SLAC-PUB-5186
September 1990
(A)

PLASMA FOCUSING AND DIAGNOSIS OF HIGH ENERGY PARTICLE BEAMS*

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ABSTRACT

Various novel concepts of focusing and diagnosis of high energy charged particle beams, based on the interaction between the relativistic particle beam and the-plasma, are reviewed. This includes overdense and underdense thin plasma lenses, an (underdense) adiabatic plasma lens, and two beam size monitor concepts. In addition, we introduce another mechanism for measuring flat beams based on the impulse received by heavy ions in an underdense plasma. Theoretical investigations show promise of focusing and diagnosing beams down to sizes where conventional methods are not possible to provide.

1. INTRODUCTION

As the energy of circular colliding beam accelerators becomes higher, one must face the limitation imposed by the energy loss to synchrotron radiation. For this reason, it is likely that future lepton colliders will be linear machines. The disadvantage of the linear scheme is that the beams are used only once, and then discarded. In order to achieve desirable luminosity, $\mathcal{L} = f_{rep}N^2H_D/4\pi\sigma_0^{*2}$, where N is the number of particles per bunch, f_{rep} the collider repetition rate, H_D the beam-beam disruption enhancement factor, and σ_0^* the rms beam radius at collision, one must either increase the beam current $f_{rep}N$ or decrease the spot size. The current is constrained by many factors, e.g., power limitations or wake-field effects in an accelerator. On the other hand, the minimum spot sizes are presently limited by the strength of conventional focusing quadrupoles. Eventually, however, one would reach the so-called *Oide* **limit,'** where the stochastic nature of the synchrotron radiation triggered at the final focusing lens imposes a strong limitation on the minimal possible beam size.

The plasma lens, which uses the self-focusing wakefields of a bunched relativistic charged particle beam in a plasma, and promises a very strong focusing, was first proposed by Chen.[2] Subsequent works[3–5] pushed the idea further. In the case of the overdense plasma lens, the beam peak density n_b is much less than the ambient plasma density n_0 it encounters as it traverses the lens. In this case, assuming that the beam length σ_z is large compared to the plasma wavelength $\lambda_p = \sqrt{\pi r_e/n_0}$ (the response of the plasma electrons to the beam is adiabatic and not oscillatory), the beam width σ is small compared to the plasma wavelength (plasma response is radial), and the ions are stationary, then the plasma -electrons move to approximately neutralize the beam charge, leaving the beam current self-pinching forces unbalanced. The focusing wakefields reduce, to a good approximation, to the magnetic self-fields of the beam.

*Work supported by Department of Energy contract DE–AC03–76SF00515.

Invited talk given at the Topical Conference on Research Trends in Nonlinear and Relativistic Effects in Plasmas, La Jolla, CA, February 5-8, 1990.

These self-fields are quite strong, but as they are dependent on the configuration of the beam density, the resulting focusing is nonlinear and prone to aberration. This requires that the lens be placed very close to the interaction point to minimize aberration effects, which in turn means that, for parameters typical of the Stanford Linear Collider (SLC) design, the plasma lens must be very dense. This dense plasma is a source of a very large background event rate, primarily due to the inelastic e-p scatterings, and is undesirable for particle physics purposes.

The background and aberration problems motivate the investigation of the underdense plasma lens.[6,7] An underdense plasma reacts to an electron beam by total rarefaction of the plasma electrons inside the beam volume, producing a uniformly charged ion column of charge density en_0. This uniform column produces linear, nearly aberration-free focusing. For positron beams, however, plasma electrons do not behave simply, and the focusing is not linear. The luminosity enhancement is thus achieved by the disruption of the larger positron beam by the smaller electron beam. This process has been termed "bootstrap disruption,"[6] as it involves a cascade of beam-dependent focusing effects; the pre-focusing of the electron beam by its own self-fields and the subsequent strengthened disruption of the positron beam by the electron beam. Simulations' indicate that luminosity enhancement by a factor 3-5 may be possible above conventional focusing schemes for the SLC design parameters, with large reductions in background event rates from similar overdense plasma lens schemes.

