# The Electric Dipole Moment of the Neutron in the Skyrme Model ${ }^{\star}$ 

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#### Abstract

We use the Skyrme model to estimate the contribution of the QCD $\theta$-angle to the neutron's electric dipole moment $\mathcal{D}_{n}$, and find $\mathcal{D}_{n}=2 \times 10^{-16} \theta e \mathrm{~cm}$. The same method may also be used to estimate the contributions to $\mathcal{D}_{n}$ of higher-dimension $C P$-violating operators.


## 1. INTRODUCTION

The current experimental bound on the electric dipole moment of the neutron [1],

$$
\begin{equation*}
\left|\mathcal{D}_{n}\right|<1.2 \times 10^{-25} e \mathrm{~cm} \tag{1}
\end{equation*}
$$

provides one of the most sensitive constraints on $C P$-violating extensions of the standard model. However, the strong interactions are an obstacle to improving the constraints from $\mathcal{D}_{n}$. The essential problem is to calculate the neutron dipole moment induced by a given $C P$-violating operator, where the operator is generated by short-distance physics and is expressed in terms of quark and gluon fields. In some cases, it is possible to make a current algebra calculation of contributions that diverge in the chiral limit [2] so that they are formally dominant, but for most operators one has to resort to a non-relativistic approximation [3] or simply to a "naive dimensional analysis" [4]. Lattice calculations are still far from practicality [5].

We propose to calculate the electric dipole moment of the neutron within the Skyrme model. It was first demonstrated in refs. [6, 7] that the static properties of baryons calculated within the $S U(2)$ Skyrme model are in reasonable agreement with experiment. Later, the model was extended to the three-flavor case [8]. Recently, there has been much progress in this direction [9,10], particularly in the treatment of the $S U(3)$ symmetry breaking. Though the model has its shortcomings (as does the more general $N_{C} \rightarrow \infty$ approach [11]), it provides a consistent framework for such calculations, and the numerical results can probably be trusted to within a factor of two or so, much better than most methods mentioncd above.

In this paper we calculate the contribution to $\mathcal{D}_{\boldsymbol{n}}$ from the QCD $\theta$-angle. ${ }^{\dagger}$ This is the simplest and most straightforward application of our program: A nonzero $\theta$-angle is equivalent via a chiral rotation to a phase in the quark mass matrix [13]. In the low-energy chiral Lagrangian the effect of $\theta$, to first order in the quark

[^0]masses and in $\theta$, is just an imaginary contribution to the meson mass matrix. The classical Skyrme solution shifts slightly due to the new mass term, thus acquiring a $C P$-violating admixture that leads to an electric dipole moment.

In the concluding section, we will briefly discuss the prospects for generalizing the above procedure to other $C P$-violating quark and gluon operators.

## 2. the $U(3)$ Skyrme model

In the Skyrme model, baryons of QCD are solitons in a $G_{L} \times G_{R}$ chiral effective theory of pseudoscalar mesons. To satisfactorily discuss the neutron electric dipole moment induced by $\theta$ we need to take into account contributions from the entire pseudoscalar meson nonet; thus we take $G_{L}$ and $G_{R}$ to be $U(3)$. (In the $S U(2)$ case (pions only) the would-be $C P$-violating term vanishes identically. The $S U(3)$ case turns out to be adequate numerically; however, including the $\eta^{\prime}$ as well allows one to study the formal effects of turning off the $U(1)_{A}$ anomaly.) The Lagrangian of the $U(3)$ Skyrme model is

$$
\begin{equation*}
\mathcal{L}=\mathcal{L}_{K}+\mathcal{L}_{S}+\mathcal{L}_{M}+\mathcal{L}_{A}+\mathcal{L}_{W Z} \tag{2}
\end{equation*}
$$

