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RECENT DEVELOPMENTS IN HEAVY FLAVOR THEORY*

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Before anything else, the reader should be forewarned that this is not a review. I choose to focus upon new work by Isgur and Wise (which I abbreviate as Wisgur), about which I am especially enthusiastic. This work has stimulated considerable activity, and I fear that even within this topic I may omit some contributions. In anticipation of this, I offer apologies in advance to those whose work escapes mention here.

What did Wisgur do and why might it be important?

Isgur and Wise[1] consider the limit of QCD in the formal limit when the masses M_c, M_b, ... are taken to infinity with, say, the ratios held fixed. In such a limiting situation the theory still exists. The situation is similar to quantum electrodynamics of atoms when nuclear masses are set to infinity; nothing terrible happens. But in this limit theoretical predictions for weak matrix elements greatly simplify, and it is this feature which makes the approach potentially interesting. And it is important to stress that the limiting theory is not a nonrelativistic limit; relativistic effects can be and are included.

To me the real potential importance of the Wisgur approach is that it affords a model–independent starting point for the consideration of, say, weak decay amplitudes of b–hadrons. Already we see all too many experimental measurements limited by theoretical systematic errors. Production of even more specific models by the theorists can only worsen the situation by increasing even further the spread of predictions, unless objective criteria can be brought to bear on discriminating between them.

Of course, the simplicity of the infinite mass limit does not guarantee that it is a close approximation to reality. That must be determined by an analysis of the corrections to the limit. While there is much still to be done, I believe that it may be possible to classify them in a systematic way. If this is possible, then model–dependence is relegated to the magnitudes of the correction terms. This would indeed be a big improvement over the existing situation.

The physics of the infinite mass limit

The basic physical ideas underlying the Wisgur approach have been in the folklore for a long time. Briefly they are as follows:

1. As $M_b \to \infty$ the b–quark and the hadron containing it become cannonballs. Once set into motion, their velocity is difficult to change. Only perturbative processes such as hard gluon emission or electroweak interactions are effective modifiers of velocity. Therefore the velocity becomes a good quantum number as far as the nonperturbative aspects of QCD are concerned.[2] (Note that onium states are not included here; the two heavy quarks orbit about each other and are not in velocity eigenstates.)

2. As mentioned already, QCD exists in this limit, and indeed simplifies greatly. An effective field–theory formalism has been worked out,[3] following the approach of Lepage and Thacker[4] originally used for quantum electrodynamics, providing reasonably firm underpinnings to the phenomenology to be discussed below.

3. The spin of the b–quark decouples from the dynamics in the infinite mass Wisgur limit, because the QCD hyperfine coupling of the heavy–quark spin to the light–quark spectator system scales inversely with heavy–quark mass.

4. It follows that, in the limit, members of a hyperfine multiplet become degenerate in mass. Consequently there is a new spin symmetry present in

the limit. For example, the vector B^* becomes degenerate with the pseudoscalar B; again the approach to degeneracy scales inversely with mass, in accordance with the trend seen experimentally.

5. There also is a flavor symmetry, because as the masses of, say, b, c, and t quarks go to infinity the flavor labels become irrelevant as far as the strong dynamics is concerned. Hence there is a combined flavor $\times$ spin symmetry reminiscent of the Wigner symmetry of nuclear physics—hence the abbreviation Wisgur.

Semileptonic form factors

As an example of the method, we choose the semileptonic decay of the bottom baryon Λ_b, because it is especially simple:

$$\Lambda_b \to \Lambda_c e^- \overline{\nu}_e \ . \tag{1}$$

In general, this transition is characterized by six invariant form factors, three vector and three axial. But in the infinite–mass limit there remains only one normalized form factor.[5] The matrix element is

$$\begin{aligned} &\langle \Lambda_c, v' \,|\, J_\mu \,|\, \Lambda_b, v \rangle = \\ &\qquad \sqrt{\frac{M_c M_b}{4E_c E_b}} \ \left[\overline{u}(v')\gamma_\mu(1-\gamma_5)u(v)\right] \ F_\Lambda(w) \ . \end{aligned} \tag{2}$$

Furthermore, this universal form factor is identical to the elastic form factor[6] of the Λ_b. This is true despite the fact that the weak transition involves a timelike momentum transfer to the lepton system, while the latter involves a spacelike momentum transfer. What matters is the invariant velocity change $w = v \cdot v'$, where the four velocity is defined as usual as $v_\mu = P_\mu / M$.

