# **QUASI-ISOCHRONOUS STORAGE RING**<sup>\*</sup>

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## Abstract

We study the single particle and collective dynamics of a storage ring where the momentum compaction is made small to make it nearly isochronous, to reduce the bunch length and increase the peak current. We find that a quasi-isochronous ring makes it possible to obtain a bunch length in the millimeter range and to increase the beam longitudinal brilliance by more than one order of magnitude.

## 1. Introduction

The beam energy spread,  $\sigma_e$ , for an electron beam in a storage ring is determined by the emission of synchrotron radiation or by the microwave instability. The bunch length  $\sigma_L$  is proportional to the relative energy spread, average beam energy,  $E_s$ , and to the ring momentum compaction factor,  $\alpha$ , and inversely proportional to the synchrotron tune,  $\nu_s$ 

$$\sigma_L = \frac{\alpha R}{\nu_s} \sigma_e \tag{1}$$

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where R is the average ring radius. The synchrotron tune,  $\nu_s$ , is proportional to the square root of the momentum compaction,  $\alpha$ , so the bunch length can be reduced by reducing the momentum compaction

$$\sigma_L \propto \sqrt{\alpha}$$
 . (2)

In this paper, we study the possibility of designing storage rings with  $\alpha$  reduced by two or more orders of magnitude with respect to existing rings, thus making the bunch length shorter by a factor of ten or more. We will with an appropriate design, show that this bunch length reduction can be achieved while simultaneously increasing the beam peak current. With this approach one can increase the six-dimensional beam phase density, extending the range of applications of storage rings.

For an electron-positron collider, reducing the bunch length allows us to decrease the beta function at the interaction point,  $\beta^*$ . The luminosity would then be increased by a factor of 10. For a damping ring of a linear collider, the bunch length can be matched to the phase acceptance of the linac, thus avoiding the need of longitudinal bunch compression. A quasi-isochronous ring can be a good driver of a FEL, providing a large beam longitudinal brilliance, and a large FEL gain.

There have been other studies which have looked at storage rings or synchrotrons with  $\alpha = 0$ , i.e., operating at transition energy. For the most part, these studies have dealt with proton rings.<sup>1,2,3</sup> In the past, there have also been proposals to use isochronous electron storage rings. Kenneth Robinson<sup>4</sup> proposed this as a system to measure the speed of light to high accuracy. David Deacon proposed the use of an isochronous ring as a driver of a FEL.<sup>5</sup> At the BNL workshop on small emittance rings some analysis was done on the dynamics of an isochronous ring.<sup>6</sup> However, none of these papers presents the analysis of the single particle and collective beam dynamics needed to establish the possibility of operating a storage ring at or near transition energy. This is what we propose to do in this paper.

We will first discuss the basic concept and some of the main properties of a quasi-isochronous storage ring. Next, the single particle dynamics will be discussed. We will establish lower limits to the value of the momentum compaction needed to provide a large enough stable phase-space area to have a good quantum fluctuations beam lifetime. Next, the effect of coherent beam instabilities will be discussed where we show that one can increase the peak current,  $I_p$ , by lowering  $\alpha$ . Finally, some of the possible applications of such a ring will be shortly discussed.

# 2. Isochronous Storage Rings

This discussion of isochronous storage rings will be preceded by a short summary of the general equations of motion for the longitudinal degree of freedom of a storage ring. Then, the differences between conventional rings and isochronous rings will be more clearly illustrated. The main difference between a conventional and isochronous storage ring lies in the longitudinal beam dynamics; the transverse beam dynamics are not influenced, except for the synchro-betatron coupling effects, and this will not be discussed here. In particular, the synchrotron oscillation frequency is assumed to be zero or very small. Defining what we mean by "very small" is one of the key questions to be discussed here.

