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Supersymmetry Breaking in String Theory*

LANCE J. DIXON

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94309

ABSTRACT

I briefly review the problems with previous investigations of supersymmetry breaking in string theory — at tree-level, at one-loop, and non-perturbatively. A variant of the original non-perturbative scenario is proposed, in which gaugino condensation takes place in two different strongly-interacting hidden-sector gauge groups. In the new scenario it is possible to generate a large hierarchy of mass scales and to simultaneously stabilize the dilaton at a large expectation value (weak coupling). However, it is still uncertain whether supersymmetry is broken in such a vacuum.

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1. INTRODUCTION

Two of the major uncertainties in extracting low-energy predictions from superstrings involve the choice of vacuum state and the mechanism of supersymmetry breaking. Here I will address the second problem, under the assumption that it can be separated from the first problem. In string theory, many important questions are entangled with supersymmetry (SUSY) breaking, such as:

(a) Can SUSY breaking take place at all?

(b) Can it explain the hierarchy of mass scales M_W/M_{Pl}?

(c) Can it fix the gauge coupling constants to some realistic values that also correspond to a weakly-coupled string?

(d) Can it explain the vanishing of the cosmological constant?

(e) Can it feed into the "observable sector" in a phenomenologically viable way?

I will present a scenario[1] in which the answer to (a), (b) and (c) appears to be yes. First, however, I review earlier work on SUSY breaking in string theory, in order to indicate how problems encountered in that work may be circumvented by the new scenario.

2. REVIEW

Supersymmetry breaking in string theory is made difficult by the existence of massless fields ϕ_i with potentials that are initially flat, $V(\phi_i) \equiv 0$, but that become non-flat in the SUSY breaking process. The non-flat potentials typically have only pathological vacua where fields run off to infinity. The prime example of this phenomenon is the dilaton field ϕ, which couples at zero-momentum to the Euler character of the world-sheet:] $\chi = \int d^2\sigma \sqrt{g^{(2)}} R^{(2)}(\sigma)$. Therefore the Polyakov amplitude for a surface with n handles — corresponding to a scattering amplitude at n loops in string perturbation theory — comes with a factor $e^{2(1-n)\phi}$. The effective Lagrangian for ϕ and all the other massless fields has the form

$$\mathcal{L}_{\text{eff}}(\phi, \partial\phi, \ldots) = e^{2\phi}\hat{\mathcal{L}}_{\text{tree}} + \hat{\mathcal{L}}_{1-\text{loop}} + e^{-2\phi}\hat{\mathcal{L}}_{2-\text{loop}} + \ldots, \tag{1}$$

where $\hat{\mathcal{L}}_{\text{tree}}, \hat{\mathcal{L}}_{1-\text{loop}}, \ldots$ depend only on $\partial\phi$ and the other fields. The vacuum expectation value (VEV) of the dilaton is identified[3] as the string coupling constant g_s,

$$e^{2\langle\phi\rangle} \equiv \frac{4\pi}{g_s^2}. \tag{2}$$

A cosmological constant $\Lambda_{\text{n-loop}}$ corresponds to a c-number contribution to $\hat{\mathcal{L}}_{\text{n-loop}}$ in (1), and generates a dilaton potential $V(\phi) \sim e^{-2n\phi}$ (after rescaling $\mathcal{L}_{\text{eff}}$ by $e^{-2\phi}$ to make the dilaton kinetic term in $\hat{\mathcal{L}}_{\text{tree}}$ canonical). This runaway potential leads to an infinite VEV for ϕ, which also means that the string coupling constant vanishes, $g_s = 0$. Certainly such a string vacuum (a free theory) does not describe the real world.

Tree-Level Breaking

The most severe way to break SUSY in string theory is at tree-level, that is, spontaneously. In ten dimensions, the 0(16) x 0(16) heterotic string[4] and related models with tachyons[4,5] are vacua with spontaneously broken supersymmetry. They all have $\Lambda_{1-\text{loop}} \neq 0$ (in fact, $\Lambda_{1-\text{loop}}$ is infinite due to tachyons for all except the 0(16) x 0(16) vacuum), so the dilaton runs off to infinity. Even had the dilaton VEV been stabilized, the SUSY breaking scale would have been of order the Planck scale M_{Pl}, leaving no hope for low-energy SUSY to explain why M_W/M_{Pl} is so tiny. One could try to stabilize the dilaton by shifting the tree-level vacuum "off-shell" (away from a conformal field theory) in order to cancel tree-level effects against one-loop effects? But here such a cancellation requires the dilaton VEV to be of order one, and so $g_s \sim 1$, a regime where the perturbative analysis is unreliable.