with

$$
\begin{align*}
\mathcal{L}_{K} & =\frac{F_{\pi}^{2}}{16} \operatorname{tr}\left(\partial_{\mu} U \partial^{\mu} U^{\dagger}\right) \\
\mathcal{L}_{S} & =\frac{1}{32 e^{2}} \operatorname{tr}\left\{\left[\left(\partial_{\mu} U\right) U^{\dagger},\left(\partial_{\nu} U\right) U^{\dagger}\right]^{2}\right\} \\
\mathcal{L}_{M} & =\frac{F_{\pi}^{2}}{16} \operatorname{tr}\left(M U+M^{\dagger} U^{\dagger}-M-M^{\dagger}\right)  \tag{3}\\
\mathcal{L}_{A} & =\frac{a F_{\pi}^{\prime 2}}{64 N_{C}}\left[\operatorname{tr}\left(\ln U-\ln U^{\dagger}\right)\right]^{2}
\end{align*}
$$

The $U(3)$ matrix $U$ transforms like $U \rightarrow L U R^{\dagger}$ under $U(3)_{L} \times U(3)_{R}$; its expansion in terms of the meson fields $\phi_{0}, \phi_{a}(a=1, \ldots, 8)$ is

$$
\begin{equation*}
U=\exp \left(\sqrt{\frac{8}{3}} \frac{i}{F_{\pi}} \phi_{0}\right) \exp \left(\frac{2 i}{F_{\pi}} \phi_{a} \lambda_{a}\right) . \tag{4}
\end{equation*}
$$

The mass term $\mathcal{L}_{M}$ explicitly breaks the axial symmetry $U(3)_{A}$ and some of the
vector symmetry $S U(3)_{V}{ }^{\ddagger}$ The anomaly term $\mathcal{L}_{A}$ breaks only $U(1)_{A}$ [15]. We will not need the explicit form of the Wess-Zumino term $\mathcal{L}_{W Z}$ here; however, its contribution to the electromagnetic current is important and will be given later.

The matrix $M$ can be diagonalized by an $S U(3)_{L} \times S U(3)_{R}$ transformation. In order to use the usual spherically symmetric ansatz for the Skyrmion, we need $M$ to preserve $S U(2)_{V}$. Thus, we use:

$$
\begin{equation*}
M=\operatorname{diag}\left(M_{1}+i \lambda_{\theta}, M_{1}+i \lambda_{\theta}, M_{3}+i \alpha \lambda_{\theta}\right) \tag{5}
\end{equation*}
$$

The imaginary part $\operatorname{Im} M$ signals breaking of $P$ and $C P$ due to $\theta$. In the absence of the $S U(3)$ singlet field $\phi_{0}$, choosing $\operatorname{Im} M \propto 1$ (i.e. $\alpha=1$ ) prevents the octet of approximate Goldstone bosons $\phi_{a}$ from acquiring vacuum expectation values [13]. Here, because of the $\phi_{0} \phi_{8}$ mixing term in $\mathcal{L}_{M}$, in order to keep $\left\langle\phi_{8}\right\rangle=0$ we need to choose instead

$$
\begin{equation*}
\alpha=\frac{M_{3}+a}{M_{1}+a} . \tag{6}
\end{equation*}
$$

(In the limit $a \rightarrow \infty$, we get $\alpha \rightarrow 1$, as expected.) With this choice of $\alpha$, only $\phi_{0}$ needs be shifted:

$$
\begin{equation*}
\phi_{0} \rightarrow \phi_{0}-\delta, \quad \text { where } \delta=\frac{\sqrt{6}}{4} \frac{\lambda_{\theta} F_{\pi}}{a+M_{1}} \tag{7}
\end{equation*}
$$

The mass terms for the meson fields are:

$$
\begin{align*}
& \mathcal{L}_{\text {mass }}(\phi)=-\frac{1}{2} M_{1} \pi_{i} \pi_{i}-\frac{1}{4}\left(M_{1}+M_{3}\right) K_{\ell} K_{\ell}-\frac{1}{2} \Phi^{T} \cdot M_{\Phi} \cdot \Phi,  \tag{8}\\
& \Phi=\binom{\phi_{8}}{\phi_{0}}, \quad M_{\Phi}=\frac{1}{3}\left(\begin{array}{cc}
M_{1}+2 M_{3} & \sqrt{2}\left(M_{1}-M_{3}\right) \\
\sqrt{2}\left(M_{1}-M_{3}\right) & 2 M_{1}+M_{3}+3 a
\end{array}\right) . \tag{9}
\end{align*}
$$

