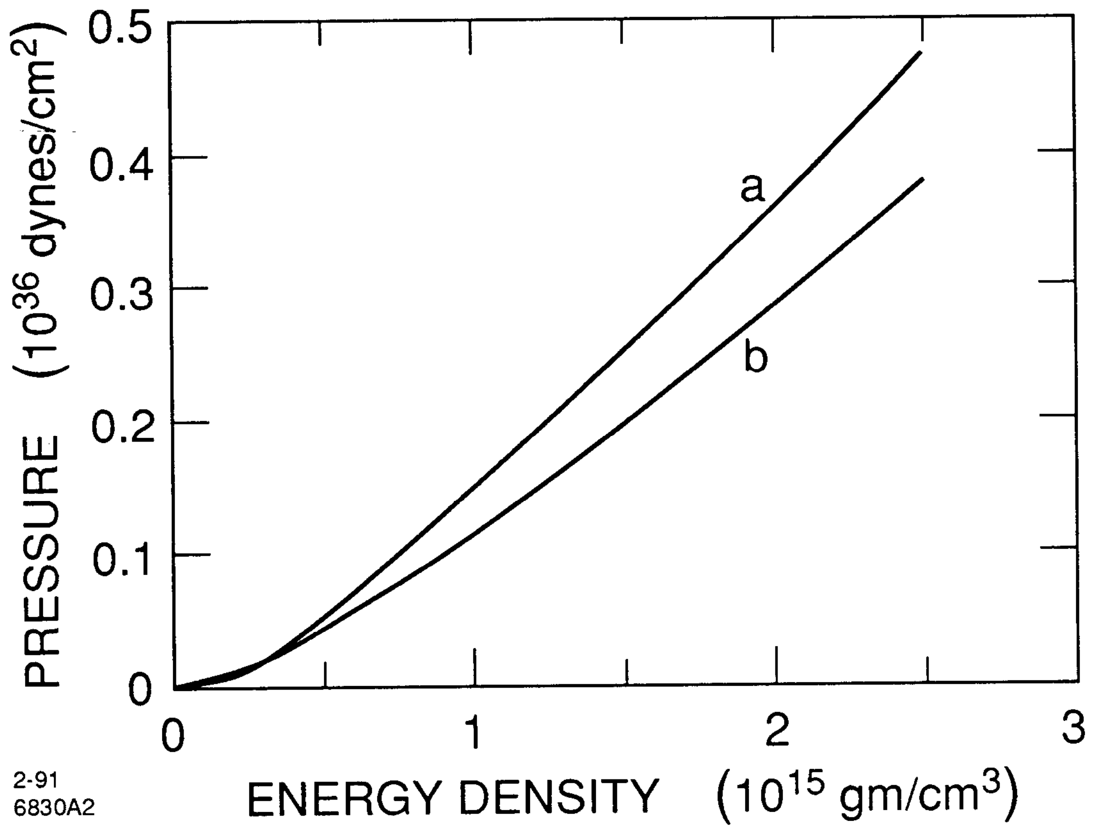
0.2
0.4
0.6
0.8
a
b
c
0
1
2
3
ENERGY DENSITY $(10^{15}$ gm/cm$^3)$
2-91
6830A1

Fig. 1



0.5
0.4
0.3
0.2
0.1
0
PRESSURE (10^36 dynes/cm^2)
a
b
0
1
2
3
2-91
6830A2
ENERGY DENSITY (10^15 gm/cm^3)

