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DISCRETE GRAVITY*

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INTRODUCTION

General relativity is an *incomplete* theory because it is scale invariant, resting as it does only on the two fundamental constants G and c. It can be enriched, as Wheeler (1) showed long ago, by coupling it to the source-free Maxwell field to form, among other things, *geons*, but this does not change the dimensional situation. To complete the theory, the currently available candidates for dimensional constants are all *quantized*, for example the elementary unit of electric charge, the proton or electron mass (to mention the only two elementary masses that are *stable* within current experience), h/e from the Josephson effect, the quantum of action h, the quantum of angular momentum, $\hbar$, ... Once a third dimensional constant is chosen in a fundamental theory we are required to compute all the others as dimensionless ratios. That this task is not so difficult as one might imagine is evidenced by our summary Table. If we choose $\hbar$ as our third dimensional constant, our unit of mass $[M]$ is the Planck mass $M_P = [\hbar c/G]^{\frac{1}{2}}$, our unit of length $[L]$ is $L_P = \hbar/M_P c$, and our unit of time $[T]$ is $T_P = \hbar/M_P c^2$.

Wheeler's approach (2) is to reduce everything to geometry in terms of the Planck length. Time, as is usual in special and general relativity — and has now been made a matter of definition by the *Conf. Generale de Poids et Mesures*— is linearly related to length by $[T] = [L]/c$. Volume, $[L^3]$, is related to space-time curvature $[R] = [L]$ by the curvature induced by mass of uniform density and specified radius. The new feature compared to classical treatments is that it is now possible to calculate the entropy of a black hole using Planck's constant, G and c. This turns out to be proportional to the area, $[L^2]$, completing Wheeler's geometrical picture (2, p. 220). Our equivalent choice of units emphasizes the connection to elementary particle physics expressed by the relation $\frac{\hbar c}{G m_p^2} = (M_{Planck}/m_{proton})^2$.

CONSTRUCTING A BIT-STRING UNIVERSE

The quantum theory of gravitation and elementary particles which we are in the process of constructing comes from interweaving several different lines of research, the earliest of which started with Bastin and Kilmister in the 50's and led to the discovery of the *combinatorial hierarchy* — i.e. the terminated sequence $3, 10, 137, 2^{127} + 136$ — by Parker-Rhodes in 1961. This discovery was reported by Bastin (3), and further developed by Bastin, et.al.(4). The most fundamental recent development, which has also shed new light on the work of Stein, Gefwert, Manthey and Etter, is McGoveran's (5) *ordering operator calculus.* Some physical consequences have been published by Noyes and McGoveran (6), and the theory is undergoing rapid development.

The common thread which unites this work is the representation of the fundamental entities by *bit-strings*:

$$\mathbf{a}(S) = (..., b^a_s, ...)_S;\ b^a_s \in 0, 1;\ s \in 1, 2, ...S;\ 0, 1, ..., S \in \ \textit{ordinal integers} \quad (1)$$

which can combine by *discrimination* (XOR) symbolized by "$\oplus$":

$$\mathbf{a} \oplus \mathbf{b} = (..., b^{a\oplus b}_i, ...)_S = \mathbf{b} \oplus \mathbf{a};\ b^{a\oplus b}_i = (b^a_i - b^b_i)^2 \quad (2)$$

or *concatenation* symbolized by "$||$":

$$\mathbf{a}(S_a)||\mathbf{b}(S_b) = (....b_i....)_{S_a}||(....b^b_j...)_{S_b} = (......, b^{a||b}_k, ...)_{S_a+S_b}$$

$$b^{a||b}_k = b^a_i,\ i \in 1, 2, ..., S_a;\ b^{a||b}_k = b^b_j,\ j \in 1, 2, ..., S_b, k = S_a + j \quad (3)$$

Disagreement as to the proper foundations for the theory stem from different assumptions about how the symbols "0" and "1" are to be generated or constructed in the first place, how the two operations themselves are generated or constructed, and how they are to be interleaved to generate strings of sufficient complexity to model physical cosmology and elementary particle physics. These differences will be actively discussed next week at the twelfth annual international meeting of the **Alternative Natural Philosophy Association** (*ANPA 12*) to be held at the Department of History and Philosophy of Science, Free School Lane, Cambridge, 14-17 September 1990. Anyone here who is interested is cordially invited to attend.

We will ignore these foundational differences here and take as our model the class of algorithms called *program universe* (see 5, pp 87-88). These pick two arbitrary strings from a universe containing strings of length S, discriminate them, and if the result is not the null string ($b^0_s = 0$ for all s) adjoin it to the universe; else we concatenate an arbitrary bit, separately chosen for each string, to the growing end of each string. If we think of this bit-string universe as a block of strings of length S and height H, the second operation (called *TICK*) amounts to adjoining an arbitrary column (Bernoulli sequence) and hence $S \to S+1$. The first operation (called *PICK*) generates a string from the extant content and adds it as a new horizontal row ($H \to H+1$). I am still amazed that this simple algorithm can be used to construct the rich structures given in our summary Table!

