SLAC-PUB-5225
April 1990
T

Do Weak Interactions Become Strong at 10 TeV?*

ROBERT SINGLETON JR.

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94309

LEONARD SUSSKIND* AND LÁRUS THORLACIUS†

Department of Physics
Stanford University, Stanford, California 94305

ABSTRACT

Simple model calculations indicate that instanton estimates of high-energy scattering amplitudes are unreliable when the number of final state particles becomes large. This casts doubt on recent claims that the cross section for baryon number violation becomes large at high energy.

Submitted to Nuclear *Physics B*

⋆ **Work supported by the Department of Energy, contract** DE–AC03–76SF00515.
† **Supported in part by NSF grant** PHY89-17438

1. Baryon Number Violation and Instantons

Years ago 't Hooft pointed out that baryon number is not strictly conserved in the electroweak theory [1]. He used semi-classical techniques to estimate amplitudes which contribute to baryon number violation and showed that they are non-zero but too small to be observed. The processes he considered involve a minimum of twelve fermions (in a three generation theory) and the system has to pass between gauge sectors of different winding number. That requires tunneling under a high barrier and the amplitudes are suppressed by a factor of $e^{-2\pi/\alpha_w}$. Recently this problem was revisited by Ringwald [2] and Espinosa [3]. They used instanton methods to compute amplitudes with some W, Z and Higgs bosons in the final state, in addition to the required minimum number of fermions, and found that the cross section for such inclusive processes increases fast with the number of particles in the final state. The total baryon number violating cross section then rises rapidly with energy as higher thresholds for multi-particle production are passed. In a subsequent paper McLerran *et al.* [4] obtained the following energy dependence for this total cross section:

$$\sigma_{tot} \sim e^{-4\pi/\alpha_w}\, e^{\alpha_w A E^2/M_w^2}\,, \tag{1.1}$$

where A is some constant of order unity. Here and throughout we take $M_h \sim M_w$ for simplicity. This expression includes the usual instanton suppression, but if the energy increases beyond $E \sim M_w/\alpha_w$ the suppression factor is overwhelmed, and the cross section becomes large. Eventually it violates the upper bound set by unitarity. Of course, the analysis leading to (1.1) is based on a number of assumptions, many of which fail before such high energy is reached. As an example, radiative corrections are no longer small when the number of gauge and Higgs bosons in the final state becomes approximately $1/\alpha_w$. This was pointed out by Ringwald in ref. [2] and he made no serious attempt to extrapolate his results to energies where unitarity would be violated. McLerran *et al.* [4] tried to go beyond the usual semi-classical approximations and present an estimate of the total cross section at energies above $E \sim M_w/\alpha_w$. They claim to have included the 'back-reaction' of the large number of external particles on the instanton. Yet, they still find a rapidly rising total cross section with energy and conclude that fermions become strongly interacting at energies above $E \sim M_w/\alpha_w \sim 10$ TeV. If this surprising prediction turns out to be true, it would provide an arena for experimental study of new non-perturbative physics at future colliders such as the SSC.

The energy at which the cross section is expected to become large is $\sim M_w/\alpha_w$ which is also the height of the tunneling barrier separating adjacent perturbative vacua. One might be tempted to argue that the instanton suppression is overcome because there is enough energy available to get across the barrier. Thus the fields no longer have to tunnel through and the reaction can readily proceed [5]. Such reasoning would, however, almost certainly be incorrect. Consider a process involving two incoming fermions with total center of mass energy of order the barrier height. Each fermion is dressed with bosonic fields, which are of strength $\sim \sqrt{\alpha_w}$. For baryon number violation to happen without tunneling the system must pass over the tunneling barrier. Since the available energy is just about the height of the barrier, this can only be done by passing over the minimum of the saddle, through a field configuration called a sphaleron. A sphaleron is a coherent state characterized by gauge fields of strength $1/\sqrt{\alpha_w}$ with Fourier modes corresponding to wavelength $\sim 1/M_w$. The dominant decay mode of a real sphaleron is to produce $\sim 1/\alpha_w$ soft bosons and the amplitude for decay into a small number of particles is exponentially suppressed in comparison [6]. This would seem to support the above picture because the instanton induced cross section becomes large precisely when the final state has a large number of particles. On the other hand, for $\alpha_w \ll 1$ there is a very big mismatch between the sphaleron and the initial field configuration. The point is that, although the system has sufficient kinetic energy to climb the barrier, it corresponds to motion in the wrong direction in configuration space. It is as though we tried to kick a ball over a hill by kicking it in the wrong direction. The energy is there but unless it can somehow be redirected it is useless. Since the theory is renormalizable, and the coupling between modes is weak, it is unlikely that the energy gets redirected in the required way in a very short time. After $t \sim 1/M_w$ the high-frequency modes will have separated to a distance greater than the size of the sphaleron and cannot reassemble. Thus the mismatch between the initial field configuration, with a few high-energy particles, and the final state with a large number of soft bosons, leads one to believe that this sort of amplitude should be highly suppressed. Even if a sphaleron configuration readily decays into a large number of particles the high-energy scattering amplitudes remain small simply because the incoming high-energy particles are unlikely to form a sphaleron.

