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Plane Wave Gravitons, Curvature Singularities And String Physics *

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ABSTRACT

Bounded (compactifying) potentials arising from a conspiracy between plane wave graviton and dilaton condensates are discussed. So are string propagation and supersymmetry in space-times with curvature singularities.

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I. Introduction/Summary

The purpose of this letter is to make three observations concerning string physics in generalized plane wave graviton backgrounds. The first illustrates how these gravitons along with certain dilaton condensates may conspire to bound the motion of some of the string's transverse coordinates. The second points out that for most space-times with plane wave curvature singularities, the total mass and number of excitations for strings after graviton scattering is the same as before the scattering. The one example of a singular plane wave metric for which this is not true turns out not to be geodesically complete. Finally, even for such a space-time, one can formally find a vacuum which is space-time supersymmetric.

II. Generalities About Plane Wave Solutions

Take space-time to be D-dimensional. Plane wave solutions of Einstein's vacuum equations are given by the metric [1]

$$\begin{aligned} ds^2 &= -2dUdV + \mathcal{F}(X,U)dU^2 + (dX^i)^2 \ , \\ U &\equiv \frac{1}{\sqrt{2}}(X^0 - X^{D-1}) \ , \qquad V \equiv \frac{1}{\sqrt{2}}(X^0 + X^{D-1}) \ . \end{aligned} \tag{2.1}$$

The X^i are the transverse string coordinates and $\mathcal{F}$ is of the form $\mathcal{F} = -h_{ij}(U)X^iX^j$ with $tr(h) = 0$. For general $\mathcal{F}$, the space-time with metric (2.1) is not maximally symmetric. However, in addition to the Killing vector expressing V independence, there will be Killing symmetries corresponding to each coordinate that $\mathcal{F}$ does not depend on. Other geometrical quantities which will be needed are:

$$\begin{aligned} G^{uv} &= -1 \ , \qquad G^{vv} = -\mathcal{F}(X,U) \ , \qquad G^{ij} = \delta^{ij} \ , \\ \Gamma^i_{uu} &= \Gamma^v_{iu} = -\frac{1}{2}\partial_i\mathcal{F}(X,U) \ , \\ R_{uiuj} &= -\frac{1}{2}\partial_i\partial_j\mathcal{F}(X,U) \ . \end{aligned} \tag{2.2}$$

The Christoffel symbols may be derived from eqn. (2.1) or more directly from the action for the point-particle:

$$S_{Part} = \frac{1}{2}\int d\tau\, [-2\dot{U}\dot{V} + (\dot{X}^i)^2 + \mathcal{F}(X,U)\dot{U}^2] \ . \tag{2.3}$$

Integration over V gives the constraint, $\ddot{U} = 0$. With $\mathcal{F}$ of the plane wave form, eqn. (2.1) describes a pure gravitational wave in that the Ricci tensor vanishes (vacuum solution). In the following more general forms for $\mathcal{F}$ will be assumed. These will not have the property of vanishing Ricci tensor, but rather R_{uu} will be a constant. Thus matter must be present. Their motivation will be made clear later when solutions to string theory are considered.

Let $\mathcal{F}$ be independent of U: $\mathcal{F} = f(X)$. Solve the constraint on U by writing $U = U_0 + p^u\tau$. (U_0 does not play a role in the following and may be dropped.) Then the Hamiltonian for the transverse coordinates contains a time-independent potential

$$\mathbf{V}(X^i, p^u) = -\frac{1}{2}f(X^i)(p^u)^2 \ . \tag{2.4}$$

It is assumed that $f(X)$ is bounded from above so that $\mathbf{V}$ is bounded from below. If the initial conditions are such that $p^u = 0$, then the particle is free. The key observation is that if $\mathbf{V}(X, p^u \neq 0)$ localizes the motion of the particle in certain directions, then those coordinates map onto a compact submanifold. Next, $\mathbf{V}$ need not depend on all of the X^i, but may only depend on some subset, labelled $\{X^I\} \subset \{X^i\}$, of them. It will be assumed that f depends on some number, n, (such that $2 < n \leq (D-2)$) of the X^i; *i.e.* $f = f(X^I)$ with $I = 1, \ldots, n$.

