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GRAVITATIONAL BEAMSTRAHLUNG[a] *

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ABSTRACT

A semiclassical formalism of gravitational radiaiion from the collision of high energy e^+e^- beams is derived. The charged particles interact with the strong collective EM field of the oncoming beam and emit beamstrahlung. We show that these photons would couple to the same EM field and ***resonantly*** excite gravitational waves with much higher frequencies than those from astrophysical signals. We examine several physical examples and show that there is a finite probability for emitting such "gravitons".

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(*a*) An abbreviated version of this paper has received an Honorable Mention from the Gravity Research Foundation, 1990.

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Despite decades of effort, the long-sought-after gravitational waves (GWs) from astrophysical sources so far still evade detection.[1] A significant advance in the study of gravity will be made if gravitational waves can be generated in the laboratory. Artificial GW sources have been proposed by various people in the past,[2] but none seem to promise a strong signal. Recently, Palazzi and Fargion[3] stimulated great interest by estimating the power of the gravitational synchrotron radiation in modern high energy circular accelerators. However, the radiation power derived by the authors, which scales as γ^6, (γ being the Lorentz factor of the radiating particle) is erroneous by a factor γ^2 too large! This greatly reduces their estimate of the yield of GWs from circular accelerators. In addition, these GWs are radiated in all azimuthal angles along a large circumference, which further reduces the collectable signals.

In addition to the direct massive radiation of GWs, it was found by Gertsenshtein[5] that there can also be GWs generated by the *resonant* coupling between a propagating EM wave and a transverse background EM field. In the situation where the EM wave is generated by a charged particle interacting with a background field, the radiation could be converted directly into GWs. Pustovoit and Gertsenshtein[6] first study GW from a relativistic electron in a uniform background magnetic field of infinite extent. The result obtained in this work turns out to be pathological: since the part of the metric perturbation that is associated with the resonant excitation does not satisfy the transversality condition. This renders the independent meaning of resonant excitation ambiguous. To introduce a spatial cut-off in this case does not help since it violates Maxwell's equations on the boundary. Sushkov and Khriplovich[7] show that this difficulty can be avoided if the phenomenon occurs for a relativistic electron executing a circular orbit in a Coulomb field. However, in real situations the effect turns out to be minute: and is therefore mainly of theoretical interest. In all these works,[3-7] the calculations are strictly classical.

It happens that a very powerful laboratory EM radiation, called ***beamstrahlung***, occurs during the collision of high energy e^+e^- beams. A substantial fraction of beam energy can be lost through beamstrahlung when particles are radially bent towards the axis of symmetry by the strong collective macroscopic EM field of the oncoming beam. In the world's first linear collider, the SLC (Stanford Linear Collider), the typical beamstrahlung photon energy is $\sim 10^{-3}$ of the initial particle energy. For future linear colliders, it is found to be inevitable that the fractional photon energy is nonnegligible,[8] and the process is necessarily quantum mechanical. With its potential impacts on high energy experimentation and its challenge as a theoretical problem, the study of quantum beamstrahlung has been intensive in recent years.'

It seems therefore natural to ask if the same physical system for beamstrahlung would also be a source for abundant GWs. To address the issue, we introduce a semi-classical formalism for resonant GW excitation from quantum beamstrahlung. The radiation power spectrum of the GW is derived in a compact and closed form. In addition, we show, through numerical examples, that there is indeed a finite probability for emitting such ***gmvitons.***

To study the GWs from e^+e^- beam-beam collision at an energy scale much lower than the Planck scale, we start with the linearized Einstein equation. With the convention G = c = $\hbar$ = 1, we write

$$\Box\, \psi_{\mu\nu} = 16\pi T_{\mu\nu} \quad , \tag{1}$$

where $\psi_{\mu\nu} = h_{\mu\nu} - \eta_{\mu\nu} h/2$ is the trace-reversed metric perturbation around the flat space-time $\eta_{\mu\nu}$ with the curved metric $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, and $\eta = \mathrm{diag}(1, -1, -1, -1)$. The stress tensor has contributions from the particle and the field: $T_{\mu\nu} = T^p_{\mu\nu} + T^f_{\mu\nu}$. The later is quadratic in EM field tensors, i.e., $Tf \sim (F^b + F^0)(F^b + F^0)$.

