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Perturbative QCD Effects in Heavy Meson Decays *

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Abstract

The amplitude for the exclusive nonleptonic decay of a heavy meson into two light pseudoscalar mesons is analyzed using the factorization formalism of perturbative QCD for exclusive reactions at large momentum transfer. We show that the leading contribution to such amplitudes is proportional to $\alpha_s(Q^2)$ where Q^2 scales with the heavy meson mass squared. Branching ratios for a few B^0 decay modes are calculated.

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There is now well-established evidence for the chiral V-A symmetry and the Standard Model of weak interactions in leptonic and semileptonic interactions. However a direct calculation of the hadronic matrix elements of the quark bilinears which appear in the effective weak interaction Hamiltonian is difficult due to the nonperturbative effects arising from the strong interactions. For light hadrons, calculations [1] include the one-loop corrections to the weak Hamiltonian from perturbative QCD [2] in order to understand the $\Delta I = 1/2$ rule in kaon decays. The renormalization procedure usually introduces an additional set of operators of the gluon monopole or Penguin type [3]. In light-flavored nonleptonic meson decays both short (perturbative) and long (nonperturbative) range strong interaction corrections must be taken into account.

The situation is much simpler in the case of a heavy meson decaying into two much lighter mesons. Because of the large momentum transfers involved, the factorization formula of PQCD for exclusive reactions becomes applicable: the amplitude can be written as a convolution of a hard-scattering quark-gluon amplitude T_h, similar to that which appears in meson form factors, and meson distribution amplitudes $\phi(x, Q^2)$ which describe the fractional longitudinal momentum distribution of the quark and anti-quark in each meson. An important feature of this formalism is that, at high momentum transfer, long-range final state interactions between the outgoing hadrons can be neglected.

As in PQCD calculations of meson form factors [4, 5], the hard scattering amplitude T_h is computed by replacing each hadron by collinear quarks; to leading order in $\alpha_s(Q^2)$ the dominant contribution is controlled by single gluon exchange. We shall show that in the case of non-leptonic weak decays the mass of the heavy meson M_H^2 establishes the relevant momentum scale $Q^2 \sim M_H^2$, so that for a sufficiently massive initial state the decay amplitude is of order $\alpha_s(Q^2)$, even without including loop corrections to the weak Hamiltonian.

The approach presented here is distinct from the usual low energy constituent quark model (CQM) calculations of hadronic matrix elements of the quark current [6, 7], where the heavy mass M_H appears only as an explicit flavor symmetry-breaking term in the wave function.

The unperturbed effective weak Hamiltonian is

$$H = \frac{G_F}{\sqrt{2}} J^\mu J_\mu^\dagger, \tag{1}$$

where

$$J^\mu = V^\mu - A^\mu = (\overline{u}, \overline{c}, \overline{t})\gamma^\mu(1-\gamma_5)V \begin{pmatrix} d \\ s \\ b \end{pmatrix},$$

and V is the Kobayashi-Maskawa (K-M) mixing matrix. We develop the formulas for the matrix elements of quark currents and calculate several hadronic decay rates of the B meson.

As argued above, for the decay of a heavy state into two light mesons we can use the PQCD factorization of exclusive amplitudes at high momentum transfer and neglect all final state interactions between corresponding particles. For one of the currents we can reduce the full hadronic field of one of the light or heavy mesons to its weak decay matrix element: e.g., $< L_i|J^\mu|0 >=< L_i|A^\mu|0 >= P_i^\mu f_i$,

$(i = 1,2)$. The decay amplitude can be written as

$$M(H \to L_1 + L_2) = c_1 f_1 P_{1\mu} < L_2|J^\mu|H >$$
$$+ c_2 f_2 P_{2\mu} < L_1|J^\mu|H > + c_H f_H P_{H\mu} < L_1, L_2|J^\mu|0 >, \quad (2)$$

where the c_i's are process dependent coefficients containing a weak coupling constant with the appropriate K-M matrix elements and factors coming from a possible Fierz transformation. The matrix element of the transition current, e.g. $< L_i|J^\mu|H > = < L_i|V^\mu|H >$, has to be evaluated between the two remaining hadronic states. The decays we will consider here are the decays of the B^0 into two pseudoscalar mesons : $B^0 \to \pi^+\pi^-$, $K^+\pi^-$, D^0K^0, $D_s^+\pi^-$, because in each of these decays there is only one contributing amplitude at the tree level in the spectator approximation.

