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NEW SYMMETRIES IN HEAVY FLAVOR PHYSICS

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ABSTRACT

Isgur and Wise have found that the formal limit M_b, $M_c \rightarrow \infty$ leads to very great simplification in the general structure of the electroweak matrix elements of hadrons containing those quarks. In addition, interesting new symmetries appear in this limit. Their results are discussed, as well as some natural extensions to matrix elements of products of currents.

I. Introduction

The ideas in this talk are not mine. They stem mainly from recent work of Nathan Isgur and Mark Wise[1,2,3] who have been investigating the consequences of taking the formal limit of infinite heavy-quark mass in the study of various weak decay amplitudes. Their methods turn out to be quite powerful, and in my opinion open up the possibility of a model-independent, systematic, framework for describing most of the decay processes involving b-quarks and c-quarks. This seems to me a very significant development. Already we see important measurements limited by theoretical systematic errors. The size of these are estimated by comparison of various model calculations. If instead one can find a model-independent idealization which is simple (in this case the infinite-mass limit), then the model dependence is restricted to a set of corrections to the limiting

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case. Furthermore, it is probable that the leading corrections to the simple limit (in this case these include M^{-1} corrections and QCD corrections of order $\alpha_s(M^2)$) can at least be fully classified if not fully calculated, thereby relegating the role of models even further back in importance.

It is too early for me to tell how far one can push the approach before model–dependence becomes essential. A lot of thought needs to be put into the classification of the corrections to the infinite-mass limit, as well as completely elucidating the consequences of the limit itself. But as of now the approach seems most promising. It is my hope that I can at least communicate here why I feel so optimistic and enthusiastic about what Isgur and Wise have done.

II. The Infinite-Mass Limit In QCD

The basic physical ideas of the infinite-mass limit are familiar ones, and do not represent especially fresh news to anyone who has been involved in the field. They are simply the following:

1. As, say, $M_b \to \infty$, the velocity of the b-quark or the meson containing the &-quark is unaffected by soft, confining forces. Only a perturbative process involving very hard gluons or an electroweak transition has enough clout to modify the velocity.
2. In the limit, QCD remains well-defined. This is rather well-studied theoretically[4]; the situation is similar to the QED of atomic physics.
3. The spin of the b quark decouples from the dynamics in the limit because the hyperfine, magnetic interaction scales as M^{-1}.
4. Consequently, in the limit the $0^- B$ and $1^- B^*$ become degenerate in mass, related to each other by a symmetry operation I call Wisgur symmetry. It is quite similar to the Wigner flavor x spin symmetry of nuclear physics. (Besides, both Messrs. Wise and Isgur sport beards.)

As promised, none of this looks especially fresh. But the news is that with only this much, one can arrive at quite powerful conclusions, and build a general framework which organizes the phenomenology.

In what follows it will be important to keep in mind that the infinite-mass limit is not a nonrelativistic or static approximation to the dynamics. Relativistic motion is allowed, and therein lies the power of the method.

III. Semileptonic B-Decays into Charm

We consider here all matrix elements of vector and axial currents bilinear in b and/or c fields, taken between B, B^*, D, D^* in initial and final states, at arbitrary momentum transfer. If one writes these all out there will be about forty independent kinematic or flavor structures, each multiplied by a form factor which depends on the squared momentum transfer between initial and final mesons.

Given this input, Isgur and Wise[2] obtain the following output: in the infinite-mass limit, all forty-odd form factors are completely determined, at all momentum transfers, in terms of only one normalized function. This function is just the elastic form factor of the B meson (with respect to the vector current $\bar{b}\gamma_\mu b$):

$$\langle B' \,|\, \bar{b}\gamma_\mu b \,|\, B\rangle = (4EE')^{-1/2}(p' + p)_\mu F(\hat{t}) \tag{3.1}$$