(The index $i$ runs from 1 to $3 ; \ell$ from 4 to 7 .) Using the experimental values of

[^1]$m_{\pi}, m_{K}$ and $m_{\eta^{\prime}}$ we get:
\[

$$
\begin{equation*}
M_{1}=0.019 \mathrm{GeV}^{2}, \quad M_{3}=0.47 \mathrm{GeV}^{2}, \quad a=0.67 \mathrm{GeV}^{2} \tag{10}
\end{equation*}
$$

\]

and obtain, in reasonable agreement with experiment:

$$
\begin{equation*}
m_{\eta}=0.49 \mathrm{GeV} ; \quad \theta_{\eta \eta^{\prime}}=-20^{\circ} \tag{11}
\end{equation*}
$$

The $C P$-violating parameter $\lambda_{\theta}$ is related to $\theta$ by $[13,2]$

$$
\begin{align*}
\theta & =\arg \operatorname{det} M=\lambda_{\theta}\left[\frac{2}{M_{1}}+\frac{1}{M_{3}}\left(\frac{a+M_{3}}{a+M_{1}}\right)\right]  \tag{12}\\
\Rightarrow \quad \lambda_{\theta} & =9 \times 10^{-3} \theta \mathrm{GeV}^{2} .
\end{align*}
$$

The fact that $\lambda_{\theta}$ vanishes in the chiral limit $M_{1} \rightarrow 0$ is expected, since $\theta$ can be rotated away completely in the QCD Lagrangian if any quark mass vanishes. Since $a, M_{3} \gg M_{1}$, we have $\lambda_{\theta} \approx \frac{1}{2} m_{\pi}^{2} \theta$. If one takes into account $S U(2)$ breaking effects in deriving (12), one obtains a numerically similar value, $\lambda_{\theta} \approx 2 m_{\pi}^{2} \frac{m_{u} m_{d}}{\left(m_{u}+m_{d}\right)^{2}} \theta=$ $8.7 \times 10^{-3} \theta \mathrm{GeV}^{2}$ (for $m_{d} / m_{u}=1.76[16]$ ). This shift in $\lambda_{\theta}$ turns out to be the only $S U(2)$ breaking effect in the calculation that is proportional to the deviation of $m_{d} / m_{u}$ from 1 ; all other effects are accompanied by the additional chiral suppression factors $M_{1} / M_{3}, M_{1} /\left(e F_{\pi}\right)^{2}$ and will be neglected here.

We now turn to the effects of the $C P$-violating term in $\mathcal{L}_{M}$ on the Skyrmion. We generalize the usual ansatz $U_{0}$ for the static soliton to

$$
\begin{align*}
& U_{c}(\vec{r})=\exp \frac{i}{F_{x}}\left[\sqrt{\frac{8}{3}}\left(\phi_{0}(r)-\delta\right)+2 \lambda_{8} \phi_{8}(r)\right] \cdot U_{0}(\vec{r})  \tag{13}\\
& U_{0}(\vec{r})=\exp \left(i F(r) \tau_{i} \hat{x}_{i}\right), \quad \tau_{i} \equiv \lambda_{i}(i=1,2,3)
\end{align*}
$$

The new ansatz is still spherically symmetric under combined isospin and spatial $S U(2)$ rotations. The $C P$-violating part of $\mathcal{L}_{M}$ becomes a source term for the
$S U(2)$-singlet fields $\phi_{8}$ and $\phi_{0}$,

$$
\begin{equation*}
\delta \mathcal{L}_{C P}=\frac{\lambda_{\theta} F_{\pi}}{2 \sqrt{3}} \frac{a}{a+M_{1}} \cdot(1-\cos F)\left(\phi_{8}+\sqrt{2} \phi_{0}\right), \tag{14}
\end{equation*}
$$

which produces an order $\theta$ expectation value for them in the soliton. The function $F(r)$ remains the same as in the $S U(2)$ Skyrme model with a massive pion $[7,9]$, up to order $\theta^{2}$.