COMBINATORIAL HIERARCHY LABELS

Finite sets of non-null bit-strings which *close* under discrimination are called *discriminately closed subsets* (*dcss*). For example, two *discriminately independent* bits-strings (i.e. $\mathbf{a} \oplus \mathbf{b} \neq \mathbf{0}$) generate 3 dcss: $\{\mathbf{a}\}$, $\{\mathbf{b}\}$, $\{\mathbf{a}, \mathbf{b}, \mathbf{a} \oplus \mathbf{b}\}$. The three member set closes under discrimination because any two members discriminate to the third. Similarly 3 discriminately independent bit-strings generate 7 dcss:

$$\{\mathbf{a}\},\ \{\mathbf{b}\},\ \{\mathbf{c}\}$$

$$\{\mathbf{a}, \mathbf{b}, \mathbf{a} \oplus \mathbf{b}\};\ \{\mathbf{b}, \mathbf{c}, \mathbf{b} \oplus \mathbf{c}\};\ \{\mathbf{c}, \mathbf{a}, \mathbf{c} \oplus \mathbf{a}\} \tag{4}$$

$$\{\mathbf{a},\ \mathbf{b},\ \mathbf{c},\ \mathbf{a} \oplus \mathbf{b},\ \mathbf{b} \oplus \mathbf{c},\ \mathbf{c} \oplus \mathbf{a},\ \mathbf{a} \oplus \mathbf{b} \oplus \mathbf{c}\}$$

Clearly, given j non-null discriminately independent strings one can form $2^j - 1$ dcss. If one starts with two discriminately independent bit-strings of length 2 [(01), (10) *or* (01), (11) *or* (11), (10)] and forms the three dcss, these can be mapped by three non-singular $\mathbf{2} \times \mathbf{2}$ matrices which are discriminately independent to provide three basis elements for a new level. This mapping can be repeated using

$\mathbf{4} \times \mathbf{4}$ matrices with $7 = 2^3 - 1 < 16$ non-singular and discriminately independent exemplars, and once again using $\mathbf{16} \times \mathbf{16}$ matrices because $127 = 2^7 - 1 < 256$; however the mapping cannot be carried further because $\mathbf{256} \times \mathbf{256}$ matrices have only 256^2 discriminately independent exemplars and $256^2 << 2^{127} - 1$. This is still the simplest way to explain how the combinatorial hierarchy can be generated and why it terminates.

At ANPA 2 (1980) Kilmister (7) proposed a specific scheme for generating the combinatorial hierarchy (CH) which did not necessarily rely on bit-strings. Soon after, Noyes and Kilmister recognized that any generation scheme could go on generating bit-strings beyond those needed for the CH construction. This suggested that the early part of the string could represent a *label* corresponding to the quantum numbers of the elementary particles — which could be closed off once the labels were long enough to represent the 4 levels of the CH — concatenated with a *content* string which would represent a space-time expanding out to an event horizon given by the string length at any particular stage in the construction. In order to explore this situation Noyes and Manthey created *program universe* as described above. When the label strings have reached length 16, they can be organized into three orthogonal dimensions corresponding to the first 3 levels of the CH containing 3, 7 and 127 strings of length 16. These strings can be used to represent the fermion number, weak isospin and baryon number of the three generations of the standard model of quarks and leptons, and the confined color charges (see Noyes, 6). The next step in the construction closes with $2^{127} - 1$ strings of length 256 making a cumulative total of $N = 2^{127} + 136 \simeq 1.7 \times 10^{38}$ states available to us.

QUANTIZED SPACE-TIME

Once we have constructed the label-content concatenation, we can interpret the situations where PICK leads to a non-null string (i.e. $\mathbf{c} = \mathbf{a} \oplus \mathbf{b}$, or equivalently $\mathbf{a} \oplus \mathbf{b} \oplus \mathbf{c} = \mathbf{0}$) as the production (eg by pair annihilation or bremstrahlung) or absorption of a single label which either initiates or terminates a propagation of the label that continues for (or ends after) some finite number of TICKs. This is a discrete model for a Feynman vertex. The completed process combining two such vertices models a 4-leg diagram $\mathbf{a} \oplus \mathbf{b} \oplus \mathbf{c} \oplus \mathbf{d} = \mathbf{0}$ which we call a 4-event.