For example the $B \to D$ semileptonic form factor is

$$\langle D' \,|\bar{d}\gamma_\mu b \,|\, B\rangle = \sqrt{\frac{M_D M_B}{4E_D E_B}} \left(\frac{p'_\mu}{M_D} + \frac{p_\mu}{M_B}\right) F(\hat{t}) \,. \tag{3.2}$$

Notice the remarkable feature that the former scattering process involves a spacelike momentum transfer, while the latter decay process involves a timelike momentum transfer, since there is a lepton pair in the final state. Nevertheless the physics in the infinite mass limit is the same, because what matters is velocity transfer. The four-velocity is defined as

$$v_\mu = \frac{p_\mu}{M} = (\gamma, \boldsymbol{\gamma} \cdot \boldsymbol{v}) \ . \tag{3.3}$$

Then for the first &scattering process the squared invariant velocity transfer $\hat{t}$ is related to the usual q^2 in a simple way.

$$\hat{t} = \frac{(v - v')^2}{4} = \frac{q^2}{4M_B^2} \tag{3.4}$$

(The factor 4 is for later convenience.) But for the $B \to D$ decay the formula is

$$\hat{t} = \frac{[q^2 - (M_B - M_D)^2]}{4M_B M_D} \ . \tag{3.5}$$

We see that when q^2 takes its maximum value of the squared mass difference of B and D, $\hat{t}$ vanishes. At this kinematic endpoint, the D remains at rest (in the rest frame of the parent B) and the spectator system does not notice that the identity of the heavy quark has changed. Thus the physics does correspond to the $q^2 = 0$ limit of elastic scattering of a B meson from a lepton.

The way forty form factors are reduced to one goes roughly as follows. One first requires that there be no explicit reference in the amplitudes to heavy quark masses, only to four-velocities. This already dictates a return to the old fashioned $\sqrt{M/E}$ normalizing factors found only in Chapter 1 of Bjorken and Drell. It in addition forces relations between some of the form factors (cf. e.g. Eq.(3.2)). The second requirement uses the Wisgur spin symmetry. In the rest frame of, say, the secondary D or D^* one applies a spin rotation to the c quark; this is a symmetry operation in the infinite mass limit. This leads to relations between the $B \to D$ and $B \to D^*$ amplitudes. Formally the way this is done is by constructing S,, the spin operator of the c-quark, and applying it to the state $|D\rangle$ at rest. Because S_z is a vector operator and commutes with H, this must give $|D^*\rangle$ with a multiplier readily found to be (in magnitude) $1/2$. In this way one can change from D to D^* and back. The current operator J then becomes JS_z, which in turn can be replaced by $[J, S_z]$ and evaluated.

This game can be repeatedly played with many matrix elements and many currents until the advertised result emerges. The procedure is not without some tedium, and is fraught with hazards having to do with a myriad of sign and phase conventions. Happily there is a concise way of summarizing the whole result, a way which in turn leads to additional generalizations with little additional work. One writes[3]

$$\langle D \text{ or } D^* \,|\, J_\mu \,|\, B \text{ or } B^* \rangle = \left(\frac{M_D M_B}{4 E_D E_B} \right)^{1/2} \text{ trace } \overline{\mathcal{D}}\, J_\mu \mathcal{B} \rho \tag{3.6}$$

The trace is simply over 4 x 4 Dirac matrices. J_μ is just the Dirac matrix for the current in question. $\mathcal{B}$ is a matrix describing the combined wave function of B and B^*, defined so that (in its rest frame) it has the correct transformation properties under b–quark spin rotations:

$$\mathcal{B} = \left(\frac{\gamma \cdot P_B + M_B}{2 M_B} \right) (\gamma_5 B + \gamma_\mu B^\mu) \tag{3.7}$$

Here B labels the pseudoscalar field and B_μ the vector field, in particular its polarization state.