The factor $\frac{a}{a+M_{1}}$ in eq. (14) shows that the effects of $\theta$ vanish in the formal limit $a \rightarrow 0$. The result is expected because in this limit $U(1)_{A}$ becomes a good symmetry and can be used to rotate away $\theta$. In practice, however, $\frac{a}{a+M_{1}} \approx 1$, so we drop the factor henceforth. If we also neglect corrections of order $m_{\pi}^{2} / m_{\eta}^{2} \sim 8 \%$, then the terms quadratic in the mass eigenfields $\eta, \eta^{\prime}$ are independent of $F$, and we have

$$
\begin{align*}
& \eta(\tilde{r})=\frac{\lambda_{\theta}}{2 \sqrt{3} e} \frac{\left(\cos \theta_{\eta \eta^{\prime}}-\sqrt{2} \sin \theta_{\eta \eta^{\prime}}\right)}{m_{\eta}} \int_{0}^{\infty} d \tilde{r}^{\prime} G_{\boldsymbol{m}}\left(\tilde{r}, \tilde{r}^{\prime}\right)\left[1-\cos F\left(\tilde{r}^{\prime}\right)\right] \\
& \eta^{\prime}(\tilde{r})=\frac{\lambda_{\theta}}{2 \sqrt{3} e} \frac{\left(\sqrt{2} \cos \theta_{\eta \eta^{\prime}}+\sin \theta_{\eta \eta^{\prime}}\right)}{m_{\eta^{\prime}}} \int_{0}^{\infty} d \tilde{r}^{\prime} G_{m^{\prime}}\left(\tilde{r}, \tilde{r}^{\prime}\right)\left[1-\cos F\left(\tilde{r}^{\prime}\right)\right] \tag{15}
\end{align*}
$$

Herc

$$
G_{m}\left(\tilde{r}, \tilde{r}^{\prime}\right)= \begin{cases}\left(\frac{\tilde{r}^{\prime}}{\tilde{r}}\right) e^{-m \tilde{r}^{\prime}} \sinh (m \tilde{r}), & \tilde{r}<\tilde{r}^{\prime},  \tag{16}\\ \left(\frac{\tilde{r}^{\prime}}{\tilde{r}}\right) e^{-m \tilde{r}} \sinh \left(m \tilde{r}^{\prime}\right), & \tilde{r}>\tilde{r}^{\prime},\end{cases}
$$

is the Green's function for the radial Klein-Gordon equation, written in terms of the dimensionless quantities

$$
\begin{equation*}
m \equiv \frac{m_{\eta}}{e F_{\pi}}, \quad \tilde{r} \equiv e F_{\pi} r \tag{17}
\end{equation*}
$$

and similarly for $\eta^{\prime}$ with $m^{\prime} \equiv m_{\eta^{\prime}} / e F_{\pi}$.

Finally, the physical neutron state $|n\rangle$ is represented by a wave function in the collective coordinates $A \in S U(3)$; the $\eta^{\prime}$ is treated as heavy and therefore has no collective coordinate associated with it. One inserts

$$
\begin{equation*}
U(\vec{r}, t)=A(t) U_{c}(\vec{r}) A^{-1}(t), \quad A(t) \in S U(3) \tag{18}
\end{equation*}
$$

into the Lagrangian (2) and quantizes the resulting $\mathcal{L}(A, \dot{A})$. To first order in $\theta$, there are no corrections to the $A$-dependent terms in $\mathcal{L}$, so the usual quantization procedure is unaltered. We use the results of ref. [9] here, in which the $S U(3)$ symmetry breaking terms were treated exactly in the quantization, and the $\operatorname{SU}(3)$ baryon spectrum was fit by $e=3.87, F_{\pi}=89.0 \mathrm{MeV}$ (their fit number (1)). (Note that experimentally, $F_{\boldsymbol{\pi}}=186 \mathrm{MeV}$.)