As in the analysis of meson form factors $< M|\, V_\mu \,|M' >$ at large momentum transfer, this matrix element can be written as (see Fig. 1)

$$\begin{aligned} < L_i|V^\mu|H > \;=\; & Tr(\overline{\psi}_i T^\mu \psi_H) \\ & + Tr(\overline{\psi}_i^\alpha T^\mu_\alpha \psi_H) + Tr(\overline{\psi}_i T^\mu_\alpha \psi_H^\alpha) \\ & + Tr(\overline{\psi}_i^{\alpha\beta} T^\mu_{\alpha\beta} \psi_H) + \ldots, \end{aligned}$$

where the $\psi_i^{\alpha_1\alpha_2\cdots}$, $\psi_H^{\alpha_1\alpha_2\cdots}$ are the amplitudes for finding collinear on-shell quark, antiquark and gluons with polarizations α_i in the light and heavy meson, respectively. $T^\mu_{\alpha_1\alpha_2\ldots}$ is the collinear irreducible hard-scattering amplitude [8], which can be calculated systematically in powers of $\alpha_s(Q^2)$. Tr stands for an integration over momentum fractions and a trace over spin, flavor and color indices of quarks and gluons. We can neglect Fock states with extra $q\overline{q}$

pairs as their contribution to the matrix element is suppressed by at least two powers of $1/Q^2$ coming from additional quark propagators [8]. Using the techniques of Brodsky and Lepage [8], we rewrite the above equation in the form

$$< L_i|V^\mu|H >= Tr(\overline{\psi}_i T_h^\mu \psi_H), \tag{3}$$

with ψ now representing an effective quark-antiquark distribution amplitude, and T_h a hard scattering amplitude given as a sum of collinear skeleton graphs (Fig. 2).

The skeletons which appear in Fig. 2 have an identical graphical representation as those contributing to the pion form factor at high momentum transfer. Since $(P_H - P_i)^2 = P_j^2 = M_\pi^2(M_K^2, M_D^2) << M_H^2$ the off-shellness of the quark and the gluon propagator are proportional to M_H, and much larger then all other mass scales. After renormalization of vertices and propagators, the strong coupling becomes $\alpha_s(Q^2 \sim M_H^2)$. For Figs. 2b and 2c we have

$$Q^2 = (1-x)(1-y)M_H^2 \text{ t } (1-x)^2 M_H^2$$

where $1-x$ and $1-y$ are the momentum fractions carried by the light quarks. Using mean values $< 1-x >\sim \epsilon$ and $< y >\sim \frac{1}{2}$ and keeping only first order terms in ϵ we obtain

$$Q^2 = \frac{\epsilon}{2} M_H^2, \tag{4}$$

where ϵ is related to the position of the maximum of the distribution amplitude ψ_H in the heavy meson

$$\epsilon^2 \sim \frac{m^2 + < k_\perp^2 >}{M_H^2}, \tag{5}$$

and m is the mass of the light quark. For example, in the decay of a B, Eq. 5 gives [9, 10] $\epsilon \sim$ 0.05-0.1, $Q^2 \sim 1.6 GeV^2$, and by taking Λ^2_{QCD} = 0.01 GeV^2 we get $\alpha_s(Q^2) \sim$ 0.38 which we consider small enough to justify a perturbative expansion.