The quantity $\overline{\mathcal{D}}$ is similarly defined:

$$\overline{\mathcal{D}} = \gamma_0 \mathcal{D}^\dagger \gamma_0 \ . \tag{3.8}$$

This leaves the matrix ρ, which is the part of the matrix element having to do with the spectator system of light valence antiquark and its companion partons. It is only restricted by invariance considerations. But in this case those restrictions are powerful. The matrix ρ can depend upon initial and final velocities in the invariant combinations $\gamma \cdot v$ or $\gamma \cdot v'$ (or $v \cdot v'$). But these can be eliminated with use of the Dirac projection operators residing in $\mathcal{B}$ and $\overline{\mathcal{D}}$. So ρ can be reduced to a multiple of the unit matrix and factored out. It becomes just the invariant form factor $F(\hat{t})$. Therefore the remaining traces provide (unique) kinematic factors to preface the universal form factor. The reader is invited to try out a case or two to see the nature of the results and also to see how easy it is to get them.

The fact that the limiting result is easy to obtain does not guarantee it will be an accurate one. In this case there is every expectation that the truth is fairly close to the limit. But for me the evidence will only be fully convincing when all leading corrections are identified and classified. Already Isgur and Wise have considered leading-log QCD corrections (ignored here). But the argument of their log is M_b/M_c, which is 2.73. So this evidently invites consideration of the rest of the order α_s terms as well. All of this is well beyond the scope of this short presentation.

The above results do cry out for generalization. This is easily accomplished. For a final state containing extra particles in addition to the D or D^*, with momenta $k_1, \ldots, k_n$, simply write down the same trace but allow ρ to depend upon the momenta $k_1, \ldots, k_n$ as well as initial and final heavy-quark velocities. In such a case ρ will be a nontrivial matrix and will express the correlation structures in the final state consistent with the infinite-mass limit. Likewise if one chooses a higher order resonance in the final state, e.g. D^{**}, then one modifies the structure of the matrix $\mathcal{D}$ in the appropriate way. For example the appropriate language for D^{**} in the infinite mass limit is the hydrogenic language of $P^{1/2}$ and $P^{3/2}$, not the onium language of ${}^3P_{0,1,2}$ and ${}^1P_1$. The $P^{1/2}$ wave functions are obtained from $\mathcal{D}$ by right-multiplication with a γ_5 to account for the opposite parity. The $P^{3/2}$ states need a little extra care.

IV. Charmless Semileptonic B-Decays

Using the same language, we may rapidly review several other results[1,5] on the simpler class of semileptonic processes without a heavy quark in the final state. The simplest of these is nothing in the final state except the lepton pair. In this case the matrix element is

$$\langle 0 \,|\, (V_\mu - A_\mu) \,|\, D \rangle = \sqrt{\frac{M_D}{2E_D}} \text{ trace } \phi J_\mu \mathcal{D} = \sqrt{\frac{2M_D}{E_D}} \left[\left(\frac{P_\mu}{M_D} \right) D + D_\mu \right] \phi \,. \quad (4.1)$$

What are the messages? First of all the coupling of W to the D is related to the coupling of W to D^*. This is important for a class of nonleptonic decays such as

$B \to \pi, \rho, \ldots + D_s$, D_s^*; the relative branching fractions into D_s and D_s^* are predicted, provided the final-state interactions of D_s, D_s^* with a, ρ, . . . can be neglected. Secondly, the decay constants of B and D are related; as conventionally defined, they scale as $M^{-1/2}$. Here it is the quantity ϕ, which has dimensions of $(\text{mass})^{3/2}$, which survives in the infinite-mass limit. Its square represents the density of quark field at the origin available for annihilation with the heavy quark to form the lepton pair.

Again there is an instant generalization. For the decay of B into a dilepton plus light particles of momenta $k_1, \ldots, k_n$, simply write down the same expression as above, with ϕ depending on $k_1, \ldots, k_n$ as well as the four-velocity of the parent B. As pointed out by Wise[6], this has the interesting consequence of relating B decays into a final state $|n\rangle$ to the D decays into the same final state $|n\rangle$, provided of course that the mass W of that system is small enough to be available to the D as well as B. This should be of great help in normalizing difficult-to-calculate B branching ratios into charmless final states in terms of observed Cabibbo-suppressed semileptonic D decays.