This leaves the question of why the instanton calculation appears to give large cross sections. First of all, the Euclidean path integral, on which the instanton estimates are based, is not well suited to describe particles with high energy. On-shell Euclidean momenta are limited by the particle masses and obtaining the Minkowski space Green's function from the Euclidean one by analytical continuation can be a delicate matter. In the leading instanton approximation one is instructed to replace external fermions by their zero modes and external bosons by their value in the instanton configuration. This guarantees that the Green's function is separable, i.e. its only singularities are poles associated with on-shell external particles. The corresponding S-matrix element does not depend on energy at all. Instanton induced amplitudes can thus be generated from a point-like effective multi-particle vertex, which explains the rapid growth with energy in the instanton estimate of the total cross section. It is not clear whether this separability persists in a more careful analysis [7,8].

There is another problem with applying instanton methods to processes which involve a large number of particles in the final state. The naive instanton approximation basically computes an overlap between initial and final states. Having a large overlap between a state with $\sim 1/\alpha_w$ quanta concentrated in a volume $\sim M_w^{-3}$ in a vacuum with baryon number n and a state with very few quanta in the $n-1$ vacuum does not necessarily imply a large transition amplitude between such states. In the following section we illustrate this idea by a simple quantum mechanical system, with a double well potential, which shares many features with the more complicated field theory problem. In particular, one can set up a situation where a transition amplitude for tunneling between the two wells is clearly small, but a naive instanton estimate nevertheless yields a large result. One can of course not draw any definitive conclusions about electroweak theory from such simple model calculations, but our results indicate that naive instanton calculations do not capture the correct physics of high-energy scattering.

Better estimates are therefore needed to discuss such amplitudes. Ideally one would address the problem directly in Minkowski space, but it is not clear how to proceed there. Instead attempts have been made to improve on the Euclidean space calculations, for example by allowing for the distortion of the instanton saddle point due to large numbers of external quanta in Green's functions. The claim that the weak interactions become strong at energies above 10 TeV is backed by an argument which is meant to take that distortion into account [4]. We briefly review this 'back-reaction' argument at the end of Section 2. It involves coupling the system to a strong external field and relies on continuing the momenta of the final state bosons from the mass shell to zero. This continuation is only valid if the instanton amplitudes are really separable in the sense described above. When applied to our quantum mechanical model this back-reaction procedure leads to a considerable enhancement over the naive instanton estimate, which is already too large. Aoki [9] has considered the back-reaction of external particles on instantons from a different viewpoint. He finds a high-energy cross section, which is dominated by an instanton configuration whose scale is set by the momentum of the incident fermions, and does not grow with energy. The calculations are based on a perturbative approach [10], which is not applicable when the number of external particles is $\sim 1/\alpha_w$, and therefore they do not directly address the issue raised by McLerran *et al.* [4].

It remains a challenging problem to estimate correctly the dominant baryon number violating cross section at high energy. To get some idea of the physics involved we consider the 1+1 dimensional Abelian Higgs model with fermions. This theory has instantons which give rise to violation of fermion chirality in scattering amplitudes. The probability of such processes is supposed to rise rapidly with energy [4] in much the same way as the instanton induced cross sections in electroweak theory. Consider the scattering of two fermions, of the same chirality, producing some number of bosons. The incident high-energy fermions, each of momentum p, are Lorentz contracted as in Figure 1. Nothing happens until the fermions overlap, at which time, energy is concentrated in a spatial volume $\delta x \sim 1/p$ for a short time $\delta t \sim 1/p$. If a tunneling event is to happen it must involve field configurations of similar space-time extensions. Thus we should expect qualitatively to replace large instantons by rescaled configurations of size $1/p$. The tunneling barrier for such rescaled instantons is $\sim p/\alpha$ and is always higher than the available energy. In this picture, the baryon number violating cross section is dominated by configurations with a few high-energy jets and not by $\sim 1/\alpha$ non-relativistic bosons. The cross section will of course be exponentially suppressed.

Mueller [11] has given a similar argument in 3+1 dimensions. If the incoming particles have energy $E \gg M_w$, the interaction region, which supplies the dominant contribution to the scattering amplitude, has volume $\sim 1/E^3$. This follows from causality, Lorentz invariance and the uncertainty principle. The minimum action configuration of that scale goes through a sphaleron-like coherent state with energy $\sim E/\alpha_w$. The high-energy particles thus see a raised barrier. The exponential suppression of the amplitude does not depend much on energy at all since E scales out of the value of the minimum tunneling action.