## 3. $\mathcal{D}_{n}$ IN THE SKYRME MODEL

The electric dipole moment of the neutron is defined by:

$$
\begin{equation*}
\overrightarrow{\mathcal{D}}_{n}=\langle n| \int d^{3} r \vec{x} J_{0}^{E M}|n\rangle=\mathcal{D}_{n}\langle n| \vec{\sigma}|n\rangle \tag{19}
\end{equation*}
$$

The electromagnetic current has contributions from three of the terms in eq. (2):

$$
\begin{equation*}
J_{\mu}^{E M}=J_{\mu}^{W Z}+J_{\mu}^{K}+J_{\mu}^{S} \tag{20}
\end{equation*}
$$

With the definitions $V_{\mu} \equiv\left(\partial_{\mu} U\right) U^{\dagger}, W_{\mu} \equiv U^{\dagger}\left(\partial_{\mu} U\right)$, we have:

$$
\begin{align*}
J_{0}^{W Z} & =-\frac{N_{C}}{48 \pi^{2}} \epsilon_{i j k} \operatorname{tr}\left[Q\left(V_{i} V_{j} V_{k}+W_{i} W_{j} W_{k}\right)\right] \\
J_{0}^{K} & =-\frac{i F_{\pi}^{2}}{8} \operatorname{tr}\left[Q\left(V_{0}-W_{0}\right)\right]  \tag{21}\\
J_{0}^{S} & =-\frac{i}{8 e^{2}} \operatorname{tr}\left\{\left[Q, V_{i}\right]\left[V_{0}, V_{i}\right]-\left[Q, W_{i}\right]\left[W_{0}, W_{i}\right]\right\}
\end{align*}
$$

where the electromagnetic charge operator $Q$ is

$$
\begin{equation*}
Q=\operatorname{diag}(2 / 3,-1 / 3,-1 / 3)=\frac{1}{2}\left(\lambda_{3}+\lambda_{8} / \sqrt{3}\right) \tag{22}
\end{equation*}
$$

To calculate $\mathcal{D}_{n}$ to order $\theta$ we need the $C P$-violating part of the electromagnetic
current in the Skyrmion ansatz (13), (18),

$$
\begin{align*}
\delta J_{0}^{W Z}= & -\frac{N_{C} e^{3} F_{\pi}^{2}}{6 \sqrt{3} \pi^{2}} \operatorname{tr}\left(A^{-1} Q A \tau_{i}\right) \hat{x}_{i} \frac{\sin ^{2} F}{\tilde{r}^{2}}\left[\left(\partial_{\tilde{r}} \phi_{8}\right)+\sqrt{2}\left(\partial_{\tilde{r}} \phi_{0}\right)\right], \\
\delta J_{0}^{K}= & \frac{F_{\pi}}{2} \operatorname{tr}\left\{\left(A^{-1} Q A\right)\left[\left[i \dot{A}^{-1} A, \lambda_{8}\right], \tau_{i}\right]\right\} \hat{x}_{i}(\sin F) \phi_{8}, \\
\delta J_{0}^{S}= & \frac{F_{\pi}}{2} \operatorname{tr}\left\{\left(A^{-1} Q A\right)\left[\left[i \dot{A}^{-1} A, \lambda_{8}\right], \tau_{i}\right]\right\} \\
& \times \hat{x}_{i}\left\{2(1-\cos F)\left(\partial_{\tilde{r}} F\right)\left(\partial_{\tilde{r}} \phi_{8}\right)+(\sin F)\left[\left(\partial_{\tilde{r}} F\right)^{2}+\frac{2 \sin ^{2} F}{\tilde{r}^{2}}\right] \phi_{8}\right\} . \tag{23}
\end{align*}
$$