One issue is that when we make  $\alpha$ , the momentum compaction, small, nonlinear terms which are usually neglected can become important. To allow for this possibility, we will assume in the equation of motion that  $\alpha$  is a function of the particle energy,  $\alpha = \alpha(\delta)$ , where  $\delta = (E - E_s)/E_s$  is the relative energy deviation. Using as variables  $\delta$  and the angular distance  $\psi = \phi - \phi_s$ , from a reference particle of energy  $E_s$  and angle  $\phi_s$ , we can write the equations of motion as<sup>7</sup>

$$\psi' = \alpha(\delta)\delta \tag{3}$$

$$\delta' = \frac{eV_0}{2\pi E_s} \sin(\psi + \phi_s) - \frac{U_0}{2\pi E_s} (1 + J_\varepsilon \delta) + \text{fluctuations}$$
(4)

where  $U_0$  is the energy radiated per turn from the reference particle,  $J_{\varepsilon}$  is the radiation damping partition number and with fluctuations we indicate the term arising from quantum fluctuations in the emission of synchrotron radiation. The superscript ' implies a derivative with respect to  $\omega_0 t$ , where t is time and  $\omega_0$  is the revolution frequency around the ring of the reference particle. For a storage ring with an energy dispersion function,  $\eta$ , and a bending radius,  $\rho$ , we have<sup>7</sup>

$$J_{\epsilon} = 2 + \frac{\left\langle (1-2n)\frac{\eta}{\rho^3} \right\rangle}{\left\langle \frac{1}{\rho^2} \right\rangle}$$
(5)

where n is the bending magnet field index, and with <> we indicate an average over the ring.

The quantity  $V_0$  is the peak voltage of the RF cavity. We also define the synchrotron phase,  $\phi_s$ , so that  $eV_0 \sin \phi_s = U_0$ . In our study, we linearize the effect of the RF cavity. We have found that doing this will have a negligible effect on the dynamics of our beam particles for ring parameters which we have been using. The physical meaning of this approximation will become apparent later on. The equations of motion then become

$$\psi' = \alpha(\delta)\delta \tag{6}$$

$$\delta' = -\kappa \psi - \frac{U_0}{2\pi E_s} J_\varepsilon \delta + \text{fluctuations}$$
(7)

where  $\kappa = (eV_0/2\pi E_s)\cos\phi_s$ .

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The momentum compaction term,  $\alpha(\delta)$ , is dependent upon two quantities; the difference in velocity between the test particle and the ideal particle, and the difference in path length between the test particle and the ideal particle as they travel around the ring. The faster the test particle moves, the farther it moves, tending to decrease  $\psi$ ; however, the longer the path length, the longer it will take to move around the ring tending to increase  $\psi$ . The information concerning these effects is embodied in the momentum compaction term  $\alpha$  which is defined as

$$\alpha = \frac{\left(\frac{\Delta\psi}{2\pi}\right)}{\left(\frac{\Delta E}{E}\right)} \tag{8}$$

where  $\Delta \psi$  is the change in  $\psi$  per turn for given  $\Delta E$ . In the simplest linear approximation

$$\alpha \approx \left\langle \frac{\eta}{\rho} \right\rangle - \frac{1}{\gamma^2} = \alpha_1 \tag{9}$$

where  $\gamma$  is the ratio of the particles total energy to that of its rest energy.<sup>7</sup>

For highly relativistic particles, the first term is dominant; it is usually positive but it can be made nearly zero or negative by having in the ring regions of inverted bending,  $\rho < 0$ , or of negative dispersion,  $\eta < 0$ . The full expression for  $\alpha$  that we should use in (6), is

$$\alpha = \frac{1}{L\delta} \oint ds \left[ \sqrt{(1 + \frac{x}{\rho_s}) 2 + (x') 2 + (z') 2} - 1 \right] - \frac{1}{\gamma^2} \quad . \tag{10}$$

To simplify this initial discussion of a "quasi-isochronous ring," we will expand  $\alpha$  to first order in delta and neglect the contributions from the the betatron oscillations,

$$\alpha = \alpha_1 + \alpha_2 \delta \tag{11}$$

with  $\alpha_1$  given by equation (9). The  $\alpha_2$  term is given by

$$\alpha_2 = \frac{1}{L} \oint ds \frac{(\eta')^2}{2} \quad . \tag{12}$$

The value of  $\alpha_1$  can be adjusted to be zero or negative but, as one can see from (12),  $\alpha_2$  is always positive. Therefore one can never completely eliminate the effect of  $\alpha_2$ . If sextupoles are included in the lattice, there is a second-order dispersion term,  $\eta_2^8$ 

$$x = \eta_1 \delta + \eta_2 \delta^2 \quad . \tag{13}$$

The second-order term in the momentum compaction is then modified and instead of (12) we have

$$\alpha_2 = \frac{1}{L} \oint ds \left[ \frac{(\eta')^2}{2} + \eta_2 \right] \quad . \tag{14}$$

Making  $\eta_2$  negative can provide a possibility to decrease  $\alpha_2$ , but we have not considered this possibility in this paper. As a first step in understanding the behavior of an isochronous ring, we will study the system of equation (6) and (7) with  $\alpha$  given by (11).