For light pseudoscalar mesons L_i, the required wave function including spin factors can be written in the form [5]

$$\begin{aligned}\psi_i = \psi_i(y, P_i) &= \sum_{\lambda\lambda'} a_{\lambda\lambda'} u_\lambda(p_1 \simeq yP_i)\overline{v}'_\lambda(p_2 \simeq (1-y)P_i) \\ &= \frac{1}{\sqrt{2}}\frac{I_c}{\sqrt{3}}\phi_i(y)\gamma_5 \not{P}_i,\end{aligned}$$

where I_c is the identity in color space and we have ignored all terms proportional to M_i. For simplicity we assume the same wave function for all 0^- mesons in the lowest octet. For a heavy pseudoscalar meson the simplest amplitude contains at least two terms

$$\psi_H = \psi_H(x, P_H) = \frac{1}{\sqrt{2}}\frac{I_c}{\sqrt{3}}\phi_H(x)\gamma_5(\not{P}_H + M_H g(x)). \qquad (6)$$

In QCD the integral of the distribution amplitude is related to the meson decay constant

$$\int \phi_i(x)dx = \frac{1}{2\sqrt{3}}f_i, \int \phi_H(x)dx = \frac{1}{2\sqrt{3}}f_H,$$

For the scalar distribution amplitudes, we use [8, 11]

$$\phi_L(x) \sim x(1-x),\ \phi_H(x) \sim \frac{1}{[\frac{\epsilon^2}{(1-x)} + \frac{1}{x} - 1]^2},$$

with ϵ given by Eq. 5. The form of ϕ_H slightly differs from the one used in Ref. [8] but the numerical results are not very sensitive to the specific parametrization.

We expect both terms, in Eq. 6 to be roughly of the same magnitude [12] i.e. g(x) $\sim$ 1, which corresponds to weakly bound quarks (we are neglecting the binding effects, which are of order of few hundred MeV in comparison with the energy scale set by M_H).

At high Q^2 the effective quark distribution amplitude $\phi(x, Q^2)$ satisfies an evolution equation in $\ln Q^2$. The general solution of the evolution equation vanishes as the momentum fraction x, or $1-x$ with $0 \leq x \leq 1$ of each constituent approaches the boundary values; thus the disconnected diagram in T_h (Fig. 2a) does not contribute in Eq. 2. The leading $\alpha_s(Q^2)$ contribution comes from Figs. 2b and 2c. By Lorentz invariance, the decay amplitude can be expressed as [6]

$$\begin{aligned} < L_i|V^\mu|H > &= (P_H^\mu + P_i^\mu - \frac{M_H^2 - M_i^2}{P_j^2} P_j^\mu) F_L(P_j^2) \\ &+ \frac{M_H^2 - M_i^2}{P_j^2} P_j^\mu F_T(P_j^2), \end{aligned} \tag{7}$$

where $P_j = P_H - P_i$ and $F_{L,T}$ -denote longitudinal and transverse form factors respectively; we have $F_L(0) = F_T(0)$ and in the limit of unbroken flavor symmetry $F_L(P_j^2) = F_T(P_j^2)$. Since we are interested in processes in which squared masses in the final state are small in comparison with the squared mass of the decaying meson, we can neglect all mass terms but M_H.

To first order in $\alpha_s = \alpha_s(Q^2)$ (see Eq. 4) Figs. 2b and 2c give

$$< L_i|V^\mu|H > =$$

$$\frac{8\pi\alpha_s}{3}\Bigg(\int\limits_0^1 dx \int\limits_0^{1-\epsilon} dy\,\phi_H(x)\frac{Tr[\not{P}_i\gamma_5\gamma^\nu\not{k}_1\gamma^\mu(\not{P}_H + M_H g(x))\gamma_5\gamma_\nu]}{k_1^2 Q^2}\phi_i(y)$$
$$+\int\limits_0^1 dx \int\limits_0^{1-\epsilon} dy\,\phi_H(x)\frac{Tr[\not{P}_i\gamma_5\gamma^\mu(\not{k}_2 + M_H)\gamma^\nu(\not{P}_H + M_H g(x))\gamma_5\gamma_\nu]}{(k_2^2 - M_H^2)Q^2}\phi_i(y)\Bigg). \tag{8}$$

The integration over momentum fraction of a quark in the light meson in the interval $1-\epsilon \leq x \leq 1$ corresponds to the Drell-Yan-West [13] end-point region. It gives only small correction to the form factors of nominal order $\epsilon/\alpha_s(Q^2) \sim 13\%$. One also expects additional suppression from Sudakov form factors in this region.

