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How to Extract CP Violating Asymmetries from Angular Correlations*

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ABSTRACT

Large CP violation effects can occur for time evolved B^0 decays into definite CP eigenstates. The rates into these unique CP eigenmodes are tiny. This report advocates the use of many additional modes that are not CP eigenstates because of mixtures of angular momenta. Naively, for those modes a partial and sometimes a large cancellation of the CP asymmetry occurs. However, a detailed study of their angular correlations enables the projection onto definite CP eigenstates, and thus recovers the full CP asymmetry.

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INTRODUCTION

The focus of the study of CP violation in neutral B–decays[1] is considerably broadened if we study modes where different partial waves contribute with different CP parities.[2–6] Many such modes exist; for example, those where the B^0 decays to two particles with spin, such as ΨK^{*0} or $D^{*+}D^{*-}$. The asymmetry in the total rate from such a channel suffers from a partial cancellation or dilution of the asymmetry from the two different CP contributions. Hence such modes require an angular analysis of the decays of the spinning particles to separate out definite CP contributions and thus obtain asymmetry measurements that probe the basic Standard Model predictions.[7] Of course, if nature is kind and a single CP channel dominates the decay, then the CP asymmetry may be approximately measured without any angular analysis. However in these cases an angular analysis can be performed without any loss in statistical accuracy and without any error from the small opposite CP contribution to yield a more precise measurement of the CP asymmetry.

The particle content of all the modes discussed here is such that one can construct CP eigenstates from a superposition of helicity states, without invoking a different particle content. Thus, for example, the modes $\psi K_s \pi^0$ and $D^{*+}D^{*-}$ are considered here, but not modes such as $D^{*+}\rho^-$. This report presents several different approaches to the angular analysis. All are based on standard helicity formalism.[8–10] The merits of the various approaches depend on a number of factors, many of which are not yet known, such as the relative strengths of the different helicity amplitudes. By the time sufficient data is accumulated to attempt any of these analyses, a great deal more will be known about these factors. For any given channel the preferred method will be clear. We present here four approaches and

briefly discuss the merits of each.

The first approach analyzes events in terms of a quantity we call transversity, which characterizes the spin projections of a three body intermediate state in a direction transverse to the plane of the three body system.' This approach requires the minimum amount of angular analysis to arrive at definite CP quantities. We show that, in certain cases, moments of the data with respect to a single polar angle can achieve the required separation. This method has the advantage that it allows us to sum resonant and non-resonant contributions to certain final states, whereas the more detailed angular analysis requires reconstruction of a specific two-body parent system for the three-body state. For another simple method applicable for some modes, see Ref. 4.

The second method uses a more complete angular analysis and forms all possible independent angular moments of the data. This allows the study of additional channels not amenable to the transversity treatment. Like the transversity moment analysis it has the advantage that it allows asymmetries to be extracted without ***a priori*** knowledge of the relative strengths of the different helicity contributions. In both cases this can be done by combining results from both B^0 and $\overline{B}^0$ decays.

The remaining two methods use a maximum-likelihood fit to the angular structure of the CP-violating decay and to a set of isospin-related channels that are not influenced by CP violating effects. This can be done either using only the transversity polar angle distributions or using the full angular distributions. For a transversity analysis of this type one needs to know the relative strengths of the contributions for each possible absolute value of the transversity. This can frequently be determined from isospin-related channels.[11] For the full angular analysis one needs to determine the full set of helicity amplitudes and their rel-

ative strong-interaction phases. Again data from isospin-related channels may make this possible. This method will provide the most accurate measure of the asymmetry for those modes where sufficient data is available to determine all the necessary quantities well.

The plan of this report is as follows. In Section 2 we introduce some general notation and review the dilution of asymmetry that occurs when two different $C\bar{P}$ channels contribute to a given final state. Section 3 presents a discussion of transversity analysis, Section 4 reviews the many channels for which it can be used, and Section 5 presents as an example the transversity analysis for the case of two spin one particles. Results from more complete angular analyses are discussed in Section 6. Section 7 reviews the accuracy of the asymmetry measurements obtained by each of the methods and discusses the relative advantage of maximum-likelihood methods compared to moment analyses. Section 8 contains some concluding observations. Appendix A contains a proof that transversity is a projector for definite CP, Appendix B contains the details of the full angular analysis, and Appendix C presents a summary of an analysis of the sensitivity of results to various measurement errors.

PRELIMINARIES—DILUTION OF CP VIOLATION

This section introduces some notation and discusses the dilution in the CP violating asymmetry when the final state is a mixture of different angular momenta which contribute with different parity and hence different CP. One can most readily treat these processes using the helicity formalism, which gives a correct relativistic analysis of the angular momentum in the decay process. This is a well established formalism which provides the basis for analysis of angular structure in the

subsequent decays of the two spinning particles.[2,8,10]

We begin by discussing the results obtained for such processes without any angular analysis. We show that the asymmetries thus measured depend on the ratio of CP–even to CP-odd contributions and are diluted, that is, reduced in magnitude, relative to the asymmetry of a pure CP state. We denote a time-evolved,initially pure B^0 as $B^0_{\rm phys}$. Any rate difference between the process, $B^0_{\rm phys} \to f$, and the CP conjugated process, $\overline{B}^0_{\rm phys} \to \overline{f}$, signals CP violation. The rate difference comes about because the processes have each two interfering contributions to each partial wave or helicity amplitude, see Fig. 1. The CP violating interference term is denoted by Im λ. The rate of a B^0 to f is

$$\Gamma(B^0_{\rm phys} \to f) = \Gamma_+(1+a) + \Gamma_-(1-a)\,, \tag{1}$$

and for the $\overline{B}^0$ to $\overline{f}$ is

$$\Gamma(\overline{B}^0_{\rm phys} \to \overline{f}) = \Gamma_+(1-a) + \Gamma_-(1+a)\,. \tag{2}$$

The CP-even and CP-odd rates are parametrized by the widths Γ_+ and Γ_-, respectively. The parameter a is proportional to Im λ, and would be the asymmetry if the CP-even state dominated. The rates of Eqs. (1)–(2) could be time dependent or time integrated. In the former case[1]

$$a = -\mathrm{Im}\,\lambda \sin(\Delta m\, t)\,, \tag{3}$$

and Γ_+ and Γ_- contain a factor $e^{-\Gamma t}$, where I' is the width of the B^0.[12] In this case the analysis of angular distributions must be made for each time-bin separately,

since the asymmetry is different at different times. Because the angular dependence and the time dependence factorize this introduces no particular complication for the extraction of the CP asymmetry from the angular information; the method is the same for every time-bin data set. For an experiment which measures a time-integrated asymmetry the prediction is

$$a = \frac{-x \mathrm{Im}\,\lambda}{1+x^2}\,, \tag{4}$$

where x $\equiv$ Am/I', and $\Gamma_\pm$ denote time-integrated quantities.

The measured asymmetry is

$$\mathrm{Asym} \equiv \frac{\Gamma(B^0_{\mathrm{phys}} \to f) - \Gamma(\overline{B}^0_{\mathrm{phys}} \to \overline{f})}{\Gamma(B^0_{\mathrm{phys}} \to f) + \Gamma(\overline{B}^0_{\mathrm{phys}} \to \overline{f})} = a\,\frac{\Gamma_+ - \Gamma_-}{\Gamma_+ + \Gamma_-}\,. \tag{5}$$

The last factor gives a dilution when the final state f is an admixture of CP–even and CP-odd parities. Presently no information exists on the ratio Γ_+/Γ_- and large dilutions could occur. Study of angular distributions allows us to avoid such dilutions regardless of the Γ_+/Γ_- ratio.

Transverse Projection as an Analyzer of CP Parities

Consider the decay of a spinless neutral particle B^0 into unstable particles ***A*** and C. (We require ***A*** and C to be unstable so that spin information can be learned from their subsequent decay.) All the subsequent discussion holds equally for decays of any neutral spin zero particle, in particular for B_s and D^0 which we will discuss later. Let the particles A and C have spins s_a, s_c, helicities λ_a, λ_c, and intrinsic (reflection) parities π_a, π_c, respectively. We consider cases where C

is seen in a two body mode $C \to C_1 C_2$, with spins s_1 and s_2.

$$\begin{aligned} B^0 &\to AC \\ &\qquad \hookrightarrow C_1 C_2 \end{aligned} \tag{6}$$

A simple example to keep in mind is the case $\boldsymbol{A} = \psi$, $C = K^{*0}$, $C_1 = K_s$, $C_2 = \pi^0$.

Let us define the transverse axis as the normal to the plane containing the three particles, AC_1C_2, in either the B^0 or C rest frame (or in the $\boldsymbol{A}$ rest frame where the plane is defined by the particles C_1 and C_2). The CP-parity eigenstates of the mode AC_1C_2 can be classified by the spin projection of the particles along this transverse axis, which we call the transversity. The state of transversity τ_i of each particle is defined as that linear combination of helicity states which represents a spin component τ_i along the transverse axis in the rest frame of particle $\boldsymbol{i}$. This definition is invariant with respect to boosts between the C rest frame, in which we analyse C decay, and the particle's own rest frame, which will be used to analyse its decay.

In Appendix A we prove, using the helicity formalism, that projection onto states of definite transversity $\tau = \tau_a + \tau_1 + \tau_2$ projects out quantities of definite CP. The following argument gives a more intuitive understanding of this result. First consider three particles moving in a plane, and let the y-axis be chosen transverse to the plane. A reflection about that plane can be written as a product of a space inversion P and a $180°$ rotation about the transverse axis.

$$R_P \equiv P e^{i\pi J_y} = P_{int} \cdot e^{i\pi\tau} \tag{7}$$

where P_{int} denotes the total intrinsic parity of the three particle system, J_y denotes the projection of the total angular momentum of the three particle state on the

y axis and τ denotes sum of the transversities of the three particles.[13] The three body state can be viewed as a product of three one-body plane wave states, the reflection acts independently on each particle.

The operator R_p has been used extensively in applying the consequences of space-time symmetries to four-point functions; i.e. processes characterized by three independent momenta.[14] In a relativistic group-theoretical description, the operator R_p is seen to be the generator of the "coplanar little group"; i.e. the subgroup of the inhomogeneous Lorentz group which leaves three momenta invariant.[15]

To evaluate the action of the reflection operation on any one particle we can use the fact that reflection commutes with boosts in the plane. We thus go to the rest frame of the particle under consideration. In that frame one readily sees from the definition of the reflection operator- that its eigenvalue for the particle j is the product of intrinsic parity times $e^{i\pi\tau_j}$, where τ_j is defined as the spin projection along the transverse axis. Equation (7) simply combines the result for each of the three particles to give the eigenvalue for a three particle state of definite transversity.

In decays like $B_d \to J/\psi K_S \pi^0$ and $B_d \to \eta_c K_S \pi^0$, each of the three particles in the final state has a definite intrinsic CP. For such cases one can define the operation of the product of charge conjugation and the reflection in the plane.

$$R_{CP} \equiv CR_P \equiv CPe^{i\pi J_y} = \boldsymbol{CP} = (CP)_{int} e^{i\pi\tau} \tag{8}$$

where $(CP)_{int}$ denotes the product of the intrinsic CP of the three particles. The first equality of Eq. (8) is true since the initial state has spin zero, hence the final

state must also have spin zero and be invariant under rotations in the center of mass system, and the second equality follows from Eq. (7).

Thus, for example, any J = 0 state of the type $|(c\bar{c})K_S\pi^o; J = 0)$ in which the (CC) has a definite intrinsic ***CP*** and is in an eigenstate of transversity τ can be shown to be a CP eigenstate with CP-parity given by the relation

$$CP\,|(c\bar{c})K_S\pi^o;\ J = 0) = R_{CP}\,|(c\bar{c})K_S\pi^o;\ J = 0)$$

$$= (CP)_{int}(-1)^{\tau}\,|(c\bar{c})K_S\pi^o;\ J = 0) \tag{9}$$

The relation (8) applies to any three-body system with a well defined intrinsic CP for each particle. It also applies if particle A does not have definite intrinsic CP but decays to a state of definite CP, for example $D^0 \to \pi^+\pi^-$. In this case we define the intrinsic CP of particle A to be the CP of its decay channel. This allows a considerable extension of the class of channels that can be used for CP analysis. Modes such as $\eta_c K_S\pi^o$, with three spinless particles have τ = 0 and the CP is the intrinsic CP of the three particles. For the final state $J/\psi K_S\pi^o$, for which the intrinsic CP is odd, the total CP is odd if τ is zero and even if τ is fl. Note also that similar results apply also to all radial excitations of the charmonium states.