The simplest case is that when  $\alpha_1 = 0$ , and in addition we neglect damping. We are now reduced to the equation:

$$\delta'' + \kappa \alpha_2 \delta^2 = 0 \quad . \tag{15}$$

This equation leads to unstable solutions, described in phase space by the curve,  $(\delta')^2 + (2/3)\kappa\alpha_2\delta^3 = \text{constant}$ , and shown in Figure 1 for a particular case.<sup>6</sup>

To make the longitudinal motion stable, we must have  $\alpha_1 \neq 0$ ; this introduces an elastic-like focusing force, and thus provides a stable oscillation region near the origin ( $\delta = \psi = 0$ ). The area of the stable region depends on the relative magnitude of  $\alpha_1$  and  $\alpha_2$ , and also on  $\kappa$ , and usually can be made large enough for a convenient accelerator operation. The stable phase-space region can also be made larger, when  $\alpha_1$  is made small, by increasing the rate at which the electrons lose energy through the emission of synchrotron light; in other words, increase the damping of the system. If  $\alpha_1 = 0$  there can be no absolute stability in the system no matter how large the damping is. However, damping will slow down the rate of the instability growth. Indirectly, damping does provide stability. Without damping, one would require a larger value of  $\alpha_1$  for stability than with damping. With more damping, the smaller  $\alpha_1$  can be. A more detailed discussion on the limits of stability will now follow.

### 3. Limits of Stability

This discussion of stability limits for  $\alpha$  will involve the second-order equations of motion including damping with  $\alpha$  given by (11). We will continue now to neglect the fluctuation term. The question which we try to answer is: given a specific value for  $\alpha_2$ , what is the smallest value of  $\alpha_1$  necessary to make the equations of motion stable? The answer is that this value of  $\alpha_1$  is dependent upon both the initial value of  $\psi$  and  $\delta$  and also on the amount of damping.

Initially we chose the following values for  $\psi_0$ ,  $\delta_0$  and  $\kappa T_0$ :  $\psi_0 = 0.0001$ ,  $\delta_0 = 0.001$ and  $\kappa T_0 = -0.01$ , where  $T_0$  is the period of oscillation. The choice of  $\psi_0$  corresponds to an initial displacement of about 1 cm for a ring with a circumference of 760 m, whose parameters are given in Table 1. The choice of  $\kappa T_0$  is consistent with possible parameters for an isochronous ring.

In the case of zero damping we find that for each value of  $\alpha_2$ , there exists a certain value of  $\alpha_1$  which just makes the motion stable. In Figure 2, we plot three curves of  $\delta$  verses  $\psi$  for a specific choice of  $\alpha_1$  and  $\alpha_2$ . The middle curve is a separatrix which defines the limiting value of  $\delta$  and  $\psi$  which will result in stable motion. The middle curve is typical of all limiting cases. All choices of  $\psi_0$  and  $\delta_0$  lying inside the curve will result in closed trajectories. All choices of  $\psi_0$  and  $\delta_0$  lying outside the curve will result in open trajectories. A choice of  $\alpha_1$  greater than the limiting value will also give rise to a closed curve. This curve will lie inside this separatrix and will be smoother than that for the limiting case. But for smaller values of  $\alpha_1$ , the curve will be open and the trajectory will be unstable, similar to the outermost curve in Figure 2.

It is interesting to see what happens when  $\psi$  and  $\delta$  are plotted against time using the case of zero damping for illustrative purposes. In Figure 3, there are four plots. The first is a plot made for a choice of  $\alpha_1$  which lies far within the stability region. It appears regular and sinusoidal for both  $\psi$  and  $\delta$ . This is a direct result of the fact that the first-order term,  $\alpha_1$ , completely dominates the second-order term,  $\alpha_2$ , and the motion is therefore almost that of a simple harmonic oscillator. The second plot is for a smaller choice of  $\alpha_1$  which is also within the stability limits but is closer to the limiting value. The period of oscillation for  $\psi$  and  $\delta$  is increasing but the shape still basically resembles that of sine waves. The third plot is for a choice of  $\alpha_1$  which is very close to the stability limit. In this plot the motion is oscillatory; however, the period has increased dramatically and the curve is strongly distorted from a sinusoid. During portions of the oscillation,  $\psi$  remains close to zero and  $\delta$  is nearly constant. If  $\alpha_1$  was chosen to lie right on the limit of stability, the period of motion would increase to infinity. Finally, there is a plot for a value of  $\alpha_1$  which lies just outside the limit of stability. The motion appears to be stable for a while but at some time both  $\psi$  and  $\delta$  diverge. The farther outside the limit of stability, the shorter time it will take for  $\psi$  and  $\delta$  to diverge.

