# Amplitude Analysis of the $K \bar{K}$ System in $J / \psi$ Radiative Decay ${ }^{\star}$ 

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#### Abstract

Preliminary results of a mass independent amplitude analysis of $J / \psi$ radiative decays into $K \bar{K}$ final states are presented. A large component of spin zero is observed at the $f_{2} / \theta(1720)$ mass region; however, a small spin two component at this mass region cannot be excluded with the present statistics.


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## 1. INTRODUCTION

Evidence for the meson $f_{2} / \theta(1720)$ has been obtained from the $J / \psi$ radiative decay processes

$$
\begin{aligned}
J / \psi & \rightarrow X \gamma \\
& \longleftrightarrow \eta \eta, K K, \pi \pi
\end{aligned}
$$

by the MARK II, ${ }^{1}$ Crystal Ball, ${ }^{2}$ MARK III, ${ }^{3,4}$ and DM2 ${ }^{5}$ groups, and from the central production process $P P \rightarrow P_{f}(K \bar{K}) P_{s}$ by the WA76 group. ${ }^{6}$ The spin and parity of this meson have also been estimated by these groups. Due to the absence of this meson in $\gamma \gamma$ reactions and low $t$ hadronic processes, its characteristics are often compared with those expected for a tensor glueball. ${ }^{7}$

The MARK III group reported its result ${ }^{3}$ on the spin of the $f_{2} / \theta(1720)$ with a study based on $2.7 \times 10^{6} J / \psi$ events. The $J / \psi \rightarrow \gamma K^{+} K^{-}$events with $K^{+} K^{-}$invariant mass in the mass region of the $f_{2} / \theta(1720)$ were analysed with the assumption of either pure spin zero or pure spin two. Spin two was found to be preferred over spin zero. Since the $f_{2} / \theta(1720)$ and $f_{2}^{\prime}(1525)$ mesons overlap in this mass region, interference between intermediate states is not necessarily negligible. A MARK III analysis with $5.8 \times 10^{6} \mathrm{~J} / \psi$ events, with the interference effect included has been carried out, ${ }^{4}$ which indicated the presence of a significant spin zero contribution in the $f_{2} / \theta(1720)$ mass region. The present report presents preliminary results of an analysis similar to the one described in Ref. 4. Consistent results are obtained on the spin and helicity amplitudes of the intermediate states of the $J / \psi$ radiative decays into $K^{+} K^{-}$and $K_{s} K_{s}$ in the mass regions of the $f_{2} / \theta(1720)$ and the $f_{2}^{\prime}(1525)$.

The method used in this analysis is referred to as the moment method because the angular distribution of the events (after correction for acceptance losses) is expressed as a sum of a set of spherical harmonics and the coefficients (moments) of the spherical harmonics are measured. The helicity amplitudes can then be obtained directly from the moments. This method provides a better understanding of the data than directly fitting the helicity amplitudes to the angular distribution, because the moments are sums of products of the helicity amplitudes.


Figure 1. Invariant mass spectra of the $K \bar{K}$ systems.

In radiative $J / \psi$ decays with $K \bar{K}$ final states, only $J^{P C}=n^{++}$, with $n=$ even, are allowed for the intermediate states. This analysis considers only $n=0,2$ as the possible spin of the intermediate resonance. Objects with spin as high as 4 are considered unlikely to exist with mass less than $2 \mathrm{GeV} / \mathrm{c}^{2}$.

The data are acquired with the MARK III detector at the SPEAR storage ring at the Stanford Linear Accelerator Center (SLAC). The details of the detector have been described elsewhere. ${ }^{8}$ In this analysis the $K^{+}, K^{-}$are identified with the Time of Flight (TOF) counters and the $K_{s}$ 's, formed with two charged pion pairs, have the invariant mass ( $M_{\pi^{+} \pi^{-}}$) being consistent with that of $K_{s}$. The event selection process of previous MARK III analysis ${ }^{4}$ is reproduced and similar invariant mass spectra are obtained as shown in Fig. 1.