It should be stressed that the problem at hand, involving high-energy scattering amplitudes, is quite distinct from the question of baryon number violation at high temperature. In a heat bath at temperature T, where $M_w \ll T < M_w/\alpha_w$, the correlation length is $\sim 1/\alpha_w T$ and Fourier modes with wavelength $\sim 1/M_w$ will be thermally populated. The high-temperature field configuration thus contains the ingredients that go into making a sphaleron and the above mismatch argument does not apply. Thermally activated processes violating baryon number conservation via the sphaleron configuration can be described in terms of classical statistical physics and are only suppressed by a Boltzmann factor $e^{-M_w/\alpha_w T}$ [12,13]. The relevant kinetic theory involves classical fields, *i.e.* coherent states with $\sim 1/\alpha_w$ quanta, rather than collisions of individual quanta.

2. A Simple Model

The suggestion that the weak interactions become strong in high-energy collisions at energy above ten TeV's is based on instanton calculations involving a large number of particles in the final state. It is very surprising to find large cross sections for processes where incoming high-energy quanta produce a multitude of soft particles. Therefore, one should perhaps question the validity of the instanton approximation under these circumstances. Unfortunately we do not have an alternate method for quantitatively computing the relevant field theory scattering amplitudes. Instead, our approach will be to apply instanton methods to study tunneling in a particularly simple system, where we have a good handle on the physics, and see if they produce reliable results for amplitudes involving many. quanta in the final state.

Consider a two-dimensional quantum mechanics problem with the following Lagrangian,

$$L = \frac{1}{\alpha}\left[\frac{1}{2}\dot{x}^2 + \frac{1}{2}\dot{y}^2 - V(x,y)\right], \tag{2.1}$$

where $V(x, y)$ is a double well potential,

$$V(x,y) = \frac{\omega_0^2}{2}\frac{x^2(x-a(y))^2}{a^2(y)}. \tag{2.2}$$

The potential has two troughs, one along $x = 0$ and the other along x = u(y). The $a^2(y)$ in the denominator ensures that the oscillator frequency in the s-direction is fixed at ω_0 along the bottom of both troughs. In order to define initial and final states localized in one potential well or the other we want the wells to be separated for asymptotic y, but at the same time the two directions should only be weakly coupled. This is achieved by choosing the function u(y) as follows,

$$a^2(y) = 1 + \beta^2 y^2 \,, \tag{2.3}$$

where β is a small coefficient. The parameter $\alpha = 1/mx_0^2$ plays the role of the squared coupling constant in the field theory problem. Here m is the mass of the particle and x_0 is the minimum separation of the two troughs. The variables in the Lagrangian (2.1) have been rescaled by x_0. Since there is only weak coupling between the x and y directions the energy eigenstates are approximated by products of plane-waves along y and the usual double-well eigenstates in x. Of course the x eigenstates will also depend on y through u(y).

Consider a particle at large negative y traveling along the bottom of the trough centered around x = 0 with momentum q in the positive y direction. Classically the particle never leaves the initial well, but it can find its way to the other one by tunneling. The minimum separation between the potential wells is at y = 0 and there the barrier between them is lowest. Therefore the transition amplitude comes mostly from that region. By dialing α we can make the barrier between the wells as high as we like and the tunneling probability arbitrarily small. The idea is to use instanton methods to compute the tunneling amplitude. The system has an instanton with a Euclidean path along y = 0, which describes the tunneling of a stationary particle from the origin to x = 1. The Euclidean solution is identical to that of the one-dimensional double well instanton,

$$x_{inst}(\tau) = \frac{1}{2}\left(1 + \tanh \frac{\omega_0 \tau}{2}\right) . \tag{2.4}$$

In our case the particle has momentum q in the y-direction and strictly speaking there is no path with finite Euclidean action satisfying the appropriate initial conditions. We can, however, approximate the transition amplitude by using the one-dimensional instanton to compute an 'instantaneous' tunneling amplitude in the neighborhood of y = 0 and cutting off the zero mode integration, over the location of the center of the instanton, at $\tau \sim 1/\beta q$. This approximation is somewhat analogous to placing external momenta on the Euclidean mass-shell when computing instanton amplitudes in field theory. Momentum is not conserved in this problem so an incoming particle with some kinetic energy can tunnel into excited states of the second well. On the other hand, if the coupling between x and y is weak we would expect amplitudes, which transfer a lot of kinetic energy into x-oscillations to be suppressed. This directly corresponds to the argument we made in the previous section about mismatch between initial and final field configurations suppressing real-life scattering cross sections. The instanton approximation misses this and therefore we might expect it to be unreliable.