Turning to the space-time field theory, the equation of motion for the massless scalar field $\phi(X, U, V)$ is given by $\Delta_D\phi = 0$, [1] where Δ_D is the D-dimensional Laplace-Beltrami operator in the metric (2.1):

$$\Delta_D = \nabla_T^2 - \mathcal{F}(X,U)\frac{\partial^2}{\partial V^2} - 2\frac{\partial^2}{\partial U \partial V} \ . \tag{2.5}$$

[1] Note that the scalar curvature of the metric (2.1) vanishes. Hence the generalized Klein-Gordon operator is given by Δ_D.

A solution of this equation is found by writing $\phi(X,U,V) = \hat{\phi}(X,U)e^{ip^u V}$. Then one finds that $\Delta_D \phi = 0$ translates into the *time-dependent* Schrödinger equation

$$[-\frac{1}{2}\nabla_T^2 \; - \; \frac{1}{2}(p^u)^2 \mathcal{F}(X,U)]\hat{\phi} \; = \; -i\frac{\partial}{\partial U}\hat{\phi} \; . \tag{2.6}$$

An illustrative example is given by letting $\mathcal{F}$ be the box potential. Then the solution of eqn. (2.6) explicitly gives the expansion of ϕ found in the Scherk-Schwarz compactification [2]. A more realistic example is given by $\mathcal{F}(X,U) = -(X^I)^2$. Then eqn. (2.6) is readily interpreted as the Schrödinger equation for the harmonic oscillator with angular frequency $\omega = p^u$. $\hat{\phi}$'s dependence on X^I is given by a wave packet whose center oscillates with this frequency. The solution in the shock wave metric, where $\mathcal{F}(X,U) = -h_{ij}X^i X^j \delta(U)$, is discussed in ref. [3].

So much for the point-particle. For a given geometry to be a solution of classical string theory, the fields which define it must satisfy the conformal invariance conditions of the non-linear sigma model. It is useful to at least have the one loop β-functions at hand. They are [4]

$$\begin{aligned} \beta^G_{mn} &= R_{mn} \; - \; \frac{1}{4}H^2_{mn} \; + \; 2\nabla_m\nabla_n\Phi \; , \\ \beta^H_{mn} &= \frac{1}{2}\nabla^p H_{pmn} \; - \; \nabla^p\Phi H_{pmn} \; , \\ \beta^\Phi &= \frac{(D - D_{crit})}{3} \; + \; \alpha'[-R \; + \; \frac{1}{12}H^2 \; + \; 4(\nabla\Phi)^2 \; - \; 4\nabla^2\Phi] \; . \end{aligned} \tag{2.7}$$

Plane wave solutions of Einstein's vacuum equations have been shown [5] to be solutions of classical string theory to all orders in α'. [2] Conformal invariance (vanishing of the Ricci tensor), in the absence of a dilaton, Φ, or 2-form, H, requires that the function $\mathcal{F}(X,U)$ satisfies the equation

$$\nabla_T^2 \mathcal{F}(X,U) \; = \; 0 \; . \tag{2.8}$$

[2] This is crucial as the standard perturbative treatments proceed by first performing a normal coordinate expansion. The validity of such an expansion requires that the space-time be geodesically complete. The latter is not necessarily true for plane wave metrics.

This is quickly seen by using eqn. (2.2) in eqn. (2.7). One important feature of this requirement is that only the dependence on the transverse coordinates are constrained. $\mathcal{F}$ is left to be an arbitrary function of the light-cone time coordinate, U. This property has been exploited by various authors [5-9] to study string propagation in space-times with singularities. In fact, if $\mathcal{F}$ is taken to be of the form

$$\mathcal{F}(X^i, U) \;=\; f(X^i) r(U) \;\;, \tag{2.9}$$

then the curvature tensor is found to be proportional to $r(U)$. Hence, for various choices for the function r, the space-time may be made to be fraught with curvature singularities. A particularly interesting choice is the shock wave metric [1]. Physically, it corresponds to a string propagating in a flat space-time for times $U < 0$. At time $U = 0$, it is hit by a gravitational wave with polarization $\xi_{uu} = f(X)$. A novel feature of this metric is that non-perturbative calculations may be performed [5-9]. The S-matrix has been explicitly written down [8,9].