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Now the case with damping is considered. Again we find that for every value of  $\alpha_2$  there exists a specific choice of  $\alpha_1$  such that all larger values are stable and all smaller values are unstable. Plotting  $\delta$  versus  $\psi$  as we did before for this limiting case, we do not find a closed separatrix, but instead, a spiral which is sharp at the bottom and slowly converges to the point (0,0) as seen in Figure 4. In the case in which  $\alpha_1$  is smaller than this critical value, the motion is unbounded as in the case with zero damping. As the value of  $\alpha_1$  increases, moving the system farther into the stability region, the curve becomes smoother and rotates quicker. And in a plot of  $\psi$  and  $\delta$  versus time, one finds that the motion is that of a damped oscillator which is converging on the reference particle's coordinates.

Let us now take a further look at the limits of stability. We have found that on the boundary between stability and instability there exists one value of  $\alpha_1$  for each value of  $\alpha_2$ . For three different cases of damping we plot these values of  $\alpha_2$  against  $\alpha_1$  as seen in Figure 5. One finds that  $\alpha_1$  increases steadily with  $\alpha_2$ . Also one finds that as the damping increases the values of  $\alpha_1$  decrease, and the curve is lowered. So for stable operation of a storage ring,  $\alpha_1$  must be chosen to lie above the curves.

Finally, to get an indication of how these requirements of  $\alpha_1$  depend upon the initial values of  $\psi$  and  $\delta$ , we choose the values  $\psi_0 = 0.0003$  and  $\delta_0 = 0.003$ . Again, making a plot of  $\alpha_1$  versus  $\alpha_2$ , we find similar curves to that of the first choice of  $\psi_0$  and  $\delta_0$  as can be seen in Figure 6. But the corresponding values of  $\alpha_1$  are almost a factor of three bigger. So the "tighter" the initial conditions for the bunch, the smaller one can make  $\alpha_1$ .

We can now comment on the approximation introduced by linearizing the sine term in (4). If we assume  $\alpha_2 = 0$ , but keep the sine term, we have a stable phase space region (the RF bucket), defined by the RF voltage,  $\phi_s$ , and  $E_s$ . The linearization does not change our results if the RF bucket is larger than the stable region that we obtain when  $\alpha_1$  is small and  $\alpha_2$  is nonzero, the "drop" of Figure 2. This condition is satisfied in all our calculations.

## 4. Coherent Instabilities

A concern involving a ring having a small momentum compaction is the threshold peak current which can be stably circulated in the ring. This current is determined by collective effects such as the longitudinal microwave and fast head-tail instabilities. Some studies have been made<sup>2,3</sup> for proton or heavy ion rings regarding these effects when crossing the transition energy. These cases are somewhat different from the electron-positron collider case due to the fact that there is no damping in these machines, and that the beam energy is changing.

In determining the effects of collective instabilities, we have first considered the effect due to the longitudinal microwave instability. As is known<sup>9</sup> the threshold peak current determined by the onset of the longitudinal microwave instability is proportional to the momentum compaction,  $\alpha_1$ , when  $\alpha_2 = 0$ :

$$I_p = \frac{2\pi\alpha_1 E_s(\beta\sigma_e)^2}{\left|\frac{Z_{||}}{n}\right|} \mathcal{F}$$
(16)

where  $|Z_{\parallel}/n|$  is the ring longitudinal coupling impedance divided by the mode number,  $n, n = \omega/\omega_0, \omega_0 = c/R$ .  $\mathcal{F}$  is a form factor coming from a dispersion relation which will be given later. On the surface, equation (16) seems to indicate that smaller  $\alpha$  machines would be unable to circulate as large a peak current as larger  $\alpha$  machines.