V. Products of Currents and Sum Rules

As one goes beyond consideration of the "elastic" and mildly inelastic matrix elements and considers complex multiparticle final states, it is natural to search for regularities in matrix elements of products of currents, and for sum rules. One does not have to look very far to find them. I have thus far only looked at the $B \to D$ matrix elements; there one defines in an obvious way the structure function $W_{\mu\nu}$ from the individual matrix elements. Then one sums with care over all final states of fixed hadronic mass W, keeping the leptonic configuration fixed, in particular its mass q. When this is done the original product of traces is replaced by a complementary product:*

⋆ Actually all that is needed to reconvolute the traces into this form is to average a given configuration of momenta $k_1, \ldots k_n$ by rigid rotation over the sphere in the rest frame of the D, D^*. Likewise there exists an easy and elegant formalism to describe the available decay correlations which is related to this procedure. The factored structure survives; the heavy quark and spectator traces become only slightly more complicated.

$$W_{\mu\nu} = \sqrt{\frac{M_B}{2E_B}} \,\mathrm{trace} \left[\frac{(\gamma \cdot P_B + M_B)}{2M_B}\right] \overline{J}_\mu \left[\frac{(\gamma \cdot P_B - \gamma \cdot q + M_D)}{2M_D}\right] J_\nu \, w(\epsilon, t) \ . \tag{5.1}$$

The first trace as explicitly given above has just the structure of free b-quark decay. Its multiplier, $w(\epsilon, t)$ is the structure function of the spectator system, which contains the unknown part of the dynamics. Its definition is

$$w(\epsilon, t) = \sum_f \mathrm{trace} \left[\frac{(\gamma \cdot P_B + M_B)}{2M_B}\right] \rho(f) \left[\frac{(-\gamma \cdot P_D + M_D)}{2M_D}\right] \rho(f) \tag{5.2}$$

where the excitation energy ϵ is just the excess mass in the final state beyond that of the heavy quark, and where the sum over f is defined in accordance with the discussion above Eq. (5.1).

$$W^2 = (M_D + \epsilon)^2 = \left(P_D + \sum_i k_i\right)^2 \ . \tag{5.3}$$

In the very-large-mass limit the decay width, differential in ϵ and q^2, is then a product of the expression for the free quark (which eventually goes like M^5), multiplied by the structure function for the spectator. Upon witnessing this factorization, one may also anticipate a sum rule

$$\int_0^\infty d\epsilon \, w(\epsilon, t) = 1 \ . \tag{5.4}$$

This turns out to be a correct inference. A way to get it is to go back to the original Fubini–Furlan current-algebra method of equal time commutation relations of currents taken between states of finite momentum[7]. Some care is required; not only are there the expected contributions, but also "Z-graph" terms coming from coupling of a highly timelike current to $B - \overline{D}$ pairs. For this class of processes there is also a structure function; the bonus for having to consider this complication is that it independently satisfies a sum rule identical to the one above.

In any case the sum rules are there. It is interesting to extract the elastic contribution therefrom; one gets

$$1 = (1-\hat{t})|F(\hat{t})|^2 + \int_0^\infty d\epsilon\, w(\epsilon,\hat{t})_{\text{inelastic}} \;. \tag{5.5}$$

This shows that the continuum contribution vanishes as $\hat{t} \to 0$, as expected. But there is also the analogue of the Cabibbo-Radicati sum rule[8]

$$F'(\hat{t})\Big|_{\hat{t}=0} = \frac{1}{2}\left[1 + \int \frac{d\epsilon}{|\hat{t}|}\, w(\epsilon,\hat{t})_{\text{inel}}\right] \;. \tag{5.6}$$

This shows that the "radius" of the elastic form factor must exceed 1/2. Naive vector–dominance arguments give a radius of unity, while general analyticity arguments say the form factor is analytic in the cut $\hat{t}$ plane, with branch point * at $\hat{t} = 1$.