As mentioned earlier, ultimately the attainable beam spot size reaches a limit due to synchrotron radiation at the lens set by Oide. This limit is generic and applies also to plasma lenses. It therefore appears that the primary motivation for plasma lenses, i.e., the very strong focusing field attainable, becomes obsolete. Ironically it occurs that plasmas provide a solution to overcome this limit. The idea of ***adiabatic focusing,*** proposed by Chen, Oide, Sessler, and Yu,[8] using for example a plasma column, promises to evade the synchrotron radiation limit set by Oide. This is achieved by implementing a beam optics system where the focusing gradient is continuously and slowly increased along the direction of beam propagation, such that the p-function decreases linearly along the lens. In such a focusing system, beam particles with different energies would always oscillate within a definite envelope and eventually be focused down to within the designated size. The problem of chromatic aberration associated with conventional discrete focusing lenses, including the plasma lenses, can thus be alleviated.

In addition to its potential as focusing devices, plasmas could also provide useful diagnosis on the beam spot size through beam-plasma interaction. The information of the beam size at its final focus is indispensable for the tuning of linear colliders. The conventional method of wire scanning detects the bremsstrahlung signals from the beam intercepted by metal wires. Experience at SLC shows that such wire (with diameter $\sim$ 2-3 μm) would not sustain bunches of size smaller than $\sigma \lesssim$ l-2 μm with N $\gtrsim 1 \times 10^{10}$ particles. Since it is expected that the beam size in the next generation linear colliders will be as small as nanometers in the vertical dimension, the conventional method would become obsolete and novel approaches are required.

It has been suggested earlier[9,10] that ions, by interacting with the collective fields of the high energy beam, can provide useful information on the size of ***round*** (a, = σ_y) beams. The idea is revived more recently with the attention to ***flat*** (a, >> σ_y) beams conceived for the next generation colliders.[11,12]

In this paper, we review various novel concepts on plasma focusing and diagnosis of high energy beams, and introduce another mechanism of flat beam measurement, which is an extension from Refs. 9 and 10, based on the responses of heavy ions in an underdense plasma intercepted by the focused beam. Materials in Secs. 2-4 are largely extracted from the published works in Refs. 2, 4, 6, and 8. This is merely due to convenience in presentation, and should not imply that these are the only contributions to the subject. Other than the review of Refs. 11 and 12, the investigation on the beam size monitor in Sec. 5 has not been published elsewhere.

2. THE OVERDENSE PLASMA LENS

Assuming that the unperturbed plasma velocity v_0 is zero and the perturbed plasma density n_1 is much smaller than its unperturbed density n_0, the equation of motion and the equation of continuity can be linearized as:

$$\begin{cases} -\partial_\zeta \vec{v}_1 = -\dfrac{e}{mc}\vec{E}_1 \ , \\ -c\partial_\zeta n_1 + n_0 \nabla \cdot \vec{v}_1 = 0 \ , \end{cases} \tag{2.1}$$

where $\partial_z = \partial_\zeta$ and $\partial_t = -c\partial_\zeta$ have been used.

Combining the fluid equation with the Maxwell's equations in the Coulomb gauge, an-d assuming that the transverse motions of the bunch particles are negligibly small during the beam-plasma interaction, we obtain

$$\partial_\zeta^2 n_1 + k_p^2 n_1 = \mp k_p^2 \sigma(\vec{x}), \tag{2.2}$$

where the plasma wave number $k_p \equiv \omega_p/c = (4\pi e^2 n_0/mc^2)^{1/2}$, and $\mp e\sigma(\vec{x})$ is the density for electron $(-)$ and positron $(+)$ bunches, respectively. It can be further shown that the combination of the perturbed plasma potentials, $A_{1z} - \phi_1$, satisfies the following equation of motion:

$$(\nabla_\perp^2 - k_p^2)(A_{1z} - \phi_1) = -4\pi e n_1 \ , \tag{2.3}$$

where $\nabla_\perp^2 = \nabla^2 - \partial_\zeta^2$.