## 2. ANALYSIS

The data sample of each decay mode is divided into subsamples according to the invariant mass of the $K \bar{K}$ system. In this analysis, events with $M_{K \bar{K}}=(1.075-$ 2.075) $\mathrm{GeV} / \mathrm{c}^{2}$ are divided into ten subsamples, each covering $0.1 \mathrm{GeV} / \mathrm{c}^{2}$. The moments and the helicity amplitudes of the intermediate states in each subsample are determined and results from the two decay modes are compared. Since the mass range of the intermediate states in each subsample is small, the variation of the phase space factor is negligible. The joint decay angular distribution $W\left(\Omega_{X}, \Omega_{K}^{*}\right)$
of each subsample is determined by the matrix elements of the specific decay chains and can be writtcn as

$$
\begin{equation*}
W\left(\Omega_{X}, \Omega_{K}^{*}\right) \propto \sum_{\lambda_{\psi}, \lambda_{\gamma}}\left|\sum_{J_{X}, \lambda_{X}} a_{J_{X}, \lambda_{X}}^{\lambda_{\gamma}} \sqrt{\frac{2 \mathrm{~J}_{\mathrm{X}}+1}{4 \pi}} \mathrm{D}_{\lambda_{\psi}, \lambda_{X}-\lambda_{\gamma}}^{* 1}\left(\Omega_{X}\right) \mathbf{D}_{\lambda_{X}, 0}^{* J_{X}}\left(\Omega_{K}^{*}\right)\right|^{2} \tag{1}
\end{equation*}
$$

where $\Omega_{X}$ represents the angles of the $K \bar{K}$ system in the Lab frame, $\Omega_{K}^{*}$ represents the angles of the $K$ in $K \bar{K}$ rest frame, $\mathbf{D}^{*}$ is the complex conjugate of the $\mathbf{D}$ matrix and $a_{J_{X}, \lambda_{X}}^{\lambda_{\gamma}}$ is the helicity amplitude of the interaction when the spin of the intermediate state is $J_{X}$ and the helicities of the photon and the intermediate state are $\lambda_{\gamma}$ and $\lambda_{X}$ respectively.

The only amplitudes considered for this analysis are: $a_{0,0}$, the amplitude with $J_{X}=0, \lambda_{X}=0$, and $a_{2,0}, a_{2,1}, a_{2,2}$, the amplitudes with $J_{X}=2, \lambda_{X}=0,1,2$. The superscript on the helicity amplitudes is suppressed since the parity conservation ensures $a_{J_{X},-\lambda_{X}}^{-\lambda_{\gamma}}=a_{J_{X}, \lambda_{X}}^{\lambda_{\gamma}}$.

The angular distribution of the events can also be written as a sum of a set of spherical harmonics

$$
\begin{equation*}
W\left(\Omega_{X}, \Omega_{K}^{*}\right)=\sum_{\mathrm{j}, \mathrm{l}, \mathrm{~m}} \mathbf{T}_{\mathrm{l}, \mathrm{~m}}^{\mathrm{j}} \mathbf{Y}_{\mathrm{j}, \mathrm{~m}}\left(\Omega_{X}\right) \mathbf{Y}_{1, \mathrm{~m}}^{*}\left(\Omega_{K}^{*}\right) \tag{2}
\end{equation*}
$$

where the $T_{1, \mathrm{~m}}^{\mathrm{j}}$ are the moments of the angular distribution. From expressions (1) and (2), the relation of the helicity amplitudes and the moments is determined. ${ }^{9}$

One measures the moments as

$$
\begin{equation*}
\mathbf{N}_{\mathrm{l}, \mathrm{~m}}^{\mathrm{j}}=\sum_{\text {events }} \operatorname{Re}\left(\mathbf{Y}_{\mathrm{j}, \mathrm{~m}}^{*}\left(\Omega_{X}\right) \mathbf{Y}_{\mathrm{l}, \mathrm{~m}}\left(\Omega_{K}^{*}\right)\right) \tag{3}
\end{equation*}
$$

and obtains the helicity amplitudes $a_{J_{X}, \lambda_{X}}$ by minimizing

$$
\begin{equation*}
\chi^{2}=\sum_{\mu, \nu=1}^{10}\left(\mathbf{N}_{\mu}-\sum_{\lambda=1}^{10} \mathbf{C}_{\mu \lambda} \mathbf{T}_{\lambda}(a)\right) \mathbf{V}^{-1}\left(\mathbf{N}_{\nu}-\sum_{\lambda=1}^{10} \mathbf{C}_{\nu \lambda} \mathbf{T}_{\lambda}(a)\right) \tag{1}
\end{equation*}
$$

where $\mu, \nu, \lambda$ stand for the indices $(\mathrm{j}, 1, \mathrm{~m})$ of the moments, $\mathbf{V}$ is the covariance matrix of the moment measurements, the $\mathbf{T}_{\lambda}(a)$ 's are the expressions for the moments in


Figure 2. Projections of the efficiency functions for the event angles. Left: $J / \psi \rightarrow \gamma K_{s} K_{s}$, Right: $J / \psi \rightarrow \gamma K^{+} K^{-}$.
terms of the helicity amplitudes given by expressions (1), (2), as described above, and $\mathbf{C}$ is the correlation matrix, which corrects the obsérved moments for acceptance losses. The matrix $\mathbf{C}$ is determined by means of Monte Carlo studies.