The structure of Eq. (2) follows from the fact that the matrix element must not depend separately upon momenta and masses of the heavy hadrons, but only on their four–velocities. In addition, the Wisgur spin–flavor symmetry relates vector current matrix elements to axial–current matrix elements. The essential point is simply that the light–diquark spectator system is spinless; all the spin information remains with the heavy quarks, whose dynamics is trivial in the infinite mass limit.

There exists an immediate generalization to complex final states:

$$\Lambda_b \to \Lambda_c X\, e^- \,\overline{\nu}_e$$

$$\begin{aligned} &\langle \Lambda_c X \,|\, J_\mu \,|\, \Lambda_b \rangle = \\ &\qquad \sqrt{\frac{M_c M_b}{4E_c E_b}}\, \overline{u}(v')\gamma_\mu(1-\gamma_5)\, u(v)\, F_\Lambda(v, v', X)\ . \end{aligned} \tag{3}$$

Essentially the same structure survives; the universal form factor now depends not only upon v and v' but all variables characterizing the remaining system X. Note the implication that all spin correlations between Λ_b and Λ_c are the same as between the free quarks b and c, independent of the final state X.

Another generalization applies to charmless final states in semileptonic Λ_b decays. There the formula is[7]

$$\langle X \,|\, J_\mu \,|\, \Lambda_b \rangle = \sqrt{\frac{M_b}{2E_b}}\, \overline{\psi}(x, v)\, \gamma_\mu(1-\gamma_5)\, u(v) \tag{4}$$

where ψ is a "form factor" which transforms as a Dirac spinor and depends upon v and all the variables in the final state X.

Of course the Wisgur development began with B–meson semileptonic decays.

The formalism for that case is only slightly more complicated. One writes[8]

$$\langle D \text{ or } D^*, X \,|\, J_\mu \,|\, B\rangle = \sqrt{\frac{M_B M_D}{4E_B E_D}}\, \mathrm{Tr}\, \overline{\mathcal{D}} \mathcal{J}_\mu \mathcal{B} \varrho(v, v', X) \ . \tag{5}$$

The trace is on 4×4 Dirac matrices:

$$\mathcal{B} = \begin{cases} \gamma_5 \left(\frac{\not{P}+M}{2M}\right) & \text{for the } 0^- \ B \\ \not{\epsilon} \left(\frac{\not{P}+M}{2M}\right) & \text{for the } 1^- \ B^* \ . \end{cases} \tag{6}$$

For the case of "elastic" semileptonic transitions of B to D or D^*, the universal function ϱ is again a function only of the four–velocities v and v' and can be easily reduced to a multiple of the unit Dirac matrix.

$$\varrho = 1 \cdot F(w) \ . \tag{7}$$

Thus in the Wisgur limit all information about these elastic semileptonic decays is again determined, up to the behavior of the universal form factor $F(w)$.

Again a formalism for charmless semileptonic B–decays exists also.

$$\langle X \,|\, J_\mu \,|\, B\rangle = \sqrt{\frac{M_b}{2E_b}}\, \mathrm{Tr}\, \mathcal{J}_\mu \mathcal{B} \phi(v, X) \ . \tag{8}$$

If

$$|X\rangle = |0\rangle \tag{9}$$

then, in terms of the usual decay constant F_B,

$$\phi = \text{constant} \sim \sqrt{M_B}\, F_B \ . \tag{10}$$

Hence

$$F_B \sim M_B^{-1/2} \ . \tag{11}$$

This has been in the folklore for some time.[9]

If

$$\langle X| = \langle \pi| \tag{12}$$

then

$$\phi = \gamma_5 (A + B \not{p}_\pi) \tag{13}$$

with

$$A = A(v \cdot p_\pi) \qquad B = B(v \cdot p_\pi) \tag{14}$$

and one has the process described in terms of two form factors, each of which depends upon the variable $v \cdot p_\pi$ as shown.