High energy e^+e^-beam particles are normally in Gaussian distributions in the three dimensional space. For the ease of calculation without sacrificing the essential physics, we invoke a parabolic bunch distribution:

$$\sigma(\vec{x}) = \rho_b\left(1 - \frac{r^2}{a^2}\right)\left(1 - \frac{(\zeta + b)^2}{b^2}\right) \ , \tag{2.4}$$

where $0 \le r \le a$ and $-2b \le \zeta \le 0$. The parabolic profiles in both r and ζ directions are introduced to approximate the Gaussian profiles. The constant, ρ, can be related to the total number of particles N in the bunch:

$$\rho_b = \frac{3N}{2\pi a^2 b} \ . \tag{2.5}$$

With this density distribution, it is straightforward to find that within the bunch,

$$A_{1z} - \phi_1 = \mp \frac{8\pi e \rho_b}{k_p^2} \left[I_0(k_p r) K_2(k_p a) + \frac{1}{2}\left(1 - \frac{r^2}{a^2}\right) - \frac{2}{k_p^2 a^2} \right]$$
$$\times \left[\left(1 - \frac{(\zeta + b)^2}{b^2}\right) + \frac{2}{k_p b} \sin k_p \zeta + \frac{2}{k_p^2 b^2} \left(1 - \cos k_p \zeta\right) \right] . \tag{2.6}$$

The transverse force exerting on the beam is simply the transverse derivative of $A_{1z} - \phi_1$. Thus

$$\mathcal{F}_\perp = \frac{8\pi e^2 \rho_b}{k_p} \left[I_1(k_p r) K_2(k_p a) - \frac{r}{k_p a^2} \right]$$
$$\times \left[\left(1 - \frac{(\zeta + b)^2}{b^2}\right) + \frac{2}{k_p b} \sin k_p \zeta + \frac{2}{k_p^2 b^2} \left(1 - \cos k_p \zeta\right) \right] . \tag{2.7}$$

-Notice that the transverse force is exerting on the like particles in the same bunch, thus $\mathcal{F}_\perp$ has the same sign for both electron and positron bunches and can be verified to be always focusing. In the case where $k_p r \leq k_p a << 1$,

$$I_1(k_p r) K_2(k_p a) - \frac{r}{k_p a^2} \simeq -\frac{1}{4} k_p r \ , \tag{2.8}$$

and we have a focusing force that is linear in r. The requirement that the focusing force be linear in r, i.e., that $k_p a << 1$, can be rewritten as $n_0 << 1/4\pi r_e a^2$, where r_e is the classical electron radius. On the other hand, self-consistency in the linearized fluid theory that we employed requires that $n_0 >> n_1$. Combining these two conditions we arrive at a chain inequality which the system must satisfy:

$$\frac{1}{4\pi r_e a^2} \gg n_0 \gg \frac{3N}{2\pi a^2 b} \left[\left(1 - \frac{(\zeta + b)^2}{b^2}\right) + \frac{2}{k_p b} \sin k_p b + \frac{2}{k_p^2 b^2}(1 - \cos k_p b) \right] . \tag{2.9}$$

An interesting situation is when $k_p b >> 1$. Therefore $b >> \lambda_p/2\pi >> a$, and we have a long bunch where the longitudinal extent is much larger than the transverse extent. When this is satisfied, the focusing force is

$$\mathcal{F}_\perp(r, \zeta) \simeq \frac{3e^2 N}{a^2 b} \left(1 - \frac{(\zeta + b)^2}{b^2}\right) r \ , \qquad k_p b \gg 1 \ . \tag{2.10}$$

We see that in this limit the focusing force varies as the longitudinal profile of the beam and the maximum is at the midpoint along the bunch. This focusing force is very strong. For comparison, consider N = 5 x 10^9, and a = 100 μm, b = 1 mm. We find the corresponding G $\sim$ 720 KG/cm. In contrast, typical iron magnets (G $\sim$ 5 KG/cm) and superconducting magnets (G $\sim$ 10 KG/cm) are about 1 $\sim$ 2 orders of magnitude weaker.