The acceptance variation of the MARK III detector is illustrated in Fig. 2. The plots show the projections of the efficiency function for each of the event angles, simulated with events flatly distributed in the sequential two body decay phase space and with $M_{K \bar{K}}=1.72 \mathrm{GeV} / \mathrm{c}^{2}$. Events with the azimuthal angle $\phi_{K}^{*}$ of the $K$ in the $K \bar{K}$ rest frame near 0 or $\pi$ are likely to have tracks parallel to the beam pipe and thus be missed by the track reconstruction or Time of Flight counters.

The analysis algorithm has been testcd extensively with Monte Carlo samples of size comparable to the real data. The number of events associated with each helicity amplitude is recovered for samples generated with a single amplitude and for samples in which the helicity amplitudes are set equivalent to those of the $f_{2}^{\prime}(1525)$,


Figure 3. Testing the moment method with Monte Carlo samples. The figure shows the population of the fraction of pure spin two events misidentified as spin zero. The Monte Carlo samples are generated with $a_{20}, a_{21}, a_{22}$ in roughly equal amounts.
i.e., $a_{20}=a_{21}$ and $a_{00}=a_{22}=0$, roughly. ${ }^{3}$ The number of events incorrectly assigned to amplitudes which had zero input is less than $10 \%$ of the total number of input events. In the case where the helicity amplitudes are set similar to those of the $f_{2} / \theta(1720)$, determined under pure spin two assumption, i.e., $a_{20}, a_{21}, a_{22}$ are roughly equal, ${ }^{3}$ approximately $10 \%$ of the events will be misidentified as being spin zero. Fig. 3 shows the distribution of the fraction of these events misidentified as spin zero over the total number of the input pure spin two events. Approximately 40 independent Monte Carlo samples are tested, each sample contains $\sim 123$ events, which simulates the number of events seen in the $K_{s} K_{s}$ data sample for the mass region of $M\left(K_{s} K_{s}\right)=(1.675-1.775) \mathrm{GeV} / \mathrm{c}^{2}$. As shown in Fig. 7 below, the fraction of real events identified as spin zero at the $f_{2} / \theta(1720)$ mass region is significantly higher than the average misidentification fraction shown by the Monte Carlo studies. As discussed later, this implies that a significant spin zero enhancement at the $f_{2} / \theta(1720)$ mass region is observed.

In the studies with Monte Carlo samples, the phase angles between the helicity amplitudes cannot be determined with the available statistics. The phase angles are included as free parameters in the $\chi^{2}$ minimization, but the results will not be discussed further.


Figure 4. The measured moments (data points) and the prediction with the fit results of the helicity amplitudes (histogram) of $J / \psi \rightarrow \gamma K_{s} K_{s}$ mode.


Figure 5. The measured moments (data points) and the prediction with the fit results of the helicity amplitudes (histogram) of $J / \psi \rightarrow \gamma K^{+} K^{-}$mode.

## 3. RESULTS

The measured moments are shown as the data points in Figs. 4 and 5 for the $K_{s} K_{s}$ and $K^{+} K^{-}$modes, respectively. These data are not corrected for the detector efficiency. A normalization factor is applied to the $K_{s} K_{s}$ mode to compensate for the branching fractions of the $K^{0} \bar{K}^{0} \rightarrow K_{s} K_{s}$ and the $K_{s} \rightarrow \pi^{+} \pi^{-}$. The predictions from the fit results for the helicity amplitudes are also shown in Figs. 4 and 5 for the $K_{s} K_{s}$ and $K^{+} K^{-}$modes, respectively. The data points are consistent with the predictions.

The efficiency-corrected moments of the two decay modes are shown in Fig. 6. The helicity amplitude distributions measured with the two decay modes, after efficiency correction, are shown in Fig. 7. Clear evidence for a spin zero enhancement at the $f_{2} / \theta(1720)$ mass region (in both decay modes) is visible; however, the presence of a small spin two component cannot be excluded with the present statistics. The events at the $f_{2}^{\prime}(1525)$ mass region are predominantly identified as spin two, and the ratios of the helicity amplitudes previously measured ${ }^{3}$ at this mass region are reproduced. The consistency of the results for the two decay modes is what we expected for isospin zero intermediate states. This indicates that the acceptance correction procedures are reliable, since the respective efficiency functions are rather different (cf., Fig. 2).