For each of the three particles the polar angular distribution of its decay with respect to the transverse axis can be used to separate contributions for each $|\tau_i|$, integrated over all other decay angles. From each set of $|\tau_i|$ one can then extract a measurement of the undiluted asymmetry. These measurements can be combined to give an improved value but their errors are highly correlated and must be treated correctly, as is discussed in Section 7.

When particles C_1 and C_2 are spinless, two further classes of decays can be analysed using transversity. Table 1 summarizes the situation, similar results for the full angular analysis have been tabulated by Dell'Aquila and Nelson.[2] The first column of Table 1 defines the classes of decays of a spinless neutral particle that can be analysed for CP asymmetries using transversity projections. For each class, Table 1 defines the quantity ξ such that the CP is given by

$$\mathrm{CP} = \xi(-1)^{\tau} \tag{10}$$

Examples for each class of decay defined in Table 1 are shown in Tables 2-4. Whenever decays of the spinning particles allow projection of the magnitude of the transverse spin, the data can be separated into definite CP classes. The errors on the various transversity projections are correlated, so care must be taken when combining results.

For class 1 the CP does not depend on the spin of particle C. Thus it is not necessary to determine that C_1C_2 arise from the decay of a well-defined particle C. Hence, in this class of decays, the resonant and non-resonant production of C_1C_2 can be combined in the data sample, since all events of a given τ contribute with the same CP. This may allow the transversity analysis for such a channel in cases where the full angular analysis cannot be reliably used because of wrong spin backgrounds. This will probably be the most useful application of transversity analysis. In class 1 the particles C_1 and C_2 may have any integer spin as long as their subsequent decay allows reconstruction of their transversity.

In class 2, C_1C_2 must have a well-measured total spin (modulo 2), but not necessarily a unique parent particle C. In this situation the helicities of particles C_1 and C_2 are interchanged as well as sign-reversed under CP. Hence we must require

that particles C_1 and C_2 have spin zero in order to form definite-CP quantities using transversity projections.

For class 3, particle C must be identified as the antiparticle of A, and again the transversity analysis can only be applied when both C_1 and C_2 have spin zero.

One last comment on Table 1. Whereas X $\leftrightarrow \overline{X}$ demands that X is seen in a CP eigenmode, Y = $\overline{X}$ puts no constraints on the decay products of either X or $\overline{Y}$. For example, class (3) allows any decay mode for particle A provided it allows transversity projections to be made, and requires only that C decay to two spinless particles.

Some Modes which can be Analysed Using Transversity

Equipped with Table 1 and its interpretation, we can increase the number of modes that can be used for CP violation studies, without dependence on any specific model. The particle content of all the modes discussed here is such that one can construct CP eigenstates from a superposition of helicity states, without invoking a different particle content. Thus, for example, the modes $\psi K_s \pi^0$ and $D^{*+} D^{*-}$ are considered here, but not modes such as $D^{*+}\rho^-$. The pure CP eigenmodes of $\overline{B}_d$, such as ψK_s, D^+D^-, $D^0\rho^0$, and $D^0\pi^0$,[16] can now be augmented by the many modes given in Table 2. This Table is not exhaustive, the reader will see obvious extensions of the list presented here.

In the Standard Model with three generations of quarks, we can study the three angles of the unitarity triangle, see Fig. 2. Modes of $\overline{B}_d$ driven by the quark subprocesses

$$b \to s + \overline{q}q \qquad c + \overline{c}s \qquad c + \overline{c}d \qquad c + \overline{u}d \tag{11}$$

are all governed by $\sin(2\beta)$. The $b \to s$ transition via a penguin is denoted by

$b \to s + \bar{q}q$. The interference term is $\sin(2\alpha)$ for processes governed by the $b \to$ u + $\bar{u}d$ quark-subprocess. Again several modes can be analysed. However, for this quark subprocess, because only light quarks occur in the final state, there may be competing contributions from penguin amplitudes which have similar CKM strength but different CKM phases.[17] These must be considered in assessing the Standard Model prediction.

Some further comments on the processes listed in Table 2 follow:

For the class 1 processes, it is irrelevant whether the $K_S\pi^0$ arises from K^{*0} or non-resonant production, as discussed above. In fact, for any three body mode of class 1, AC_1C_2, there is no need to find a pseudo-two-body mode AC. The C_1C_2 could come from non-resonant as well as resonant production.

For class 2 the D^{*0} of the mode $D^{*0}\rho^0$ must be seen in a CP eigenmode. Either of two decay chains qualify:

$$D^{*0} \to \gamma\, f_D \qquad D^{*0} \to \pi^0 f_D$$

where f_D denotes any CP eigenstate produced from D^0 decay. Both processes occur through L = 1, because of parity conservation. Note however that it is important in such cases to be able to distinguish between the photon and the π^0 as these have opposite intrinsic CP, and hence give opposite CP contributions for the same transversity.[6]

In the class 3 processes $D^{*+}D^{*-}, \overline{D}^{*0}D^{*0}$ the D^*'s can be studied in all decay modes. We do not require the neutral D^0, which could arise in the decays $D^* \to \pi D^0$ or $D^* \to \gamma D^0$, to decay to a CP eigenmode. However we do need at least one of the D^*'s to decay to two spin zero particles (usually πD).

The final column of Table 2 lists a few of the many additional modes that can be analysed using full angular analysis, which we discuss in Section 6. The modes listed here are not accessible via transversity analysis alone. In contrast, any quasi–two body mode that can be analysed using transversity can also be treated by the more complete angular analysis which we will discuss later.

Table 3 presents a similar list for the decays of the $\overline{B}_s$. For all modes of B_s of the type studied here that are driven by the quark subprocesses of Eq. (11) the CP asymmetries are predicted to be tiny in the Standard Model. In contrast modes mediated by the $b \to u\bar{u}d$ subprocess have a CP asymmetry proportional to $\sin(2\gamma)$ which could be large. Any modes of the type $X^0Y^0K_s$ or $X^0Y^0(K_s\pi^0)_{K^*}$ belong to class 1 of Table 1 and can be used to study CP violating asymmetries. Here X^0Y^0 is any pair of light neutral mesons of zero total strangeness which decay in such a way that transversity can be reconstructed. The transversity analysis thus also can considerably enrich the possibilities for a measurement of $\sin(2\gamma)$. However, here again the contributions of penguin amplitudes may complicate the theoretical predictions.

Consider now $\overline{B}^0$ decays which are driven by $b \to c\bar{u}d$ and produce a neutral D. Such modes can be used for CP violation studies when this neutral D decays into a CP eigenstate.[1,6] It is therefore advantageous to increase the data sample for D^0 decays into CP eigenstates. Hence in Table 4 we list modes that can be analysed by applying the same type of transversity analysis to the D^0 decay itself. This may in turn allow significant increase in the analyzable data sample of B decays. The Mark III collaboration has already determined that the $D^0 \to \rho^0 K^{*0}$ is dominated by the s- and d-waves. That means that this mode is dominated by a single CP when the K^* decays to $K_s\pi^0$, and hence this mode can be readily

treated with this analysis without significant loss of statistical accuracy compared to a pure CP channel.

EXTRACTION OF DEFINITE CP QUANTITIES FROM TRANSVERSITY

We now turn to the transversity analysis which we present for the case of spin one for particle A. For higher spins the method is similar; the separation of each $|\tau|$ can always be made from the polar angle distribution about the transversity axis. If particles C_1 or C_2 have spin a similar analysis is needed also for their decays.

To analyse the decay of ***A*** we go to the ***A*** rest frame. In Table 5 we present the results. The first column defines two readily analysed groups of possible decays. Group (a) includes all decays of a spin 1 particle to two spinless particles and also decays of a vector particle to three pseudoscalar ones. Group (b) includes the decay of an axial-vector particle to three pseudoscalars, the decay of any spin 1 particle to a photon plus a spinless particle, and the decay of a spin one particle to a pair of negligible mass spin $1/2$ particles via a vector or axial-vector coupling. The second column presents examples for decays of particle A. (We implicitly assume that this decay proceeds through parity conserving interactions.) Columns 3 and 4 present the angular distributions for each $|\tau|$, $r_\tau(\theta)$, normalized so that

$$\int_{-1}^{1} d\cos\theta\, r_\tau(\theta) = 1 \ . \tag{12}$$

Here θ is an angle that describes the angular distribution of the decaying particle ***A,*** in the rest frame of ***A,*** relative to the transverse axis. When ***A*** decays into two particles the angle θ is the polar angle for one of the particles. When ***A*** decays to three spinless particles the angle θ is the polar angle of the normal to the plane

containing the three decay products.' In all cases all other decay angles have been integrated out.

Using the angular distributions $r_\tau(\theta)$ one can then define the quantities

$$r_\pm(\theta) = r_0(\theta)(1 \pm \xi)/2 + r_1(\theta)(1 \mp \xi)/2 \tag{13}$$

where ξ is given in Table 1. The rate for a $B^0_{\rm phys}$ to f_θ can be written as

$$\Gamma(\theta) \equiv \Gamma(B^0_{\rm phys} \to f_\theta) = \Gamma_+(1 + a)r_+(\theta) + \Gamma_-(1 - a)r_-(\theta) \ , \tag{14}$$

where a and $\Gamma_\pm$ may be time-dependent or time-integrated quantities (see Eqs. (3)–(4)). Letus now define the weighted integrals,

$$M_0 \equiv \int_{-1}^{1} d\cos\theta\, \Gamma(\theta) = \Gamma_+(1 + a) + \Gamma_-(1 - a) \ , \tag{15}$$

and

$$M_2 \equiv \int_{-1}^{1} d\cos\theta\, P_2(\cos\theta)\, \Gamma(\theta) = \Gamma_+(1 + a)\omega_+ + \Gamma_-(1 - a)\omega_- \ . \tag{16}$$

where $\omega_\pm$ are defined by

$$\omega_\pm \equiv \int_{-1}^{1} d\cos\theta\, P_2(\cos\theta)\, r_\pm(\theta) \ . \tag{17}$$

Similarly the rate for a $\overline{B}^0_{\rm phys}$ to $\overline{f}_{\overline{\theta}}$ is

$$\overline{\Gamma}(\overline{\theta}) \equiv \Gamma(\overline{B}^0_{\rm phys} \to \overline{f}_{\overline{\theta}}) = \Gamma_+(1 - a)r_+(\theta) + \Gamma_-(1 + a)r_-(\theta) \ , \tag{18}$$

The state $|\overline{f}_{\overline{\theta}}\rangle$ means the state $CP\,|f_\theta\rangle$. Hence in Eq. (18) the quantity $\overline{\theta}$ is sometimes $\pi - \theta$ and sometimes θ depending on the particle content of the state.

Since the angular dependence is such that $r_\tau(\pi - \theta) = r_\tau(\theta)$ this introduces no complication in the analysis. Thus we can extract

$$\overline{M}_0 \equiv \int_{-1}^{1} d\cos\theta\, \overline{\Gamma}(\theta) = \Gamma_+(1-a) + \Gamma_-(1+a)\ , \tag{19}$$

and

$$\overline{M}_2 \equiv \int_{-1}^{1} d\cos\theta\, P_2(\cos\theta)\, \overline{\Gamma}(\theta) = \Gamma_+(1-a)\omega_+ + \Gamma_-(1+a)\omega_-\ . \tag{20}$$

The moments $M_0, \overline{M}_0, M_2, \overline{M}_2$ derived from both the B^0 and $\overline{B}^0$ data samples, can be combined in many different ways to construct ratios which each give an undiluted measurement of the CP-violating asymmetry a. First construct the combinations

$$W_\pm = \Gamma_\pm(1 \pm a) = \mp\frac{w_\mp M_0 - M_2}{\omega_+ - \omega_-} \tag{21.}$$

from the B data and the similar quantities $\overline{W}_\pm$ obtained from the $\overline{B}$ data. These then allow two determinations of the asymmetry a,

$$a_\pm = \frac{\pm(W_\pm - \overline{W}_\pm)}{W_\pm + \overline{W}_\pm}\ . \tag{22}$$

Note that neither of these results requires prior knowledge of the ratio of Γ_+ to Γ_-. Furthermore each measures the intrinsic CP-asymmetry of the underlying quark process without dilution. To obtain the most accurate value of asymmetry from this analysis one takes a linear combination

$$a = \alpha a_+ + (1-\alpha)a_- \tag{23}$$

with α chosen to minimize the error on the result. The optimal choice of α does

depend on the actual values of the Γ's. We will return to a discussion of the best choice of α in Section 7.