String physics in the potential (2.4) will be discussed in the next section. Section IV contains observations about string propagation in space-times with curvature singularities. Supersymmetry in these space-times is discussed in section V. Section III is independent of sections IV and V.

III. Freezing Left Over String Coordinates in Background Solutions

By recalling the standard treatment [10] of the bosonic string in the light-cone gauge and from eqn. (2.1), it is seen that one can take $U \equiv U_0 + p^u \tau$. Then the Polyakov action takes the form ($\alpha' \equiv \frac{1}{2\pi}$)

$$S_{BS} \;=\; -\frac{1}{2} \int d^2\sigma [X^i \Box X_i \;-\; \mathcal{F}(X, U)(p^u)^2] \;\;, \tag{3.1}$$

with $\mathcal{F}$ satisfying eqn. (2.8) (for conformal invariance).

Take $r(U)$ in eqn. (2.9) to be the identity and do not couple to the dilaton or the 2-form. Then a class of solutions to eqn. (2.8) may be written as

$$f(X) = \left\{ \begin{array}{ll} -\rho^{-(n-2)} - h_{IJ}X^I X^J \ , & n > 2 \ , \\ \ln\rho - h_{IJ}X^I X^J \ , & n = 2 \ , \end{array} \right\} \qquad (3.2)$$
$$\rho \equiv \sqrt{X^I X_I} \ .$$

An overall normalization is suppressed. Each term is separately a solution. Without loss of generality take h_{IJ} to be symmetric. It must be traceless in order to solve eqn. (2.8). As it is a solution of Laplace's equation, f will be unbounded from both above and below. Hence the corresponding potential, $\mathbf{V}$, given by eqn. (2.4), will be unphysical.

If eqn. (2.8) were made inhomogeneous then it would be a simple matter to construct $\mathbf{V}$ so that it is bounded from below. In principle, this might be achieved through the introduction of a constant or time-dependent source. How this might be obtained is suggested by a perusal of eqn. (2.7). Indeed, when the dilaton, Φ, is coupled to the theory, the conditions for conformal invariance allow for such a solution. This is seen as follows.

With the metric (2.1) and a dilaton which depends on U, conformal invariance may be shown to be maintained to all orders in α' if eqn. (2.8) is replaced by [6]

$$\nabla_T^2 \mathcal{F} + 2\partial_u \partial_u \Phi = 0 \ . \qquad (3.3)$$

Of course, eqn. (3.1) must be modified in the usual way [10] to include the dilaton. Take $\mathcal{F}(X,U)$ to be dependent only on the X^I, *i.e.* $\mathcal{F} = f(X^I)$. As Φ can be an arbitrary function of U, take it to be quadratic in that coordinate with $\partial_u \partial_u \Phi \equiv -c/2$, where c is an arbitrary real constant for now. With these choices, eqn. (3.3) reduces to

$$\nabla_T^2 f = c \ , \qquad (3.4)$$

Then $f(X^I)$, as given in eqn. (3.2), is a solution of eqn. (3.4) provided that h_{IJ} is no longer traceless but satisfies the equation: $tr(h) = -c/2$. For simplicity take $h_{IJ} \equiv \delta_{IJ}$

with $c = -2n$. Then $\mathbf{V}$ is bounded from below and if the homogeneous solution is dropped, the minimum of the potential is at X^I=0. Including the homogeneous solution, the minima condition is solved by the sphere S^{n-1} in R^n with radius

$$\rho_0 = \left\{ \begin{array}{ll} (\frac{n-2}{2})^{\frac{1}{n}} , & n > 2 , \\ \frac{1}{2} , & n = 2 . \end{array} \right\} \tag{3.5}$$

Of course, there is a mixed possibility wherein the homogenous and inhomogeneous solutions separately depend on different numbers of the internal coordinates. Whatsmore, there are additional solutions with reduced symmetry.