The Wess-Zumino term $\delta J_{0}^{W Z}$ gives the leading $1 / N_{C}$ contribution to $\mathcal{D}_{n}$.
Partitioning the electric dipole moment into $\mathcal{D}_{n}=\mathcal{D}_{n}^{W Z}+\mathcal{D}_{n}^{K+S}$, and using eqs. (15), (19) and (23), we get:

$$
\begin{align*}
\overrightarrow{\mathcal{D}}_{n}^{W Z} & =-\frac{N_{C} \lambda_{\theta}}{27 \pi\left(e F_{\pi}\right)^{2}}\langle n| \operatorname{tr}\left(A^{-1} Q A \vec{\tau}\right)|n\rangle \\
& \times\left[\left(\cos \theta_{\eta \eta^{\prime}}-\sqrt{2} \sin \theta_{\eta \eta^{\prime}}\right)^{2} \frac{I_{m}}{m_{\eta}}+\left(\sqrt{2} \cos \theta_{\eta \eta^{\prime}}+\sin \theta_{\eta \eta^{\prime}}\right)^{2} \frac{I_{m^{\prime}}}{m_{\eta^{\prime}}}\right] \\
\overrightarrow{\mathcal{D}}_{n}^{K+S} & =\frac{\pi F_{\pi}^{2} \lambda_{\theta}}{3 \sqrt{3}\left(e F_{\pi}\right)^{5}}\langle n| \operatorname{tr}\left\{\left(A^{-1} Q A\right)\left[\left[i \dot{A}^{-1} A, \lambda_{8}\right], \vec{\tau}\right]\right\}|n\rangle \\
& \times\left[\cos \theta_{\eta \eta^{\prime}}\left(\cos \theta_{\eta \eta^{\prime}}-\sqrt{2} \sin \theta_{\eta \eta^{\prime}}\right) \frac{H_{m}}{m_{\eta}}+\sin \theta_{\eta \eta^{\prime}}\left(\sqrt{2} \cos \theta_{\eta \eta^{\prime}}+\sin \theta_{\eta \eta^{\prime}}\right) \frac{H_{m^{\prime}}}{m_{\eta^{\prime}}}\right] \tag{24}
\end{align*}
$$

where the radial integral moments of the Skyrmion $I_{m}, H_{m}$ are defined as follows:

$$
\begin{align*}
I_{m} \equiv & \int_{0}^{\infty} d \tilde{r} \tilde{r} \sin ^{2} F(\tilde{r}) \int_{0}^{\infty} d \tilde{r}^{\prime} \frac{d G_{m}\left(\tilde{r}, \tilde{r}^{\prime}\right)}{d \tilde{r}}\left[1-\cos F\left(\tilde{r}^{\prime}\right)\right] \\
H_{m} \equiv & \int_{0}^{\infty} d \tilde{r} \tilde{r}^{3}\left\{2[1-\cos F(\tilde{r})]\left(\partial_{\tilde{r}} F\right) \int_{0}^{\infty} d \tilde{r}^{\prime} \frac{d G_{m}\left(\tilde{r}, \tilde{r}^{\prime}\right)}{d \tilde{r}}\left[1-\cos F\left(\tilde{r}^{\prime}\right)\right]\right. \\
& \left.+\sin F(\tilde{r})\left[1+\left(\partial_{\tilde{r}} F(\tilde{r})\right)^{2}+2 \frac{\sin ^{2} F(\tilde{r})}{\tilde{r}^{2}}\right] \int_{0}^{\infty} d \tilde{r}^{\prime} G_{m}\left(\tilde{r}, \tilde{r}^{\prime}\right)\left[1-\cos F\left(\tilde{r}^{\prime}\right)\right]\right\} \tag{25}
\end{align*}
$$