With a limited amount of data one could alternatively begin by dividing the angular distribution into two angular bins, commonly called polar and equatorial, with a cut at some appropriate angle. Let us cut at $\theta = \pi/3$ where $\cos\theta = 1/2$ and define the equatorial and polar components E and P by the relations

$$E = 2\int_0^{1/2} d\cos\theta\,\Gamma(\theta) = \Gamma_+(1+a)e_+ + \Gamma_-(1-a)e_- \tag{24}$$

$$P = 2\int_{1/2}^{1} d\cos\theta\,\Gamma(\theta) = \Gamma_+(1 \mp a)p_+ \mp \Gamma_-(1-a)p_- \ . \tag{25}$$

where the numbers $e_\pm$ and $p_\pm$ are defined by

$$e_\pm \equiv 2\int_0^{1/2} d\cos\theta\, r_\pm(\theta) \tag{26}$$

$$p_\pm \equiv 2\int_{1/2}^{1} d\cos\theta\, r_\pm(\theta)\ . \tag{27}$$

The quantities $W_\pm$ and $\overline{W}_\pm$ can now be extracted using E and P and the corresponding quantities $\overline{E}$ and $\overline{P}$ constructed from the $\overline{B}^0$ data sample, just as was done above using M_0, M_2, $\overline{M}_0$ and $\overline{M}_2$. The asymmetries $a_\pm$ can then be determined as before. The only differences between the two procedures will be in the errors on the estimates of a, which will be reduced by the moment treatment. However the simpler binning procedure could be used for a preliminary look. If a

CP violation effect is present, it should show up as a statistically significant non-vanishing value for a even in this simpler analysis. Once such an asymmetry is found, the treatment of the data can be refined.

In either of these analyses the result for the asymmetry a does not depend on Γ_+ or Γ_-, or, in principle, on the choice of α. However the error on a can be minimized by choosing α in a way that depends on Γ_+ and Γ_-. The values of Γ_+ and Γ_- can be determined from examination of channels related to the channel under study by isospin symmetry. In many cases these channels will provide much more data than the channel used for the CP analysis and so the errors on Γ_+/Γ_- will have little effect on the error on the asymmetry.

Full Angular Analysis

For classes (2) and (3) a full angular analysis will usually prove superior to,. the simple moment treatments described above, since the error on the asymmetry measurement from a given set of data can be reduced by more fully exploiting the known angular structure to extract several measurements of the asymmetry a with different correlations among their errors. Such analysis also allows study of modes for which the transversity treatment is not applicable, for example modes where neither particle A nor C decay to two spin zero particles.

Appendix B presents the general helicity formalism and develops a method based on using the Y_{lm} functions to perform angular projections. The treatment of the case of $AC = \Psi K^{*0}$ is given as an example. The results for $D^*\overline{D}^*$ are also tabulated. The angular analysis of the decay of each particle is most simply carried out in the rest frame of that particle. One needs to specify a choice of coordinates for each decay of a spinning particle, once this is done the angular projections can

be used to separate out the combinations of helicity amplitudes that have a definite CP and hence to measure asymmetries.[2] As in the case of the transversity analysis this can be achieved by combining B^0 and $\overline{B}^0$ data, without any ***a priori*** knowledge of the various amplitudes involved. One can obtain an asymmetry measurement for each possible combination of helicities. These results can then be combined for an improved measurement as discussed in Section 7.

This analysis can be applied for any of the modes discussed previously, provided the system $(C_1\, C_2)$ has a well defined total angular momentum. In addition, modes where the transversity analysis cannot readily be used can also be analysed; some such modes are listed in Tables 2-4. For example consider the case

$$B^0 \to \Delta\overline{\Delta}$$

where each A subsequently decays to a proton and a pion. Although the proton spin cannot be measured it is still possible to use the angular projections of these decays to extract quantities of definite CP. This analysis is also presented in Appendix B. We find for this case that two definite CP quantities $Re\mathcal{G}_{3/2+}\mathcal{G}^*_{1/2+}$ and $Re\mathcal{G}_{3/2-}\mathcal{G}^*_{1/2-}$ where $\mathcal{G}_{\lambda\pm} = \frac{A_\lambda \pm A_{-\lambda}}{\sqrt{2}}$ can be isolated using angular projections. Each of these provides a possible measurement of the intrinsic CP asymmetry. This result applies for any pair of spin $3/2$ resonances, both of which decay to $p\pi$ (or $\overline{p}\pi$). For the case of two spin $1/2$ resonances which both decay to pa (or $\overline{p}\pi$) the averaging over the proton spins removes all possibility of separating the different CP contributions by angular analysis as only a uniform angular distribution survives.

MINIMIZING THE ERROR ON THE MEASURED ASYMMETRY

The methods described above each give more than one measurement of the asymmetry. With the transversity analysis we had $a_{\pm}$ or in the more general case one measurement for each set of $|\tau_i|$. Consider for instance for integer spin particles A and C the full angular analysis gives effectively $(s + 1)^2$ measurements, one from the square of each of the 2s + 1 definite CP combinations of helicity amplitudes and one from each interference term between any two such amplitudes with the same CP. Here $s = \min(s_a, s_c)$. Interference terms between opposite CP contributions depend only quadratically on the asymmetry and hence do not provide as sensitive a measurement. Furthermore, as can be seen from the example of

$$\mathrm{Im}\,\mathcal{G}_{1+}\mathcal{G}^*_{1-} = e^{-\Gamma t}\Big[\mathrm{Re}(G_{1+}G^*_{1-})\eta\ \mathrm{Re}\lambda_{KM}\ \sin\ \mathrm{Amt} \\ \bullet t\ \mathrm{Im}(G_{1+}G^*_{1-})\cos\ \mathrm{Amt}\Big] \tag{28}$$

derived in Appendix B the separation of the weak phase dependence from the strong phases is not as clean in this case. For the pure CP terms such as

$$\mathcal{G}_{1+}\mathcal{G}^*_{1+} = |G_{1+}|^2[1 - \eta\ \mathrm{Im}(\lambda_{KM})\ \sin(\mathrm{Amt})]\ e^{-\Gamma t}$$

the time and asymmetry dependence is much simpler. From such a term one can readily form the combination

$$a_{++} = \frac{(\mathcal{G}_{1+}\mathcal{G}^*_{1+} - \overline{\mathcal{G}}_{1+}\overline{\mathcal{G}}^*_{1+})}{(\mathcal{G}_{1+}\mathcal{G}^*_{1+}\ \mathrm{t}\ \overline{\mathcal{G}}_{1+}\overline{\mathcal{G}}^*_{1+})} = -\eta\mathrm{Im}(\lambda_{KM})\ \sin(\mathrm{Amt}) \tag{29}$$

In either analysis the errors on the various measurements of the asymmetry are correlated, and these correlations must be treated properly in determining the error on any value of the asymmetry extracted by combining them. All this is standard statistical analysis, we review it briefly here for completeness.

Consider first the case where we have only the two measurements $a_\pm$ extracted from the single moment transversity analysis. We choose[18]

$$\mathbf{a} = \mathbf{au+} + (\mathbf{1} - \alpha)a_- \,. \tag{30}$$

Minimizing the χ^2 with respect to α gives, for small asymmetry,

$$\alpha = \frac{\sigma_+^2 \text{ t } \rho\sigma_+\sigma_-}{\sigma_-^2 \text{ t } \sigma_+^2 \text{ t } \rho\sigma_+\sigma_-} \tag{31}$$

where $\sigma_\pm$ are the standard errors on $W_\pm$ and ρ measures the correlation of these errors. The result of this treatment is shown as the solid curve in Fig. 3 as a function of

$$\epsilon = \frac{\Gamma_+ - \Gamma_-}{\Gamma_+ + \Gamma_-} \tag{32}$$

for the case of two spin one particles one of which decays to two spin zero particles while the other decays to an e^+e^- pair e.g.ΨK^{*0}. We plot the ratio of the expected error from this analysis to that obtainable with an equal number of decays to a pure CP state.[19] For comparison we also show the errors obtained for a fixed value $\alpha = \frac{1}{2}$, this gives the upper curve on Fig. 3. One sees that, in the worst case, where Γ_+ and Γ_- are equal, this analysis requires approximately nine times more data than a pure CP channel to achieve equal accuracy for the asymmetry measurement.

This situation can be improved by making a maximum likelihood fit for the asymmetry using expression for the B-angular distribution given by Eq. (14). This analysis requires further parameters, namely the quantities $\Gamma_\pm$ which we assume can be extracted from isospin-related channels. The result of this treatment for the error on the asymmetry is shown as the dashed curve in Fig. 3.

Another way to improve the result is by making a more complete angular analysis. Again we have two options, an analysis based on moments that does not require knowledge of the relative strengths and phases of the various amplitudes, and a maximum likelihood fit to the full set of parameters. Where sufficient information is available, the latter method is superior. Figure 4 shows the result for the errors on the asymmetry from using a maximum likelihood fit to the angular dependence of the data where it is assumed that the quantities $G_{1\pm}$ and G_0 defined in Appendix B, are all known from measurements on isospin related channels. For simplicity we assumed a << 1 in making this error analysis. For comparison we plot the result against the same combination of variables as were available in the transversity analysis. The various cases shown are chosen to indicate the range of possibilities. It can be seen that even in the worst case that we studied this type of analysis provides a more accurate value for the asymmetry than the best transversity treatment. Figure 4 also shows that in the fortunate situation where a single CP contribution dominates either treatment gives accuracy comparable to that obtained for a pure CP channel.

We have also carried out a study of the sensitivity of the asymmetry measurement in a maximum-likelihood fit procedure to errors in the estimated values and phases of the various amplitudes. This analysis is summarized in Appendix C. The results are encouraging, the asymmetry measurement errors will most likely be dominated by the statistics of the channel for which the asymmetry measurement is made and is relatively insensitive to small errors in the amplitude values or phases. However, if these quantities are poorly determined, one can fall back to the moment analyses to extract asymmetry measurements that do not depend on them.

To summarize the situation we remark again that the value of the transversity analysis is greatest in the channels described by class 1 of Table 1, namely three CP-self-conjugate particles, where it allows combination of resonant and non-resonant production of the particles C_1 and C_2. It also has the feature of being particularly insensitive to the non-CP-violating asymmetries of the amplitudes that is to asymmetries between B and $\overline{B}$ data that arise from interference between the CP-odd and CP–even contributions. However whenever there is not a single dominant CP contribution the most accurate results for asymmetries will come from the use of a maximum-likelihood fit to the parameters that define the angular distributions, rather than any of the analyses which depend on projecting out specific moments to identify definite CP contributions. Whenever possible, such a treatment will include isospin-related channels in the fitting procedure. Since the isospin-related channels typically have higher rates than the CP eigenstate channels the additional parameters needed for this type of analysis will be well measured for many modes by the time one has sufficient data to measure the asymmetry, so this method will be the one used for most channels.

Summary and Conclusions

We have shown that there are many channels for which one can study CP violations in B^0 decays if one uses angular analysis to separate the different CP contributions which arise from different helicity terms. Some of these modes will compete in accuracy with the modes with unique CP which have already been much discussed. In general, to carry out the angular analysis accurately somewhat more data is needed for these modes than for the unique CP modes; in the worst case that we have analysed this requires approximately ten times as much data for

an equally accurate measurement of asymmetry, the degradation will possibly be even greater for higher spin channels.

We have presented several different approaches to the angular analysis, each of which has merit in different situations. To summarize:

Transversity analysis is most useful in the case of decays to three self-conjugate particles, class 1 of Table 1, where it allows the combination of resonant and non-resonant production of the particles C_1 and C_2. If the relative strengths of the two CP contributions $(\Gamma_\pm)$ are not known, then a moment analysis of the type described in Section 5 should be used. Whenever the values of $\Gamma_\pm$ can be determined using data from isospin related channels then a maximum-likelihood fit to the transversity polar angle distributions will provide a more accurate result. For all other modes, including those listed as class 2 and 3 in Table 1 which could be analysed using the transversity method, the full angular distribution analysis will prove superior. Again there are two choices, a moment analysis of the type described in Appendix B or a maximum-likelihood fit to the full angular distributions. Wherever sufficient information on the various helicity amplitudes can be extracted from data on isospin-related channels the latter method will again give more accurate results. Clearly what this means is that in such cases one should make a global fit of all parameters, the helicity amplitudes and the asymmetry, to the data from all related channels, to obtain the most accurate asymmetry measurement.