In order to calculate transition amplitudes into excited final states we define a Heisenberg operator,

$$\tilde{x}(\omega) = \int_{-\infty}^{\infty} dt\, (x - u(y))\, \theta(t)\, e^{i\omega t}\,. \tag{2.5}$$

Since $x(t)$ oscillates with frequency ω_0, as $t \to \infty$, $\tilde{x}(\omega)$ has a pole

$$\tilde{x}(\omega) \to \frac{1}{\omega - \omega_0}\, \overline{x}(\omega)\,. \tag{2.6}$$

If we apply $\frac{1}{\sqrt{n!}} \left[\sqrt{\frac{\omega_0}{2\alpha}}\, \tilde{x}(\omega) \right]^n$ to the ground state, and let $\omega \to \omega_0$, the system will be excited to the nth level in the second well. The factor of $1/\sqrt{n!}$ has to be included for proper normalization. In terms of a path integral the amplitude is

$$\mathcal{A}_n = \frac{(\omega-\omega_0)^n}{\sqrt{n!}} \int \mathcal{D}x \left\{ \sqrt{\frac{\omega_0}{2\alpha}} \int_0^{\infty} dt\, \left(x(t) - a\right) e^{i\omega t} \right\}^n e^{i\int dt\, L}\,. \tag{2.7}$$

where the limit of $\omega \to \omega_0$ is understood. Wick rotation gives

$$\mathcal{A}_n = \frac{(\omega-\omega_0)^n}{\sqrt{n!}} \int \mathcal{D}x \left\{ \sqrt{\frac{\omega_0}{2\alpha}} \int_0^{\infty} d\tau\, \left(x(\tau) - a\right) e^{\omega\tau} \right\}^n e^{-S_E[x]}\,. \tag{2.8}$$

Now we simply saturate this with the one-dimensional instanton (2.4). The asymptotic behavior, $x_{inst} \sim 1 - e^{-\omega_0 \tau}$, gives rise to the correct pole in the τ integral. When we have included the instanton zero-mode integral our estimate of the transition amplitude reads

$$\mathcal{A}_n \sim \frac{1}{\beta q} \frac{(-1)^n}{\sqrt{n!}} \left(\frac{\omega_0}{2\alpha}\right)^{n/2} e^{-\omega_0/6\alpha} . \tag{2.9}$$

This has the characteristic features of the instanton amplitudes of refs. [2] and [3]: exponential suppression and a factor of $1/\sqrt{\alpha}$ for each final state quantum. We find the total probability for the particle to tunnel by squaring the amplitudes (2.9) and summing over final states up to a point determined by energy conservation. In the limit of very high energy the sum gives an exponential and we find a total probability,

$$P_{tot} \sim \frac{1}{\beta^2 q^2} e^{+\omega_0/6\alpha} . \tag{2.10}$$

If α is small enough this result is considerably larger than one. Evidently we have drastically overestimated the amplitude (or perhaps we have explained cold fusion). Having a tunneling probability larger than one in this problem corresponds to instanton amplitudes violating unitarity in electroweak theory.' It is clearly incorrect to sum over all values of n in (2.9). We obtain a more reasonable answer if we only include final states with energy less than the barrier height. However, there is no physical prescription which tells us where the sum should be cut off and it is not clear how to get a useful result out of this. What is clear is that the actual tunneling probability in this problem is exponentially small. At no point in the evolution of the system does the wave function ever leak appreciably into the well at $x = u(y)$.

We have arranged our tunneling calculation in a way which brings out the similarity with instanton calculations in field theory. We now discuss the same problem from a slightly different point of view, which makes it clear what goes wrong with the instanton estimate. We will again do an approximate calculation which assumes that tunneling occurs only when the two wells are close together. This time, we will use the Hamiltonian, $H_0 = \frac{\alpha}{2} p_x^2 + \frac{1}{\alpha} V(x, 0)$, and evaluate the amplitude $\langle\psi_f|\, e^{-H_0T}\,|\psi_i\rangle$. The tunneling probability will be proportional to $\beta^{-2}q^{-2}\left|\langle\psi_f|\, e^{-H_0T}\,|\psi_i\rangle\right|^2$. The amplitude can be obtained by inserting a complete set of H_0 eigenstates. If we take $T \gg \omega_0^{-1}$ the contribution from excited eigenstates is heavily suppressed. There are two low-lying eigenstates with energies $E = \frac{\omega_0}{2} \pm Ke^{-S_0}$. Here S_0 is the Euclidean action associated with the instanton and the factor K is obtained from the path-integral determinant at the instanton saddle point [14]. The small energy difference gives rise to beats between the wells. We can assume that T is short compared the characteristic time for these beats to-avoid tunneling back into the original well. The two low-lying eigenstates are denoted $|-\rangle$ and $|+\rangle$, and they are the spatially even and odd combinations of the particle ground-states in the two wells:

$$
\begin{aligned}
|-\rangle &= \frac{1}{\sqrt{2}}\left(|0\rangle + |0'\rangle\right),\\
|+\rangle &= \frac{1}{\sqrt{2}}\left(|0\rangle - |0'\rangle\right).
\end{aligned}
\tag{2.11}
$$

The dominant contribution to the tunneling amplitude comes from.these states,

$$
\begin{aligned}
\langle\psi_f|\, e^{-H_0T}\,|\psi_i\rangle &\simeq \left(\langle\psi_f|-\rangle\,\langle-|\psi_i\rangle + \langle\psi_f|+\rangle\,\langle+|\psi_i\rangle\right) e^{-\omega_0 T/2\alpha}\\
&= \left(\langle\psi_f|0\rangle\,\langle 0|\psi_i\rangle + \langle\psi_f|0'\rangle\,\langle 0'|\psi_i\rangle\right) e^{-\omega_0 T/2\alpha}\,.
\end{aligned}
\tag{2.12}
$$