There are also non-supersymmetric compactifications of superstrings to less than ten dimensions, such as "twisted tori"[7] and toroidal compactifications of non-supersymmetric ten-dimensional vacua.[8] The one-loop cosmological constant $\Lambda_{1-\text{loop}}(R_i)$ is now a function of the radii R_i of the tori, which can be tuned to special values R_i^c so that

$$\Lambda_{1-\text{loop}}(R_i^c) = 0. \tag{3}$$

However, the radii also represent massless fields with flat potentials at tree-level in

g_s, and $\Lambda_{1-\mathrm{loop}}(R_i)$ represents a one-loop potential for them, so a stable vacuum at one-loop also requires

$$\partial_{R_i} \Lambda_{1-\mathrm{loop}} \,|_{R_i^c} = 0, \qquad \text{for all } R_i. \tag{4}$$

Vacua satisfying (3) generally fail to satisfy (4), and so the radii fields run away.*

One-Loop Breaking

Henceforth, let the vacuum state be four-dimensional (4d), and let supersymmetry be unbroken at tree-level. Then the effective Lagrangian is supersymmetric, and the dilaton can be organized into a chiral superfield,

$$S(x,\theta) \;=\; (e^{2\phi}(x) + ib(x)) \;+\; \theta\psi_S(x) \;+\; \theta\theta F_S(x). \tag{5}$$

A Peccei-Quinn symmetry for the axion, b(s) $\rightarrow$ b(z) + const., ensures that there is no superpotential for S (no F-term) at any order in string perturbation theory.[9] Generically, 4d vacua have other massless chiral superfields $T_i(x,\theta)$, called moduli, that result from continuous parameters in the compactification (like the torus radii R_i) and that have no superpotential at string tree-level. The same argument of ref. 9 shows that the T_i also acquire no superpotential perturbatively.

Even though F-terms cannot be generated for S, T_i in string perturbation theory, a Fayet-Iliopoulos D-term can be generated at one-loop.[10] This happens whenever there is a $U(1)$ factor in the 4d gauge group with a trace anomaly, $\sum_i Q_i \neq 0$, where the sum is over all left-handed, massless fermions in the spectrum, with $U(1)$ charges Q_i. In field theory the trace anomaly leads to a quadratically divergent D-term for the $U(1)$.[11] In string theory the one-loop D-term is finite and calculable,[12] and results in a non-vanishing two-loop cosmological constant $\Lambda_{2-\mathrm{loop}} \sim D^2_{1-\mathrm{loop}}$.

$\star$ Even if it happened somehow that both (3) and (4) were satisfied at one-loop, the problem would reappear at two-loops, and so on.

As in the tree-level case, $\Lambda_{2-\mathrm{loop}} \neq 0$ means that the dilaton can run off to infinity. This time, however, there is another possibility:[13,10,12] The one-loop D-term can be cancelled against a tree-level D-term induced by giving VEV's to charged, massless scalars A_i with no superpotential, which "happen" to be present in the massless spectrum! The tree-level plus one-loop D-term is

$$D = (\mathrm{Re}S) \sum_i Q_i A_i^\dagger A_i + c \sum_i Q_i, \qquad c > 0. \tag{6}$$

The solution to $D = 0$ with g_s nonzero has $\langle A_i \rangle \sim (\mathrm{Re}S)^{-1/2} \sim g_s$. Note that the shift of VEV's is only O(g) in this case, in contrast to the tree-level case where it was O(1); this makes the shifted vacuum perturbatively reliable here. However, supersymmetry is not broken in this vacuum (at least at one-loop, and most likely to any finite order), the string coupling constant g_s is still not determined, and hence there is still a "dilaton" S' which is a mixture of S and the A;, with (S') $\sim g_s^{-2}$.