However, the value of  $|Z_{\parallel}/n|$  is dependent upon the length of the bunch. The wavelengths of modes which are important in contributing to the longitudinal microwave instability are the size of the bunch length or smaller. The impedance of the ring should therefore be evaluated using only these modes. In general, the impedance will decrease at smaller wavelengths (SPEAR scaling), and this can compensate the decrease of  $\alpha$ .<sup>9</sup>

In addition to this, there is another effect which is important in determining the threshold peak current. One can tolerate an instability growth so long as it is smaller then the damping rate. In other words, we redefine the threshold peak current as the peak current which corresponds to an instability growth equal to the damping rate. As we will see from the following analysis, the growth rate of the instability is a function of  $\alpha_1$  and tends to zero as  $\alpha_1$  tends to zero. Hence, for rings with small  $\alpha_1$  and large damping we can circulate larger currents.

Using this definition we calculate the threshold peak current as a function of the damping rate including the effect of the  $\alpha_2$  term in the equation of motion. This is done using the coasting beam approximation and following the usual Vlasov equation approach.<sup>10</sup> The distribution of the particles is a function of the angle  $(\psi)$ , the energy deviation  $(\delta)$  and time (t). Due to the periodic nature of the collider one can make a separation of variables and write the distribution as

$$f(\psi,\delta,t) = \frac{N}{2\pi} \sum_{-\infty}^{\infty} f_n(\delta,t) e^{-in(\psi+\omega_0 t)} \quad .$$
(17)

Furthermore, an assumption is made that the distribution function of energy and time,  $f_n$ , is separable and can be written:

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$$f_n(\delta, t) = \overline{f}_n(\delta) e^{i\nu_n t} \quad . \tag{18}$$

Using this expression for the distribution it is clear that the variable  $\nu_n$  contains information concerning the stability of the distribution. In general,  $\nu_n$  can be written in terms of its real and imaginary parts.

$$\nu_n = \nu_{nR} + i\nu_{nI} \quad . \tag{19}$$

If  $\nu_{nI}$  is positive the distribution is a stable function of time. If  $\nu_{nI}$  is negative the distribution is an unstable function of time. Writing down the Vlasov equation an expression can be found for  $f_n(\delta, t)$ . In our calculation, this was done keeping terms up to first order in the impedance:

$$f_n(\delta, t) = \frac{\omega_0 e^2 \beta c}{2\pi E_s} Z_n(\omega) \lambda_n(t) N \frac{df_0}{d\delta} \frac{1}{i(\nu_n + n\omega_0 \alpha_1 \delta + n\omega_0 \alpha_2 \delta^2)}$$
(20)

where  $\omega_0$  is the design orbital frequency of particles in the machine,  $f_0$  is the normalized zeroth-order distribution and N is the number of particles in the ring.  $\lambda_n(t)$  is given by the expression:

$$\lambda_n(t) = \frac{1}{R} \int_{-\infty}^{\infty} f_n(\delta, t) d\delta$$
(21)

Integrating (21) over  $\delta$ , one removes the energy dependence and ends up with a dispersion relation

$$1 = -\frac{ieI_p}{4\pi\alpha_1 E_s \sigma_e^2} \frac{Z_{\parallel}}{n} \int_{-\infty}^{\infty} d\Delta \frac{\frac{df_0}{d\Delta}}{a + ib + \Delta + d\Delta^2}$$
(22)

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$$a = \frac{\nu_{nR}}{\sqrt{2}n\sigma_e\omega_0\alpha_1} \tag{23}$$

$$b = \frac{\nu_{nI}}{\sqrt{2}n\sigma_e\omega_0\alpha_1} \tag{24}$$

$$d = \frac{\alpha_2}{\sqrt{2}\sigma_e \alpha_1} \tag{25}$$

$$\Delta = \frac{\delta}{\sqrt{2}\sigma_e} \qquad . \tag{26}$$

This differs from the usual expression of the dispersion relation only for the  $\delta^2$  term in the denominator of the integral.