The factor of $e^{-\omega_0 T/2\alpha}$ comes from the zero point energy and can be absorbed into the normalization. The instanton amplitude is thus simply given by the overlaps of the initial and final states with the ground states of the two wells. If the initial and final states are $|0\rangle$ and $|0'\rangle$ respectively then the instanton amplitude is particularly simple:

$$
\langle 0'|\, e^{-H_0T}\,|0\rangle \sim \langle 0'\,|0\rangle \sim e^{-S_0} = e^{-\omega_0/6\alpha}. \tag{2.13}
$$

This agrees with our previous result. We can also consider more general amplitudes where the particle starts out in the ground state of one well but tunnels to some excited state in the other well. At each energy level of a double well system there is an almost degenerate pair of eigenstates, one even and the other odd, and these can be combined to form a state localized in one well. Such states can be approximated by harmonic oscillator states centered in the given well (at least for the low-lying levels) and then the overlaps can be obtained in a straightforward way,

$$\langle n' | e^{-H_0 T} |0\rangle \sim \langle n'|0\rangle = \frac{(-1)^n}{\sqrt{n!}} \left(\frac{\omega_0}{2\alpha}\right)^{n/2} e^{-\omega_0/4\alpha} . \tag{2.14}$$

Note that the exponential suppression appears to be greater now than in (2.13). The harmonic oscillator wave-functions fall off more rapidly under the barrier than the actual eigenfunctions of the double well system so we systematically underestimate the overlaps. Squaring the amplitudes and summing over final states gives an estimate for the probability of finding the particle in the second well. However, it is now clear that we have to cut the sum off before we reach harmonic oscillator states with energies of order the barrier height $\sim \omega_0^2/32\alpha$. If we fail to do so we will indeed find a large 'probability' for the particle to tunnel. For small α the initial wave-function is mostly found in the first well and is likely to stay there throughout. Its tail extending into the other well is exponentially suppressed. What is wrong with the 'probability' found above is that for high excitations the wave-functions centered in the second well are peaked far from its center at x = 1. For n $\sim \omega_0/2\alpha$ the wave-function ψ_n is actually concentrated in the first well! This is illustrated in Figure 2 where we have graphed the ground state wave function of the first well and an excited one of the second well, with the parameters arbitrarily chosen to correspond to $\omega_0/\alpha \sim 50$. Large overlaps of the initial state with excited final states, which are peaked in the original well, reflect the fact that the particle is unlikely to tunnel at all, and in particular, do not imply large transition amplitudes.

We can give another related example where instanton methods overestimate a physical effect. Consider a conventional double well problem in one dimension,

$$L = \frac{1}{\alpha}\left[\frac{1}{2}\dot{x}^2 - \frac{\omega_0^2}{2}x^2(x-1)^2\right] . \tag{2.15}$$

We will compute the contribution to the ground state energy from processes in which the particle tunnels from the ground state of one well to an excited state of the second well and back. The energy shift is

$$\delta E_0 = \sum_n \frac{| \mathcal{A}_n |^2}{\omega_0/2 - \omega_n} . \tag{2.16}$$

If we try to use the instanton estimate (2.9) for the transition amplitudes and then sum over all n we find an absurdly large result,

$$\begin{aligned} \delta E_0 &\sim - \sum_n \frac{1}{n!}\left(\frac{\omega_0}{2\alpha}\right)^n \frac{1}{(n-\frac{1}{2})\omega_0} e^{-\omega_0/3\alpha} \\ &\sim -e^{\omega_0/6\alpha} . \end{aligned} \tag{2.17}$$

Again, this demonstrates that instanton methods are unreliable in quantum mechanics when highly excited states are involved.

When the number of quanta in the final state is $\sim 1/\alpha$ the path integral saddle point is no longer dominated by the action and the instanton gets distorted. In the electroweak theory this can occur at an energy $\sim M_w/\alpha_w$ and improved calculations are needed to obtain reliable estimates at that energy and higher. McLerran et al. [4] addressed this back-reaction problem by coupling the Higgs field to an external source and studying the generating functional $Z[J]$. In general there are also gauge bosons in the final state, but for simplicity we concentrate on amplitudes involving Higgs bosons only. The idea is to use analytic properties of $Z[J]$, as a function of the external source field J(x). Correlation functions involving n external Higgs particles are obtained by taking n derivatives of $Z[J]$, and for large n they are determined by the asymptotic behavior of the power series expansion of $Z[J]$. McLerran et *al.* assume that the Higgs particles can be placed at zero four-momentum. Then a constant external source J can be considered and the calculation is simplified enormously. A constant external source distorts the scalar potential and shifts the Higgs expectation value. At a critical value, J_c, a local minimum of the potential becomes unstable and $Z[J]$ develops an imaginary part. The location of the singular point limits the radius of convergence of the power series expansion of $Z[J]$, and we can estimate the limiting behavior of its terms as $(J/J_c)^n$ for large n. Once J_c has been determined, the n Higgs particle Green's function is estimated as

$$\mathcal{A}_n \sim \zeta(n)\, n!/\mid J_c \mid^n , \tag{2.18}$$

where $\zeta(n)$ is some function of n that grows slower than exponentially [4]. McLerran et *al.* applied their method to estimate baryon number violating amplitudes and found an even larger answer than a naive extrapolation of Ringwald's instanton results.