Non-Perturbative Breaking

Non-perturbative SUSY breaking seems to have the best chance of generating a large hierarchy through factors like those occurring in instanton calculations, $\exp(-c/g_s^2)$, if the string coupling g_s can be fixed to a small value. Since there is currently no framework for calculating non-perturbative effects in the full string theory, one has to assume that non-perturbative effects in the low-energy effective field theory dominate over "stringy" non-perturbative effects.

The first attempts to break SUSY non-perturbatively in string theory invoked gaugino condensation in the hidden Es in Calabi-Yau compactifications of the heterotic string.[14,15] Up to an overall constant C_G, the gaugino condensate for

† I know of no general argument for the existence of such fields, but empirically they are always present. Note that their charge has to be *opposite* in sign to the trace anomaly.

supersymmetric pure Yang-Mills theory with gauge group G follows from renormalization group invariance and dimensional analysis,

$$\langle \lambda^\alpha \lambda_\alpha \rangle = C_G \ \Lambda_G^3 = C_G \left[M \left(\frac{1}{g^2(M)} \right)^{b_1/2b_0^2} \exp\left(\frac{-8\pi^2}{b_0 g^2(M)} \right) \right]^3 \left(1 + O(g^2)\right) . \tag{7}$$

Here Λ_G is a dynamically generated mass scale, approximated by the scale at which the two-loop running coupling blows up, and M will be set to $1/\sqrt{\alpha'}$ for the application to string theory. The renormalization group constants $b_{0,1}$ are defined by

$$\beta(g) = -\frac{b_0}{(4\pi)^2} g^3 - \frac{b_1}{(4\pi)^4} g^5 + \quad \cdots . \tag{8}$$

Now $\lambda^\alpha \lambda_\alpha$ is the *lowest* component of a chiral, composite superfield $W^\alpha W_\alpha$ (W_α is the supersymmetric field strength), so its VEV normally would not break supersymmetry. But in string theory the dependence of the condensate (7) on the gauge coupling translates into dependence on the dilaton superfield-S, and in fact it generates (as will be shown below) a superpotential for S which could break supersymmetry,[16,14,15]

$$W(S) \sim M^3 S \exp\left(-\frac{6\pi}{b_0} S \right) . \tag{9}$$

However, for weak-coupling (large S) the potential

$$V(S, \bar{S}) = e^{K(S,\bar{S})} \left[g^{S\bar{S}} D_S W \bar{D}_{\bar{S}} \bar{W} - 3|W|^2 \right] \tag{10}$$

is monotonic and leads once again to a runaway dilaton. In ref. 15 it was proposed to stabilize the dilaton by an additional c-number term in the superpotential, resulting from a VEV for the antisymmetric tensor field strength $H_{\mu\nu\lambda} = \partial_\mu B_{\nu\lambda} + \ldots$ on the internal Calabi-Yau space, $W(S) \to W(S) + c$ with $c = \langle H_{ijk} \rangle$. Unfortunately, c is quantized to be of order 1 in Planck units,[17] so the stabilization takes place at around the Planck scale and for $\langle S \rangle \sim 1$. Thus the semi-classical analysis is unreliable, and in any case it does not predict a large hierarchy. Attempts"] to stabilize the dilaton through a dependence of the (radiatively-corrected) potential $V(S, T_i)$ on the moduli T_i tend to result in the moduli running away.

In ref. **19** it is argued on general grounds that the results reviewed above are generic, i.e. that string theory has *no* acceptable weakly-coupled vacua. However, possible loopholes to the argument are also given. A scenario for a weakly-coupled vacuum will now be presented that relies on loophole #1 of the first ref. 19; specifically it relies on the existence of small (combinations of) discrete, non-dynamical parameters in the theory.

3. A NEW SCENARIO

Consider now a variant of the above non-perturbative scenario, in which the hidden sector is more complicated than just a single SUSY pure Yang-Mills theory! Indeed, many 4d string vacua have hidden sectors that are fragmented into the. products of several simple Lie algebras, and they may contain charged matter supermultiplets as well. Perhaps the combination of gaugino condensates from the different gauge group factors can stabilize the dilaton VEV at a perturbatively reliable (large) value. For definiteness and simplicity let the hidden sector be SUSY pure Yang-Mills with gauge group $G_1 \times G_2$, where G_1 and G_2 are simple. Then the relative phase between the two condensates (XX), and (XX), can adjust to -1 to minimize the dilaton potential. Clearly, in order for this scenario to work the two running couplings $g_1(\mu)$, $g_2(\mu)$ should become strong at roughly the same scale, M_I. Also, $M_I << M_{\rm Pl}$ is desired in order to generate a large hierarchy. Thus the couplings should be slightly different at $M_{\rm Pl}$, and should have slightly different ,&functions.