From Eq. 7,8 the form factors are given by

$$\begin{aligned}
F_T(P_j^2) &= \frac{P_{j\mu}}{M_H^2} < L_i|V^\mu|H > \\
&= \frac{2}{3}\frac{16\pi\alpha_s}{M_H^2}\int\limits_0^1 \int\limits_0^{1-C} dy\,\phi_H(x)\Bigg[\frac{2g(x) - y - (1-y)\frac{P_j^2}{M_H^2}}{(1-x)(1-y)^2(1-\frac{P_j^2}{M_H^2})} \\
&- \frac{\frac{P_j^2}{M_H^2}}{(1-x)(1-y)(1+\frac{P_j^2}{M_H^2})}\Bigg]\phi_i(y),
\end{aligned}$$

$$\begin{aligned}
F_L(P_j^2) &= -\frac{P_j^2}{(P_j^2 - M_H^2)^2}(P_{H\mu} + P_{i\mu} - \frac{M_H^2}{P_j^2}P_{j\mu}) < L_i|V^\mu|H > \\
&= \frac{2}{3}\frac{16\pi\alpha_s}{M_H^2}\int\limits_0^1 dx \int\limits_0^{1-t} dy\,\phi_H(x)\Bigg[\frac{2g(x) - y - (1-y)\frac{P_j^2}{M_H^2}}{(1-x)(1-y)^2(1-\frac{P_j^2}{M_H^2})^2}
\end{aligned}$$

$$+ \quad \frac{\frac{P_j^2}{M_H^2}}{(1-x)(1-y)(1-\frac{P_j^4}{M_H^4})}\Bigg] \phi_i(y).$$

For g(x) = g = $\frac{1}{2}$ the above expressions simplify considerably. The dependence of both form factors on g is quite similar, but the magnitude is strongly g-dependent. To see how large this dependence could be, we examine B^0 decay modes listed previously. For example, for $B^0 \to \pi^-\pi^+$, in the Δb = 1, As = AC = 0 sector from Eq. 2 we have

$$M(B^0 \to \pi^-\pi^+) = \kappa\frac{1}{3}f_{B^0}P_{B^0\mu} < \pi^-\pi^+|V^\mu|0 > \\ + \kappa f_{\pi^+}P_{\pi^+\mu} < \pi^-|V^\mu|B^0 >, \kappa = \frac{G_F}{\sqrt{2}}V_{ud}^*V_{ub}, \tag{9}$$

where we have used the fact that $< \pi^+|V^\mu|B^0 >= 0$ in a spectator model. The first term in the above expression, corresponding to the annihilation of the heavy meson is proportional to $[(M_{\pi^+}^2 - M_{\pi^-}^2)F_\pi(M_B^2) - M_B^2F_-(M_B^2)]$where F_π and F_- are the pion form factors

$$< \pi, P'|V^\mu|\pi, \mathbf{P} >= F_\pi((P'-P)^2)(P'+P)^\mu+F_-((P'-P)^2)(P'-P)^\mu.$$

Because of isospin symmetry F_- = 0, and this term gives zero contribution; the only contribution comes from the second term in Eq. 9. Similarly, in the other decays the matrix element of the current always vanishes for a transition to one of the two light mesons, whereas the annihilation contribution is in general very small [6] and can be neglected. Our predictions for the branching ratios are given in Table 1. In Fig. 3 the branching ratios are shown as a function of

Decay mode	Width $[\frac{G_F^2}{16\pi} M_{B^0}^3]$	Width $[10^4 \xi^2\ /sec]$ $g = \frac{1}{2}$	$g = 1$	B.R. $[10^{-8}\xi^2]$ $g = \frac{1}{2}$	$g = 1$
$B^0 \to \pi^+\pi^-$	$f_\pi^2 \vert V_{ud}^* V_{ub}\vert^2 F_T^2(M_\pi^2)$	6.14	180.44	5.77	169.53
$B^0 \to K^+\pi^-$	$f_K^2 \vert V_{us}^* V_{ub}\vert^2 F_T^2(M_K^2)$	0.44	13.37	0.42	12.56
$B^0 \to D^0 K^0$	$\frac{1}{9} f_D^2 \vert V_{cs}^* V_{ub}\vert^2 F_T^2(M_D^2)$	1.40	62.45	1.32	58.67
$B^0 \to D_s^+\pi^-$	$f_{D_s}^2 \vert V_{cs}^* V_{ub}\vert^2 F_T^2(M_{D_s}^2)$	12.30	576.18	11.56	541.34

