SLAC-PUB-5345
September 1990
(A)

COMPUTER DETERMINATIONS OF THE PROPERTIES OF WAVEGUIDE LOADED CAVITIES*

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Abstract

Two recently developed computer methods to determine the resonant frequency and Q_{ext} of waveguide loaded cavities are described. We present an application of these methods to a cavity of simple form with a range of coupling apertures. The methods are found to be in good agreement with each other and also with experiment.

1. Introduction

In this paper we describe methods to determine the properties of accelerator cavities which are loaded by external waveguides designed to damp higher order beam induced modes. The methods use MAFIA or ARGUS computations of the properties of the resonant modes of the combination of shorted sections of the loading waveguides coupled via apertures to the accelerator cavities. We begin with a review of the method of Kroll and Yu,[1] which makes use of the relation between the frequencies of the modes and the lengths of the shorted waveguide sections. A useful extension of this method[2] (which reduces the number of lengths required by a factor two) will be described. It makes use of the fact that the derivative of the frequency with respect to the shorted waveguide length can be computed from stored energy and field strength data provided by these computer codes. Some comparison of the method with experimental determinations will also be given. As will be shown by examples, the Kroll-Yu method is also well suited to experimental determinations of cavity properties, especially in situations where more standard methods are difficult to apply.

2. Basic Approach

We consider a cavity coupled to a waveguide through an aperture. We wish to determine the resonant frequencies and decay constants of various modes when the waveguide is terminated by a matched load. We neglect all internal losses so that the decay constant is directly related to the external Q of the cavity (referred to as Q henceforth). These quantities can be directly related to the mode spectrum of the coupled cavity-waveguide system formed by shorting the waveguide at a distance D from the output plane, by studying the dependence of the mode spectrum upon the distance D. A typical example of this dependence is illustrated in Fig. 1, where the frequency f is normalized to the frequency of the uncoupled cavity mode and the abcissa r refers to D normalized to one-half the cutoff wavelength of the waveguide. Since all of the curves refer to a single cavity resonance, we refer to them as branches. The following formula provides an excellent four-parameter representation of these curves in the vicinity of the resonant frequency of the cavity.

$$\tan\left[k(\omega)D + \chi(u) + \chi'(u)(\omega - u)\right] = \frac{u}{2Q(\omega - u)}\ , \quad (1)$$

Here $k(\omega)$ is equal to $2\pi/\lambda_g$ as usual. The parameter u represents the resonant frequency of the cavity coupled through the output and waveguide to a matched load, while the parameter Q represents the associated external Q. The remaining two parameters, $\chi(u)$ and $\chi'(u)$, parameterize the effect of distant cavity resonances. Theoretical background for Eq. (1.1) is provided in the Appendix. As noted by Kroll and Yu, the four parameters may be determined by computing four frequency-length pairs in the vicinity of the cavity resonance (as identified by inspection of field plots) and requiring that Eq. (1.1) be satisfied for each pair. A minimum

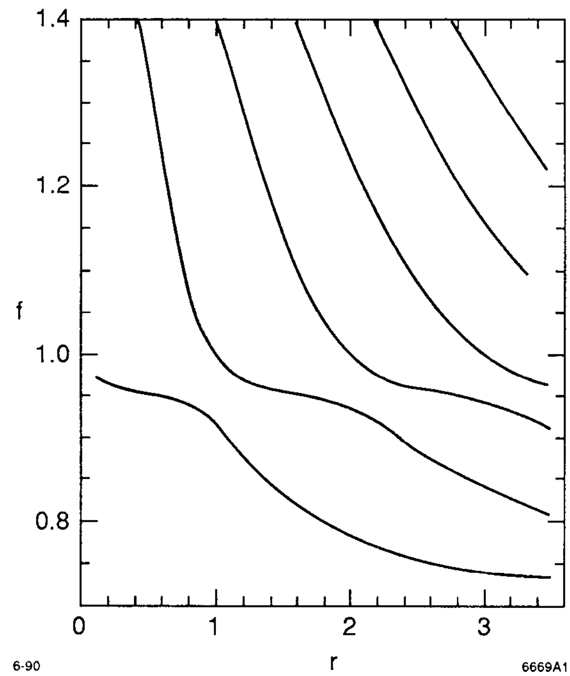

Fig. 1. Mode frequencies versus waveguide length.

of four computer runs at four different lengths is required to produce four reasonably placed points on a single branch. On the other hand, if the lengths are chosen in the vicinity of an avoided crossing region, where two branches are adequately close to the resonance, only two lengths are required and two frequency-length pairs are taken from each of the two branches. This reduces the amount of computing required by a factor two, since the computer run typically yields all the modes of interest at a specified length in a single run.