Why should $g_1(M_{\rm Pl})$ differ from $g_2(M_{\rm Pl})$? Each tree-level gauge coupling in string theory is given by $g_i = g_s/\sqrt{k_i}$, where the positive integer k_i is the level of the Kac-Moody algebra of world-sheet currents J"(z) that generate the gauge symmetry G_i.[20] At one-loop, g_i also depends on the spectrum of charged, massive string excitations that have been integrated out to obtain $\mathcal{L}_{\text{eff}}$. The tree-level plus one-loop result is

$$\frac{8\pi^2}{g_i^2(M_{\text{Pl}})} = \frac{8\pi^2 k_i}{g_s^2} + \frac{1}{2}\Delta_i \quad , \tag{11}$$

where Δ_i is given by an integral over the modular domain for the world-sheet torus, which can be computed for "exactly solvable" 4d string vacua.[21] The identification $4\pi/g_s^2 \equiv \text{Re}S$ means that the S-dependence of holomorphic expressions like the superpotential W(S) are obtained by the replacement

$$\frac{8\pi^2}{g_i^2(M_{\text{Pl}})} \rightarrow 2\pi k_i S + \frac{1}{2}\Delta_i + O(S^{-1}). \tag{12}$$

It is also important to know the group-dependence of the constant prefactor C_G in the gaugino condensate (7). 0 ne way to determine the constant C_N for $SU(N)$ is to use induction on N.[22] Skipping all the details, the result is $C_N = (C_{N-1})^{1-1/N}$. The large N behavior of this result (which turns out to be the regime of interest) is $C_N \rightarrow$ constant. This behavior can be checked by summing planar diagrams with $g^2 \sim 1/N$ to get $\langle\lambda\lambda\rangle \sim N f(g^2N)$. Using also the renormalization-group-invariant expression (7), with $b_0 = 3N$, $b_1 = 6N^2$ for $SU(N)$, one does get $C_N \approx$ constant.

The dilaton superpotential induced by gaugino condensation is W(S) = b_0 $\langle\lambda\lambda\rangle$. (This equation is related to the trace/axial anomaly equation, $T^\mu_\mu + i\partial \cdot J_A = b_0(F^2_{\mu\nu} + iF\tilde{F})$, by a supersymmetry transformation; both equations follow from conservation of the super-stress-tensor.) With the replacement (12), the superpotential generated by the hidden sector gauge group $SU(N_1)_{k_1} \times SU(N_2)_{k_2}$ is

$$W_{\text{tot}}(S) = aM^3\ S\left[N_1k_1e^{-\Delta_1/2N_1}e^{-2\pi\frac{k_1}{N_1}S} - N_2k_2e^{-\Delta_2/2N_2}e^{-2\pi\frac{k_2}{N_2}S}\right], \tag{13}$$

where the numerical constant a has not been computed. Ignoring gravitational corrections, the potential for S is just $V(S, \bar{S}) = F_S^2 = |\partial_S W|^2$ and is minimized (at V = 0) by setting $F_S = \partial_S W = 0$, or

$$S = \frac{1}{2\pi}\left(\frac{k_1}{N_1}-\frac{k_2}{N_2}\right)^{-1}\left[\ln\left(\frac{k_1^2}{k_2^2}\right)+\frac{\Delta_2}{2N_2}-\frac{\Delta_1}{2N_1}\right]\left[1+O\left(\frac{k_1}{N_1}-\frac{k_2}{N_2}\right)\right] . \quad (14)$$