The choice of this criterion is not arbitrary. McGoveran (5, Theorem 13) has shown that any discrete space of D "homogeneous and isotropic" dimensions synchronized by a universal ordering operator can have no more than *three* indefinitely continuable dimensions; three separate out and the others "compactify" after a surprisingly small number of constructive operations. The proof rests on the fact that if we consider D independently generated Bernoulli sequences (i.e. arbitrary sequences of the symbols 0, 1), Feller (8) has shown that the probability that after n synchronized trials all will have accumulated the same number of "1" 's is less than $n^{-\frac{1}{2}(D-1)}$. It can be shown that the requirement that $D+1$ strings of length n discriminate to the null string is equivalent to Feller's condition. Consequently the probability of continued sequences of events involving D labels vanishes like $n^{-\frac{3}{2}}$ for $D = 4$, and increasingly rapidly for higher numbers. Applying McGoveran's Theorem to the label space allows us to understand why there are only three asymptotically conserved quantum numbers. We have mentioned fermion number, weak isospin and baryon number in making connection between the first three levels of the hierarchy and the standard model. Once we have made this identification, the colored quarks and gluons *have* to be confined independent of any "dynamical mechanism".

To map bit-strings onto integer and half-integer coordinates first note that the Hamming measure $a := \Sigma_{s=1}^{S} b_s^a$ takes the null string as the "reference ensemble" in McGoveran's definition of *attribute distance* (see 5). We restore the symmetry between the symbols "0" and "1" by using for our measure the signed coordinate $q_a(S)$ defined by

$$-\frac{S}{2} \leq q_a := a - \frac{S}{2} \leq +\frac{S}{2} \tag{5}$$

There are $2S$ such integrally spaced coordinates for S even and $2S+1$ for S odd. These integer *or* half-integer coordinates can be related to the usual angular momentum "space quantization" of elementary quantum mechanics by defining

$$J(S)cos\ \theta_a := q_a;\ J^2(S) := \frac{S}{2}(\frac{S}{2}+1) \tag{6}$$

Then integer steps correspond to "rotations" leave the string length and hence J^2 invariant. Alternatively we can define

$$\tau(S)cosh\ \xi_a := q_a;\ \tau^2(S) := \frac{S}{2}(\frac{S}{2}+1) \tag{7}$$

with $\beta_a =:= tanh\ \xi_a := \frac{2a}{S} - 1$ and "Lorentz transformations" which leave $\tau^2(S)$ invariant. Extending these definitions to 3+1 dimensions for 4-events as defined above, we find that we can map the content strings (space-time) onto the C4 Clifford algebra (quaternions) in Greider's (9) formulation of non-interacting relativistic quantum mechanics for particles and fields. This fact can be used to establish the "Poincaré invariance" of our representations in the context of our integer restrictions that make all 4-vector components signed integers or half-integers. Applied to our finite label space, this mapping also can be used to establish the conservation of fermion number, weak hypercharge and baryon number across the intervals connecting two scattering events.

GRAVITATIONAL STABILIZATION OF THE PROTON

In order to connect our dimensional constants to quantized particle physics, we assume that N states of mass m are bound together by Newtonian gravitation to form the largest possible mass allowed within their common Compton wavelength $\hbar/mc$. Adapting an argument given by Dyson (10) for quantum electrodynamics to gravitation (Noyes, 11) we take $NGm^2/r = NGm^2/(\hbar/mc) = mc^2$. Trying to add one more particle will create a free particle of energy mc^2 in addition to this "Laplacian black hole"; in other words, this small black hole is indubitably unstable against Hawking radiation once we try to go from N to $N+1$. Hence the largest possible mass for an *elementary system* is indeed the Planck mass.

If we take this maximum number N to be the terminating cardinal of the CH, $m = (\hbar c/G)^{\frac{1}{2}}/(2^{127}+136)$ and we find that m is equal to the proton mass to an accuracy of about 1%. The unit of *particle* mass for our theory can be taken to be the proton mass. (A correction we will not have time to discuss here brings Newton's constant G computed from m_p, c and $\hbar$ into agreement with experiment, as noted in the Table.) Our interpretation of this calculation is that the mass of the proton is due to its gravitational self-energy, necessarily finite in our theory. For us, as for Wheeler (2), both black holes and the Hawking radiation are basic; the two approaches are closer than one might think at first glance.