The square of the background field, F^0F^0, bears no relation to the motion of the particle, and we shall ignore it in the following. There is also no need to discuss the square of the beamstrahlung field, F^bF^b, since almost everywhere $F^b << F^0$ except at small distance from the particle. But this has been taken into account in the mass renormalization, and thus is already contained in T^p. So the contribution from Tf is simply $F^bF^0 + F^0F^b$. It can be shown[10] that for strong and extended background fields the contribution to GWs is dominated by the resonant excitation over the direct massive radiation. Therefore in the following we shall concentrate on the cross terms from Tf only.

It is found[9,11] to be physically reasonable to model the fields of the generally Gaussian distribution of ultra-relativistic ($\gamma >> 1$) particles in a bunch by an effective constant transverse magnetic field. Let us assume that the effective field has a strength B pointing along y-axis and a length L along z-axis with abrupt cut-offs at both ends:

$$\vec{\mathbf{B}} = B\left[\theta(z = -L/2) - \theta(z = +L/2)\right]\hat{e}_y \quad . \tag{2}$$

This. satisfies Maxwell's equations on the boundary to the accuracy of the order $1/\gamma$, as the collective field is composed of relativistic charges. From straightforward calculations, the stress tensor can be reduced to

$$T_{\mu\nu} = \frac{B}{4\pi}\begin{pmatrix} B_2 & -E_3 & 0 & E_1 \\ -E_3 & B_2 & -B_1 & 0 \\ 0 & -B_1 & -B_2 & -B_3 \\ E_1 & 0 & -B_3 & B_2 \end{pmatrix} \equiv \frac{B}{4\pi}Q_{\mu\nu} \quad , \tag{3}$$

where the elements in the above matrix are components of the radiation field.

From Eq. (1) we find

$$\psi_{\mu\nu}(\vec{\mathbf{R}}) = -\frac{4}{R}e^{ikR}T_{\mu\nu}(\vec{\mathbf{k}}) \tag{4}$$

in the wave zone, where $\vec{R} = R\vec{n}$ and R is the distance to the observation point. The GW radiation power is[12]

$$\begin{aligned} W_G(\omega) &= -\frac{R^2}{32\pi}\int d\Omega \left[\partial_0\psi^{\mu\nu}\partial_i\psi_{\mu\nu} - \frac{1}{2}\partial_0\psi^{\mu}_{\mu}\partial_i\psi^{\nu}_{\nu}\right]n^i \\ &= -\frac{\omega^2}{2\pi}\int d\Omega\, T^{\mu\nu}(\vec{\mathbf{k}})T_{\mu\nu}(-\vec{\mathbf{k}}) \end{aligned} \quad , \qquad (5)$$

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where n_i is the ith component of $\vec{\mathbf{n}}$. Our task is therefore to find the stress tensor in the momentum space.

It has been shown by Baier and Katkov[13] that in the ultrarelativistic limit, the quantum phenomenon of charged particle radiation in a background EM field can be reduced to a semiclassical one. Namely, under a well-defined transcription, the transition amplitude of the current operator has a classical meaning. In fact, this approach has been successfully applied by several authors in Ref. 9 in the-studies of quantum beamstrahlung. Symbolically, we shall then write the Maxwell equation as

$$\Box A_\mu = -4\pi\langle j_\mu\rangle \ , \qquad (6)$$

where $j_\mu = e\bar{\psi}\gamma_\mu\psi$ is the current operator of the Dirac field. The solution in momentum space is simply

$$A_\mu(\vec{\mathbf{q}}) = \frac{4\pi}{\omega^2 - \vec{\mathbf{q}}^2}\langle j_\mu(\vec{\mathbf{q}})\rangle \quad , \qquad (7)$$

from which $\vec{\mathbf{E}}(\vec{\mathbf{q}})$ and $\vec{\mathbf{B}}(\vec{\mathbf{q}})$ can be trivially deduced:

$$\begin{pmatrix} \vec{\mathbf{E}}(\vec{\mathbf{q}}) \\ \vec{\mathbf{B}}(\vec{\mathbf{q}}) \end{pmatrix} = \frac{4\pi i}{\vec{\mathbf{q}}^2 - \omega^2}\begin{pmatrix} \omega\overrightarrow{\langle \mathbf{j}(\vec{\mathbf{q}})\rangle} - (\vec{\mathbf{q}}^2/\omega)[\vec{\mathbf{n}}\cdot\overrightarrow{\langle \mathbf{j}(\vec{\mathbf{q}})\rangle}]\,\vec{\mathbf{n}} \\ \vec{\mathbf{q}}\times\overrightarrow{\langle \mathbf{j}(\vec{\mathbf{q}})\rangle} \end{pmatrix} \quad . \qquad (8)$$

Combining with the Fourier transform of the background field, we obtain $T_{\mu\nu}(\vec{\mathbf{k}})$ from Eq. (3) through convolution:

$$T_{\mu\nu}(\vec{\mathbf{k}}) = \frac{1}{4}LB\frac{k}{k_3}\left[1 - \frac{\sin(k_3L)}{k_3L}\right]Q_{\mu\nu}[\overrightarrow{\langle \mathbf{j}(\vec{\mathbf{k}})\rangle}] \quad , \qquad (9)$$

where all the elements in $Q_{\mu\nu}$ are properly replaced by the expressions in Eq. (8). It can be shown, by insering $T_{\mu\nu}$ into Eq. (4), that $\psi_{\mu\nu}$ indeed satisfies the transversality condition: $\partial^{\mu}\psi_{\mu\nu} = 0$. Thus the physical meaning of this process is unambiguous.

Inserting Eq. (9) into Eq. (5), we find

$$W_G(\omega) = -\frac{1}{16\pi}\omega^2 L^2 B^2 \int d\Omega \frac{k^2}{k_3^2}\left[1 - \frac{\sin(k_3 L)}{k_3 L}\right]^2 n_\perp^2 \sum_\lambda |\langle \epsilon_\lambda^\mu j_\mu \rangle|^2 \quad , \tag{10}$$

where ϵ_λ^μ is the polarization of the EM radiation. To the accuracy of the order $1/\gamma$, and with the fundamental constants restored, we obtain the major result of this paper:

$$W_G(\omega) = \frac{\pi}{4}\frac{Gm^2}{e^2}\left(\frac{L}{\lambda_c}\frac{B}{B_c}\right)^2\left[1 - \frac{\sin(\omega L)}{\omega L}\right]^2 W_{EM}(\omega) \quad , \tag{11}$$

where λ_c is the Compton wavelength, $B_c \equiv m^2c^3/e\hbar \sim 4.4 \times 10^{13}$ Gauss is the Schwinger critical field, and W_{EM} the power spectrum for single-photon process in quantum beamstrahlung. The square bracket represents the form factor from the Fourier spectrum of the background field. We see that this form factor is essentially of the order unity for wavelengths $\lambda \lesssim 2L$, where the last zero at $\sin(2\pi L/\lambda) = \sin\pi$ occurs. Beyond this wavelength the GWs are largely suppressed.

The radiation power per electron can be easily calculated. In terms of the wavelength of radiation, the EM beamstrahlung can be classified into the coherent and the incoherent regimes. For wavelengths longer than the radiating beam, the radiation is coherent where the entire bunch population radiate together. For wavelengths shorter than the bunch, the radiation is essentially incoherent. In the situation where the length of the radiating beam is equal to that of the target beam, i.e., $l = L$, we find

$$W_G = W_G^c + W_G^i, \tag{12}$$

where the coherent contribution is

$$W_G^c \simeq \pi \frac{Gm^2}{e^2}\left(\frac{L}{\lambda_c}\frac{B}{B_c}\right) \frac{\alpha m c^3}{\lambda_c} N\left(\frac{\lambda_c}{\gamma l}\right)^{4/3} \Upsilon^{2/3} \quad , \quad \mathrm{VT} \quad ,$$

and the incoherent contribution is

$$W_G^i \simeq \frac{\pi}{6}\frac{Gm^2}{e^2}\left(\frac{L}{\lambda_c}\frac{B}{B_c}\right)^2 \frac{\alpha m c^3}{\lambda_c} \begin{cases} \Upsilon^2 \quad , & \Upsilon \lesssim 0.2 \quad , \\ 0.2\Upsilon \quad , & 0.2 \lesssim \Upsilon \lesssim 22 \quad , \\ 0.556\Upsilon^{2/3} \quad , & 22 \lesssim \Upsilon \quad . \end{cases}$$