This discussion makes no distinction between a time-integrated experiment or one that measures time dependencies of the ***B-*** and B-decays. In the latter case the angular structure and the time dependence factorize in a simple way, as demonstrated in Appendix B. In a time-dependent experiment one simply performs the

angular analysis for each time bin separately, or in a maximum-likelihood fitting procedure one includes the predicted time dependence in the fitting prescription, and treats the data as a function of time as well as angles.

However the angular analysis so enriches the sample of modes to study that we expect it will play an important role in the extraction of the CP violating physics at a B factory. Among the many channels listed in Tables 2 and 3 there well may be-some that provide as accurate or more accurate asymmetry measurements as the unique-CP modes. Since we do not yet have much information on branching fractions to the various modes it is too early to be certain which of the many modes will provide the best measurements. Hence we have simply presented summary tables of some of the modes which, according to the Standard Model, will measure the various angles of the Unitarity Triangle. We have not found any one mode for which the currently measured branching fractions suggest it would be markedly superior to the unique-CP modes, but several may be comparable, especially in the fortunate circumstance that a single CP channel dominates the process. Our knowledge of these branching fractions will certainly be much better by the time any B-factory capable of measuring CP-asymmetries is built, so at that time it will be obvious which of these modes is most readily used, and which method of analysis to apply to it.

APPENDIX A

HELICITY AND TRANSVERSITY

This Appendix gives the general proof that transversity projections can be used to select definite CP quantities. When a spin zero particle B, at rest, decays into two particles (A and C), they must have equal helicities (X). Now we consider the case where the particle C decays to two integer spin particles C_1 and C_2, which have spins s_1 and s_2 and helicities λ_1 and λ_2. In the rest frame of particle C we can write the state formed by the decay of B as

$$|f(\theta)\rangle = \sum_{\lambda,\lambda_1,\lambda_2} \sqrt{\frac{2s_c+1}{4\pi}} A_{\lambda,\lambda_1,\lambda_2} D^{s_c^*}_{\lambda,\lambda_1-\lambda_2}(R_C)\,|\lambda,\lambda_1,\lambda_2;\theta\rangle\,, \tag{A.1}$$

where we define

$$|\lambda,\lambda_1,\lambda_2;\theta\rangle = |A(\lambda;0,0), C_1(\lambda_1;\theta,0)C_2(\lambda_2;\pi-\theta,\pi)\rangle: \tag{A.2}$$

We use Jackson conventions to define our angles and axes.* In addition we have chosen to define the C-decay angles so that $\phi_{C_1} = 0$, thus $R_C = (0,\theta,0)$. The choice $\phi_{C_1} = 0$ is made event by event without any loss of generality. It is a convenient choice for the transversity discussion since it identifies the y-axis of these coordinates with the direction transverse to the plane. In Appendix B we will use a different convention for ϕ in the full angular analysis. Of course these choices are merely a matter of convenience for each analysis and have no physical content.

It is important to note that for three self-conjugate particles we can here avoid the assumption of a specific particle C and simply sum over all possible angular momenta for the system C_1C_2 in its rest frame, in which case λ is the projection of

this angular momentum along the direction opposite to particle ***A.*** Then Eq.(A.1) generalizes to

$$|f(\theta)\rangle = \sum_{\lambda,\lambda_1,\lambda_2} \sum_J \sqrt{\frac{2J+1}{4\pi}} A^J_{\lambda,\lambda_1,\lambda_2} D^{J*}_{\lambda,\lambda_1-\lambda_2}(R_C)\,|\lambda,\lambda_1,\lambda_2;\theta\rangle\,. \tag{A.3}$$

We now wish to use the decays of A,C_1 and C_2 to analyse their transversity. To do this for each particle we go to its rest frame. The transversity for each particle is defined as the spin component along the y-axis in the particle's rest frame. With the choice ϕ_{C_1} = 0 all three y-axes are parallel. However we must choose the same direction for the definition of transversity for all three particles, so that we can define the total transversity as the sum of the three projections. We will fix this as the direction of the positive y-axis for the decay of particle C. Then we can relate transversity states to helicity states for a particle of integer spin s by

$$\begin{aligned} |s,\tau\rangle_{y_c} &= \sum_\lambda D^s_{\lambda,\tau}(\kappa\pi/2,\pi/2,0)\,|s,\lambda\rangle_z \\ &= \sum_{\lambda\geq 0}[1-\tfrac{1}{2}\,\delta_{\lambda,0}]D^s_{\lambda,\tau}(\kappa\pi/2,\pi/2,0) \\ &\quad\times\{|s,\lambda\rangle_z + (-1)^{s-\lambda-\tau}\,|s,-\lambda\rangle_z\}\,. \end{aligned} \tag{A.4}$$

The rotations are defined with respect to the axes just described, and κ = **1** for particle C_1 and κ = -1 for particles ***A*** and C_2 in order to achieve the required matching of positive transversity direction. The phase factor on the negative helicity term arises from redefining the D-function for $-\lambda$ in terms of that for λ.

Now let us first consider decays in which the three particles ***A,*** C_1 and C_2 all are neutral bosons self-conjugate under CP (class 1 in Table 1). Then

$$CP\,|\lambda,\lambda_1,\lambda_2;\theta\rangle = \xi(-1)^{s_a+s_1+s_2-\lambda-\lambda_1-\lambda_2}\,|-\lambda,-\lambda_1,-\lambda_2;\theta\rangle\,, \tag{A.5}$$

where ξ is the product of the intrinsic CP-parities of the three particles. Notice

that the angle θ is unchanged under CP, since it is defined to be the angle between particle C_1 and the direction opposite particle $\boldsymbol{A}$ in the C rest frame, and hence is unaltered by the reversal of all momenta. Now we use the property of the D-function

$$D^s_{\lambda,\mu}(0,\theta,0) = d^s_{\lambda,\mu}(\theta) = (-1)^{\lambda-\mu} d^s_{-\lambda,-\mu}(\theta) \tag{A.6}$$

torewrite Eq. (A.3) as

$$\begin{aligned} |f(\theta)\rangle = \frac{1}{\sqrt{2}} \sum_{\lambda,\lambda_1,\lambda_2 \geq 0} & [1-\tfrac{1}{2}\,\delta_{\lambda,0}][1-\tfrac{1}{2}\,\delta_{\lambda_1,0}][1-3\,\delta_{\lambda_2,0}] \\ \Big\{ & \left|\mathcal{S}^+_{\lambda,\lambda_1,\lambda_2}\right\rangle + \left|\mathcal{S}^+_{\lambda,\lambda_1,-\lambda_2}\right\rangle + \left|\mathcal{S}^+_{\lambda,-\lambda_1,\lambda_2}\right\rangle + \left|\mathcal{S}^+_{\lambda,-\lambda_1,-\lambda_2}\right\rangle \\ & + \left|\mathcal{S}^-_{\lambda,\lambda_1,\lambda_2}\right\rangle + \left|\mathcal{S}^-_{\lambda,\lambda_1,-\lambda_2}\right\rangle + \left|\mathcal{S}^-_{\lambda,-\lambda_1,\lambda_2}\right\rangle + \left|\mathcal{S}^-_{\lambda,-\lambda_1,-\lambda_2}\right\rangle \Big\}, \end{aligned} \tag{A.7}$$

where

$$\begin{aligned} \left|\mathcal{S}^\pm_{\lambda,\lambda_1,\lambda_2}\right\rangle = \sum_J \sqrt{\frac{2J+1}{4\pi}}\, d^J_{\lambda,\lambda_1-\lambda_2}(\theta) \times \\ \mathcal{G}^{J\pm}_{\lambda,\lambda_1,\lambda_2}\{|\lambda,\lambda_1,\lambda_2;\theta\rangle \pm (-1)^{\lambda+\lambda_1+\lambda_2}|-\lambda,-\lambda_1,-\lambda_2;\theta\rangle\} \end{aligned} \tag{A.8}$$

and we have introduced the amplitudes

$$9^{J\pm}_{\lambda,\lambda_1,\lambda_2} = \frac{A^J_{\lambda,\lambda_1,\lambda_2} \pm A^J_{-\lambda,-\lambda_1,-\lambda_2}}{\sqrt{2}} \tag{A.9}$$

which correspond to definite CP contributions. Under CP

$$CP\left|\mathcal{S}^\pm_{\lambda,\lambda_1,\lambda_2}\right\rangle = \pm\xi(-1)^{s_a+s_1+s_2}\left|\mathcal{S}^\pm_{\lambda,\lambda_1,\lambda_2}\right\rangle, \tag{A.lO}$$

where we have used Eq. (A.5).

Now let us project out the contribution obtained by requiring the transversities τ_a, τ_1, τ_2 for the particles A, C_1, C_2. We can write the result in the form

$$A(\tau_a, \tau_1, \tau_2) = \langle \tau_a, \tau_1, \tau_2; \theta | f(\theta) \rangle$$
$$= (1 + (-1)^{S+\tau})\mathcal{G}^+ + (1 - (-1)^{S+\tau})\mathcal{G}^-, \tag{A.11}$$

where $S = s_a + s_1 + s_2$, $\tau = \tau_a + \tau_1 + \tau_2$, and

$$\begin{aligned}\mathcal{G}^\pm = \sum_{\lambda,\lambda_1,\lambda_2 \geq 0} \rho_{\lambda,\lambda_1,\lambda_2} \sum_j \sqrt{\frac{2j+1}{4\pi}} \\ \Big\{ d^j_{\lambda,\lambda_1-\lambda_2}(\theta)\, \mathcal{G}^{j\pm}_{\lambda,\lambda_1,\lambda_2} \\ + (-1)^{s_2-\lambda_2-\tau_2} d^j_{\lambda,\lambda_1+\lambda_2}(\theta)\, \mathcal{G}^{j\pm}_{\lambda,\lambda_1-\lambda_2} \\ + (-1)^{s_1-\lambda_1-\tau_1} d^j_{\lambda,-\lambda_1,-\lambda_2}(\theta) \mathcal{G}^{j\pm}_{\lambda,-\lambda_1,\lambda_2} \\ + (-1)^{s_a+\lambda-\tau_a} d^j_{-\lambda,\lambda_1-\lambda_2}(\theta)\, \mathcal{G}^{j\pm}_{-\lambda,\lambda_1,\lambda_2} \Big\} .\end{aligned} \tag{A.12}$$

Here ρ is

$$\begin{aligned}\rho_{\lambda,\lambda_1,\lambda_2} = \frac{1}{\sqrt{2}} \{[1 - \tfrac{1}{2}\delta_{\lambda,0}][1 - \tfrac{1}{2}\delta_{\lambda_1,0}][1 - \tfrac{1}{2}\delta_{\lambda_2,0}]\}^2 [1 + \delta_{\lambda_0}\delta_{\lambda_{10}}\delta_{\lambda_{20}}] \\ (-1)^{\lambda+\lambda_2} (i)^{\lambda+\lambda_1+\lambda_2} d^{s_a}_{\lambda,\tau_a}(\pi/2) d^{s_1}_{\lambda_1\,\tau_1}(\pi/2) d^{s_2}_{\lambda_2,\tau_2}(\pi/2) .\end{aligned} \tag{A.13}$$

In Eq. (A.11) we have used the fact that

$$d^s_{\lambda,\tau}(\pi/2) = (-1)^{s-\tau} d^s_{-\lambda,\tau}(\pi/2). \tag{A.14}$$

Equation (A.11) clearly shows that for any fixed τ we have projected out a definite CP contribution. Combining Eq. (A.12) and Eq. (A.10) we see that the non-vanishing contributions all have CP parity $\xi(-1)^\tau$.

Examination of Eq. (A.ll) clearly shows that only the absolute values of τ, τ_1, τ_2 need be definite, since they are all integers. This then indicates that the simplest experimental procedure to separate definite CP quantities will be to integrate over the azimuthal dependence of the decays with respect to the transverse axis and to project for definite $|\tau_i|$ using the distribution polar-angle about this axis. We thus need only take non-trivial moments of a single angular dependence for each particle to reconstruct the magnitude of its transversity. We then can combine B and $\overline{B}$ data to obtain a measurement of the CP asymmetry for each set of $|\tau|$, $|\tau_1|, |\tau_2|$, as discussed for the example in Section 5. These results can then be combined to yield an improved estimate as discussed in Section 7.

If C_1 and C_2 are not self-conjugate particles, as in classes 2 and 3 of Table 1, then Eq. (A.5) does not apply since CP interchanges particles. However, if we require both C_1 and C_2 to be spin zero particles, then the transversity of particle A will again allow separation of CP–odd and CP-even contributions. The proof can readily be seen from the case discussed above,with the sums over J reduced to the single term $J = s_C$ and with $s_1 = \lambda_1 = 0$ and $s_2 = \lambda_2 = 0$.