With these basic results there exists a rather long list of applications. Those enumerated below may not exhaust the list. But for better or worse here they are:

1. As noted above, there is only one normalized form factor in elastic semileptonic B decays.[1]
2. The same holds for Λ_b elastic decays.[5]
3. The formalism generalizes to complicated final states.[10]
4. The spin correlations in the Λ_b semileptonic decays to general final states containing a Λ_c are universal and the same as at the quark level.[7]
5. Pseudoscalar decay constants of heavy mesons (which determine the pure leptonic decay rates) scale inversely as the square root of the mass. This

result emerges already in the nonrelativistic approximation and has been known for some time.

6. Generalizations[11] exist for "penguin" processes such as

$$b \to s e^+ e^- \qquad \text{or} \qquad b \to s\gamma \ . \tag{15}$$

7. Results exist for somewhat more complicated "elastic" processes such as[5]

$$\Omega_b \to \Omega_c e^- \overline{\nu}_e \tag{16}$$

and[12]

$$\overline{B} \to D^{**}\, e^- \overline{\nu}_e \ . \tag{17}$$

8. Nonleptonic decays involving two heavy mesons in the final state have been considered,[13] such as

$$B \to \overline{D} D_s \ . \tag{18}$$

9. Elastic electron–positron pair production of heavy mesons (e.g. $\overline{B}B^*$) has also been studied[14] as well as extensions to baryon–antibaryon production.[15]
10. The semileptonic decay of a D into a general (charmless) final state is related to the semileptonic decay of a B into the identical final state,[16] using the Wisgur flavor symmetry. This may be important in normalizing various rare decay branching ratios of B's into charmless final states.
11. A similar statement applies to charmless decays of the Λ_b.
12. In the factorization approximation there exist a number of direct generalizations of the above items to nonleptonic decays.[17]

Semileptonic inclusive properties and sum rules

If many final–state channels are kinematically open, we may anticipate that inclusive techniques analogous to structure–function analyses for deep–inelastic processes become of relevance. This is not quite the case for b–to–c transitions, but is definitely relevant for charmless final states in b–decays. It turns out that, although the physics here is a little different, sum rules exist[18] and some statements can be made.

Again the semileptonic baryon decays provide the simplest example. Consider the inclusive decay

$$\Lambda_b \rightarrow \Lambda_c X e^- \overline{\nu}_e \ . \tag{19}$$

In the infinite–mass limit, the general matrix element could be written down; it was exhibited in Eq. (3). Therefore an inclusive sum can be carried out, yielding

$$\frac{d\Gamma}{dq^2 d\epsilon} \xrightarrow[M_b \rightarrow \infty]{} \left(\frac{d\Gamma_0}{dq^2}\right) \cdot f(v \cdot v', \epsilon) \tag{20}$$

where $d\Gamma_0/dq^2$ is the spectrum given in the free–quark "spectator" approximation and

$$\begin{aligned} W &= M_{\Lambda_c} + \epsilon = \text{final–state mass of } \Lambda_c + X \\ q &= \text{dilepton mass} \ . \end{aligned} \tag{21}$$

The structure function is simply[19]

$$f(w, \epsilon) = \sum_X |F_X(w)|^2 \, \delta(E_X - M_c - \epsilon) \tag{22}$$

with the sum taken in the rest frame of Λ_c. Just by looking at the above expression, one is tempted to write a sum rule

$$\int_0^\infty d\epsilon\, f(\epsilon, w) = 1 \tag{23}$$

which guarantees that the "spectator model" for inclusive decays comes out correctly in the infinite–mass limit. It turns out that the inference is correct.[18,19] It can be derived using ancient techniques of current algebra, although I am sure that more modern methods will be brought to bear on the problem as well.[20]

The physics of this sum rule is similar to the physics of an atom which has been excited due to a sudden acceleration of its nucleus. The sum of all the excitation probabilities must give unity. Notice that this physics is distinct from deep–inelastic scattering, which in the atomic analogy is what happens when one of the electrons in the cloud is suddenly accelerated by an external probe.