Solving the equation of motion for the string coordinates is tractable if the homogeneous solution in eqn. (3.3) is dropped. Then the solution for the X^I closed string coordinates may be written as

$$X^I(\tau,\sigma) = \sum_n \frac{1}{\omega_{pn}}[\chi^I_n \cos(\omega_{pn}\tau) + \tilde{\chi}^I_n \sin(\omega_{pn}\tau)]e^{i2n\sigma} , \tag{3.6}$$

where $\omega_{pn} \equiv \sqrt{(p^u)^2 + 4n^2}$. The hermitian conjugates are $(\chi^I_n)^\dagger = \chi^I_{-n}$ and $(\tilde{\chi}^I_n)^\dagger = \tilde{\chi}^I_{-n}$. The total momentum of the closed string is $P^I = (\tilde{\chi}^I_0 - \chi^I_0)$. The remaining X^i coordinates have the usual flat space-time solutions.

Had $\mathcal{F}(X,U)$ been taken to be of the form given in eqn. (2.9) with $r(U)$ arbitrary, the dilaton would have a generalized U-dependence. Given eqns. (3.3) and (3.4) the latter is dictated by $\partial_u\partial_u\Phi = -\frac{c}{2}r(U)$. A similar result is obtained with coupling to the anti-symmetric tensor rather than the dilaton. As shown in ref. [6], the $\partial_u\partial_u\Phi$ term in eqn. (3.3) will then be replaced by the square of the anti-symmetric tensor, A_{ij}. Take the latter to be a function only of U: $A_{ij}(U)$. Let $A_{IJ} \equiv 0$, then eqn. (3.4) is obtained along with $r(U) = cA^2(U)$. A_{IJ} is taken to be zero so that the X^I equation of motion remains the same as in the dilaton case. An important point is that one or both of these backgrounds must be included. The pure metric background does not lead to a potential

which is bounded from below. Time-dependent compactifications have been considered in ref. [11] and references therein.

IV. String Propagation in Singular Backgrounds

Let $\mathcal{F}$ be of the form given in eqn. (2.9) and U be given by $U = p^u \tau$ in the light-cone gauge. Then the X^I equation of motion, derived from (3.1), is

$$\Box X^I = \frac{1}{2}\partial^I f(X) r(U)(p^u)^2 \quad . \tag{4.1}$$

The prototypical metric for the study of the propagation of strings in singular backgrounds is the shock wave metric. It is given by eqn. (2.1) with $r(U) = \delta(U)$. In this metric, space-time is flat for $U \neq 0$. In these regions, one expands the closed string solutions to eqn. (4.1) as

$$X^I_{(\lessgtr)}(\tau,\sigma) = x^I_{(\lessgtr)} + p^I_{(\lessgtr)}\tau + i\frac{1}{2}\sum_{n\neq 0}\frac{1}{n}(\alpha^I_{n,(\lessgtr)}e^{i2n\sigma} + \tilde{\alpha}^I_{n,(\lessgtr)}e^{-i2n\sigma})e^{-i2n\tau} \quad . \tag{4.2}$$

Integrating eqn. (4.1) over $\int_0^\pi d\sigma e^{i2m\sigma}\int_{0^-}^{0^+} d\tau$, leads to expressions for the $\alpha^I_{m,>}$ oscillators in terms of the $\alpha^I_{m,<}$ oscillators [8]. These expressions may be summarized as ($\alpha^I_0 = \frac{1}{2}p^I$)

$$\alpha^I_{m,>} = S^\dagger \alpha^I_{m,<} S \quad , \tag{4.3}$$

with

$$S = \exp\left[i\frac{1}{2\pi}p^u\int_0^\pi d\sigma f(X(0,\sigma))\right] \quad . \tag{4.4}$$

One verifies that $S^\dagger S = 1$. The center-of-mass coordinates is unaffected by the interaction. An expression similar to eqn. (4.3) holds for the $\tilde{\alpha}$ oscillators. Note that the p^u operator in S (or $S^\dagger$) annihilates the vacuum.