Here N is the total number of particles in the radiating beam. Υ is the so-called *beamstrahlung* parameter, defined as $\Upsilon \equiv \gamma B/ B_c$. Physically, Υ << 1 corresponds to the classical regime of radiation, while Υ >> 1 corresponds to the extreme quantum regime. It can be verified that at low energies the radiation power is mostly contributed from the coherent radiation, whereas at high energies it is predominantly from the incoherent one.

For the SLC, $\Upsilon \sim 10^{-3}, \gamma = 10^5$, $L \sim 0.3$ cm, and N = 5 x 10^{10}, we find the power to be $W_G \sim 2 \times 10^{-21}$ eV/sec. The corresponding dimensionless strain, $h \equiv (4\pi G L^2 \gamma^2 N W_G / c^5 R^2)^{1/2}$, is $\sim 3 \times 10^{-41}$ at a distance R = 1 m, which is very weak. For the next generation colliders in the 1 TeV range (TLC), it is conceivable that $\Upsilon \sim 10$, $L \sim 100\,\mu$m, N $\sim 10^{10}$. The power then increases to $W_G \sim 5 \times 10^{-12}$ eV/sec, and $h \sim 2 \times 10^{-39}$. Although from the look these dimensionless strains are much weaker than those typical of astrophysical GW signals, but the frequencies in our case are much higher. For SLC, the typical incoherent radiation frequency is $\omega^i \sim \Upsilon\gamma mc^2/\hbar \sim 10^{23}$/sec, and for TLC $\omega^i \sim 10^{27}$/sec. It is unclear whether in this ultra -high frequency regime, the notion of strain still has a physical meaning. This leads us to a different perspective on the phenomenon.

Although in our calculation the spacetime perturbation is a classical quantity, we may heuristically discuss the gravity "quanta" with energy $\hbar\omega$ when the radiation frequencies are high. After all, the source of the GW excitation in our process, i.e. the photon from beamstrahlung, was quantum mechanical in the first place. Thus we assume that the coefficient in Eq. (11) that converts W_{EM} into W_G is also the probability of converting a photon with frequency ω into a *graviton* with the same frequency. In this regard, it is more probable to generate gravitons from the coherent photons than the incoherent ones, as the former is generally more abundant in number. For SLC, although $W_G^c/W_G^i \sim 6 \times 10^{-5}$, the number of photons is mainly from coherent radiation: $n^c/n^i = (W^c/W^i)(2\pi c/l\omega^i) \sim 8 \times 10^6$. Putting together all parameters, we find the probability of exciting a graviton per collision to be $N_G \sim n^c NL/2c \sim 5 \times 10^{-23}$. With a collision repetition rate $f \sim 100/\mathrm{sec}$, this is certainly hopeless. The situation improves somewhat in TLC, where $W_G^c \sim 4 \times 10^{-20}$ eV/sec and $N_G \sim 5 \times 10^{-21}$/collision. With a repetition rate $f \sim 3000$ /sec, the yields are still disappointingly low. These colliders, however, are not designed for dedicated GW generation.

One way to improve the situation is to collide asymmetric beams where the radiating beam is sufficiently shorter than, but not necessarily as energetic as, the target beam. Notice that $W_G^c \propto \gamma^{-2/3}$ for the radiating beam, whereas W_G^i increases as positive powers of γ. Thus there exists an optimum value for γ where it is low enough such that the radiation is primarily coherent, yet high enough such that the avalable beam energy is maximized.