Assuming given rings characteristics, a specific energy for the particles and a gaussian distribution in energy, one has three unknown parameters,  $\nu_{nR}$ ,  $\nu_{nI}$ , and  $I_p$ . The dispersion relation, being complex, yields two equations. To solve for any two of these unknown variables, one must assume a certain value for the third. In the limiting case of the threshold peak current without damping, one sets  $\nu_{nI} = 0$  and then determines  $I_p$  and  $\nu_{nR}$ . This is the conventional threshold peak current. In our calculation  $\nu_{nI}$  was set to several different positive values corresponding to machines with different damping times. For each of these values of  $\nu_{nI}$ ,  $I_p$  and  $\nu_{nR}$  were determined. The result can be seen in Figure 7 where we have used for the impedance the "broad-band" resonator model<sup>9,10</sup>

$$Z_{\parallel}(\omega) = \frac{R_s}{\left[1 + iQ\left(\frac{\omega_r}{n\omega_0} - \frac{n\omega_0}{\omega_r}\right)\right]} \quad .$$
(27)

where,  $R_s$  is the shunt impedance of the ring, we assume a Q = 1 resonator, and the critical frequency  $\omega_r$  corresponds to the vacuum pipe cutoff,  $\omega_r = c/b$ , b being the pipe radius. We also evaluate this impedance at a mode number or frequency, corresponding to the bunch length,  $n\omega_0 = c/\sqrt{2\pi}\sigma_L$ , assuming that  $\sigma_L < b$ . Notice that with this assumption, and for short bunch length, the impedance scales as  $\sigma_L/b$ , and Z/n as  $\sigma_L^2$ . This scaling will change when the impedance becomes of the order of the vacuum impedance,<sup>11</sup>  $Z/n \sim 300b/R$ . In our calculation we assume to be in the region of frequencies larger than the pipe cutoff and smaller than that where the vacuum impedance becomes the dominant term.

The results in Figure 7 show that in the absence of damping, the threshold peak current is independent of  $\alpha$  as is expected. But as the amount of damping increases, the threshold peak current for the small momentum compaction becomes greater then that of the large momentum compaction ring. In the case of extreme damping

$$I_p \propto \frac{1}{\alpha_1} \quad . \tag{28}$$

There is no strong dependence on  $\alpha_2$ .

This result can be understood rather easily. In the limit of no damping, using the impedance (27) and evaluating it at the frequency  $\omega = c/\sqrt{2\pi\sigma_L}$  with  $\sigma_L = (\alpha_1 R\sigma_e/\nu_s)$ , one can see from (16) that  $I_p$  is independent of  $\alpha_1$ . This makes the two curves in Figure 7 almost equal when  $1/\tau_D$  becomes small. In the limit of large damping rates since the instability growth time is proportional to the momentum compaction, the peak current is inversely proportional to the damping time and thus is inversely proportional to the momentum compaction. This calculation has been done using the coasting beam and broad-band impedance model. In order to make a better determination of the effects of the microwave instability, a calculation of the bunched beam longitudinal microwave instability needs to be done, and the effect of different impedances needs to be analyzed.

We can use the same impedance model to look at the scaling of the fast head-tail instability. The expression for the threshold peak current<sup>12</sup> is

$$I_p \propto \frac{\nu_s \nu_\beta E_s}{e Z_\perp} \tag{29}$$

where  $Z_{\perp}$  is the transverse impedance of the ring,  $\nu_{\beta}$  is the betatron tune of the ring and  $\nu_s$  is the synchrotron tune of the ring. In the high-frequency region, above the pipe cutoff, the transverse impedance can be assumed to be proportional to the longitudinal impedance,  $Z_{\perp} = 2(R/b) 2(Z_{\parallel}/n)$ . The synchrotron tune,  $\nu_s$ , is proportional to  $\sqrt{\alpha}$ . Using our SPEAR scaling argument again we find that the transverse impedance,  $Z_{\perp}$ , is proportional to  $\alpha$ . The threshold peak current as a function of  $\alpha$  is then

$$I_p \propto \frac{1}{\sqrt{\alpha}}$$
 . (30)

The small momentum compaction ring seems to be advantageous over a conventional ring for circulating large currents. For damping times on the order of milliseconds (the magnitude of damping which would be desirable for a B or  $\Phi$  factory) one can have a much larger peak current in a small  $\alpha$  ring then in a conventional ring.