## 4. CONCLUSION

In this preliminary analysis, a large component of spin zero is observed at the $f_{2} / \theta(1720)$ mass region. A small spin two component at this mass region cannot be excluded with the present statistics. Results from the $K^{+} K^{-}$and $K_{s} K_{s}$ modes are consistent with each other.

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Figure 6. The efficiency-corrected moment measurements.


Figure 7. The helicity amplitude distributions. Left: $J / \psi \rightarrow \gamma K_{s} K_{s}$, Right: $J / \psi \rightarrow \gamma K^{+} K^{-}$. A large component of spin zero is visible at the $f_{2} / \theta(1720)$ mass region in both decay modes, along with a small spin two component.

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9. The expressions of the moments in terms of the helicity amplitudes are below. These include corrections to some minor errors that appeared in the expressions in Ref. 4.

$$
\begin{aligned}
& \mathbf{T}_{0,0}^{0}=\left|a_{0,0}\right|^{2}+\left|a_{2,0}\right|^{2}+\left|a_{2,1}\right|^{2}+\left|a_{2,2}\right|^{2} \\
& \mathbf{T}_{0,0}^{2}=\frac{\sqrt{5}}{10}\left[\left|a_{0,0}\right|^{2}+\left|a_{2,0}\right|^{2}-2\left|a_{2,1}\right|^{2}+\left|a_{2,2}\right|^{2}\right] \\
& \mathbf{T}_{2,0}^{0}=\frac{\sqrt{5}}{5}\left[2 \sqrt{5} \operatorname{Re}\left(a_{0,0} a_{2,0}^{*}\right)+\frac{5}{7}\left(2\left|a_{2,0}\right|^{2}+\left|a_{2,1}\right|^{2}-2\left|a_{2,2}\right|^{2}\right)\right] \\
& \mathbf{T}_{2,0}^{2}=\frac{1}{5}\left[\sqrt{5} \operatorname{Re}\left(a_{0,0} a_{2,0}^{*}\right)+\frac{5}{7}\left(\left|a_{2,0}\right|^{2}-\left|a_{2,1}\right|^{2}-\left|a_{2,2}\right|^{2}\right)\right] \\
& \mathbf{T}_{2,1}^{2}=-\frac{\sqrt{3}}{10}\left[\sqrt{5} \operatorname{Re}\left(a_{0,0} a_{2,1}^{*}\right)+\frac{5}{7}\left(\operatorname{Re}\left(a_{2,0} a_{2,1}^{*}\right)-\sqrt{6} \operatorname{Re}\left(a_{2,1} a_{2,2}^{*}\right)\right)\right]=\mathbf{T}_{2,-1}^{2} \\
& \mathbf{T}_{2,2}^{2}=\frac{\sqrt{6}}{10}\left[\sqrt{5} \operatorname{Re}\left(a_{0,0} a_{2,2}^{*}\right)-\frac{10}{7} \operatorname{Re}\left(a_{2,0} a_{2,2}^{*}\right)\right]=\mathbf{T}_{2,-2}^{2} \\
& \mathbf{T}_{4,0}^{0}=\frac{1}{7}\left[6\left|a_{2,0}\right|^{2}-4\left|a_{2,1}\right|^{2}+\left|a_{2,2}\right|^{2}\right] \\
& \mathbf{T}_{4,0}^{2}=\frac{\sqrt{5}}{70}\left[6\left|a_{2,0}\right|^{2}+8\left|a_{2,1}\right|^{2}+\left|a_{2,2}\right|^{2}\right] \\
& \mathbf{T}_{4,1}^{2}=-\frac{\sqrt{3}}{14}\left[\sqrt{6} \operatorname{Re}\left(a_{2,0} a_{2,1}^{*}\right)+\operatorname{Re}\left(a_{2,1} a_{2,2}^{*}\right)\right]=\mathbf{T}_{4,-1}^{2} \\
& \mathbf{T}_{4,2}^{2}=\frac{3 \sqrt{2}}{14} \operatorname{Re}\left(a_{2,0} a_{2,2}^{*}\right)=\mathbf{T}_{4,-2}^{2}
\end{aligned}
$$


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