The two collective-coordinate matrix elements appearing in eq. (24) have been calculated in ref. [10] as a function of the $S U(3)$ symmetry breaking parameter $\omega^{2}$
that enters the wave functions. Fit number (1) in ref. [9] has $\omega^{2}=5.3$, which gives [10]

$$
\begin{equation*}
\langle n| \operatorname{tr}\left(A^{-1} Q A \tau_{3}\right)|n\rangle=0.24, \quad\langle n| \operatorname{tr}\left\{\left(A^{-1} Q A\right)\left[\left[i \dot{A}^{-1} A, \lambda_{8}\right], \tau_{3}\right]\right\}|n\rangle=0.091 / \beta^{2} \tag{26}
\end{equation*}
$$

where the moment of inertia $\beta^{2}=3.55 \mathrm{GeV}^{-1}$ is also taken from ref. [9]. Evaluating the integrals in eq. (25) numerically, we find

$$
\begin{equation*}
I_{m}=-1.00, \quad I_{m^{\prime}}=-0.57, \quad H_{m}=+12.0, \quad H_{m^{\prime}}=+7.3 \tag{27}
\end{equation*}
$$

so that

$$
\begin{align*}
\mathcal{D}_{n}^{W Z} & =+0.57 \times 10^{-16} \theta \mathrm{ecm} \\
\mathcal{D}_{n}^{K+S} & =+1.3 \times 10^{-16} \theta \mathrm{ecm} \tag{28}
\end{align*}
$$

Our final result is then:

$$
\begin{equation*}
\mathcal{D}_{n}=+2 \times 10^{-16} \theta e \mathrm{~cm} \tag{29}
\end{equation*}
$$

## 4. DISCUSSION AND CONCLUSIONS

Formally, the leading contribution in $1 / N_{C}$ to $\mathcal{D}_{n}$ comes from the Wess-Zumino term: $\mathcal{D}_{n}^{W Z} / \mathcal{D}_{n}^{K+S} \propto N_{C}$. One might worry about the fact that $\mathcal{D}_{n}^{K+S}$ is numerically larger than $\mathcal{D}_{n}^{W Z}$ while being suppressed by $1 / N_{C}$. However, the same situation is encountered in the calculation of the electric charges of the proton and neutron, as pointed out in ref. [17]:

$$
Q^{W Z}(p)=Q^{W Z}(n)=1 / 5 ; \quad Q^{K+S}(p)=4 / 5, \quad Q^{K+S}(n)=-1 / 5
$$

Since the final electric charges are correct without further $1 / N_{C}$ corrections, it is reasonable to assume that the most important contributions to $\mathcal{D}_{n}$ are included in our calculation. Still, one must acknowledge that at present it is unknown how to systematically calculate $1 / N_{C}$ corrections in the Skyrme model.

Let us compare our result (29) with the most singular contribution to $\mathcal{D}_{n}$ in the chiral $m_{\pi} \rightarrow 0$ limit, identified in ref. [2],

$$
\begin{equation*}
\mathcal{D}_{n}=\frac{g_{\pi N N} \bar{g}_{\pi N N}}{4 \pi^{2} M_{N}} \ln \left(M_{N} / m_{\pi}\right)=+3.6 \times 10^{-16} \theta e \mathrm{~cm} \tag{30}
\end{equation*}
$$

Here $M_{N}$ is the nucleon mass, and $g_{\pi N N}\left(\bar{g}_{\pi N N}\right)$ is the pseudoscalar coupling ( $C P$-violating scalar coupling) of the pion to the nucleon. Numerically the two results are in broad agreement (including the sign). On the other hand, their formal properties are somewhat different. The chiral estimate of $\bar{g}_{\pi N N}$ in ref. [2] shows its dependence on $m_{\pi}$ and $N_{C}$ to be $\bar{g}_{\pi N N} \sim m_{\pi}^{2} N_{C}^{1 / 2}$, and we also have $g_{\pi N N} \sim N_{C}^{3 / 2}, M_{N} \sim N_{C}, e F_{\pi} \sim 1$ (see ref. [18]); thus the chiral estimate (30) yields $\mathcal{D}_{n} \sim\left(m_{\pi}^{2} \ln m_{\pi}^{2}\right) N_{C}$. However, if we consider eq. (24), and note that both $I_{m}$ and $H_{m}$ are finite as $m_{\pi} \rightarrow 0$, we see that our estimate has $\mathcal{D}_{n} \sim m_{\pi}^{2} N_{C}$.