The extension of their method which we report here replaces two of the frequency-length pairs with a determination of the derivative of frequency with respect to length at the remaining two frequency-length pairs. By differentiating Eq. (1.1), we obtain

$$\frac{d\omega}{dD} = -k(\omega)\left[\frac{\omega D}{kc^2} + \chi'(u) + \frac{2Qu}{4Q^2(\omega - u)^2 + u^2}\right]^{-1} \quad (2)$$

where c represents the velocity of light. The four parameters are now determined by requiring that both Eqs. (1.1) and (1.2) be satisfied for the two frequency length pairs. The derivative which appears in Eq. (1.2) is obtained from the computer runs via the formula

$$\frac{d\omega}{dD} = -\frac{\omega\int\left(\mu_0 H^2 - \epsilon_0 E^2\right)dS}{2\int \mu_0 H^2 dV}\ , \quad (3)$$

which follows from cavity resonator perturbation theory. The volume integral in Eq. (1.3) is proportional to the stored energy of the mode, which is one of the standard outputs of a typical computer run. The surface integral is carried out over the shorting plane at the end of the waveguide. Since the field values are also available as output, it can be readily computed. In the case of standard waveguides, the surface integral can be determined from field values at single points; there is, of course, no electric field contribution in the case of TE waveguide modes.

*Work supported by Department of Energy contracts DE-AS03-89ER40527 and DE-AC03-76SF00515.

Contributed to the Linear Accelerator Conference, Albuquerque, New Mexico, September 10–14, 1990.

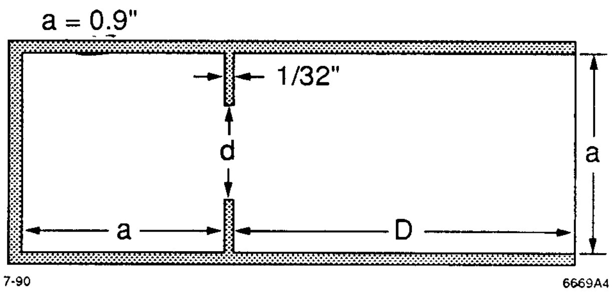

Fig. 2. Test cavity with iris and waveguide output.

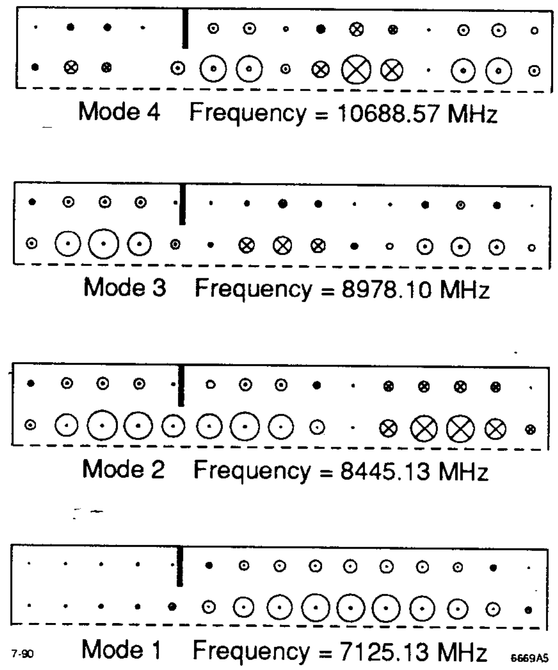

Fig. 3. Electric field plots for first four modes.

If it is clear from an examination of the field plots associated with a single computer run that two branches from the same cavity mode are present (as is often the case), then that run is sufficient to determine the properties of the resonance. If only one branch is recognizable, then a second run at a different length will provide a second point on the same branch, and the determination of cavity properties can be made from the two runs. This latter procedure may also be preferable when the frequency separation of the points on the two branches is too large.

3. Example

We illustrate the procedure for the simple cavity shown in cross section in Fig. 2. The cavity height is chosen to be small as the modes of interest to us here have frequencies independent of cavity height. In Fig. 3 we show electric field plots of the first four modes for the case $d/a = 0.5$ at $D = 2.0$ inches. Since we are studying the lowest cavity mode, which is symmetric with respect to the center of the iris, only half of the cavity-waveguide combination is shown. The curves of Fig. 1 imply that no matter what value is chosen for D, at least one mode will be close to a cavity resonance. Examination of the field plots shows that it is the third mode which is closest to the cavity resonance, that modes 2 and 4 are different branches of the same resonance, and that mode 2 is clearly the closer of the two. Since the two branches nearest resonance are clearly identified, a single length determination of cavity properties using modes 2 and 3 can proceed. The result obtained is frequency = 8769.07 MHz and $Q = 34.542$.