Here SUSY is unbroken in the flat limit (since $F_S = 0$), but should be broken by gravitational corrections. To confirm this expectation, one should properly derive the effective supergravity theory for the dilaton superfield (and possibly other superfields such as the moduli T_i) interacting with a composite chiral superfield that represents the gaugino condensate, following ref. 23; this remains to be done. A 'naive' approach[16] is to simply replace XX by its VEV in the classical supergravity Lagrangian[24] and to leave $W = 0$; using this approach here one would conclude that SUSY is unbroken. However, another approach[15] is to incorporate the effects of the gaugino condensate by substituting the effective superpotential (here, $W_{\text{tot}}(S)$ from eq. (13)) into the supergravity Lagrangian; in this case the result now depends on properties of the other massless chiral superfields (T_i, etc.). If there is only one such field, with a no-scale Kähler potential[25] as suggested by dimensional reduction of tree-level ten-dimensional string theory[26], then one finds (just as in ref. 15) that SUSY is broken, with vanishing cosmological constant in this approximation. The scale of supersymmetry breaking is

$$M_{\text{SUSY}} \sim \langle W_{\text{tot}} \rangle \sim M_I^3/M_{\text{Pl}}^2 , \quad (15)$$

where M_I^3 = (XX). It is clearly important to establish which approach (if either) is correct in the present circumstances.

Assume henceforth that SUSY is broken at the scale (15). For supersymmetry to explain the M_W/M_{Pl} hierarchy, M_{SUSY} should be around a TeV, or $M_I \sim 5 \times 10^{13}$ GeV. The desired value of S = $(\alpha_{\text{GUT}}(M_{\text{Pl}}))^{-1}$ depends somewhat on whether there are additional, light fields transforming under the standard model gauge group, but roughly S $\sim$ 30 to 40 is desired (assuming level 1 for the standard model Kac-Moody algebras).

To get a sufficiently small M_I and large S, the parameters N_i, k_i have to be "discretely fine-tuned" to make $\frac{k_1}{N_1} - \frac{k_2}{N_2} << 1$. (The fine-tuning circumvents the argument of ref. 19 for a strongly-coupled vacuum.) However, in string theory the degree of possible fine-tuning is restricted by the total Virasoro central charge available. Using the Sugawara formula, and taking the standard model to be level 1, gives

$$c_{\text{hidden}} \equiv c_{G_1} + c_{G_2} = \frac{d_{G_1} k_1}{k_1 + C(G_1)} + \frac{d_{G_2} k_2}{k_2 + C(G_2)} \leq 18. \qquad (16)$$

This restriction makes it difficult to accommodate higher-level Kac-Moody algebras $(k > 1)$ in this scenario. Finally, in the absence of a specific 4d string vacuum, a guess must be made for Δ_1 and Δ_2. Here it is assumed that $\Delta_1/2N_1 = -3/2$, $\Delta_2/2N_2 = +3/2$. Also $a \approx 1$ is assumed.

The "best" results obtained in this scenario so far are for $SU(9)_1 \times SU(10)_1$ ($c_{\text{hidden}} = 17$):

$$S \approx 43, \qquad M_I \sim 5 \times 10^{14} \text{ GeV}. \qquad (17)$$

For $SU(8)_1 \times SU(9)_1$ ($c_{\text{hidden}} = 15$) one gets:

$$S \approx 34, \qquad M_I \sim 1 \times 10^{15} \text{ GeV}. \qquad (18)$$

And the best result incorporating a level-2 group, $SU(9)_2 \times SU(4)_1$ ($c_{\text{hidden}} = 17.5$), seems to be untenable:

$$S \approx 9, \qquad M_I \sim 9 \times 10^{16} \text{ GeV}. \qquad (19)$$

It is interesting that very few choices of $G_1 \times G_2$ (if any!) can give a sufficiently small scale M_I.

Obviously much remains to be done in examining the details of the scenario: Attempts should be made to build actual string models, and to evaluate Δ_i for them; then the gravitino mass can be calculated, as well as the communication of SUSY breaking to the observable sector. There is always the caveat that the above results may be washed out by stronger, "stringy" nonperturbative effects. If one could somehow take the limit N_1, $N_2 \to \infty$ while holding any other non-perturbative effects fixed — and assuming the latter effects to behave like $\exp(-\text{const.}/g_s^2)$ — then the gaugino condensation effects would dominate. Unfortunately no such limit exists. If this problem is neglected, however, a mechanism has been identified for fixing the dilaton VEV at a large value (weak coupling) and perhaps breaking supersymmetry at a phenomenologically interesting mass scale.

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