The amount of excitation energy ϵ to expect can be rather easily estimated. In the rest frame of the final D, the spectator light quark system initially approaches at the velocity of the &meson. But the b-meson disappears and the d-quark stays behind. The spectator then breaks up into light hadrons which have maximum Lorentz gamma on average about the same as the gamma of the original B. From this one sees that the total energy ϵ should not be much different from γ Gev; it scales in proportion to $\hat{t}$, with coefficient of order one Gev. Examination of practical decay processes then shows that the sum rule should always converge within the allowed kinematic region.

The timelike sum rule is interesting, in that the elastic contribution is suppressed by the kinematic factor $(1-\hat{t})$ near threshold. This is the onium region, and I suspect there

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⋆ There emerges a serious problem at this point, which was pointed out to me by Mark Wise. Consider the elastic form factor of the D^*, which may be considered for these purposes as a (very) loosely bound state of a pion and a D. The system clearly has a large radius; formally this is related to the existence of an "anomalous threshold" (a branch cut at $\hat{t} \ll 1.$). This problem occurs because the hyperfine interval is of order the pion mass, insufficiently far away from the infinite–mass limit of zero. While this clearly requires careful attention, the anomalous threshold contribution is unique and well defined. So there is some reason to expect that even though the correction may be significant it still can be accurately taken into account.

is interesting singular behavior in both the elastic form factor and inelastic structure function near $\hat{t} = 1$.

I think that in the long run these sum rules should help greatly in extracting an accurate value of V_{cb} from semileptonic decay data, with the extant M^5 ambiguity resolved (namely, which mass M one should use in the formula for the semileptonic decay width: b–quark or B-meson). But again its resolution will require careful attention to the corrections to the infinite mass limit. It is premature (at least for me!) to anticipate how-this will all turn out.

VI. Is the Strange Quark Heavy Enough?

Now and then one makes believe that the strange quark belongs to the heavy-quark family and halfway gets away with it, e.g. in the treatment of the ϕ as part of the onium family. While this is very dangerous territory, it seems to me that it will be useful to apply the infinite-mass limits-to final states with strangeness. One cannot expect high accuracy, but at least this may be a first step toward organizing the. phenomenology along the lines given above. In particular, one should be able to liberate the sum rules from their infinite-mass-limit origins.

The reason this is important is clear. There is the class of decays $b \rightarrow s + \psi$ vital to CP phenomenology, as well as penguin processes $b \rightarrow s + \gamma/\text{dilepton}$. Furthermore, if the hadronic matrix elements can be tamed, then broken SU(3) may be enough to take one to the charmless, nonstrange final states as well. Not only are these important for the determination of V_{ub}, but they also become important for nonleptonic processes such as $B \rightarrow \pi + \pi$, $\rho + \pi$, etc. These are excellent CP-violation candidates, and I believe that the decay amplitudes are accurately described using the "factorization" hypothesis. Factorization implies that there is negligible final state interaction between the system of hadrons on one side of the W and those on the other. This is reasonable when the system made by the virtual W is of low mass, because its Lorentz gamma is large and at birth it is a small color-dipole, which only grows into a complex, interacting structure after it has left the environment of the spectator system[9].

For the case of $b \rightarrow s$ semileptonic transitions, it is interesting to compare the available ranges of $\hat{t}$ with the safer case of $b \rightarrow c$ transitions. In the former case, the maximum value of $\hat{t}$ is 2.2 and for typical decays it is more like 1.6. For final states containing charm, the maximum value of $\hat{t}$ is only 0.3. In this latter case, one should expect that at least half of the final states should reside in D and D^* alone, but that there is room for some higher excitations. For strange final states the form factors are of course very important, but not overwhelmingly so. Estimation of the magnitude of the excitation energy ϵ needed to saturate the sum rules gives a value of one to two Gev at typical $\hat{t}$, small enough to provide saturation within the allowed kinematic region.