As an example, we consider a high current target beam with L = 10 cm, which has an effective field strength comparable to that in the TLC, but could be at a lower beam energy. Next we assume that low energy radiating beams with $\gamma = 2 \times 10^3$, N = 5×10^{11}, and $l = 100\ \mu$m can be achieved. The choice of γ comes close to the optimum condition for the given B field strength. The conversion probability is $P \sim 1.3 \times 10^{-30}$. Note that in the case when $l << L$, the coherent radiation power is roughly 3/2 times larger than that appeared in Eq. (12), which was for equal beams. Thus we have $W^c_G/W^i_G \sim 9N(\lambda_c/\gamma l\Upsilon)^{4/3} \sim 0.2$, where $W^c_G \sim 2 \times 10^{-10}$ eV/sec. The typical graviton energy in this example is $\sim$ 0.01 eV. To estimate the yield, one observes that the effective collision time in this case is not $L/2c$. When the target is so long that the entire beam energy is lost through EM radiation before the end of collision, the effective collision time is $\tau \sim \gamma mc^2/W_{EM}$, in this case $\sim 1 \times 10^{-12}$ sec. The probability of graviton excitation is then $N_G \sim 9 \times 10^{-9}$/collision. With a repetition rate of $f \sim 4000$/sec, we have $N_G \sim 10^3$/year.

Our formula can also be applied to physical systems other than the bona *fides* e^+e^- beam-beam collisions. For example, the interaction between a relativistic electron beam and a plasma has a direct analogy to the beam-beam interaction.[14] When the plasma is underdense than the beam, the beam-plasma interaction results in a total rarefaction of plasma electrons from the beam channel, leaving an ion column behind the track. In general, laboratory plasmas (and therefore the ion column) may not be as dense as colliding e^+e^- beams, thus the effective field is lower. But the GW conversion probability can nevertheless be compensated by the longer target attainable using plasmas. For our purpose, we may consider a two-stage process: First a precursing beam that establishes the ion column. This is then followed by a radiating beam before the ion column is degraded. From Eq.(12), we see that a precursing beam with $\gamma_p = 1 \times 10^3$, $\sigma_p = 1\mu$m, l_p = 3 mm, and $N_p = 5 \times 10^{10}$, would penetrate a plasma of density $n_p = 5 \times 10^{18}/\text{cm}^3$ by $L \sim 9$ m, before it loses all its energy through the coherent EM radiation. The effective field strength is then $B/B_c \sim 6.5 \times 10^{-8}$, and the conversion probability is $P \sim 6.8 \times 10^{-31}$. If a radiating beam with $\gamma = 1 \times 10^4, N = 5 \times 10^{11}, l = 100\ \mu$m, and the same cross section impinges the ion column, we will have $W^c_G \sim 1.2 \times 10^{-12}$ eV/sec, $W^i_G \sim 5.5 \times 10^{-13}$ eV/sec, and the effective collision time is $\tau \sim 2 \times 10^{-9}$ sec. This gives $N_G \sim 1.2 \times 10^{-7}$/collision. With a repetition rate $f \sim 4000$/sec, one expects $N_G \sim 1.4 \times 10^4$/year.

It is not clear, however, how these events could be detected. Detailed discussion on the experimental possibility is beyond the scope of this paper. Yet some comments may help to elucidate the issue. The conventional Weber resonant bar method[15] and the laser interferometry approach[16] do not seem to be applicable in our frequency regime. In searching for a detector that would work in the high frequency regime and could overcome the faintness of the signal, two major facts about gravitational beam-strahlung may be helpful: The gravitons in this case are essentially mono-energetic, and they are emitted with time intervals that are commensurate to the periodicity of collisions. In this regard, one possibility is to use crystals with a high quality factor-such that graviton-induced phonon excitations with the proper frequency can last long enough for a resonant build-up of the signals. Evidently, more studies are necessary for this idea, or any other approach, to be justified.

To sumarize, in this paper we have introduced a semiclassical formalism for the resonant excitation of high frequency GWs from quantum beamstrahlung. This is theoretically interesting for its own right. For it leads to the argument, though only heuristic, for exciting quantized gravitons. When applied to specific physical systems, we show that there is indeed a finite probability for emitting gravitons from asym-metric e^+e^- collisions, and its variation like beam-plasma interaction. The limitation on the yields, however, is not fundamental. With advances on the accelerator tech-nology, especially that of high current, low emittance beams, the graviton production -rate may be further improved.

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