QUANTUM GEONS

Looking at our interpretation of the labels (6) in more detail, we see that electromagnetism enters only after we have constructed the third level of the CH; this is where we have the first opportunity to interpret the cumulative cardinal 137 as a first calculation of $\hbar c/e^2$. As was discovered by Parker-Rhodes (12) and afterwards argued by us (Bastin, et. al., 4) once we have accepted the proton mass (now gravitationally generated) as specifying our local unit of mass, we can calculate the electron mass as due to its electromagnetic self-energy and obtain the surprisingly accurate result given in the Table. This calculation is reviewed below. Before our construction reaches level 3, we have only the 3+7=10 states of the first two levels of the CH. These cannot as yet refer to electromagnetism. For massless content strings, We interpret these ten labels as two chiral neutrinos, two chiral photons, five chiral gravitons, and the ubiquitous "interaction" represented by the anti-null string $b_s^1 = 1$ for all s. This string couples *strings* of any composition into a possible metric relationship. We interpret this string as an early version of the "Newtonian" interaction which ties all identifiable objects together. For massless content strings it will have a "coupling constant" of 1/10, which will become weaker and weaker as more and more degrees of freedom are constructed until the closure of the hierarchy labels allows us to interpret it as "Newtonian gravitation".

Because we start out with massless states, one would think that only two chiral gravitons are allowed. But thanks to the "gravitational" self-interaction, we can form massive objects ("quantum geons") and hence macroscopic orbits relative to which all five states of the chiral gravitons with spin 2 can be defined. Note that this is basically the same argument we used to correctly calculate the precession of the perihelion of Mercury in the paper presented at the first conference in this series (Noyes, 13). As our construction proceeds, we will get one of these "quantum geons" with relative probability 1/10 compared to the probability of getting "visible matter" of 1/127. Therefore our candidate for "dark matter" should be 12.7 times as prevalent as visible matter, which is consistent with current observations.

COSMOLOGICAL CONSEQUENCES

Thinking about this construction, we realize that there will be N^2 initial scattering events which conserve baryon number, providing our universe with this number of baryons, and hence about 1% of the closure mass. Our 256^2 initial and 256^2 final states would, in the absence of further information, be equally divided between baryons and anti-baryons, i.e. on the average contain an equal number of zero's and one's, leading to baryon number zero for the universe. However, the asymmetry inherent in our construction stemming from the special role played by the null string in discrimination and the CH requires us to start the labels with a one, rather than a zero. This asymmetry will persist throughout the statistical "averaging" which follows. In our theory strings with an odd number of "1" 's correspond to fermions; we expect $1/256^4$ baryons per photon in our universe, which is about right (see Table).

Our time steps are of length h/m_pc^2 once the universe is dilute enough so that we can make a linear local connection between space and time, and recognize electromagnetic processes as improbable by about one part in 137 compared to "first law motion". It takes at least N^2 events (TICKs) after the label strings have closed to construct content strings (space-time) which has these properties and the gravitational scale for stabilizing m_p at the value which freezes the time-step. Using a linear time scale (i.e. backward extrapolation from this stage of the construction), this marks a transition between and "optically thick" and an "optically thin" universe. We call this backward extrapolation to the start of the content string — label string boundary "fireball time". Using the linear scale gives us 3.5 million years. This is consistent with our other numbers and the currently observed $2.7^o K$ cosmic background radiation.

Having established our gravitational-cosmological framework, the constructive enterprise can now address more local questions about particle masses and coupling constants. After protons, the other easily recognized stable mass value is the electron mass, so our next step is to calculate their ratio.

THE PROTON-ELECTRON MASS RATIO

An elementary starting point for the calculation of the electron-proton mass ratio is the assumption that, just as we have seen that the proton mass can be generated gravitationally, the electron mass can be generated electromagnetically. Although we could talk about this as the self-energy of the electron due to its interaction with vacuum fluctuations — whose only constituents we can recognize at this point in the construction are proton-antiproton pairs, the coulomb interaction and/or gamma rays — it is simpler to calculate the mass of the electron as generated by its charge by taking some appropriate finite statistical average over its electrostatic self-energy

$$m_e c^2 = < e^2/r > \tag{8}$$

Our unit of length for a spherically symmetric system is the proton Compton radius $h/2m_p c$. The system has spherical symmetry and the calculation occurs before we have enough information about other quantum numbers to add any additional degrees of freedom. Consequently we cannot use the corrected (or empirical) fine structure constant, but must use the combinatorial hierarchy value 137 to define our unit if charge, i.e. $e^2 = \hbar c/137$. Since the fluctuations involve both charged (eg proton-antiproton pairs) and neutral (eg γ-rays) particles, the charge fluctuations are independent of the space-fluctuations, but must conserve charge, i.e. $e \rightarrow$

$xe+(1-x)e$ where x is some statistical variable; the contribution of the fluctuations outside of the range $0 \leq x \leq 1$ must cancel by symmetry. Hence

$$< e^2 >= \frac{\hbar c}{137} < x(1-x) > \tag{9}$$

For the space fluctuations we scale by the proton Compton radius and conclude that

$$< 1/r >= \frac{2m_p c}{h} < 1/y > \tag{10}$$

with $0 \leq 1/y \leq 1$ and hence that

$$\frac{m_p}{m_e} = \frac{137\pi}{< x(1-x) >< 1/y >} \tag{11}$$

This completes our dimensional analysis.