Physically, this self-focusing effect arises because the electrons in the plasma are either expelled (for the case of interacting with an electron bunch) or pulled (for the case of interacting with a positron bunch) by the leading particles in the bunch, while on this time scale the ions in the plasma are essentially stationary. As a result, the trailing particles in the same bunch experience an attractive force due to the access charges in plasma within the volume of the bunch. Large self-pinching of the beam is thus induced. This effect has been observed in computer simulations,[13,3] and in experiments.[14,15]

3. THE UNDERDENSE PLASMA LENS

In the underdense regime, the electron beam experiences a linear, nearly aberration-free focusing. Simulations have shown that one needs to have $n_b \geq 2n_0$ to produce linear focusing over most of the bunch? When this condition is satisfied, the beam p-function as a function of the distance down the beam-line s can be described by the third order linear differential equation:

$$\beta'' + 4K\beta' + 2K'\beta = 0 , \tag{3.1}$$

where $\beta = \sigma^2/\epsilon_0$, ϵ_0 is the unnormalized transverse emittance and $K = 2\pi r_e n_0/\gamma$ is the focusing strength of the lens. To solve Eq. (3.1) we must first integrate through the S-function in ***Ii"*** at the start of the lens to obtain $\Delta\beta_0'' = -2K\beta_0$. The other two initial conditions are just continuity requirements $\beta' = \beta_0'$, and $\beta = \beta_0$. Assuming the electron bunch to have a cylindrically symmetric bi-Gaussian distribution of rms length σ_z, then we can define the phase space density parameter $\zeta = Nr_e/\sqrt{8\pi}\epsilon_0\gamma\sigma_z$, and the focusing strength of an underdense plasma lens is, with $\beta = \beta_0$ and $n_0/n_b = 1/2$ at the start of the lens, $K = \zeta/\beta_0$. Using the initial conditions, we integrate Eq. (3.1) once to obtain

$$\beta'' + 4K\beta = 2/\beta_0^* + 2\zeta , \tag{3.2}$$

where β_0^* is the minimum p-function achieved in the absence of the plasma lens. The solution for the p-function inside the lens is easily found from Eq. (3.2) to be

$$\beta = \frac{\beta_0}{2} + \frac{1}{2K\beta_0^*} + \left(\frac{\beta_0}{2} - \frac{1}{2K\beta_0^*}\right)\cos\nu(s-s_0) + \frac{2s_0}{\nu\beta_0^*}\sin\nu(s-s_0) , \tag{3.3}$$

where $\nu^2 = 4K$.

It is straightforward to show from the above considerations that the maximum reduction in β^* that one can achieve with this lens occurs when one places the entrance of the plasma at a position $-s_0 >> \beta_0^*$. This reduction is given by

$$\frac{\beta^*}{\beta_0^*} = \frac{1}{1 + K\beta_0^*(\beta_0 - \beta_1)} \simeq \frac{1}{1+\zeta\beta_0^*} \tag{3.4}$$