Similar considerations can also be applied to our quantum mechanical problem. In particular we can estimate the effect of adding a constant source term to the Euclidean action. The story is much the same as in the field theory and we will not present the analysis here. We find that this method leads to an even larger result than (2.10). A qualitative physical explanation of this over-estimate can be obtained by following the time evolution of the transition amplitude into an excited state with $n \geq 1/\alpha$. This is shown schematically in Figure 3. At early times the particle is concentrated in the well centered about x = 0 and the wave function overlap with highly excited states in the other well is small. As the particle approaches the origin, where the potential barrier is lowest, the overlap grows, but at late times the overlap becomes minute once again. Since the overall tunneling probability is non-vanishing there is a small amplitude to find the particle in the second well at late times and a characteristic e^{-iE_nT} phase develops. This is represented by the oscillating piece (which is not drawn to scale) in the figure. Evaluating zero-momentum amplitudes corresponds to taking the total area under the curve whereas an on-shell calculation would identify the small oscillating piece at late times.

3. Conclusion

We see no reason to believe that the weak interactions become strong at energies of a few TeV's. There is no obvious physical process in a weakly coupled theory by which incoming high energy particles can produce a sphaleron-size coherent state of non-relativistic W, Z and Higgs bosons. Scaling considerations can be used to argue that weak interaction baryon number violation is exponentially suppressed in high-energy scattering at all energies and that the cross section is dominated by processes involving a few high-energy jets [11].

The large cross sections predicted in ref. [4] were obtained using an instanton approximation. The analysis in the previous section illustrates that one must be very careful in using instantons to calculate transition amplitudes to states with large numbers of quanta in the final state. In our quantum mechanical model large n excitations about one vacuum make substantial excursions into the other vacuum configuration. Naive instanton calculations compute this overlap rather than the actual transition amplitude. Of course, our model is very simple and our results do not rigorously exclude the possibility that large cross sections occur in the real problem. New tools for calculation are called for, to place upper bounds on baryon number violating scattering amplitudes in electroweak theory and lay this question to rest.

Acknowledgements: We thank members of the theory group at the Stanford Linear Accelerator Center for many helpful discussions. We are also grateful to M. Voloshin for confusing us, W. Fischler for listening, B. Warr for advising us not to publish and S. Bahcall for culinary support.

REFERENCES

1. G. 't Hooft, *Phys. Rev. Lett.* 37 (1976), 8, *Phys. Rev.* **D14** (1976), 3432, *Phys. Rev.* **D18** (1978), 2199.

2. A. Ringwald, *Nucl. Phys.* B330 (1990), 1.

3. 0. Espinosa, *High Energy Behavior of Baryon* and *Lepton Number Violating Scattering Amplitudes and Breakdown of Uniturity in the Standard Model,* Caltech preprint, CALT-68-1586, November 1989.

4. L. McLerran, A. Vainshtein and M. Voloshin, *Electroweuk Interactions Become Strong at Energy above* $\sim$ *10 Te V*, University of Minnesota preprint, TPI-MINN-89/36-T, November 1989.

5. H. Aoyama and H. Goldberg, *Phys. Lett.* **188B** (1987), 506.

6. P. Arnold and L. McLerran, *Phys.* Rev. D37 (1988), 1020.

7. V. Zakharov, *Classical Corrections to Instanton Induced Interactions,* University of Minnesota preprint, TPI-MINN-90/7-T, January 1990.

8. P. Arnold and M. Mattis, *Baryon Violation at the SSC? Recent Claims Reexamined,* Los Alamos preprint, LA-UR-90-1218, April 1990.

9. K. Aoki, *On Fermion Number Violation at High Energies,* UCLA preprint, UCLA/90/TEP/7, January 1990.

10. K. Aoki and P. Mazur, *Effects ojjermion back reaction on instuntons,* UCLA preprint, UCLA/89/TEP/67, December 1989.

11. A. Mueller, *On the High Energy Behavior of S-Matrix Elements in the Electroweak Theory,* Columbia preprint, CU-TP-454, February 1990.

12. V. Kusmin, V. Rubakov and M. Shaposhnikov, *Phys. Lett.* **155B** (1985), **36.**

13. M. Dine, 0. Lechtenfeld, B. Sakita, W. Fischler and J. Polchinski, *Baryon Number Violation at High Temperature in the Standard Model,* City College preprint, CCNY-HEP-89/18, December 1989.