For B–meson decays there are analogous results. Again

$$\frac{d\Gamma}{dq^2 d\epsilon} = \left(\frac{d\Gamma_0}{dq^2}\right) \cdot f(w, \epsilon) \; . \tag{24}$$

This time the structure function has a more complicated definition, involving a trace over the Dirac matrices introduced earlier. It will not be written down here. But again an analogous sum rule exists.

$$\frac{1}{2}\,(1+w)\,|F(w)|^2 + \int_{\epsilon>0}^{\infty} d\epsilon\, f(w, \epsilon) = 1 \; . \tag{25}$$

Here we have isolated the "elastic" contribution, which is adorned with a kinematic factor linear in $w = v \cdot v'$. The first derivative of this relation at $w = 1$ gives an analogue of the Cabibbo–Radicati sum rule of old.[21] The squared radius of the elastic form factor is proportional to the amount of decay into "continuum" states such as D^{**} and beyond.[12]

Similar considerations are possible for the charmless semileptonic decays as well, and work on this is in progress in collaboration with Isi Dunietz and Josep Taron. In this case, charmless baryon decays again can be written in the form

$$\frac{d\Gamma}{d\epsilon\, dW^2} = \left(\frac{d\Gamma_0}{d\epsilon}\right) \cdot f(\epsilon, W^2)\ . \tag{26}$$

The invariant variables this time are

$$\epsilon = v \cdot W = \text{hadron energy in } \Lambda_b \text{ rest frame} \tag{27}$$

and

$$W^2 = \text{squared final–state hadron mass.} \tag{28}$$

We are still working on the form of the relevant sum rules.

Corrections to the limiting behavior

The Wisgur limiting theory (note: do *not* call it a model!) may be simple, but that does not guarantee its accuracy. One must classify the corrections and estimate their size. I believe that there is still much which needs to be done; maybe not all leading corrections are as yet identified. Those which are include the following:

1. $\alpha_s(M^2)$ *corrections*: For b–to–c transitions, there are finite renormalization factors[1] coming from virtual processes occurring between the b–mass and

c–mass scales, and other velocity–transfer–dependent renormalizations[22] occurring at scales lower than the charmed–quark scale:

$$\left[\frac{\alpha_s(M_b^2)}{\alpha_s(M_c^2)}\right]^{-\frac{6}{25}} \cdot \left[\frac{\alpha_s(M_c^2}{\alpha_s(\mu^2)}\right]^{d(w)} \quad . \tag{29}$$

These have the typical renormalization group structure emergent from leading–logarithm summations, with the added novelty of a velocity–dependent anomalous dimension (which vanishes at $w = 1$). Other non–logarithmic corrections have been examined[22] and claimed to be small. As yet, there are no results out regarding radiative corrections to the inclusive sum rules.

2. *Power–law corrections*: The techniques of Lepage and Thacker[4] of integrating out high momentum degrees of freedom and replacing the original Hamiltonian with an effective Hamiltonian leads to a quite simple form of M^{-1} expansion:

$$\begin{aligned} H_{\text{eff}} &= b^\dagger D_0 b + b^\dagger \frac{|\boldsymbol{D}|^2}{2M} b + \frac{\kappa}{2M} b^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{B} b \\ &\quad + \mathcal{O}\left(\frac{1}{M^2}\right) \quad . \end{aligned} \tag{30}$$

This form is relevant to the heavy–quark degree of freedom in its rest frame, and exhibits the terms expected in a non–relativistic approximation (in that frame). This allows a systematic starting point for the evaluation of the kinematic corrections to the infinite–mass limit.