where β_1 is the p-function at the exit of the plasma lens at $s = s_1$. For SLC design parameters (ϵ_n = 3 x 10^{-5}m-rad, σ_z = 1mm, β_0^* = 7mm, $\gamma = 10^5$, and ***N = 5*** x 10^{10}) we have ζ = 9.4 x 10^2m^{-1}, and a possible reduction in β of 1/7.5. It is also interesting to note that according to this formula, one should never back off of the focus, i.e., make β_0^* larger, as the ultimate β^* attainable -is inversely proportional to $\zeta + (1/\beta_0^*)$. This implies one should minimize β_0^*. It also says that if $\zeta\beta_0^* < 1$ then plasma lens is irrelevant, as it is not strong enough to overcome the inherent divergence in the beam. If one only reduces the spot size σ_-^* of the electron beam in the collisions and leaves the positron beam spot size σ_0^* unchanged, then the possible luminosity enhancement due to the lens H_L (excluding depth of focus and disruption effects) is easily shown to be

$$H_L = \frac{2(\sigma_0^*)^2}{(\sigma_-^*)^2 + (\sigma_0^*)^2} = \frac{2\beta_0^*}{\beta_-^* + \beta_0^*} \quad , \tag{3.5}$$

which is strictly less than two. For example, an electron spot size reduction of $\sigma_-^*/\sigma_0^* = 0.4$ gives a luminosity enhancement of 1.73. This is a very modest number; it is boosted, however, by the bootstrap disruption enhancement.

Previous calculations of the luminosity enhancement due to beam-beam disruption have treated symmetric beams. It has been found[16] that the disruption luminosity enhancement is influenced by two factors: the strength of the pinch, represented by the disruption parameter D,

$$D = \frac{N r_e \sigma_z}{\gamma {\sigma_0^*}^2} = \frac{N r_e \sigma_z}{\gamma \beta_0^* \epsilon_0} \quad , \tag{3.6}$$

and the effects of the inherent divergence of the beam, represented by the divergence parameter $A = \sigma_z/\beta_0^*$. The disruption enhancement H_D is a strongly decreasing function of A when $A > 1$, due to the effects of depth of focus and inherent beam divergence, and a monotonically increasing function of D. Since both D and A are inversely dependent on β_0^*, there exists a maximum luminosity for some value of β_0^*. This is also true for bootstrap disruption enhancement, H_B.

Since simulations have shown that the underdense plasma lens can focus positrons: albeit with strong aberrations, it is interesting to see what sort of luminosity enhancements are ultimately possible using two underdense lenses. A theory of aberration-prone focusing has been developed in Ref. 4. In terms of the quantity called the aberration power P, the transformations of the initial transverse phase space parameters $(\alpha_0, \beta_0, \epsilon_0)$ by an aberration-prone thin lens are

$$\alpha = (\alpha_0 + \beta_0/f)/P, \; \beta = \beta_0/P, \; \epsilon = \epsilon_0 P \quad , \tag{3.7}$$

where j is the lens focal length, $\alpha = -2\beta'$, and $P = \sqrt{1+(\beta_0\eta/f)^2}$. The parameter η corresponds to the rms variation of the focusing strength K in the lens. Simulations have shown that for a mildly underdense lens, $\eta \simeq 0.28$ for positron focusing. Note that in this model the aberration results in an emittance blowup which is dependent on the strength of the lens. The total reduction in spot size is thus

$$\frac{\sigma^*}{\sigma_0^*} = \left[\frac{\beta^*\epsilon}{\beta_0\epsilon_0}\right]^{1/2} = \frac{P}{\sqrt{P^2 + (\alpha_0 + \beta_0/f)^2}} \quad . \tag{3.8}$$

Using this model and the computational results from Ref. 8, Chen et al.,[6] simulate the collision of an electron beam focused by an underdense plasma lens to 0.4 of its original spot size with a positron beam focused, with aberrations, by a mildly underdense plasma lens, to 0.6 of the conventionally achieved spot size. All other parameters are taken from the SLC design. The luminosity is found to be 1.5×10^{31} cm^{-2} sec^{-1} and the total enhancement is approximately five. The physical parameters involved in this configuration are shown in Table I.