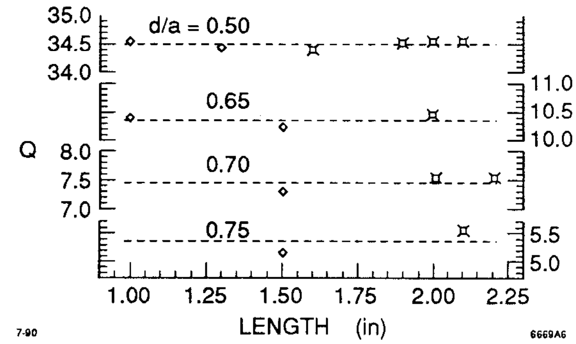

Fig. 4. Computed Q values for test cavity.

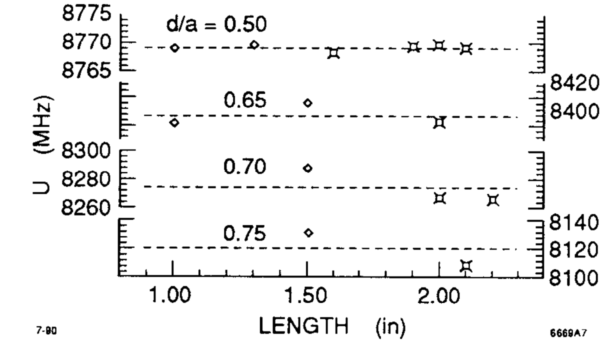

Fig. 5. Computed frequency values for test cavity.

In order to investigate the stability of results with respect to the waveguide length D chosen, we carried out computations at a number of different values. We also investigated a number of different iris openings d/a. The results for Q and frequency are shown in Figs. 4 and 5. Note that the points arising from the first two modes are displayed with a different symbol than those arising from modes 2 and 3. The horizontal lines represent arithmetic averages of the determinations for each d/a value, and are inserted to assist an assessment of the variation. A more thorough investigation of the dependence of results upon values for D and d/a has been carried out for an analytic approximation to the cavity of Fig. 2 with zero iris thickness. The analytic approximation also permits exact evaluations of the resonant frequency and Q to be compared with the single length determinations. Excellent agreement was found.[2]

Although single length, two-branch determinations proved to be satisfactory for this model at all lengths tested, there may be circumstances in which a two-length single branch determination is preferable. Accordingly, we used the data obtained to investigate the length dependence of the above results, to also test their consistency with two-length single-branch determinations. We were also able to compare results with those obtained from the Kroll-Yu method. A representative set of comparisons is shown in Table I. The method employed for each case should be clear from the branch and length lists.

4. Comparison with Experiment

In order to compare the results from Table I with experimentally determined values, a cavity conforming to the configuration of Fig. 2 was constructed from standard 0.9 in by 0.4 in waveguide and provided with a set of interchangeable irises with the d/a values listed in Table I. Measurements were carried out with the assistance of a Hewlett-Packard network analyzer. The standard detuned short

Table I

d/a	Frequency (MHz)	Q	Branches	Lengths (inches)
0.5	8769.07	34.54	2,3	2.0
	8769.09	34.52	2	1.3,1.9
	8768.93	34.63	2,3	2.0,2.1
	8768.84	34.59	2	1.3,1.6,1.9,2.0
0.65	8405.01	10.41	2,3	2.0
	8407.99	10.39	2	1.5,2.0
	8403.95	10.34	2,3	1.5,2.0
0.70	8266.35	7.54	2,3	2.0
	8270.48	7.48	2	1.5,2.0
	8263.87	7.48	2,3	1.5,2.0
0.75	8109.44	5.57	2,3	2.1
	8114.20	5.47	2	1.5,2.1
	8109.52	5.55	2,3	1.5,2.1

method of Q and frequency measurement was attempted, using a $0.26 \times 0.90 \times 0.72$ inch aluminum insert attached to the rear cavity wall to detune the cavity. Unfortunately, it produced adequate detuning to carry out the measurement only for the case $d/a = 0.5$, so that we are able to report detuned short method results for this case only. As an alternative, we carried out measurements for all four d/a values using the Kroll-Yu method. That is to say, we measured the phase of the reflection coefficient referred to the iris plane and used the Kroll-Yu four-parameter formula. The results obtained are shown in Table II and should be compared to Table I and Figs. 4 and 5.

Table II

d/a	Frequency (MHz)	Q	Method
0.50	8739	33.9	Detuned Short
	8750 ± 1	35.3 ± 1	Kroll-Yu
0.65	8375 ± 20	10.1 ± 0.8	Kroll-Yu
0.70	8210 ± 40	7.7 ± 0.7	Kroll-Yu
0.75	8040 ± 30	5.5 ± 0.2	Kroll-Yu

The uncertainties shown in Table II are crudely based upon variation observed with different selections of the four frequency phase pairs, and do not include an estimate of other sources of error. These variations are larger than those shown with different selections among the MAFIA computed pairs, and may reflect a lack of precision in the measurements.