## 5. Possible Applications of a Quasi-Isochronous Ring

#### 5.1. Collider

As was mentioned earlier, one of the possible uses of a quasi-isochronous ring would be as a high-luminosity collider. This is of interest for Z-Factories, B-Factories and  $\Phi$ -Factories. The collider luminosity,  $\mathcal{L}$ , can be written as

$$\mathcal{L} = \frac{fN^2}{4\pi(\sigma_x^*\sigma_y^*)} = \frac{fN^2}{4\pi\sqrt{\beta_x^*\beta_y^*\varepsilon_x\varepsilon_y}} \quad . \tag{31}$$

N is the number of particles in each bunch assumed to be the same for each beam, f is the frequency of collisions,  $\sigma_x^*$  and  $\sigma_y^*$  are the transverse dimensions of the beam at the interaction point,  $\beta_x^*$  and  $\beta_y^*$  are the beta functions at the interaction point and  $\varepsilon_x$  and  $\varepsilon_y$  are the x and y emittances of the beam in the ring. The vertical emittance is determined by the coupling and is proportional to  $\varepsilon_x$ . There is an effective limit to how small one can make the  $\beta^*$ s so as to increase  $\mathcal{L}$ .

$$\beta_{\min}^* \approx \sigma_L \quad . \tag{32}$$

Making  $\beta^* < \sigma_L$  would not result in an increased luminosity. In addition the beambeam tune shift puts a limit on the ratio  $N/\varepsilon_x$ . If one wants to increase  $\mathcal{L}$  without increasing fN, i.e., the beam power, one would have to or decrease  $\sigma_L$  and  $\beta^*$ .

Assuming one can achieve such a small bunch length, it then is necessary to have a focusing system which can provide a  $\beta^*$  about equal to  $\sigma_L$  in the millimeter range. In a recent paper by Robert Palmer<sup>13</sup> concerning the prospects for high-energy electron-positron linear colliders, he discusses different methods of producing small beta functions at the interaction point. He provides a formula for determining  $\beta^*$ using conventional focusing techniques.

$$\beta^* = T \frac{E_s/e}{c} \frac{S}{B^*} \sqrt{\frac{\varepsilon}{\beta^*}} \sigma_e^2 \tag{33}$$

where T and S are constants whose values each are about 10 and  $B^*$  is the maximum quadrupole pole tip field in Tesla. Using this formula and assuming a large value of the emittance,  $10^{-6}$ m-rad, a  $B^*$  of 1T, a beam energy,  $E_s$ , of 5GeV and a relative energy deviation,  $\sigma_e$ , of  $10^{-3}$  we find that we can produce a  $\beta^*$  of 0.1 mm. Thus it seems possible to provide millimeter  $\beta^*$ .

Assuming we can provide such a small beta function, a tentative list of parameters for a B factory using a quasi-isochronous ring is given in Table 1. Conventional B factory designs<sup>14</sup> require several Amperes of current to reach a luminosity of  $10^{34}$  cm<sup>-2</sup> s<sup>-1</sup> while a quasi-isochronous collider needs only a few hundred milliamperes. This is because conventional colliders have an  $\alpha_1$  of about  $10^{-3}$  while our value for  $\alpha_1$  is  $3.5 \times 10^{-5}$  which allows for a reduced bunch length of about 7.5. The smaller current of a quasi-isochronous ring, reduces the RF power, the synchrotron radiation power density on the vacuum chamber and associated vacuum problems, the problem of controlling the multibunch instability, thus making the collider technology much easier. Alternatively for the same beam current one can increase the luminosity by another factor of ten.

Our choice of  $\alpha_1$  was made by choosing an appropriate value from Figure 5 which was consistent with our other quasi-isochronous collider parameters. As a result of this smaller bunch length, the beta function of the quasi-isochronous ring is smaller, which in turn provides a bunch width considerably smaller than that of the conventional ring. The total current can then be reduced for the same luminosity.

Our justification of this difference lies in the fact that the ratio of the two values of the momentum compaction is about 650. By the relationship given in the previous section, the bunch length could be reduced by a factor on the order of 25. Thus a reduction in  $\sigma_L$  of 7.5 would be very reasonable.

## 5.2. DAMPING RINGS, LIGHT SOURCES AND FREE ELECTRON LASERS

Another possible use of a quasi-isochronous ring would be for a damping ring or a synchrotron light source. The main advantage of a ring of this type would be the short bunches which it would naturally produce. For a damping ring, the beam would not have to be "artificially" bunched after it exits the ring before it goes through another accelerating section. Short bunches, of a few picoseconds, would also be useful for light sources where there are some experiments where time-of-flight measurements are important.