Table I A set of plasma lens parameters for SLC

Plasma Lens Parameters	Electrons	Positrons
n_0 [cm^{-3}]	1.5×10^{15}	4.8×10^{16}
l[cm]	5.0	0.33
Beam Parameters		
N	5×10^{10}	5×10^{10}
$\mathcal{E}$ [GeV]	50	50
ϵ_0 [m-rad]	3×10^{-10}	3×10^{-10}
σ_z [mm]	1.0	1.0
Beam Optics Parameters		
s_0 [cm]	20.0	1.3
β_0^* [mm]	7.0	7.0
ϵ[m-rad]	3 x10-10	4.2x10-l'
β^* [mm]	1.12	1.84
η	0	0.28
P	1.0	1.39
f [cm]	7.5	1.1
Luminosity Enhancement		
$\mathcal{L}_{00}$ [10^{30}cm^{-2}]	1.76	1.76
H_D	1.73	1.73
$\mathcal{L}_0(=H_D\mathcal{L}_{00})$ [10^{30}cm^{-2}]	3.0	3.0
$\sigma_\pm^*/\sigma_0^*$	0.4	0.6
H_B	5.0	5.0
$\mathcal{L}(=H_B\mathcal{L}_0)$ [10^{30}cm^{-2}]	15.0	15.0

This treatment of the positron focusing is approximate, but gives qualitative insight into the role aberrations play in this scheme.

4. ADIABATIC FOCUSING

In general, in a focusing (or defocusing) environment, a particle with coordinate y satisfies the equation of motion

$$\frac{d^2y}{ds^2}+K(s)y = 0 \quad , \tag{4.1}$$

and the well-known solution is [17]

$$y(s) = \sqrt{\epsilon\beta(s)}\ \cos[\psi(s) + \phi] , \tag{4.2}$$

where ϵ is the emittance and

$$\frac{d\beta}{ds} = -2\alpha(s) \quad ; \qquad \psi(s) = \int^{s} \frac{ds'}{\beta(s')} \quad .$$

In an adiabatic focusing, we demand that the change in β, occurring in a length given by β, is small compared to β. For the sake of simplicity, we shall assume that

$$\frac{d\beta}{ds} = \text{constant} . \tag{4.3}$$

Hence we take

$$\beta(s) = \text{PO} - 2\alpha_0 s , \tag{4.4}$$

where α_0 is the initial condition and a constant of the system that characterizes the amount of adiabaticity.

Since $\alpha(s) = \alpha_0$ = constant, we have $d\alpha/ds = 0$, and the focusing strength along the channel varies as

$$K(s) = \frac{1+\alpha_0^2}{\beta^2} = \frac{1+\alpha_0^2}{(\beta_0 - 2\alpha_0 s)^2} . \tag{4.5}$$

Notice that the focusing strength scales inverse quadratically with $\beta(s)$. The phase advance, on the other hand, varies as

$$\psi(s) = \frac{1}{2\alpha_0} \ln \frac{\beta_0}{\beta_0 - 2\alpha_0 s} \quad . \tag{4.6}$$

For a particle with less energy than the design energy E_0 , i.e., $E = (1-\delta)E_0$, where $\delta << 1$, thefocusingforce K is larger by an amount $1/(1+\delta)$. According to Eq. (4.1), the matched p-function for the lower-energy particle becomes $\bar{\beta}(s) = \sqrt{1-\delta}\beta(s)$, and the a-function is also reduced to $\bar{\alpha}(s) = \sqrt{1-\delta}\alpha(s)$. The mismatched p-function can be shown to be

$$\widetilde{\beta}(s) = \beta(s)[1 - \delta \sin^2 \bar{\psi}(s)] \leq \beta(s) , \tag{4.7}$$

where $\bar{\psi}(s) \equiv \psi(s)/\sqrt{1-\delta}$.

Thus, the amplitude of the lower-energy particle never exceeds that of the reference particle. If one chooses the design energy of the focuser at the maximum energy of the incoming beam, the entire beam is expected to be focused. This ***achromatic*** nature of the focuser will hold true for a particle, which emits radiation while traversing the focuser, and is the very basis of the adiabatic focuser concept .