The statistical calculation invokes the three degrees of freedom of space. For us, thanks to McGoveran's Theorem (5, Theorem 13) that in any discrete theory such as ours, space can have at most three asymptotic dimensions and one universal ordering operator — which in our cosmology is isomorphic to the time scale of the expanding event horizon — the expectation values are calculated with three degrees of freedom. Since the probability of finding a fluctuation falls off like $1/y$ for the coulomb interaction along any radius, we use this as our weighting factor and obtain

$$< 1/y >= \frac{\int_0^1 [1/y](1/y)^3 d(1/y)}{\int_0^1 (1/y)^3 d(1/y)} = \frac{4}{5} \tag{12}$$

Since the effective source of the interaction with the vacuum fluctuations is proportional to $(xe) \times (1-x)e$ and must end in a sink with the same strength we take the weighting factor for the calculation of $< e^2 >$ to be $x^2(1-x)^2$ and for one degree of freedom find that

$$K_1 = \frac{\int_0^1 [x(1-x)]x^2(1-x)^2 dx}{\int_0^1 x^2(1-x)^2 dx} = \frac{3}{14} \tag{13}$$

Once the charge has separated into pieces with charge ex and $e(1-x)$ the effective squared charge in the interaction is either e^2x^2 or $e^2(1-x)^2$, so we can write the recursion relation

$$K_n = \frac{\int_0^1 [x^3(1-x)^3 + K_{n-1}x^2(1-x)^4]dx}{\int_0^1 x^2(1-x)^2 dx} = \frac{\int_0^1 [x^3(1-x)^3 + K_{n-1}x^4(1-x)^2]dx}{\int_0^1 x^2(1-x)^2 dx}$$

$$= \frac{3}{14} + \frac{2}{7}K_{n-1} = \frac{3}{14}\Sigma_{s=0}^{n-1}[\frac{2}{7}]^s \tag{14}$$

Putting all this together we find that

$$\frac{m_p}{m_e} = \frac{137\pi}{\frac{3}{14} \times [1 + \frac{2}{7} + (\frac{2}{7})^2] \times \frac{4}{5}} = 1836.151497.... \tag{15}$$

This completes our gravitational-electromagnetic unification at the level of the static (Newtonian and Coulomb) interactions exemplified experimentally by the two stable particles with masses m_p and m_e whose masses we have calculated relative to the Planck scale.

WEAK-ELECTROMAGNETIC UNIFICATION

Our connection between quantum numbers and space-time requires that $G_F m_p^2 \approx [\sqrt{2}(265)^2]^{-1}$, which is good to better than 7%, and McGoveran's correction (see Table) brings this reasonably close to the empirical value, as does his correction of our original estimate $sin^2\theta_{Weak} = 0.25$. The definitions of coupling constants and our bit string representation of the quantum numbers require, at this level of accuracy, that

$$M_Z^2 = M_W^2/cos^2\theta_{Weak} \tag{16}$$

We have seen above how the electromagnetic interaction of the electron with the vacuum fluctuations dominated by proton-antiproton pairs explains m_p/m_e in terms of a statistically calculable geometrical factor. But since the electron also couples to the vacuum fluctuations of the $W-\bar{W}$ and $Z-\bar{Z}$ via the massless neutrino in the same geometrical fashion, self-consistency requires that the calculation using the Fermi interaction rather than α must lead to the same electron mass. Chasing this through, we find that

$$M_W^2 = \frac{\pi\alpha}{G_F\sqrt{2}sin^2\theta_{Weak}} \tag{17}$$

Note that we achieve a good first approximation ("tree level" in the conventional jargon) to weak-electromagnetic unification *without* invoking gauge bosons. In fact, if a negative prediction counts as a prediction, I will stick my neck out and assert that the Higgs boson will not appear during the next decade in any non-controversial form.

SEWGUT

The research goal of many contemporary elementary particle physicists is to find, establish or create a strong, electromagnetic, weak, gravitational unified theory (SEWGUT). For many theorists, the gravitational aspect of a research program aimed at this goal ("quantum gravity") is both the most challenging technically and the most difficult conceptually. Thanks to the CH and the *ordering operator calculus* we have been able to pick up the stick by that end, and construct a *particle* theory in agreement with experiment to first order in $e^2/\hbar c$ for electromagnetic effects, in $G_{Fermi}m_p^2/\hbar c$ for weak effects, in $sin^2\theta_{Weak}$ for weak-electromagnetic unification, and in $G_{Newton}m_p^2/\hbar c$ for gravitational effects. We have also shown that the gross cosmological consequences of our theory are at least roughly in accord with current observational facts as conventionally interpreted. This closes off our theory at the other end of the gravitational scale. What remains is to connect all this up with the strong interactions (*quantum chromodynamics* or QCD) self-consistently.