3. *Anomalous–threshold effects*: The Wisgur limit requires that the hyperfine splittings tend to zero. In some cases this may mean they be small compared to 300 MeV, the most typical scale in strong interactions. But there are cases where the splitting may need to be small in comparison to the pion mass,

something definitely not true for the charm system. For example, in the Wisgur limit the elastic form factor of a D^* should be the same as that of the D. But the D^*, even if considered stable, will have an anomalous threshold in its form factor[23] (coming from a $\pi - D$ "weak–binding" configuration) which is absent for the D. The anomalous threshold contribution is in any case precisely defined, so hope remains that the correction will be controllable. But this question definitely needs study. Maybe the semileptonic decays of the D are the place to start. There the $K - K^*$ mass difference is definitely too large for the limit. And the trend of the data[24] is not in accordance at present with what one would anticipate from a naive (unjustified) application of Wisgur to the kaon system. (However, model calculations[25] are not so far away from what Wisgur predicts, and they are in trouble with the data too.)

Concluding remarks

I have in this contribution concentrated on the work of Isgur and Wise and developments which have followed from it. It is appropriate, before concluding, to mention some of the important prehistory. This is not meant to be comprehensive; I am not a very good historian of the literature.

We mentioned that the consequences of a nonrelativistic approximation to the heavy quark dynamics, such as the mass dependence of decay constants, has been known for some time, especially within the lattice–QCD community. And an important precursor to Wisgur is contained in work of Voloshin and Shifman,[26] which as I understand it was important in the development of Wise and Isgur's subsequent contributions.

The technology of the trace formalism goes all the way back (at the least) to the work of Delbourgo, Salam, and Strathdee[27] on $\widetilde{U}(12)$, where attempts to relativize the SU(6) nonrelativistic quark–model were being made. In the interim, this formalism was used especially by the Mainz group[28] in their descriptions of baryon decays containing heavy quarks. They used "constant–velocity" wave functions for light and heavy quarks, and the Wisgur results are implicitly realized.

Also, the general results for bottom semileptonic decays are contained in many model calculations, and indeed comparison of the model results with those of the Wisgur limit show quite good agreement. But again the important point is that results in the Wisgur limit represent model–independent statements which are implied by QCD alone.

We conclude with a guess as to the most important consequences for experiment of all this for the near future:

1. The best measure of V_{cb} will be via the exclusive semileptonic decay of B into D. The form factor slope as well as the branching ratio is what is needed. However, all the sundry supplementary measurements of the B to D^* semileptonic processes, as well as the B decays to higher states will be an essential complement, to make sure that the theory is working right. The situation is not so different from kaon semileptonic decays, where K_{e3} decays give the best estimate of the Cabibbo angle, but all the other measurements are vital in building one's confidence that the theory is correct.

2. The best measure of V_{ub} may come via ratios of bottom semileptonic decays to Cabibbo–suppressed semileptonic decays into common final states:

$$\left|\frac{V_{ub}}{V_{cd}}\right|^2 \approx \frac{d\Gamma(B \to Xe\nu)}{d\Gamma(D \to Xe\nu)} \times (\text{known kinematic factors}) \ . \tag{31}$$

3. There exist many predictions for interesting nonleptonic decay processes provided the factorization hypothesis works. For example[29]

$$\frac{\Gamma(\overline{B} \to D^{*}\pi^{-})}{\Gamma(\overline{B} \to D\pi^{-})} = 1 \; . \tag{32}$$

   It is therefore especially important to test that hypothesis in as model independent a way as possible, and as much as possible.

4. As we have seen, there exist especially strong predictions for baryon decays. I would hope that these results may provide special impetus for experimental pursuit of these interesting processes.

5. It will be of interest to test the sum rules.

To me the most difficult theoretical issues remain charmless nonleptonic decays, and the implications—if any—for the corrections to the vacuum insertion approximation to $B - \overline{B}$ mixing. And the whole subject of corrections to the Wisgur limit remains at present a problematic one. Nevertheless, I remain optimistic that this development will lead to much less model dependence than we now have, with the long–range prospect being reduced theoretical systematic errors in determining some of the most fundamental parameters of the standard model.

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