5. Current Applications and Future Work

The methods described in this paper are being applied to the design and experimental analysis of accelerator cavities with heavily damped higher order modes. The experimental application of the method to the SLAC radial slot structure is described elsewhere in these proceedings[3] and will therefore not be repeated here. Application to the circumferential slot designs described there are in progress.

Preliminary work directed towards the design of accelerator cavities for a $B\bar{B}$ factory storage ring has indicated a need to deal with the problem of close resonances. We have recently developed an implementation procedure for the two-resonance representation[see Eq. (A5)] mentioned in the Appendix. It requires computer runs for at least two waveguide lengths. Initial results have been encouraging, and we hope to report in detail after we have more experience with the method. There is also a need for dealing with multiple non-symmetric outputs and with multimode waveguide loading.

Appendix
Theoretical Background and Derivation of Eq. (1.1)

Following Kroll and Yu, we consider the boundary value problem presented by the cavity with its waveguide output, which we now consider to be infinitely long. We assume perfect conductor boundary conditions on all of the walls. As one proceeds along the waveguide towards infinity, the fields are required to approach those of the principal mode (assumed here to be the only one which can propagate without attenuation), propagating towards infinity. The eigenmodes of such a system are complex, with positive imaginary part, corresponding to oscillations which are exponentially damped in time. Writing this eigenvalue as $u + jv$, we identify u with the resonant frequency of the waveguide loaded cavity and $u/2v$ with the cavity Q.

The z dependence of the electric field between the waveguide origin and the shorting plane is of the form

$$e^{jkz} + R\,e^{-jkz}$$

where $k = 2\pi/\lambda_g$, z is distance along the waveguide axis, and R is the reflection coefficient referred to the waveguide origin plane. Since it must vanish at $z = D$, the shorting plane position, it must also be proportional to

$$2j\sin(kz - \psi - n\pi) \;=\; (e^{jkz} - e^{2j\psi}e^{-jkz})\,e^{-j\psi},$$

where $\psi = k(\omega)D$. Comparing the two forms, we see that $R = -\exp(2j\psi)$. We now observe that because the eigenfrequency corresponds to a situation in which there is an outgoing wave but no incoming wave, the reflection coefficient must have a pole there. This, combined with the fact that R must have absolute value one for real values of ω, means that it may be written

$$R(\omega) \;=\; -\frac{\omega - u + jv}{\omega - u - jv}\,e^{-2j\chi(\omega)} \;=\; -e^{2j\psi}, \qquad (A1)$$

where $\chi(\omega)$ is a real function, analytic at $\omega = u + jv$. It represents nonresonant effects, effects of distant resonances, and effects associated with the mode structure of the waveguide. Taking the logarithm of both sides of Eq. (A1) we find

$$\psi(\omega) \;=\; \tan^{-1}\left(\frac{v}{\omega - u}\right) - \chi(\omega) + n\pi \quad . \qquad (A2)$$

We shall assume that $\chi(\omega)$ can be adequately represented for real values of its argument in the vicinity of the resonance by the first two terms of its power series expansion about u. Thus, we write

$$\chi(\omega) \;\cong\; \chi(u) + \chi'(u)(\omega - u) \quad . \qquad (A3)$$

Equation (A3) is the basic approximation upon which the method is based. Taking the tangent of Eq. (A2), using Eq. (A3), and the expressions for ψ and Q yields Eq. (1.1).

The choice of representation of $R(\omega)$ by Eq. (A1) is not unique. There may be situations in which it is useful, for instance, to exhibit two resonances. The resonances may be too close to the frequency of interest to make the approximate Eq. (A3) adequate in itself. Taking account of the fact that $R(\omega)$ has a pole in the complex plane at both resonances, we replace Eq. (A1) with

$$R(\omega) \;=\; -\frac{\omega - u_1 + jv_1}{\omega - u_1 - jv_1}\;\frac{\omega - u_2 + jv_2}{\omega - u_2 - jv_2}\,e^{-2j\chi(\omega)} = -e^{2j\psi}. \qquad (A4)$$

Correspondingly, Eq. (A2) becomes

$$\psi(\omega) \;=\; \tan^{-1}\left(\frac{v_1}{\omega - u_1}\right) + \tan^{-1}\left(\frac{v_2}{\omega - u_2}\right) - \chi(\omega) + n\pi. \qquad (A5)$$

The function $\chi(\omega)$ is now analytic at both poles, and a linear approximation analogous to Eq. (A3) would normally be employed.

The guidance and assistance of W. R. Fowkes and T. Lee in carrying out the experiment is gratefully acknowledged.

References

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