One could also use a quasi-isochronous storage ring as a driver for a Free Electron Laser. The advantage which a quasi-isochronous ring would have is that it could produce beams with a large longitudinal brilliance, more than what has been achieved in conventional rings. The longitudinal Brilliance,  $\mathcal{B}_L$ , is a measure of the quality of the beam and is defined as<sup>15,16</sup>

$$\mathcal{B}_L = \frac{Nec}{\sqrt{2\pi}\varepsilon_L} = \frac{I_p}{\gamma\sigma_e} \tag{34}$$

where  $\varepsilon_L$  is the normalized longitudinal emittance. The power in an FEL is a function of the energy spread and peak current, and it scales like the square root of the longitudinal brilliance.<sup>16</sup> Thus a quasi-isochronous ring can produce a larger gain and extend the operation region of an FEL to shorter wavelengths.

#### 6. Conclusions

We have shown that one can have stable motion in a ring with values of  $\alpha_1$ which are many orders of magnitude smaller than  $\alpha_2$  and much smaller than used in existing rings. Since the size of the bunch is related to the momentum compaction, one can decrease the bunch length by decreasing the momentum compaction factor. In fact, in the case where  $\alpha_2 = 0$ ,  $\sigma_L$  is proportional to  $\sqrt{\alpha_1}$ . In the case where  $\alpha_2 \neq 0$ , one can still use this relationship to obtain an estimate of the bunch length, the reason being that for stable motion, the  $\alpha_1$  term  $(\alpha_1 \delta)$  is larger than the  $\alpha_2$  term  $(\alpha_2 \delta^2)$ . A simulation was done including the effect of fluctuations in the photon emission spectrum and the results confirm this statement. For a "normal" ring  $\alpha_1$  is  $10^{-2} \sim 10^{-4}$  giving  $\sigma_L \sim 1$  cm. In our case, we can have  $\alpha_1 \sim 10^{-5}$  which means a bunch length,  $\sigma_L \sim 1$  mm. Also, our first look at the effects of other many particle instabilities seems to indicate threshold current limits for the small momentum rings are no worse and can even be better then that of conventional rings.

A small  $\sigma_L$  allows us to use a small  $\beta^*$ , increasing the luminosity of a collider for a given beam average current. It might therefore be possible to build high-luminosity colliders using smaller beam currents. But before we can make definitive statements, more work has to be done. A more general form of  $\alpha$  will have to be used, including the effect of betatron oscillations and higher-order terms in  $\delta$ . We also need to look at the effect that higher-order lattice elements, sextupoles and octupoles, have on the expression of the momentum compaction. Finally, a "real" machine needs to be designed to see if these parameters for  $\alpha_1$  and  $\alpha_2$  are possible. We expect to report in the near future the results of this additional work.

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Table 1
Quasi-Isochronous B Factory Parameters

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760
5
$10^{34}$
0.48
0.05
1
194
120
1.6
$2.56\times 10^{10}$
3.5
3.6
0.68
$2 \times 10^{-8}$
1.3
$1.1\times 10^{-5}$
$3 \times 10^{-5}$

19

## **Figure Captions**

1. An example of unstable motion for the case of  $\alpha_1 = 0$  and zero damping.

2. An example of three curves in energy angle phase space for a specific value of  $\alpha_1$  and  $\alpha_2$  in the absence of damping. Outermost curve lies outside the region of stability, the innermost curve lies within the stability region, and the middle curve lies on the limit of stability, which lies on the stability limit for the case of zero damping.

- 3.  $\delta/\delta_0$  and  $\psi/\psi_0$  versus number of revolutions around the ring: Three examples within the stability region and one outside for the case of zero damping.
- 4. An example which lies on the limit of stability for the case of finite damping.
- 5. Three curves which define the stability limits for  $\alpha_1$  shown for three different values of damping with  $\delta_0 = 0.001$  and  $\phi_0 = 0.0001$ .
- 6. Three curves which define the stability limits for  $\alpha_1$  shown for three different z-values of damping with  $\delta_0 = 0.003$  and  $\phi_0 = 0.0003$ .
- 7. Microwave instability threshold peak current as a function of inverse damping time. The peak current is in arbitrary units.



Fig. 1



Fig. 2



Fig. 3



Fig. 4



Fig. 5



Fig. 6



Fig. 7