Fig. 2

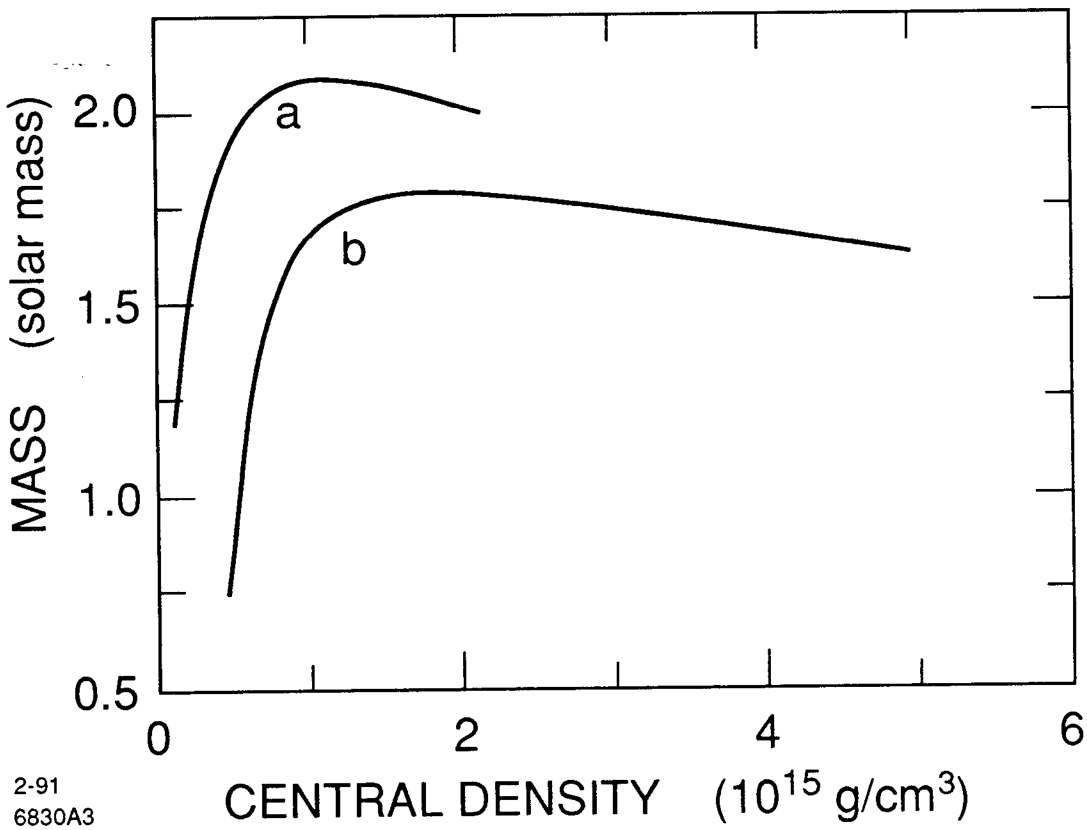

2.0
1.5
1.0
0.5
0
2
4
6
a
b
MASS (solar mass)
CENTRAL DENSITY (10^15 g/cm^3)
2-91
6830A3

Fig. 3

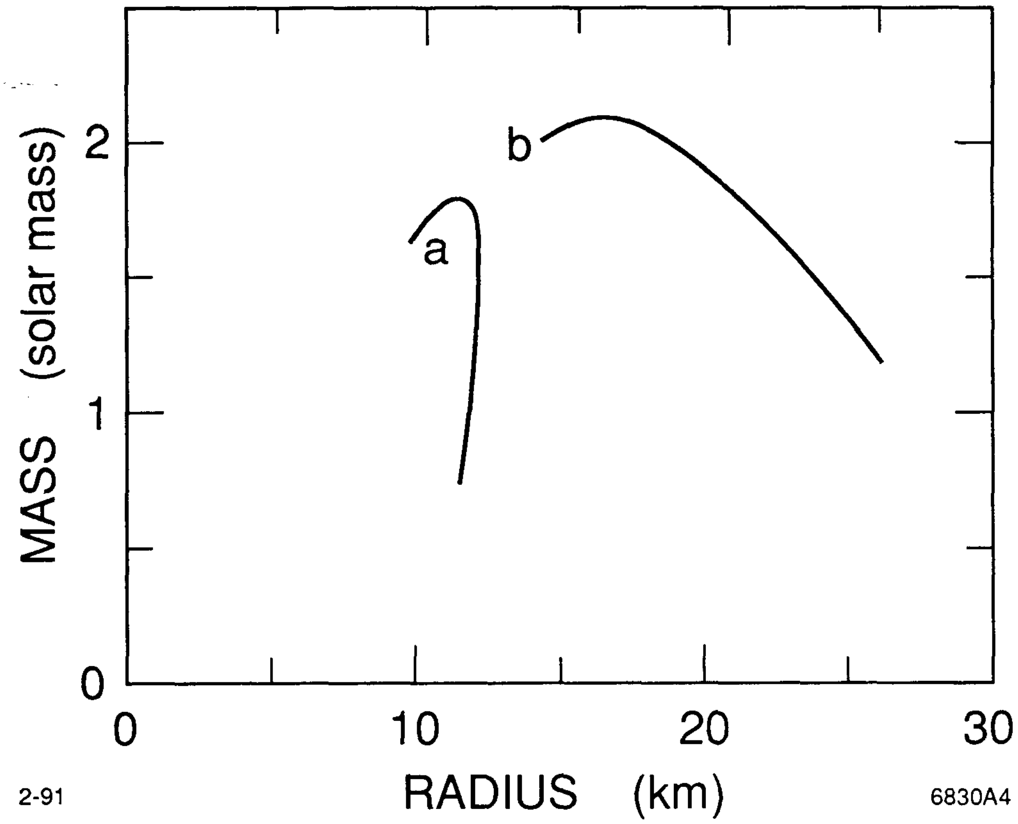


Fig. 4