Now, well defined geodesics are not sufficient to ensure string propagation. One must check that observables such as the number and mass operators are well behaved. According to ref. [7] the total mass diverges. From eqn. (4.3) it is concluded that this is *not* the case. The "in" number operator is given by $N_< = \sum_{n=1}^{\infty} \alpha^i_{-n,<}\alpha^i_{n,<}$. From eqn. (4.3) it follows that the "out" number operator is $N_> = S^\dagger N_< S$. Beyond this, the number operator at the nth mode obeys the relation $\langle N_{n,>}\rangle = \langle N_{n,<}\rangle$. Hence the total mass which is proportional to $\sum_n n\langle N_n\rangle$, is conserved. Consequently, one has

$$\begin{aligned} \langle N_>\rangle &= \langle N_<\rangle \ , \\ \langle M^2_>\rangle &= \langle M^2_<\rangle \ . \end{aligned} \tag{4.5}$$

So strings propagate through a shock wave singularity.

There is an even simpler explanation for this result. This explanation also has the virtue of generalizing the discussion to arbitrary $r(U)$. Lorentz boost to the X^D-velocity, $\tanh\xi$. Then the coordinates U and V boost to $U \to e^{-\xi}U$ and $V \to e^{\xi}V$. The $\mathcal{F}$ independent part (Minkowski metric) of ds^2 in eqn. (2.1) is invariant under this transformation. The remaining part, call it ds^2_F, transforms as

$$ds^2_F \to e^{-2\xi} f(X) r(e^{-\xi}U) dU^2 \ . \tag{4.6}$$

For the shock wave metric, this expression vanishes when one takes $\xi \to \infty$.[3]

As singular metrics are of current interest, generalize $r(U)$ to $r(U) = U^l$. Then by taking $\xi \to \pm\infty$ the metric ds^2 becomes Minkowski except for the case $l = -2$. It will vanish in general unless $r(U)$ contains a factor which is homogeneous with degree -2.

The number operator is a Lorentz invariant quantity. Thus its computation in the boosted metric will give the same answer as in the metric (2.1). Hence, as the boosted metric is Minkowski, except for $l = -2$, the number operator is the same for all times U.

[3] This analysis builds on an observation made by E. Witten as referenced in ref. [11].

The p^u operator drops out of the interacting Hamiltonian for the $l = -2$ case. An explicit calculation must be done in this case. Similarly, the mass operator in the metric (2.1) is the same (except for $l = -2$) as that in Minkowski space-time.

It can be shown that the metric (2.1) with $r(U) = r_0^2 U^{-2}$ is singular even in the sense of general relativity. First consider the geodesic motion of the particle. As in ref. [7] take the sandwich wave [1] for which $r(U)$ differs from the identity only over a finite U interval. $f(X^i)$ is of the form $f(X^i) = -\sum_i^{(D-3)}(X^i)^2 + (D-3)(X^{D-2})^2$. Let $r(U)$ be given by

$$r(U) = \left\{ \begin{array}{ll} \frac{1}{4}U^{-2} \ , & |U| < T \ , \\ 0 \ , & |U| \geq T \ . \end{array} \right\} \tag{4.7}$$

The geodesic equations of the first $(D-3)$ X^i-coordinates are then $\ddot{X}^i + \frac{1}{4\tau^2}X^i = 0$ for $|U| < T$ and $\ddot{X}^i = 0$ otherwise. In the three distinct regions, these $X^i(\tau)$ are given by

$$X^i(\tau) = \left\{ \begin{array}{ll} x^i_< + p^i_< \tau \ , & U \leq -T \ , \\ \alpha^i \sqrt{|\tau|} \ , & |U| < T \ , \\ x^i_> + p^i_> \tau \ , & U \geq T \ , \end{array} \right\} \tag{4.8}$$

with $U = p^u \tau$.[4] The α^i's are, in principle, determined in terms of $x^i_<$ and $p^i_<$. It is possible to show that a trajectory incident from $\tau = -\infty$ cannot smoothly traverse the $\tau = 0$ singular point. This is seen by attempting to match the solutions at this barrier. One finds that this is not possible. Note that the equation of motion is in Schrödinger form with zero energy. Thus one can intuit that there can be no barrier penetration in this case.