Table 1: Decay widths and branching ratios for B^0 decay modes in terms of $\xi = 10|V_{ub}/V_{cb}|$. The units are given in square brackets.

the parameter g and compared to those obtained from CQM calculations and the available experimental data. All theoretical results are upper limits due to the present bound for the ratio $|V_{ub}/V_{cb}| \leq 0.1$. We see that the calculated branching ratios lie much below both the experimental upper limits and other theoretical calculations.

We have shown that in the nonleptonic decays of a heavy meson the decay amplitude can be separated into long range and hard (T_h) parts, and that QCD perturbative techniques can be used for T_h. We have applied this method to some decays of the B^0. The approximations made here are even more valid for T^0 (top quark meson). We are examining higher order (in α_s) corrections to the B^0 decays discussed here.

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Figure Captions

Fig. 1 Expansion of $< L_i|V^\mu|H >$ in terms of $q\bar{q}g^n$ distribution amplitudes. Solid, wavy and dashed lines represent quarks, gluons and external current respectively.

Fig. 2 Expansion of T_h in powers of $\alpha_s(Q^2)$. Disconnected diagram vanishes after integration with collinear meson distribution amplitudes.

Fig. 3 Branching Ratios for four B^0 decay modes. Dots are the PQCD predictions. Experimental upper limits Ref. [14]; other theoretical estimations Ref. [6, 7].