The three axes in our label space, which we have chosen to name fermion number, weak isospin and baryon number relate to the first three levels of the combinatorial hierarchy and provide precisely the quantum numbers needed for describing the first generation of the standard model of quarks and leptons, as has been known for some time (6). Other conserved quantum numbers such as electric charge, lepton number, or weak hypercharge correspond to rotations and renamings in the 3-dimensional label space. Color confinement occurs naturally, thanks to McGoveran's Theorem, since the three axes mentioned exhaust the absolutely conserved quantum numbers. This is our version of "compactification". Our original bit-string representation of the first three levels of the combinatorial hierarchy used up $2 + 4 + 8 = 14$ of the sixteen slots available. Unfortunately we did not see in time that these provide a natural way to close off this structure with three generations, so we did not "predict" the width of the Z_0 before it was measured. But this clue has led to new results.

With this much solidly established, we can, tentatively, follow up our clue about the second and third generations by suggesting that the muon mass $m_\mu = 3 \times 7 \times 10\ m_e \approx 210\ m_e$, and (less clearly) that the τ-lepton mass $m_\tau \approx 21 m_\mu$. The first prediction can be checked by chasing through the consequences in $\pi - \mu$ and $\pi - e$ decay lifetimes, the Goldberger-Trieman relation, and all that. In principle these are now all calculable, finite and if they don't come out approximately right will give us a lot of headaches. Should this happen the discrepancies could be serious enough to cause me to abandon the whole scheme — as would a failure to get a good approximation for the Lamb shift to the next order in α.

But the most exciting prospect, if we have really cracked the generation problem, is to calculate the "mass" of the top quark. A start has been made. We have seven color states (three colors: r, y, b; three anticolors: $\bar{r}, \bar{y}, \bar{b}$; black: $r \oplus y \oplus b$) and the colorless state $\mathbf{0} = r \oplus \bar{r} = y \oplus \bar{y} = b \oplus \bar{b} = \bar{r} \oplus \bar{y} \oplus \bar{b}$ (see 6). Consequently these seven states occur seven times as often as the colorless state, and the analog of $e^2/\hbar c$ for QCD in our theory is 7.

In order to make use of this *strong* coupling constant, we need a connection between masses and coupling constants (Noyes, 14) which allows coupling constants to be greater than unity. $f^2 > 1$ is one way to characterize "strong interactions"; little technical use can be made of theories specified in this way using finite empirical constants within the framework of second quantized relativistic field theory. Thanks to our mapping onto finite particle number relativistic quantum mechanics, which will be discussed in more detail at ANPA 12 next week, we have an alternative approach. The basic S-Matrix point of view associates a bound or resonant state of any two-particle system with a pole at invariant 4-momentum squared s_0 in the two-particle momentum-space wave function $\phi(s, s_0)$ whose residue defines the "coupling constant" f^2. In the narrow width approximation this translates to

$$\phi(s, s_0) = \frac{f^2 \mu}{s - s_0} \tag{18}$$

Assuming the state contains only two particles of mass m_1, m_2 yields the normalization condition

$$\int_{(m_1+m_2)^2}^{\infty} ds |\phi(s, s_0)|^2 = 1 \tag{19}$$

which forces us, for dimensional reasons, to include some mass μ in the definition of the residue if we wish (in analogy with $e^2/\hbar c$) to keep the coupling constant f^2 dimensionless. By performing the integration we obtain a simple connection between masses and coupling constants

$$(f^2 \mu)^2 = (m_1 + m_2)^2 - s_0 \tag{20}$$

Note that the *magnitude* of f^2 is not seriously restricted by this algebraic connection until we have inserted more information. We assert that this starting point is *non-perturbative* and rests only on *unitarity* and relativistic quantum mechanics in a finite particle number space.

Before proceeding further, we provide two specific examples where this simple (some may say *simple-minded !*) approach obviously "works". It is currently unfashionable to think of the pion as a "bound state" of a nucleon-antinucleon pair, even though Fermi and Yang (15) provided a good model for the quantum numbers of the pion in this way. These quantum numbers are still useful for large parts of nuclear and low energy particle physics. If we wish to use Eq.20 to supply the dynamics, we must take $m_1 \simeq m_N = m_{\bar{N}} \simeq m_2$ and $s_0 = m_\pi^2$. Since we know the coupling to be strong, we must take $\mu = m_\pi$ rather than the only available alternative within the system (i.e. m_N), and find that if $m_N/m_\pi \simeq 7$ then $G^2_{\pi N\bar{N}} \simeq 14$ or visa versa, which is a good starting point for low energy nuclear phenomenology.