APPENDIX B

Full Angular Analysis and Time Dependence

In this Appendix we will present a method for using the full angular distribution to define a set of moments from which all measurable combinations of helicity amplitudes can be extracted. The method is a standard helicity analysis which we present here for completeness. We analyse here the B^0 decays into two spin one particles, one of which decays to two spin zero particles and the other to an e^+e^- pair, for example the mode ψK^* where $\psi \to e^+e^-$ and $K^* \to K_s\pi^0$. A similar

analysis for B^0 decays into two spin 1 particles which each subsequently decay to two spin zero particles is also presented. An analysis for the case of two spin $3/2$ particles is also briefly discussed. We further present here the explicit structure of the time dependence of the various quantities that can be measured and discuss the extraction of time-dependent CP-asymmetries.

The first step in this analysis requires the definition of some conventions. We use here the conventions of Jackson for the definition of the rotation D-functions. The decay angles for the process $B \to \psi\, K^{*0}$, $\psi \to e^+e^-$, $K^{*0} \to K_s\pi^0$ are shown in Fig 5. We assume the B, $\Upsilon(4s)$, J/ψ and the $\boldsymbol{K^*}$ are in the plane of the paper. The Z axis in the respective helicity frames are opposite to the parent particle. The Y axes are in the direction of the cross production of the Z axis of the parent and the Z axis of the helicity frame. This causes the Y axis of the $\boldsymbol{K^*}$ to be out of the paper and the Y axis of the ψ to be into the paper. Hence the X axes are both pointing upward. This will cause the ϕ angle of the e^- and the π^0 to be going in opposite directions such that their sum will yield the relative angle between the two decay planes. In this drawing neither the e^+, e^-, π^0 nor the K^0 need to lie in the plane of the paper.

The matrix element of the decay of $B \to \psi\, K^{*0}$, $K^{*0} \to K_s\pi^0$ can be written using the helicity formalism as

$$|M|^2 = \sum_{\alpha=\pm1} \left| \sum_{\lambda=0,\pm1} \sqrt{\frac{2J_\psi+1}{4\pi}} \sqrt{\frac{2J_K+1}{4\pi}} \times A_\lambda D^{J_\psi *}_{\lambda,\alpha}(R_\psi) D^{J_K *}_{\lambda,0}(R_K) \right|^2 . \tag{B.1}$$

The amplitudes A_λ in (B.l) contain implicit time dependence which we will discuss later. The important point is that the time dependence and the angular dependence

factorize in this way, so one can perform the angular analysis for each time bin and thus extract time-dependent asymmetries. For the Jackson convention $R = (\phi, \theta, 0)$ (this differs from the Jacob-Wick' where $R = (\phi, \theta, -\phi)$). Expanding Eq. (B.l) gives

$$|M|^2 = \frac{2J_\psi + 1}{4\pi} \frac{2J_K + 1}{4\pi} \sum_{\alpha=\pm1} \sum_{\lambda,\lambda'=0,\pm1} A_\lambda A^*_{\lambda'} \tag{B.2}$$
$$D^{J_\psi *}_{\lambda,\alpha}(R_\psi) D^{J_\psi}_{\lambda',\alpha}(R_\psi) D^{J_K *}_{\lambda,0}(R_K) D^{J_K}_{\lambda',0}(R_K)$$

Changing the charge conjugate to real

$$D^{J*}_{M'M}(R) = (-1)^{M'-M} D^{J}_{-M'-M}(R)$$

and inserting the double D summations

$$D^{J_1}_{M_1'M_1}(R) D^{J_2}_{M_2'M_2}(R) =$$
$$\sum_{J_3=|J_1-J_2|}^{J_1+J_2} \langle J_1 M_1, J_2 M_2 \mid J_3 M_3 \rangle \langle J_1 M_1', J_2 M_2' \mid J_3 M_3' \rangle D^{J_3}_{M_3' M_3}(R) \tag{B.3}$$

gives

$$|M|^2 = \frac{2J_\psi + 1}{4\pi} \frac{2J_K + 1}{4\pi}$$
$$\sum_{\lambda,\lambda'=0,\pm1} A_\lambda A^*_{\lambda'} \sum_{\alpha=\pm1} (-1)^\alpha \sum_{J_L, J_R=0,1,2} \tag{B.4}$$
$$\langle 1\alpha, 1-\alpha \mid J_L 0 \rangle \langle 1\lambda, 1-\lambda' \mid J_L M_L' \rangle D^{J_L}_{-M_L',0}(R_\psi)$$
$$\times \langle 10, 10 \mid J_R 0 \rangle \langle 1\lambda, 1-\lambda' \mid J_R M_R' \rangle D^{J_R}_{-M_R',0}(R_K)$$

After a little rearranging, we have

$$|M|^2 = -2\left(\frac{3}{4\pi}\right)^2 \sum_{\lambda,\lambda'=0,\pm1} A_\lambda A^*_{\lambda'} \sum_{J_L,J_R=0,2}$$

$$\langle 11, 1-1 \,|\, J_L 0\rangle \langle 1\lambda, 1-\lambda' \,|\, J_L \lambda-\lambda'\rangle D^{J_L}_{\lambda'-\lambda,0}(R_\psi) \tag{B.5}$$

$$\times \langle 10, 10 \,|\, J_R 0\rangle \langle 1\lambda, 1-\lambda' \,|\, J_R \lambda-\lambda'\rangle D^{J_R}_{\lambda'-\lambda,0}(R_K)$$

The J_L = 1 terms vanish because of the sum over α on the Clebsch-Gordan coefficient $\langle 1\alpha 1-\alpha \,|\, J_L 0\rangle$ and the J_R = 1 terms vanish because of the coefficient $(1010 \,|\, J_R 0\rangle$. We now simplify with the following relations

$$D^L_{M,0}(\phi,\theta,\chi) = \sqrt{\frac{4\pi}{2L+1}}\, Y^*_{LM}(\theta,\phi)$$

where $Y^*_{LM} = (-1)^M Y_{L-M}$ and $\int Y^*_{\ell m}(\Omega) Y_{\ell' m'}(\Omega)\, d\Omega = \delta_{\ell\ell'}\delta_{mm'}$, to obtain

$$|M|^2 = -2\left(\frac{3}{4\pi}\right)^2 \sum_{\lambda,\lambda'=0,\pm1}$$

$$\times\, A_\lambda A^*_{\lambda'} \sum_{J_L,J_R=0,2} \frac{4\pi}{\sqrt{2J_L+1}\sqrt{2J_R+1}} \tag{B.6}$$

$$\times \langle 11, 1-1 \,|\, J_L 0\rangle \langle 1\lambda, 1-\lambda' \,|\, J_L \lambda-\lambda'\rangle Y^*_{J_L,\lambda'-\lambda}(\Omega_\psi)$$

$$\times \langle 10, 10 \,|\, J_R 0\rangle \langle 1\lambda, 1-\lambda' \,|\, J_R \lambda-\lambda'\rangle Y^*_{J_R,\lambda'-\lambda}(\Omega_K)\,.$$

Let us now define the moments

$$T_{J_L J_R M} = \iint |M|^2 Y_{J_L,M}(\Omega_\psi)\, Y_{J_R,M}(\Omega_K)\, d\Omega_K\, d\Omega_\psi \tag{B.7}$$

and thus

$$|M|^2 = \sum_{J_L=0,2} \sum_{J_R=0,2} \sum_{M=0,\pm1,\pm2} T_{J_L J_R M}\, Y^*_{J_L,M}(\Omega_\psi)\, Y^*_{J_R,M}(\Omega_K) \quad . \tag{B.8}$$

and $T_{J_L J_R -M} = T^*_{J_L J_R M}$. The relation between the helicity amplitudes and the

moments is

$$T_{J_L J_R - M} = \frac{-9}{2\pi} \frac{1}{\sqrt{2J_L+1}} \frac{1}{\sqrt{2J_R+1}} \sum_{\lambda,\lambda'=0,\pm 1} \langle 11, 1-1 \,|\, J_L 0\rangle \langle 1\lambda, 1-\lambda' \,|\, J_L M\rangle \times \langle 10, 10 \,|\, J_R 0\rangle \langle 1\lambda, 1-\lambda' \,|\, J_R M\rangle A_\lambda A^*_{\lambda'} . \tag{B.9}$$

Depending on the relative strengths of different CP contributions the various moments $T_{J_L\ J_R M}$ will show different asymmetries. Linear combinations of moments can always be found which give undiluted asymmetry measurements. The various moments are given in Table 6, where we have defined the definite CP quantities $\mathcal{G}_{\lambda\pm} = (A_\lambda \pm A_{-\lambda})/\sqrt{2}$ and $\mathcal{G}_0 = \mathcal{G}_{0+}/\sqrt{2} = A_0$. Table 7 presents the results of a similar analysis for the decay into two spin 1 particles which each in turn decay to two spin zero particles; for example, the mode $D^{*+}D^{*-}$, where both D^*'s decay to $D\pi$. Clearly in either case we can extract the quantities

$$\mathcal{G}_{1+}\mathcal{G}^*_{1+}, \quad \mathcal{G}_{1-}\mathcal{G}^*_{1-}, \quad \mathcal{G}_0\mathcal{G}^*_0, \quad \mathrm{Re}\,\mathcal{G}_{1+}\mathcal{G}^*_0,$$

and

$$\mathrm{Im}\,\mathcal{G}_{1+}\mathcal{G}^*_{1-} \text{ and } \mathrm{Im}\,\mathcal{G}_{1-}\mathcal{G}^*_0.$$

The first four of these are each definite CP quantities, combining B and $\overline{B}$ data they can each be used to give an asymmetry measurement. The last two quantities represent interference terms between CP odd and CP even amplitudes, which have a more complicated time dependence. They depend only quadratically on the CP asymmetry and so are less sensitive for small asymmetries.

To display the time-dependent phase structure explicitly we introduce the parametrization

$$\mathcal{G}_{\lambda\pm}(t) = g\, G_{\lambda\pm}\, e^{-imt}\, e^{-\Gamma t/2} \left[\cos \frac{\mathrm{Amt}}{2} \pm i\eta\, \lambda_{KM} \sin \frac{\mathrm{Amt}}{2} \right] \tag{B.lO}$$

where $\eta = \xi\,(-1)^s$ and ξ is given in Table 1. For the mode $\psi K_s \pi^0$ we have $\eta = -\xi = +1$. The quantity g is the phase of the CKM matrix elements and the quantity G_j contains any phases from final state interactions and other strong interaction effects as well as the magnitude of the time-zero amplitude. The CP violating quantities are contained in the λ_{KM}, in the standard model $\lambda_{KM} = e^{2i\phi}$ where $\phi = -\beta$ or α or $-\gamma$ is one of the angles of the unitarity triangle, see Fig. 2.

The equivalent quantities for the $\overline{B}_{\mathrm{phys}}$ decays are

$$\begin{aligned} \overline{\mathcal{G}}_{\lambda\pm}(t) = {} & \pm\eta g^* G_{\lambda\pm}\, e^{-imt} \\ & \times e^{-\Gamma t/2} \left[\cos \frac{\mathrm{Amt}}{2} \pm i\eta\, \lambda_{KM}^{-1} \sin \frac{\mathrm{Amt}}{2} \right] \end{aligned} \tag{B-11}$$

One then sees that

$$|\mathcal{G}_{\lambda\pm}|^2 = |G_{\lambda\pm}|^2\, e^{-\Gamma t}\, (1 \mp \eta\, \mathrm{Im}\, \lambda_{KM} \sin \mathrm{Amt}) \tag{B.12}$$

and the equivalent quantity extracted from B-decays give a simple time dependent asymmetry. For example

$$a(1+,1+) = \frac{\mathcal{G}_{1+}\mathcal{G}_{1+}^* - \overline{\mathcal{G}}_{1+}\overline{\mathcal{G}}_{1+}^*}{\mathcal{G}_{1+}\mathcal{G}_{1+}^* + \overline{\mathcal{G}}_{1+}\overline{\mathcal{G}}_{1+}^*} = -\eta\, \mathrm{Im}\, \lambda_{KM} \sin \mathbf{Amt} \tag{B.13}$$

Similarly the interference term between two same-CP amplitudes gives a direct

asymmetry measurement, for example

$$\mathrm{Re}\mathcal{G}_{1+}\mathcal{G}_0^* = \mathrm{Re}(G_{1+}G_0^*)(\,1 - \eta\,\mathrm{Im}\,\lambda_{KM}\,\sin\,\mathrm{Amt})\,e^{-\Gamma t}\,. \tag{B.14}$$

Thus we have several asymmetry measurements, one for each possible pair of same CP contributions. A best asymmetry can be obtained by minimizing the error on an arbitrary linear sum of these values. This requires some knowledge of the relative sizes of the various $\mathcal{G}$'s. The even-odd interference terms are less readily used. We find

$$\begin{aligned}\mathrm{Im}\,\mathcal{G}_{1+}\mathcal{G}_{1-}^* = e^{-\Gamma t}\Big[&\mathrm{Re}(G_{1+}G_{1-}^*)\eta\,\mathrm{Re}\lambda_{KM}\,\sin\,\mathrm{Amt}\\ &+\,\mathrm{Im}\,(G_{1+}G_{1-}^*)\,\cos\,\mathrm{Amt}\Big]\end{aligned} \tag{B.15}$$

which is not, particularly useful for extracting the value of λ_{KM}.