There is a similar discrepancy between the form of the chiral and Skyrme estimates of the isovector electric mean square radius of the nucleon [7]. We suggest that the explanation for the disagreement given in ref. [7], namely the non-commutativity of the chiral $m_{\pi} \rightarrow 0$ limit with the large- $N_{C}$ limit, applies here as well. That is, suppose that the true answer for $\mathcal{D}_{n}$ were of the form $\left(m_{\pi}^{2} \ln m_{\pi}^{2}\right) N_{C} \cdot\left(m_{\pi}^{2}+1 / N_{C}\right) /\left(m_{\pi}^{2} \ln m_{\pi}^{2}+1 / N_{C}\right)$. Then, performing the $m_{\pi} \rightarrow 0$ limit first, at fixed $N_{C}$, we would get $\left(m_{\pi}^{2} \ln m_{\pi}^{2}\right) N_{C}$ as in ref. [2], while performing the $N_{C} \rightarrow \infty$ limit first, with fixed $m_{\pi}$, we would find a behavior of the form $m_{\pi}^{2} N_{C}$.

The different formal properties in the two limits reflect different mechanisms for production of $\mathcal{D}_{n}$. The dominant term in the chiral limit is obtained when the neutron dissociates into a proton and a pion; whereas the leading Skyrme

[^2]contributions have no quantized pion fluctuations in them at all. In ref. [12] the Skyrme model was used to estimate $\bar{g}_{\pi N N}$, which was then inserted into the chiral formula (30) to obtain $\mathcal{D}_{n}$. However, our calculation of $\mathcal{D}_{n}$ was performed entirely within the Skyrme model, and in that context all virtual pion contributions are subleading in $1 / N_{C}$. (Also, the fact that the static classical solution $U_{0}$ deforms in the presence of $\theta$ is not taken into account in the calculation of $\bar{g}_{\pi N N}$ in ref. [12].)

The procedure for calculating $\mathcal{D}_{n}$ demonstrated in this paper can be generalized to other operators. One has to (a) identify the equivalent operator in the effective Skyrme Lagrangian, and (b) calculate the effects of this term on the Skyrmion. It is step ( $a$ ) which poses the greater theoretical challenge. For the four-quark operators, step (a) may be performed in the vacuum-insertion approximation, which is valid in the large- $N_{C}$ limit and which allows the four-quark operators to be represented by meson bilinears with known coefficients in the Skyrme Lagrangian. (However, the reliability of the approximation has been questioned in the context of $K$ decays [11].) We are presently studying the applicability of the general procedure to other classes of $C P$-violating operators as well.

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[^0]:    $\dagger$ A previous attempt was made in this direction in ref. [12].

[^1]:    $\ddagger$ A more complete treatment of $S U(3)$ symmetry breaking would include also higherderivative terms such as $\operatorname{tr}\left(M \partial_{\mu} U U^{\dagger} \partial^{\mu} U\right)$, which is responsible for $F_{K} \neq F_{\pi}[14,10]$.

[^2]:    $\star$ Actually, both estimates have hidden in them an extra $\sim 1 / N_{C}$ suppression factor related to the $1 / N_{C}$ behavior of the $U(1)_{A}$ anomaly. In our calculation this $1 / N_{C}$ was hidden in the ratio $\frac{a}{a+M_{1}}$ in eq. (14) - note that $a$ should be written as ( $3 / N_{C}$ ) a, so that $N_{C} \rightarrow \infty$ corresponds to $a \rightarrow 0$. Including the extra factor, the neutron dipole moment approaches a constant in the large- $N_{C}$ limit, even though $C P$-violation in the meson sector is vanishing like $1 / N_{C}$; this is possible because the neutron is made up of $\sim N_{C}$ mesons.