As another example, take the proton and the electron bound electromagnetically by $f^2 = e^2/\hbar c = \alpha \simeq 1/137$. To reduce this to a "single particle problem" including recoil effects we take the free system mass to be the reduced mass "$m_1 + m_2$" $\rightarrow m_{ep} = \frac{m_e m_p}{m_e + m_p}$, which implies that $s_0 = (m_{ep} - \epsilon_{Bohr})^2$ and use this also as the reference mass μ. This gives us the relativistic Bohr formula

$$(m_{ep} - \epsilon)^2[1 + \alpha^2] = m_{ep}^2 \tag{21}$$

first derived by Bohr (16) in 1915. This was Sommerfeld's (17) starting point for calculating the fine structure spectrum of hydrogen, including the formula which is still correct to order α^2. For those of you who are puzzled by how a calculation based on the relativistic mass increase of the electron in elliptical orbits could lead to a result in agreement with experiment a decade before the discovery of spin and quantum mechanics, let alone Dirac's theory, see Biedenharn's (18) paper.

Our way of arriving at the connection between masses and coupling constants contains some puzzles when we look at it in configuration rather than momentum space. Non-relativistically, if we talk about the radial wave function $u_\gamma(r) = N\, e^{-\gamma r}$, Eq.' s 18 and 19 correspond to the asymptotic normalization $\int_0^\infty u_\gamma^2(r)dr = 1$ and hence $N^2 = 2\gamma = 2(2\mu\epsilon)^{\frac{1}{2}}$ with μ the reduced mass and ϵ the binding energy. This model asserts that most of the time the particles are "outside the range of forces" and hence that only the asymptotic region contributes significantly to the probability of encountering either of the particles when the system is probed. We claim this is a proper way to define a low energy coupling constant in a relativistic theory, when we contemplate calculating corrections due to additional degrees of freedom (real or virtual particle creation) as the energy of the probe is raised. Application to the "infinite range" coulomb interaction is also justified, since the scale is set by the excitation of higher levels, which is of order α^2. That we can get the higher levels themselves by replacing α^2 by $(\alpha/n)^2$ is somewhat more troublesome until we realize that the infinite range of the coulomb interaction always makes the asymptotic region the most important one if we go to high enough resolution.

What most of you will find bizarre is that McGoveran derived this formula directly from our discrete theory, and was able to extend it to the Sommerfeld formula when a second degree of freedom is present, without any "space-time" or "momentum space" considerations (McGoveran, 19). Most people find it incredible that the calculation at the same time corrects our "first order" result (6) that $\hbar c/e^2 = 137$ to a value in much better agreement with experiment (cf. Table). We will not have time to justify this result here.

We have a second way to calculate the mass of the pion, which goes back to our version (11) of Dyson's argument (10) applied to electromagnetism rather than gravitation. Consider an assemblage of N_e charged particle pairs each of mass m in a volume whose average radius is the pair-creation radius $\hbar/2mc$ and whose electrostatic energy is

$$N_e\frac{e^2}{r} = N_e\frac{e^2}{(\hbar/2mc)} = N_e\frac{e^2}{\hbar c}(2mc^2) \approx \frac{N_e}{137}(2mc^2) \tag{22}$$

Thus when the number of pairs exceeds 137, we have enough energy to create another pair. Dyson used this fact to argue that the QED renormalized perturbation series with $e^2 \to -e^2$ begins to diverge beyond 137 terms, and hence that the series is not uniformly convergent. I prefer to interpret the result as saying that we cannot *count* more than 137 charged particle pairs within their own Compton radius. If we take the smallest known stable mass — i.e. the electron mass m_e — for m we have an explanation for the termination of growth of the system. At the level of analysis we are invoking 2×137 particles, half electrons and half positrons, within their individual Compton radius are indistinguishable from a neutral pion with $m_{\pi^0} < 274 m_e$. The system is electrostatically bound. Of course this assemblage is unstable against 2γ decay, but if we add an electron plus an anti-neutrino (or a positron and a neutrino) to the assemblage the lifetime becomes much longer and the sum of the masses of the constituents comes close to $m_{\pi^\pm}$. For recent corrections due to McGoveran which bring these original estimates for m_{π^0} and $m_{\pi^\pm}$ into agreement with experiment, see the Table. We also note that this gives us a start toward understanding why the range of nuclear forces is half the classical electron radius $e^2/m_e c^2$, and the dimensional memnonic

$$e^2/m_e c^2 \ (nuclear) = \alpha(\hbar/mc) \ (QED) = \alpha^2(me^2/\hbar^2) \ (atomic) \tag{23}$$

which I learned from Joe Weinberg in 1947.