Experimentally we obtain the moments by weighting the experimental events by the $Y_{\ell m}$. For example the T_{222} moment is extracted from the data by calculating

$$T_{222} \cong \frac{1}{N_{\mathrm{evt}}}\sum_{i=1}^{N_{\mathrm{evt}}} Y_{2,2}(\Omega_\psi^i)\,Y_{2,2}(\Omega_K^i)\,. \tag{B.16}$$

The A4 = $1,2$ terms will have a $\phi_\psi + \phi_K$ dependence, with our definition of axes this is the phase between the planes of the two two-particle decay states in the B rest, frame. To predict the time dependence of the moments one needs to substitute Eq. (B.lO) in Eq. (B.9). The relevant time-dependent expression has the form

$$\begin{aligned}A_\lambda A_{\lambda'}^* = \tfrac{1}{2}\,e^{-\Gamma t}\Big\{&[G_{\lambda+}G_{\lambda'+}^* + G_{\lambda-}G_{\lambda'-}^*]\\ &+ [G_{\lambda+}G_{\lambda'-}^* + G_{\lambda-}G_{\lambda'+}^*]\,\cos(\mathrm{Amt})\\ &+ i\eta[G_{\lambda+}G_{\lambda'-}^* - G_{\lambda-}G_{\lambda'+}^*]\mathrm{Re}(\lambda_{KM})\,\sin(\mathrm{Amt})\\ &- \eta\,[G_{\lambda+}G_{\lambda'+}^* - G_{\lambda-}G_{\lambda'-}^*]\mathrm{Im}\,(\lambda_{KM})\sin(\Delta m t)\Big\},\end{aligned} \tag{B.17}$$

and $G_{0+} = \sqrt{2}G_0$. Thus we see that the general moment has three terms with

distinct time dependent behaviors $e^{-\Gamma t}$, $e^{-\Gamma t}$ cos(Amt), and $e^{-\Gamma t}$ sin(Amt). Extracting the moments requires convolution with the relevant resolution function.

A similar analysis for the decay

$$B^0 \to \Delta \quad \overline{\Delta}$$
$$\hookrightarrow p\pi \qquad \hookrightarrow \overline{p}\pi$$

can be carried out. In this case the proton (or antiproton) helicity is not observed. Summing over the possible helicity values once again eliminates odd values for J_L and J_R. The distinct moments are given in Table 8, in addition $T_{200} = T_{020}$ and $T_{220} = \frac{1}{5} T_{000}$. From this one can identify the definite CP quantities

$$\mathrm{Re}\,(\mathcal{G}_{(3/2)+}\mathcal{G}^*_{(1/2)+}) \quad \text{and} \quad \mathrm{Re}\,(\mathcal{G}_{(3/2)-}\mathcal{G}^*_{(1/2)-})$$

and thus this mode can be used to measure the CP violating asymmetry. A similar analysis can be applied to any spin $3/2$

$$B \to \Lambda \quad \overline{\Lambda} \qquad \text{or} \qquad \Sigma \quad \overline{\Sigma}$$
$$\hookrightarrow p\pi \quad \hookrightarrow \overline{p}\pi \qquad\qquad \hookrightarrow p\pi \quad \hookrightarrow \overline{p}\pi$$

channels. For two spin $1/2$ particles which each decay strongly to a nucleon and a spin zero meson only T_{000} survives after summing over nucleon and antinucleon spins, hence one cannot construct undiluted CP asymmetries in these cases. We have not studied the situation for weak decays of such particles.

APPENDIX C

SENSITIVITY ANALYSIS

For the maximum likelihood fits we have assumed that the amplitudes of the decay, except for the CP violating part,, are understood from study of the untagged and isospin related channels. The question of the sensitivity of the results to this assumption naturally arises; specifically an error on the even to odd ratio (Γ_+/Γ_-) in the transversity case or the amplitudes $(\mathcal{G}_{1+}, \mathcal{G}_0, \mathcal{G}_{1-})$ in the case of the full angular distribution analysis will lead to an error in the measured value of asymmetry.

The transversity analysis is relatively simple. We parameterize the error as follows:

$$\delta a/a = (1/a)(da/d\epsilon) \times \delta\epsilon \tag{C.1}$$

where $\epsilon = (\Gamma_+ - \Gamma_-)/(\Gamma_+ + \Gamma_-)$. Evaluating the derivative numerically we find $|(1/a)(da/d\epsilon)| < 1.2$ for all ϵ.

For ψK^* one can estimate that the data sample that will be available for evaluating ϵ from untagged and isospin related channels will be ≈ 20 times larger than the tagged sample. This implies that typical errors on ϵ will be $\approx \sqrt{20}$ smaller than the error on a. Thus considering Eq. (C.1) it is clear that the effect, of a typical $\delta\epsilon$ on δA is negligible.

In the case of the full angular analysis the situation is more complex. Three amplitudes $(\mathcal{G}_{1+}, \mathcal{G}_0, \mathcal{G}_{1-})$ are needed. Two of the amplitudes ($\mathcal{G}_{1+}$ and $\mathcal{G}_0$) are CP even and the third $(\mathcal{G}_{1-})$ is CP odd. We parameterize the errors on these amplitudes by rotations between the magnitudes of two amplitudes and by errors on the relative phases. For example our estimate for the magnitudes of the $\mathcal{G}_{1+}$

and $\mathcal{G}_0$ amplitudes might be related to the true values as follows:

$$|\widehat{\mathcal{G}}_{1+}| = |\mathcal{G}_{+1}| \cos(x) + |\mathcal{G}_0| \sin(x) \tag{C.2}$$

$$|\widehat{\mathcal{G}}_0| = |\mathcal{G}_0| \cos(x) - |\mathcal{G}_{1+}| \sin(x) \tag{C.3}$$

where the $\widehat{\mathcal{G}}$'s represent the estimated values and the plain $\mathcal{G}$'s represent the true values. Similar relationships can be used to quantify the possible experimental confusion between $|\mathcal{G}_{1+}|$ and $|\mathcal{G}_{1-}|$, and between $|\mathcal{G}_0|$ and $|\mathcal{G}_{1-}|$. Figure 6 shows $(1/a)(da/dx)$ for each possible angle of confusion. Note that confusion between the two CP even states $(\mathcal{G}_{1+}$ and $\mathcal{G}_0)$ has little effect but that confusion between either CP-even and the CP-odd amplitudes typically produces noticeable effects. Thus it appears that the overall CP-even to CP-odd ratio is the most sensitive parameter. As seen above it should be possible to determine this parameter to an accuracy much better than needed using the untagged and isospin related channels. We have also investigated the effect of phase errors in $\mathcal{G}_{\lambda\pm}$ and we find them to be small. For example a phase error of 30 degrees changes the asymmetry by only 0.003 when the true asymmetry is 0.15 and $\Gamma_+ = \Gamma_-$ (the worst case).

The final analysis will probably be a maximum likelihood fit of all the parameters (the three amplitudes and the CP violating asymmetry) to all the data samples (tagged, untagged, isospin related). This analysis of sensitivity of the measured asymmetry to assumed values of the parameters indicates that the resulting errors will be only marginally worse than single parameter analysis used in this paper for illustrative purposes.

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FIGURE CAPTIONS

1) Schematic representation of two paths (a) from B^0 to the final state j, direct or via mixing to the $\overline{B}^0$ followed by decay, and (b) from $\overline{B}^0$ to the final state $\overline{f}$, the CP conjugate of j, via direct decay or via mixing to B^0 followed by decay.

2) The unitarity triangle for the three generation standard model, showing the definitions of the angles α, β and γ and some processes that could be used to measure each angle.

3) The ratio of the expected error on the CP-violating asymmetry extracted using transversity from the mixed CP state ψK^* to that obtainable with an equal number of decays to a pure CP state. The curve labelled "α = 1/2" is based on equal weightings of $a_\pm$, while that labelled "α fit" uses the optimal choice for each ϵ. The lowest curve is obtained from a maximum likelihood fit to the asymmetry, assuming Γ_+/Γ_- is known.

4) The ratio of expected error on asymmetries obtained using maximum likelihood fits for a mixed CP (ψK^*)channel to that for a pure CP channel with equal number of decays. The top curve is the transversity based result, shown also on Fig. 3. The remaining curves represent different assumed values for $\mathcal{G}_{1+}/\mathcal{G}_0$, with $\mathcal{G}_{1+}$ and $\mathcal{G}_{1-}$ taken to be relatively real.

5) Schematic drawing of the kinematics of B^0 production and decay showing definitions of the various axes and angles. Each decay is considered in the rest-frame of the parent particle.

6) The fractional derivative of the asymmetry with respect to an angle, (x), which describes the confusion between the amplitudes. The derivative was obtained numerically.

TABLE CAPTIONS

1: The CP parity for the mode AC_1C_2 with transversity τ. The first column defines possible classes. The symbol X $\leftrightarrow$ $\overline{X}$ denotes that particle X is either a CP eigenstate or is observed via its decay into a CP eigenstate. $\eta(X)$ denotes the intrinsic CP-parity of particle X.

2: Examples of B_d modes, which are mixtures of CP eigenstates, that can be studied with an angular analysis. Here f_D denotes any CP eigenmode of D^0 and $(C_1C_2)_C$ denotes particles C_1 and C_2 coming from a parent particle C.

3: Examples of $\overline{B}_s$ modes which are admixtures of CP eigenstates that can be studied with an angular analysis.

4: D^0 Modes

5: Angular structure as a function of the polar angle about the transverse axis.

6: Moments for $B^0 \to (e^+e^-)_\psi(K_s\pi^0)_{K^*}$ in terms of helicity amplitudes.

7: Moments for the B^0 decay to two spin one particles, each of which subsequently decays to two spin zero particles.

8: Moments for $B^0 \to (p\pi)_\Delta(\overline{p}\pi)_{\overline{\Delta}}$ in terms of helicity amplitudes.

Table 1

The CP parity for the mode AC_1C_2 with transversity τ. The first column defines possible classes. The symbol X $\leftrightarrow \overline{X}$ denotes that particle X is either a CP eigenstate or is observed via its decay into a CP eigenstate. $\eta(X)$ denotes the intrinsic CP-parity of particle X.

CLASS	Example AC_1C_2	CP Parity $\equiv \xi\ (-1)^{\tau}$
(1) $A \leftrightarrow \overline{A}$ $C_1 \leftrightarrow \overline{C}_1 \quad C_2 \leftrightarrow \overline{C}_2$	$\psi K_S \pi^0$	$\eta(A)\eta(C_1)\eta(C_2)(-1)^{\tau}$
(2) $A \leftrightarrow \overline{A} \quad c \leftrightarrow \overline{C}$ $C_1 = \overline{C}_2$ $S_{C_1} = S_{C_2} = 0$	$D^{*0}(\pi^+\pi^-)_\rho$	$\eta(A)(-1)^{s_c+\tau}$
(3) $A = \overline{C}$ $S_{C_1} = S_{C_2} = 0$	$D^{*+}D^{*-}$	$(-1)^{s_c+\tau}$

Table 2

Examples of $\overline{B}_d$ modes, which are mixtures of CP eigenstates, that can be studied with an angular analysis. Here f_D denotes any CP eigenmode of D^0 and $(C_1C_2)_C$ denotes particles C_1 and C_2 coming from a parent particle C.