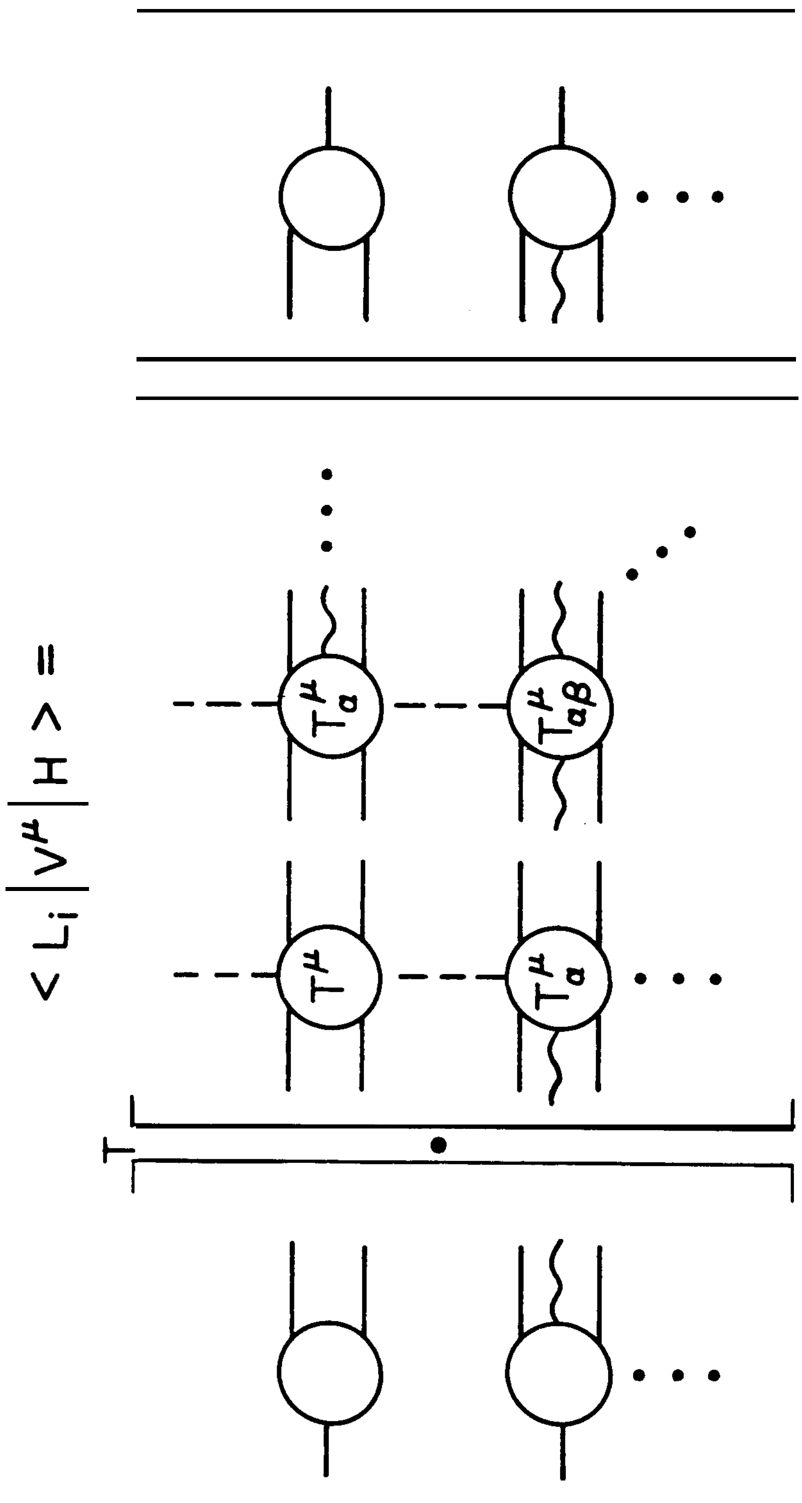

Figure 1

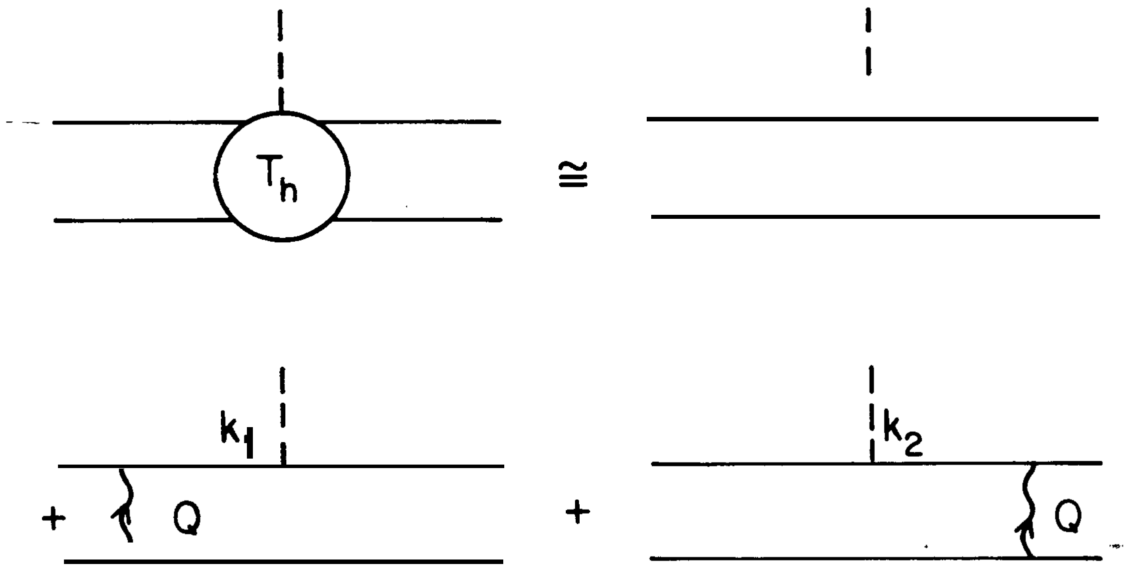

Figure 2

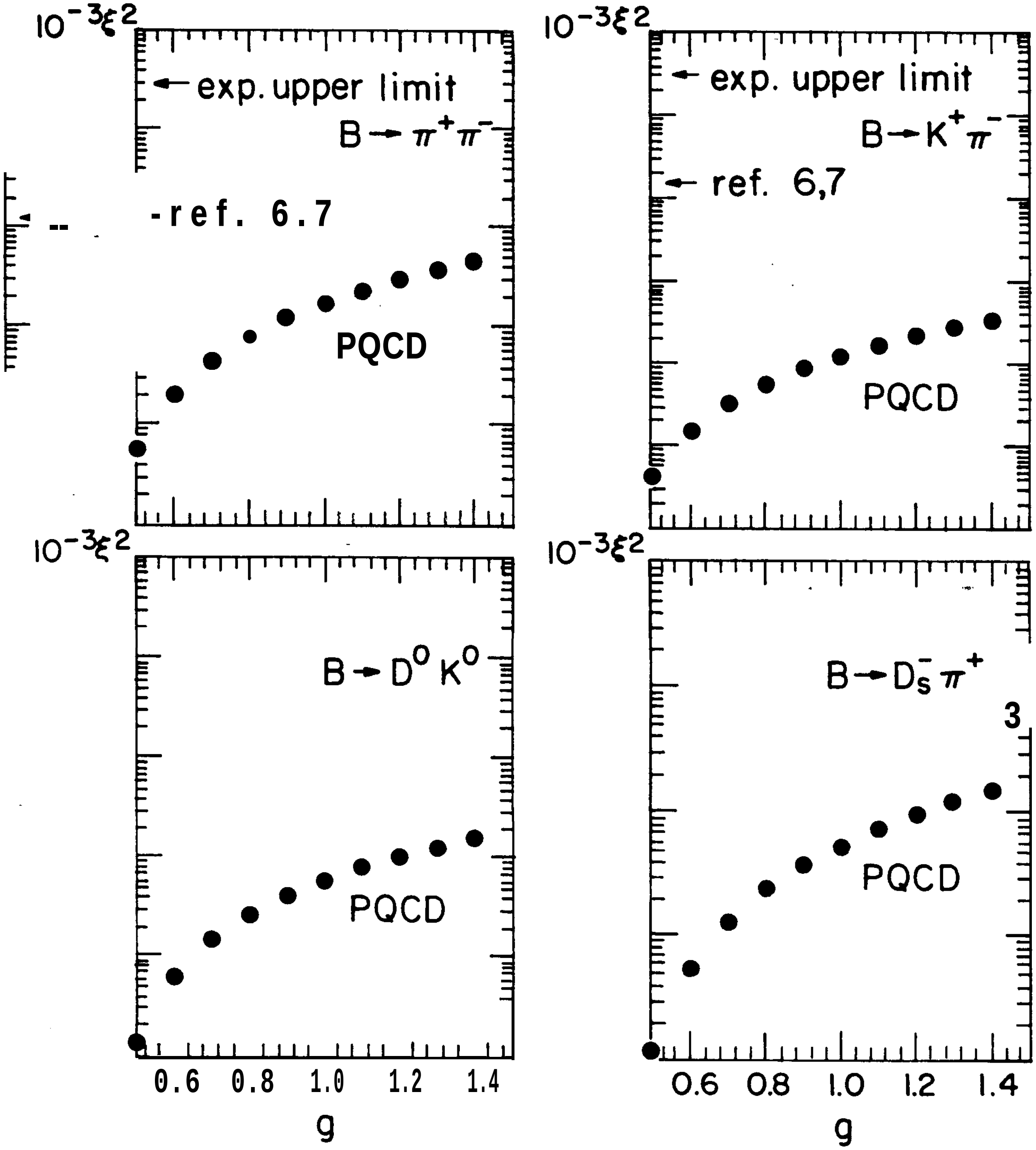
$10^{-3}\xi^2$
← exp. upper limit
$B \to \pi^+\pi^-$
-ref. 6.7
PQCD
$10^{-3}\xi^2$
← exp. upper limit
$B \to K^+\pi^-$
← ref. 6,7
PQCD
$10^{-3}\xi^2$
$B \to D^0 K^0$
PQCD
0.6 0.8 1.0 1.2 1.4
g
$10^{-3}\xi^2$
$B \to D_s^- \pi^+$
3
PQCD
0.6 0.8 1.0 1.2 1.4
g

Figure 3