Invoking our original S-matrix argument appropriately rewritten for massless constituents, this gives us $7m_\pi = m_N$, which is clearly consistent both with our calculation of the pion as 137 electron-positron pairs *and* with our calculation of $G^2_{\pi N \bar{N}} = 14$. The theory is starting to meet self-consistency checks.

The next step is to note that we are now in a position both to calculate the nucleon mass from a relativistic version of the Chew-Low bootstrap *and* from a constituent quark model starting from massless quarks, using a version of finite particle number relativistic scattering theory which I have been developing along another line of enquiry. This should give us some clues as to the relationship between current and constituent quark masses and the pressing problem of modeling "hadronization" in a simple way. If the weak interaction sector involving $\pi - \mu - e$ works out all right, we can then bring in the strange quark and the weak K-decays to sort out the states, and go on from there to u, d, s strong interaction dynamics. Charm and beauty should follow in due course. Then on to the top!

We conclude with the conjecture that following through the implications of our construction will lead to a theory which — at least to first order in $e^2/\hbar c$, $G_{Fermi}m_p^2/\hbar c$ and $G_{Newton}m_p^2/\hbar c$— that provides a self-consistent unification of strong, electromagnetic, weak *and* gravitational interactions (SEWGUT).

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Table of Results, June, 1990

General structural results

- 3+1 asymptotic space-time
- combinatorial free particle Dirac wave functions
- supraluminal synchronization and correlation *without* supraluminal signaling
- discrete Lorentz transformations for event-based coordinates
- relativistic Bohr-Sommerfeld quantization
- non-commutativity between position and velocity
- conservation laws for Yukawa vertices and 4- events
- crossing symmetry, CPT, spin and statistics

Gravitation and Cosmology

- the equivalence principle
- electromagnetic and gravitational unification
- the three traditional tests of general relativity
- event horizon
- zero-velocity frame for the cosmic background radiation
- mass of the visible universe: $(2^{127})^2 m_p = 4.84 \times 10^{52}\ gm$
- fireball time: $(2^{127})^2 \hbar / m_p c^2 = 3.5$ million years
- critical density: of $\Omega_{Vis} = \rho/\rho_c = 0.01175$ $[0.005 \leq \Omega_{Vis} \leq 0.02]$
- dark matter = 12.7 times visible matter [10??]
- baryons per photon $= 1/256^4 = 2.328... \times 10^{-10}$ $[2 \times 10^{-10}?]$

Unified theory of elementary particles

- quantum numbers of the standard model for quarks and leptons with confined quarks and exactly 3 weakly coupled generations
- gravitation: $\hbar c / G m_p^2 = [2^{127} + 136] \times [1 - \frac{1}{3 \cdot 7 \cdot 10}] =$
  $1.70147...[1 - \frac{1}{3 \cdot 7 \cdot 10}] \times 10^{38} = 1.6934... \times 10^{38}$ $[1.6937(10) \times 10^{38}]$
- weak-electromagnetic unification:
  $G_F m_p^2 / \hbar c = (1 - \frac{1}{3 \cdot 7})/256^2 \sqrt{2} = 1.02\ 758... \times 10^{-5}$ $[1.02\ 684(2) \times 10^{-5}]$;
  $sin^2 \theta_{Weak} = 0.25(1 - \frac{1}{3 \cdot 7})^2 = 0.2267...$ $[0.229(4)]$
  $M_W^2 = \pi\alpha / \sqrt{2} G_F sin^2 \theta_W = (37.3\ Gev/c^2 sin\ \theta_W)^2$; $M_Z cos\ \theta_W = M_W$
- the hydrogen atom: $(E/\mu c^2)^2 [1 + (1/137 N_B)^2] = 1$
- the Sommerfeld formula: $(E/\mu c^2)^2 [1 + a^2/(n + \sqrt{j^2 - a^2})^2] = 1$
- the fine structure constant: $\frac{1}{\alpha} = \frac{137}{1 - \frac{1}{30 \times 127}} = 137.0359\ 674...$ $[137.0359\ 895(61)]$
- $m_p/m_e = \frac{137\pi}{\frac{3}{14}\left(1 + \frac{2}{7} + \frac{4}{49}\right)\frac{4}{5}} = 1836.15\ 1497...$ $[1836.15\ 2701(37)]$
- $m_\pi^\pm / m_e = 275[1 - \frac{2}{2 \cdot 3 \cdot 7 \cdot 7}] = 273.1292...$ $[273.12\ 63(76)]$
- $m_{\pi^0}/m_e = 274[1 - \frac{3}{2 \cdot 3 \cdot 7 \cdot 2}] = 264.2\ 1428..$ $[264.1\ 160(76)]$
- $(G_{\pi N}^2 m_{\pi^0})^2 = (2m_p)^2 - m_{\pi^0}^2 = (13.86811 m_{\pi^0})^2$

  [( )] = empirical value (error) or range