In the string theory, $X^i(\tau, \sigma)$ is given by a solution of the equation

$$\ddot{X}^i_n + (4n^2 + \frac{r_0^2}{\tau^2})X^i_n = 0 \ , \tag{4.9}$$

[4] If $r(U) = -m(m+1)U^{-2}$ for $|U| < T$ then a solution for $X^i(\tau)$ in this region is $X^i(\tau) = \alpha^i \tau^{m+1}$.

for the closed string in the $|U| < T$ region. This follows from eqn. (4.1) with X_n defined by $X^i(\tau,\sigma) \equiv \sum_n X^i_n(\tau)e^{i2n\sigma}$. Unlike the particle case, the energy in this Schrödinger like equation is non-zero for $n \neq 0$. However, the problem of the lack of a smooth trajectory across the singular point persists as the equation for the zero-mode, X^i_0, is the same as that of the particle. For completeness, the solution of eqn. (4.9), with $n \neq 0$, is given in terms of Bessel functions of order $q = \sqrt{\frac{1}{4} - r_0^2}$. Simplification is achieved when r_0^2 is taken to be $r_0^2 = \frac{1}{4}$ with $q = 0$.

To summarize this section, the mass (number) operators for the string propagating in the metric (2.1) are the same as those of Minkowski space-time. The only exception to this rule is when $\mathcal{F}$ contains a term which is homogeneous in U with degree -2. The simplest example for which this is the case, namely $\mathcal{F} = f(X)U^{-2}$, already leads to a space-time which is geodesically incomplete.

V. Supersymmetry And Singular Space-times

In compactifying superstrings one usually requires that low energy D=4, N=1 supersymmetry survives so that the gauge hierarchy problem may be solved. Thus, one would like to know if the metric (2.1) admits a space-time supersymmetric vacuum. (It has been shown, in ref. [8], to be a classical solution of the NSR string theory.) From the discussion in the previous section, one might expect that it does. This is because one can boost the metric to Minkowski form. That is, except for the singular case mentioned above.

Go to a vielbein basis wherein $G_{mn} = e_m{}^{\tilde{m}} e_n{}^{\tilde{n}} \eta_{\tilde{m}\tilde{n}}$. The $\tilde{m}$ and $\tilde{n}$ indices denote tangent space directions. Then one derives from eqn. (2.1):

$$e_i{}^{\tilde{j}} = \delta_i{}^{\tilde{j}} \ , \qquad e_u{}^{\tilde{v}} = e_v{}^{\tilde{u}} = 1 \ , \qquad e_u{}^{\tilde{u}} = -\frac{1}{2}\mathcal{F}(X,U) \ , \tag{5.1}$$

as the only non-zero components. The inverses of these objects are

$$e_{\tilde{i}}{}^{j} = \delta_{\tilde{i}}{}^{j} \ , \qquad e_{\tilde{v}}{}^{u} = e_{\tilde{u}}{}^{v} = 1 \ , \qquad e_{\tilde{v}}{}^{v} = \frac{1}{2}\mathcal{F}(X,U) \ . \tag{5.2}$$