Understanding this class of matrix elements will certainly require interplay between theory and experiment, but it seems to me there is enough in the theory to support a relatively model-independent attack.

VII. Asymptotic Constituent Quarks and Scattering Theory

There is a by-product of this line of study which I find interesting, which has to do with soft strong interactions. Consider the scattering of a B or B^* off of a D or D^* with possible production of light hadrons $k_1, \ldots k_n$, but ignoring perturbative, hard-gluon contributions. Then the final velocities of outgoing heavy mesons will be the same as the initial velocities in the infinite-mass limit. Furthermore the heavy-quark dynamics becomes trivial in the limit. Consequently the interesting physics is reduced to the interaction of constituent (anti-) quarks of specified velocity. These quarks can exchange a significant amount of momentum without altering their velocity. (This is possible because in the infinite-mass limit one has no way of specifying the mass of the constituent quark!) And they can produce mesons as well, of course. The kinematics in this limit is at first a little unusual. For elastic processes the usual variables s and t are traded in for q^2, the momentum transferred from one projectile to the other (it is essentially still t), and $v \cdot v'$. There is no crossing symmetry in the limit. One may write the scattering amplitude using the trace formalism. The interesting part containing the quark-quark dynamics has all the richness found in the general description of proton–proton scattering, with the additional feature of two color channels to consider.

Why is this interesting? Clearly constituent quarks play an important role in the nonperturbative descriptions of hadrons. But their introduction is in general at a highly phenomenological level. Here one has a formal way of defining an asymptotic state of a single constituent quark and a way of discussing its structure which is precisely and operationally defined. For example its charge form factor and magnetic moment are determined by matrix elements of light-quark weak currents between B and/or B^* states with equal initial and final state velocities:

$$\langle B \text{ or } B^* \,|\, J_\mu \,|\, B \text{ or } B^* \rangle = \left(\frac{M_B}{2E_B}\right) \text{ trace } \overline{\mathcal{B}}\mathcal{B}\rho_\mu(v,q) \qquad (v_\mu \cdot q^\mu = 0)\,. \tag{7.1}$$

Here ρ_μ can be written out in terms of the usual kinematic structures (convection current, magnetic, axial vector, induced pseudoscalar,. . .). Likewise one may define the deep-inelastic structure functions of isolated constituent quarks in a similar way. While applications are sparse because of the absence of external beams of heavy-flavor mesons, there are of course familiar applications in the pionic or photonic transitions from B^* to B or D^* to D. The physics is not new; but the conceptual simplification is to me significant.

In particular, there is a topical issue which can be expressed in this language. Consider B or B^* elastic scattering with $\overline{B}$ or $\overline{B}^*$ at low relative velocity. This is really scattering of a constituent quark with a constituent-antiquark at low velocity. There are two color channels to consider, singlet and octet. The octet channel is the one of most contemporary interest; if there is attraction then it might lead to extra resonant structures of a non-onium nature in the upsilon region. If the $\Upsilon(4S)$ were to mix with such a structure, then there might be a chance to interpret the direct decay of the $\Upsilon(4S)$ into energetic ψ's (as reported recently[10] by the Cornell collaboration) as the annihilation of the b and $\bar{b}$ from color octet into two gluons, one of which fragments into a ψ. Such a picture is not far from ideas of Lipkin, Marciano, and Ono[ll].

In any case, it seems to me that if, as a theorist, one chooses to study constituent quarks at a fundamental level, then the infinite-mass limit provides an ideal context for doing so in the simplest possible way.

VIII. Conclusions

While this talk has been long on optimism and short on concrete results, I think that within a year or two it is quite possible that the language used in describing heavy quark decay phenomenology will shift away from comparison of data with Model A or Model T, and instead be phrased in a language which deals with the importance of a correction of Type X or Type Y. If so, it seems to me that in the long run the hopes of reducing the theoretical systematic errors in this field are greatly enhanced.

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