Quark-subprocess	Class (1)	Class (2)	Class (3)	Full Angular Analysis
$b \to c\bar{c}s$	$\psi K_s\pi^0, \psi'' K_s\pi^0$	$(K_s\pi^0)_{\overline{K}^*}(D\overline{D})_{\psi''}$		$\psi(\rho K_S)_{\overline{K}_1}$
$b \to c\bar{c}d$	$\omega\pi^0 f_D, \rho^0\pi^0 f_D$ $\omega\omega f_D, \rho^0\rho^0 f_D$ $\omega\pi^0 f_{D^*}, \rho^0\pi^0 f_{D^*}$	$f_{D^*}(\pi^+\pi^-)_{\rho^0}$		$\omega(\gamma f_D)_{D^*}$ $a_1(\gamma f_D)_{D^*}$
$b \to c\bar{c}d$	$\psi\rho^0\omega$ $\psi\rho^0\pi^0$ $\psi\omega\pi^0$	$\psi(\pi^+\pi^-)_{\rho^0}$	$(\pi D)_{D^*}(\pi\overline{D})_{\overline{D}^*}$ $(\gamma\overline{D})_{\overline{D}^*}(\pi D)_{D^*}$ $(\gamma D)_{D^*}(\pi\overline{D})_{\overline{D}^*}$	$(\gamma D)_{D^*}(\gamma\overline{D})_{\overline{D}^*}$ $\psi\omega$ ψa_1
$b \to uiid$	$\omega\omega\rho^0, \omega\rho^0\pi^0$ $\omega\omega\pi^0, www$	$\omega(\pi^+\pi^-)_{\rho^0}$ $a_1^0(\pi^+\pi^-)_{\rho^0}$	$\rho^+\rho^-$ $\rho^0\rho^0$	ww $a_1^+a_1^-, a_1^0a_1^0$ $\omega a_1^0, \Delta\overline{\Delta}$

Table 3

Examples of $\overline{B}_s$ modes which are admixtures of CP eigenstates that can be studied with an angular analysis. Here $f_{D^{*0}}(f_D)$ denotes any CP eigenmode of $D^{*0}(D^0)$.

Quark-subprocess	Class 1	Class 2	Class 3	Full Angular Analysis
$b \to c\bar{c}s, s$	$\psi\phi\pi^0$	$\psi(K^+K^-)_\phi, \psi''\phi, \ldots$	$\phi\phi$	$(\gamma D_s^+)_{D_s^{*+}}(\gamma D_s^-)_{D_s^{*-}}$
$b \to c\bar{u}d$	$f_{D^{*0}}\pi^0 K_s$			$(\gamma f_D)_{D^{*0}}(\rho^0 K_s)_{K_1}$
$b \to c\bar{c}d$	$\psi\pi^0 K_s, \psi''\pi^0 K_s$			$\psi(\rho^0 K_s)_{K_1}$
$b \to u\bar{u}d$	$\omega\pi^0 K_s, \rho^0\pi^0 K_s, a_1\pi^0 K_s$ $\omega\omega K_s, \omega\rho^0 K_s$			$\omega(\rho^0 K_s)_{K_1}, \rho^0 K_1, a_1 K_1, \ldots$

Table 4

D^0 Modes which are admixtures of CP eigenstates that can be studied with an angular analysis.

Class 1	Class 2	Class 3	Full Angular Analysis
$\omega\pi^0 K_s, \rho^0\pi^0 K_s$ $\rho^0(K_s\pi^0)_{K^{*0}}$	$\phi(\pi^+\pi^-)_{\rho^0}, \rho^0(K^+K^-)_\phi, \omega(K^+K^-)_\phi$	$K^{*0}\overline{K}^{*0}, K^{*-}K^{*+}, \rho^+\rho^-$ $\rho^0\rho^0$	ww

Table 5

Angular structure as a function of the polar angle about the transverse axis.

Group		Example	$r_1(\theta)$	$r_0(\theta)$
(a)	$1 \to 0+0$ or $1^- \to 3(0^-)$	$D^* \to \pi D$ $w \to \pi^+\pi^-\pi^0$	$\frac{3}{4}\sin^2\theta$	$\frac{3}{2}\cos^2\theta$
(b)	$1^+ \to 3(0^-)$ or $1 \to \gamma + 0$ or $1 \to \frac{1}{2}+\frac{1}{2}$	$f_1 \to \eta\pi\pi$ $D^* \to \gamma D$ $\Psi \to e\text{-}e^+$	$\frac{3}{8}(1+\cos^2\theta)$	$\frac{3}{4}\sin^2\theta$

Table 6

Moments for $B^0 \to (e^+e^-)_\psi(K_s\pi^0)_{K^*}$ in terms of helicity amplitudes.

J_L	J_R	M	$4\pi T_{J_L J_R M}$	$4\pi T_{J_L J_R M}$
0	0	0	$2(A_1A_1^* + A_{-1}A_{-1}^* + A_0A_0^*)$	$2(\mathcal{G}_{1+}\mathcal{G}_{1+}^* + \mathcal{G}_{1-}\mathcal{G}_{1-}^* + \mathcal{G}_0\mathcal{G}_0^*)$
0	2	0	$\frac{2}{\sqrt{5}}(2A_0A_0^* - A_1A_1^* - A_{-1}A_{-1}^*)$	$\frac{2}{\sqrt{5}}(2\mathcal{G}_0\mathcal{G}_0^* - \mathcal{G}_{1+}\mathcal{G}_{1+}^* - \mathcal{G}_{1-}\mathcal{G}_{1-}^*)$
2	0	0	$\frac{1}{\sqrt{5}}(A_1A_1^* + A_{-1}A_{-1}^* - 2A_0A_0^*)$	$\frac{1}{\sqrt{5}}(\mathcal{G}_{1+}\mathcal{G}_{1+}^* + \mathcal{G}_{1-}\mathcal{G}_{1-}^* - 2\mathcal{G}_0\mathcal{G}_0^*)$
2	2	0	$-\frac{1}{5}(A_1A_1^* + A_{-1}A_{-1}^* + 4A_0A_0^*)$	$-\frac{1}{5}(\mathcal{G}_{1+}\mathcal{G}_{1+}^* + \mathcal{G}_{1-}\mathcal{G}_{1-}^* + 4\mathcal{G}_0\mathcal{G}_0^*)$
2	2	-1	$-\frac{3}{5}(A_0A_{-1}^* + A_1A_0^*)$	$-\frac{3\sqrt{2}}{5}(\mathrm{Re}\,\mathcal{G}_{1+}\mathcal{G}_0^* + i\mathrm{Im}\,\mathcal{G}_{1-}\mathcal{G}_0^*)$
2	2	-2	$-\frac{6}{5}A_1A_{-1}^*$	$-\frac{3}{5}\left\{\lvert\mathcal{G}_{1+}\rvert^2 - \lvert\mathcal{G}_{1-}\rvert^2 + 2i\mathrm{Im}(\mathcal{G}_{1-}\mathcal{G}_{1+}^*)\right\}$

Table 7

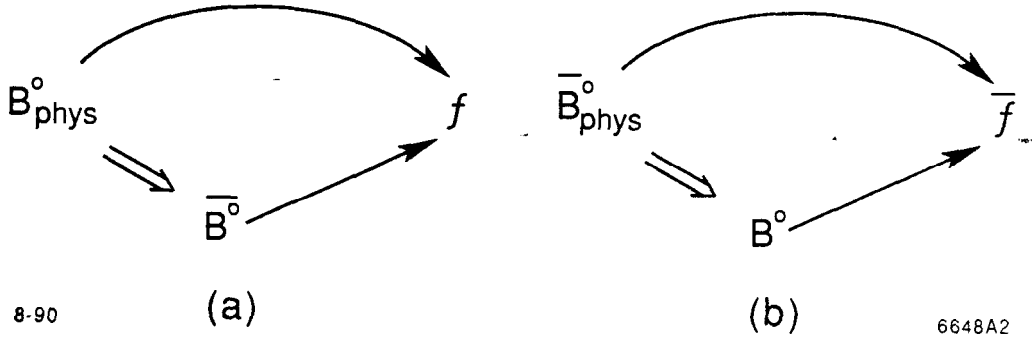
Moments for the B^0 decay to two spin one particles, each of which subsequently decays to two spin zero particles.

J_L	J_R	M	$4\pi T_{J_L J_R M}$	$4\pi T_{J_L J_R M}$
0	0	0	$(A_1 A_1^* + A_{-1} A_{-1}^* + A_0 A_0^*)$	$(\mathcal{G}_{1+}\mathcal{G}_{1+}^* + \mathcal{G}_{1-}\mathcal{G}_{1-}^* + \mathcal{G}_0\mathcal{G}_0^*)$
0	2	0	$\frac{1}{\sqrt{5}}(2A_0A_0^* - A_1A_1^* - A_{-1}A_{-1}^*)$	$\frac{1}{\sqrt{5}}(2\mathcal{G}_0\mathcal{G}_0^* - \mathcal{G}_{1+}\mathcal{G}_{1+}^* - \mathcal{G}_{1-}\mathcal{G}_{1-}^*)$
2	0	0	$\frac{1}{\sqrt{5}}(2A_0A_0^* - A_1A_1^* - A_{-1}A_{-1}^*)$	$\frac{1}{\sqrt{5}}(2\mathcal{G}_0\mathcal{G}_0^* - \mathcal{G}_{1+}\mathcal{G}_{1+}^* - \mathcal{G}_{1-}\mathcal{G}_{1-}^*)$
2	2	0	$\frac{1}{5}(A_1A_1^* + A_{-1}A_{-1}^* + 4A_0A_0^*)$	$\frac{1}{5}(\mathcal{G}_{1+}\mathcal{G}_{1+}^* + \mathcal{G}_{1-}\mathcal{G}_{1-}^* + 4\mathcal{G}_0\mathcal{G}_0^*)$
2	2	-1	$\frac{3}{5}(A_0A_{-1}^* + A_1A_0^*)$	$\frac{3\sqrt{2}}{5}(\mathrm{Re}\,\mathcal{G}_{1+}\mathcal{G}_0^* + i\mathrm{Im}\,\mathcal{G}_{1-}\mathcal{G}_0^*)$
2	2	-2	$\frac{6}{5}A_1A_{-1}^*$	$\frac{3}{5}\{\lvert\mathcal{G}_{1+}\rvert^2 - \lvert\mathcal{G}_{1-}\rvert^2 + 2i\mathrm{Im}\,(\mathcal{G}_{1-}\mathcal{G}_{1+}^*)\}$

Table 8

Moments for $B^0 \to (p\pi)_\Delta(\overline{p}\pi)_{\overline{\Delta}}$ in terms of helicity amplitudes.

J_L	J_R	M	$\frac{4\pi}{4}\, T_{J_L J_R M}$
0	0	0	$A_{3/2}A^*_{3/2} + A_{-3/2}A^*_{-3/2} + A_{1/2}A^*_{1/2} + A_{-1/2}A^*_{-1/2}$
0	2	0	$\frac{1}{\sqrt{5}}(-A_{3/2}A^*_{3/2} - A_{-3/2}A^*_{-3/2} + A_{1/2}A^*_{1/2} + A_{-1/2}A^*_{-1/2})$
2	2	1	$\frac{2}{5}(A_{-3/2}A^*_{-1/2} + A_{1/2}A^*_{3/2})$
2	2	2	$\frac{2}{5}(A_{-3/2}A^*_{+1/2} + A_{-1/2}A^*_{3/2})$
0	0	0	$\lvert\mathcal{G}_{3/2+}\rvert^2 + \lvert\mathcal{G}_{3/2-}\rvert^2 + \lvert\mathcal{G}_{1/2+}\rvert^2 + \lvert\mathcal{G}_{1/2-}\rvert^2$
0	2	0	$\frac{1}{\sqrt{5}}(-\lvert\mathcal{G}_{3/2+}\rvert^2 - \lvert\mathcal{G}_{3/2-}\rvert^2 + \lvert\mathcal{G}_{1/2+}\rvert^2 + \lvert\mathcal{G}_{1/2-}\rvert^2)$
2	2	1	$\frac{2}{5}\Big[\mathrm{Re}(\mathcal{G}_{1/2-}\mathcal{G}^*_{3/2-}) + \mathrm{Re}(\mathcal{G}_{3/2+}\mathcal{G}^*_{1/2+}) + i\,\mathrm{Im}(\mathcal{G}_{1/2-}\mathcal{G}^*_{3/2+}) - i\,\mathrm{Im}(\mathcal{G}_{3/2-}\mathcal{G}^*_{1/2+})\Big]$
2	2	2	$\frac{2}{5}\,\mathrm{Re}(\mathcal{G}_{3/2+}\mathcal{G}^*_{1/2+}) + i\,\mathrm{Im}(\mathcal{G}_{3/2+}\mathcal{G}^*_{1/2-}) + i\,\mathrm{Im}(\mathcal{G}_{1/2+}\mathcal{G}^*_{3/2-}) - \mathrm{Re}(\mathcal{G}_{3/2-}\mathcal{G}^*_{1/2-})$