Expressions for the spin-connections are determined by the torsion free constraint on the geometry. Having determined these, one finds the covariant derivatives to be

$$\begin{aligned} \nabla_u &= \partial_u + \frac{1}{2}\partial^{\tilde{i}}\mathcal{F}(X,U)M_{\tilde{i}\tilde{u}} \ , \qquad \nabla_v = \partial_v \ , \qquad \nabla_i = \partial_i \ , \\ \nabla_{\tilde{u}} &= \partial_v \ , \qquad \nabla_{\tilde{v}} = \partial_u + \frac{1}{2}\mathcal{F}(X,U)\partial_v + \frac{1}{2}\partial^{\tilde{i}}\mathcal{F}(X,U)M_{\tilde{i}\tilde{u}} \ , \qquad \nabla_{\tilde{i}} = \partial_{\tilde{i}} \ . \end{aligned} \tag{5.3}$$

M is the Lorentz generator acting on the tangent space indices. Adopt the solution wherein the 3-form vanishes and the dilaton is a constant. The conditions for unbroken D=10, N=1 supersymmetry have been reviewed in ref. [10]. The are all identically satisfied except for the vanishing of the supersymmetry transformation of the gravitino which requires

$$\nabla_m \epsilon = 0 \ . \tag{5.4}$$

The vacuum supersymmetry transformation of the dilatino automatically vanishes as all of the components of the field strength of the gauge field are zero in the vacuum. This follows from the Bianchi Identity: $dH = tr(R \wedge R) - tr(F \wedge F)$. The left-hand-side of the latter equation vanishes (as $H = 0$). Due to the form of the curvature given in eqn. (2.2), $R \wedge R$ vanishes identically. Hence a solution of the Bianchi Identity is given by $F = 0$. Eqn. (5.4) is solved if the supersymmetry parameter, ϵ, is a constant which satisfies the condition $\Gamma_{\tilde{u}}\epsilon = 0$. This follows as in evaluating $M_{\tilde{i}\tilde{u}}$ on spinors, one finds that it is proportional to $\Gamma_{\tilde{i}}\Gamma_{\tilde{u}}$. Using the inverse of the flat metric, this requirement may also be written as $\Gamma^{\tilde{v}}\epsilon = 0$. This is solved by the eight dimensional spinors of $SO(8)$. It so happens that this last equation is also the light-cone gauge condition in the Green-Schwarz superstring.

The metric (2.1) admits a supersymmetric vacuum. Since the U dependence of $\mathcal{F}$ has not entered into this analysis, supersymmetry is present even in the singular case where $\mathcal{F}$ is a homogeneous function of U of degree -2.

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REFERENCES

1. R. Penrose, *Rev. Mod. Phys.* **37** (1965) 215.
2. E. Cremmer and J. Scherk, *Nucl. Phys.* **B118** (1977) 61; J. Scherk and J. H. Schwarz, *Nucl. Phys.* **B153** (1979) 61.
3. C. Klimčík, *Phys. Lett.* **208B** (1988) 373.
4. For a review, see C. Callan and L. Thorlacius, Princeton and SLAC preprint # Print-89-0232 (March 1989).
5. D. Amati and C. Klimčík, *Phys. Lett.* **219B** (1989) 443.
6. R. Güven, *Phys. Lett.* **191B** (1987) 275; M. Fabbrichesi and G. Veneziano, *Phys. Lett.* **233B** (1989) 135; H. J. de Vega and N. Sanchez, *Phys. Lett.* **244B** (1990) 215.
7. G. T. Horowitz and A. R. Steif, *Phys. Rev. Lett.* **64** (1990) 260; *Phys. Rev.* **D42** (1990) 1950; preprint # UCSB-TH-35 (June 1990); A. R. Steif, *Phys. Rev.* **D42** (1990) 2150.
8. D. Amati and C. Klimčík, *Phys. Lett.* **210B** (1988) 92.
9. H. J. de Vega and N. Sánchez, *Nucl. Phys.* **B317** (1989) 706.
10. M. B. Green, J. H. Schwarz and E. Witten, **Superstring Theory**, Cambridge Univ. Press (1987), Vol. 1.
11. R. E. Rudd, Princeton preprint # PUPT-1178 (April 1990).
12. P. M. Schwarz, CalTech preprint # CALT-68-1674 (October 1990).