14. S. Coleman, *The Uses of Instantons,* in *Aspects of* **Symmetry,** *Selected Erice lectures,* Cambridge University Press, 1985.

FIGURE CAPTIONS

1) Lorentz contracted fermion world lines.

2) Overlap of simple harmonic oscillator wave functions centered in different wells.

3) Schematic large n transition amplitude.

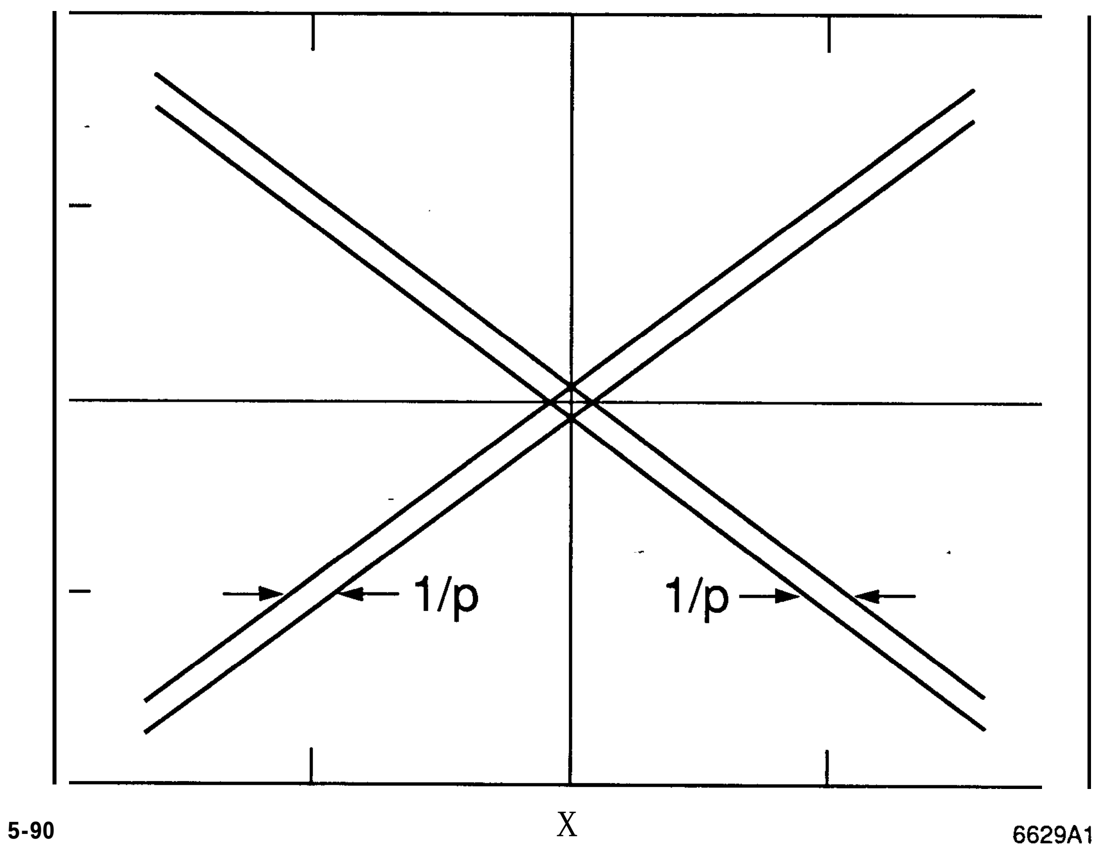

Fig. 1

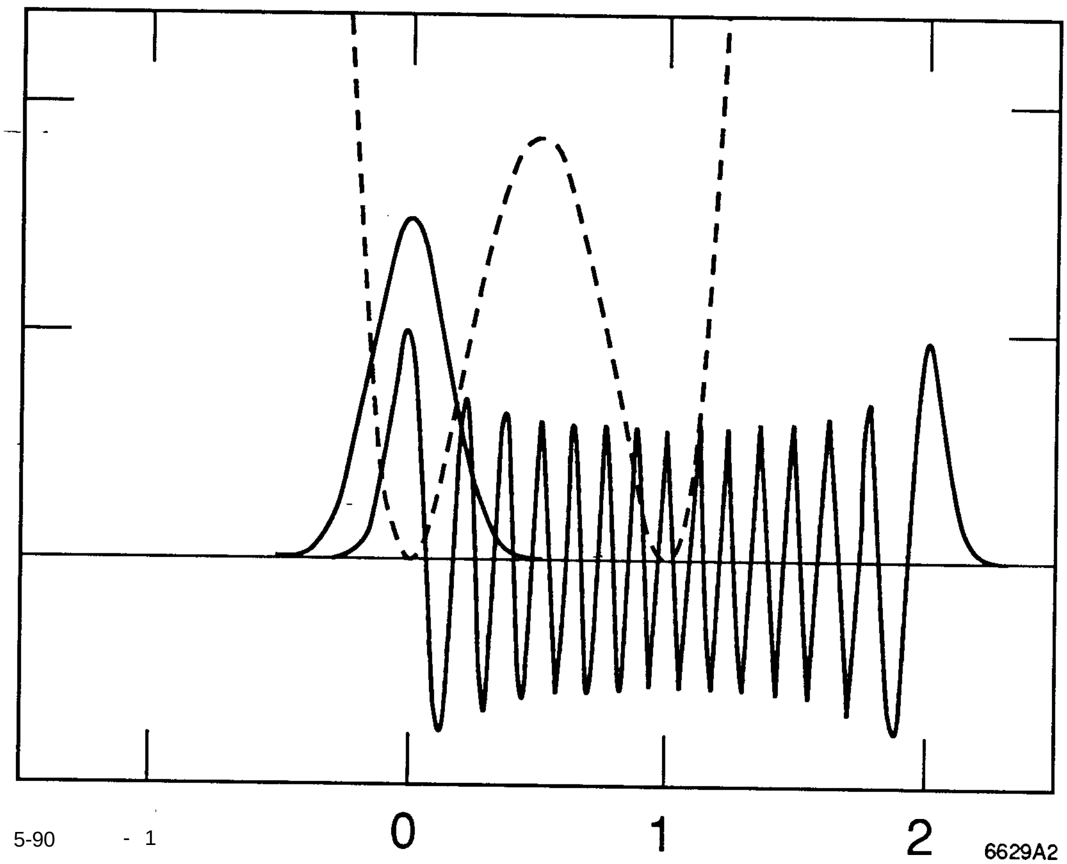

Fig. 2

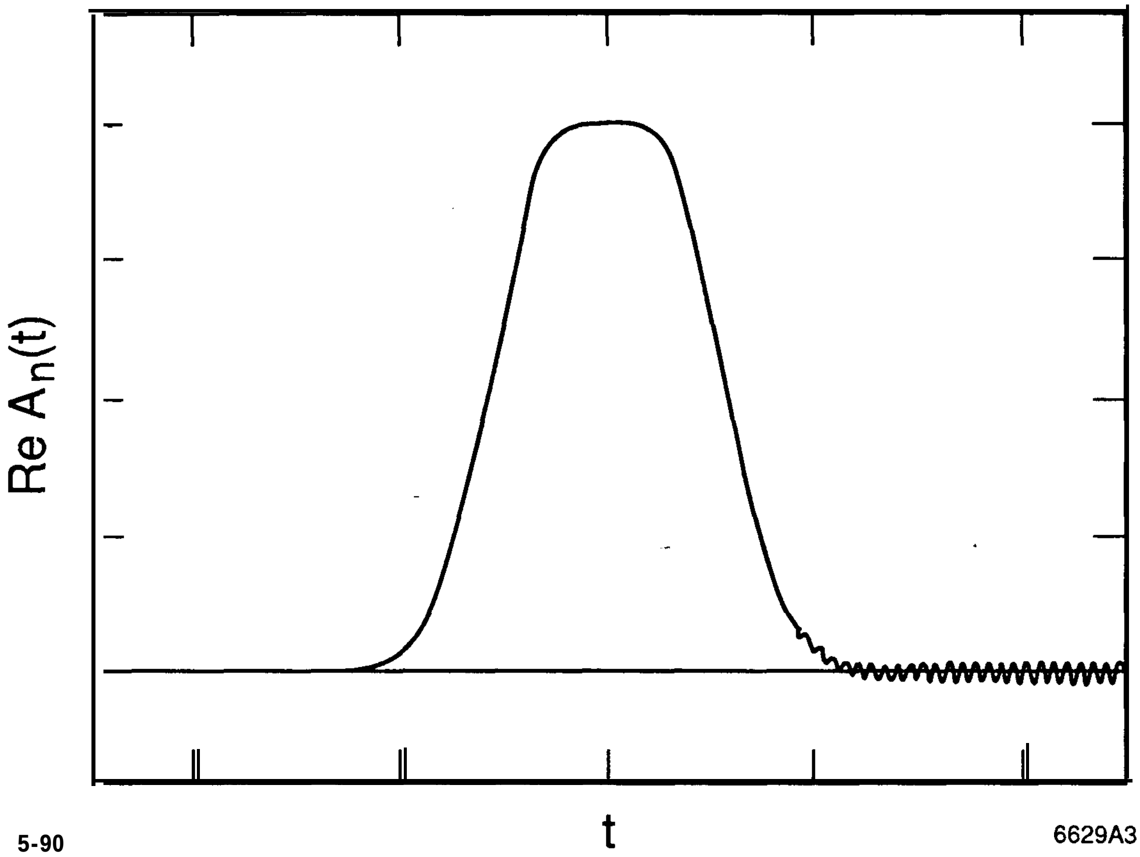

Fig. 3