B^{o}_{phys}
f
$\overline{B}^{o}$
(a)
$\overline{B}^{o}_{phys}$
$\overline{f}$
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(b)
8-90
6648A2

Fig. 1

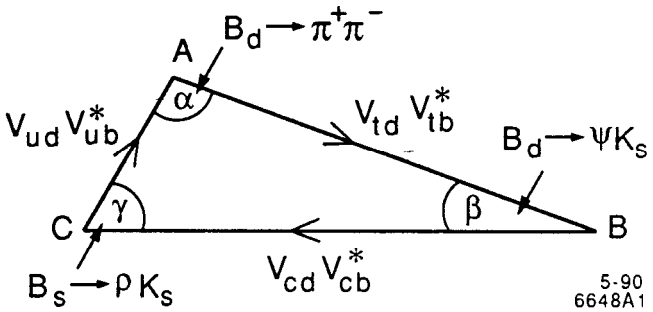
$B_d \to \pi^+\pi^-$
A
α
$V_{ud} V_{ub}^*$
$V_{td} V_{tb}^*$
$B_d \to \psi K_s$
C
γ
β
B
$B_s \to \rho K_s$
$V_{cd} V_{cb}^*$
5-90
6648A1

Fig. 2

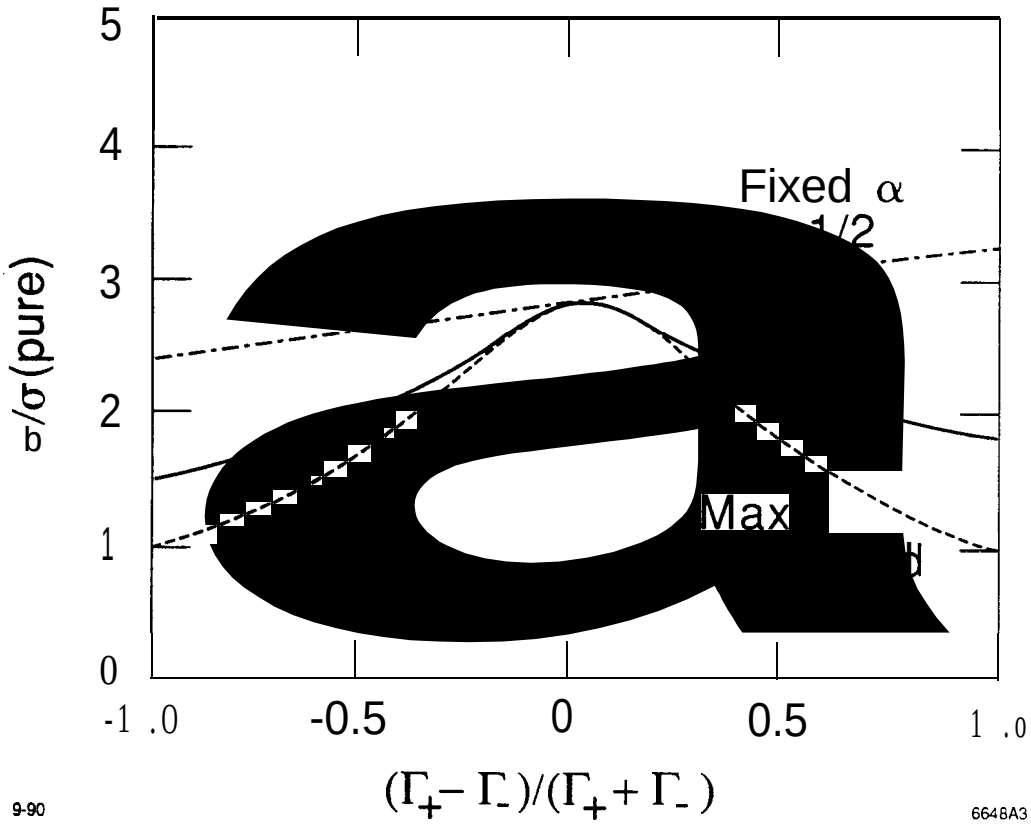
5
4
3
2
1
0
Fixed α
1/2
Max
-1.0
-0.5
0
0.5
1.0
$(\Gamma_+ - \Gamma_-)/(\Gamma_+ + \Gamma_-)$
9-90
6648A3

Fig. 3

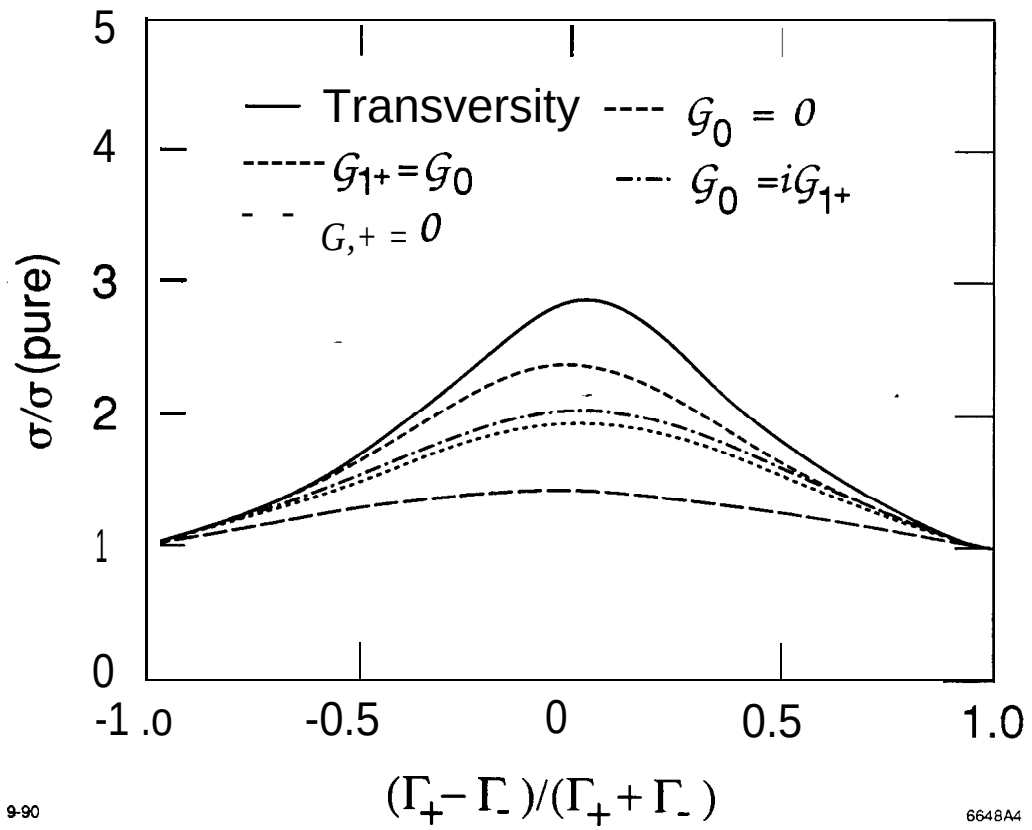
Transversity
$\mathcal{G}_0 = 0$
$\mathcal{G}_{1+} = \mathcal{G}_0$
$\mathcal{G}_0 = i\mathcal{G}_{1+}$
$G_{,+} = 0$
σ/σ (pure)
5
4
3
2
1
0
-1.0
-0.5
0
0.5
1.0
$(\Gamma_+ - \Gamma_-)/(\Gamma_+ + \Gamma_-)$
9-90
6648A4

Fig. 4

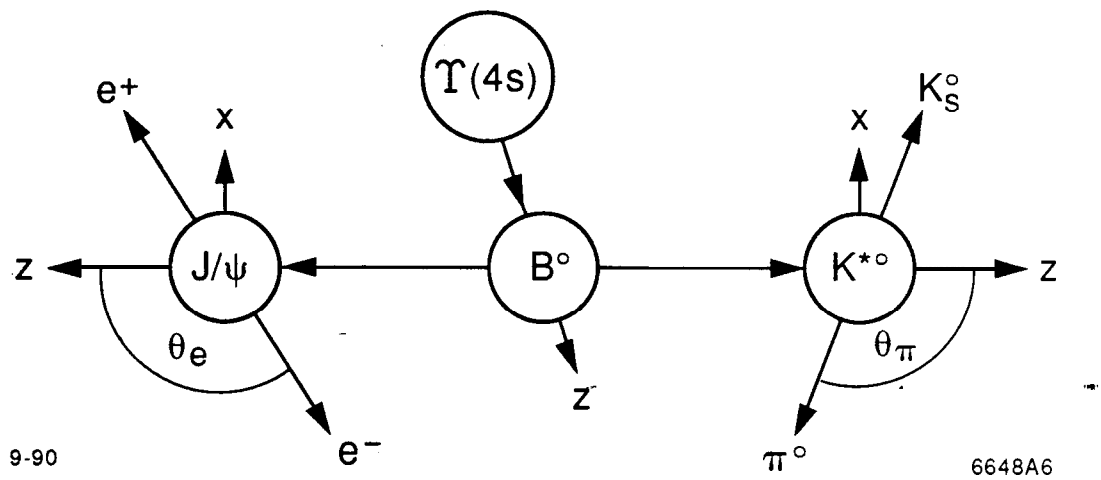

Fig. 5

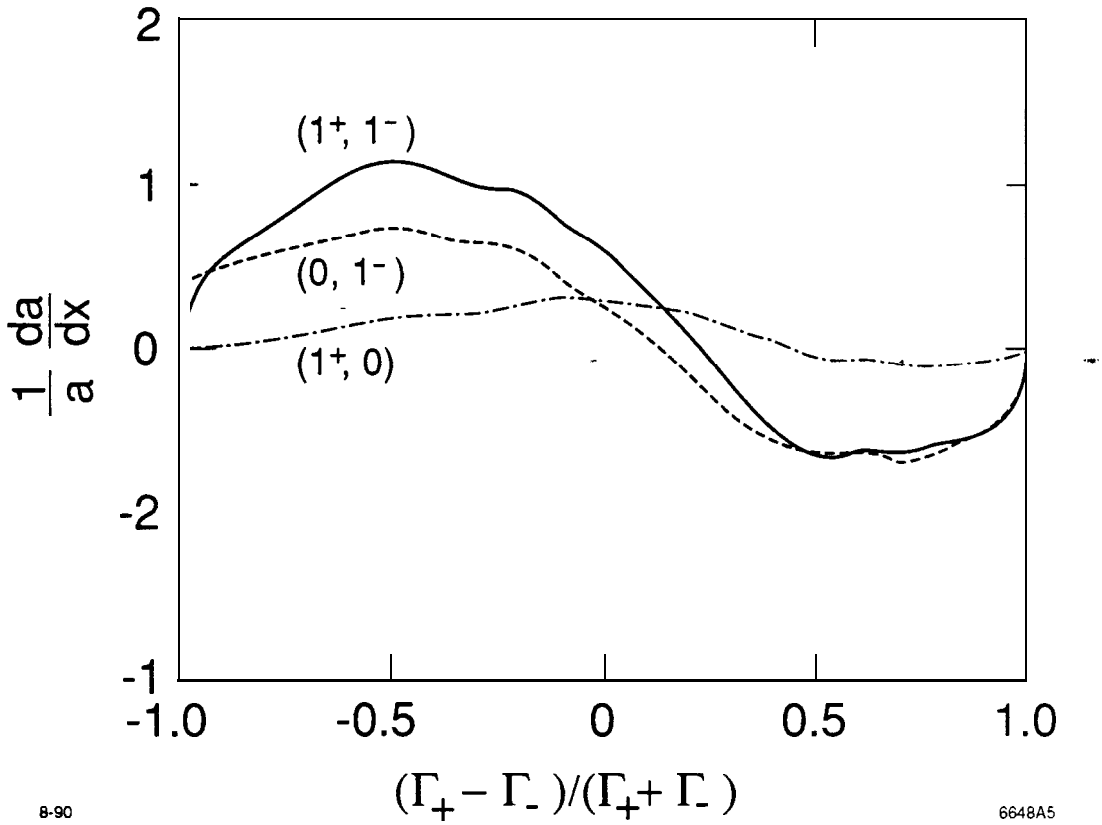

Fig. 6