Insensitive to the energy variation as it is, an adiabatic focuser would render useless if a large fraction of the beam energy is lost during the process. The rate of energy loss of a relativistic electron due to synchrotron radiation is well-known.[18]

In order to perform simple analytic calculations, it is convenient to approximate the exact formula by the following expressions in the ***classical,*** the ***transition,*** and the ***quantum*** regimes: [19]

$$\frac{d\gamma}{ds} = -\frac{2}{3}\frac{\alpha}{\lambda_c}\begin{cases} \Upsilon^2 , & \Upsilon \lesssim 0.2 , \\ 0.2\Upsilon , & 0.2 \lesssim \Upsilon \lesssim 22 , \\ 0.556\Upsilon^{2/3} , & 22 \lesssim \Upsilon , \end{cases} \tag{4.8}$$

where γ is the Lorentz factor of the electron, α the fine structure constant, and $2\pi\lambda_c$ the Compton wavelength. We see that the energy loss is uniquely determined by the parameter Υ, which is Lorentz invariant and defined as

$$\Upsilon \equiv \gamma\frac{B}{B_c} = \frac{\gamma^2\lambda_c}{\rho} . \tag{4.9}$$

Here B is the local field strength, $B_c = m^2c^3/e\hbar \simeq 4.4 \times 10^{13}$ Gauss is the Schwinger critical field, and ρ is the instantaneous radius of curvature of the particle.

Since $1/\rho = K(s)y$, with the help of the relation $\sigma = \langle y^2 \rangle = \beta\epsilon$, where ϵ is the emittance of the beam, one could calculate the energy loss in terms of α_0, β_0, and the resulting β, in the three regimes. It is found[9] that there exists an optimal value of α_0 for attaining a desired β-function with minimum energy loss:

$$\alpha_0 = \begin{cases} \frac{1}{\sqrt{3}} , & \text{(classical)} , \\ 1 , & \text{(transition)} , \\ \sqrt{3} , & \text{(quantum)} . \end{cases} \tag{4.10}$$

We see that the optimal situation does not rely on the condition $\alpha_0 << 1$, in the strict sense of the word ***adiabatic.***

It should, in principle, be possible to set up an adiabatic focuser where the increase of its focusing strength varies in accordance with the three different optimum values given above. But the focuser may be experimentally more convenient if α_0 is fixed throughout the system. If a focuser covers all three regimes of radiation, an obvious compromise would be $\alpha_0 = 1$. With this choice there will be about 15% additional radiation in the classical regime and about 30% more in the quantum regime. Alternatively, since the radiation loss occurs primarily near the end of an adiabatic focuser, a choice of α_0 according to the final regime is most advisable.

With the choice of $\alpha_0 = \sqrt{3}$, we find that there exists a ***critical emittance*** above which a beam can never be focused so tight as to enter the quantum regime of radiation:

$$\epsilon_c \equiv \frac{3^{3/2}\cdot 15^3}{2^3\cdot 4^2\cdot 22}\frac{\lambda_c}{\alpha^3} = 6.17 \times 10^{-6} \text{ m} , \tag{4.11}$$

which depends only on fundamental physical parameters.

If the actual normalized emittance ϵ_n in the system can indeed be lower than the above critical value, then the requirement that the fractional energy loss be much less than unity translates into a limit on the final beam size,

$$\sigma_{ad} >> 1.39 \times 10^{-8}\left(\frac{\epsilon_n}{\epsilon_c}\right)^{2/3} \exp\left\{ -1.12\left(\frac{\epsilon_n}{\epsilon_c}\right)^{-1/3}\right\} \text{ m} . \qquad (4.12)$$

This is to be compared with the Oide limit on beam size at the focus which can be expressed as

$$\sigma \gtrsim 3.4 \times 10^{-4} \epsilon_n^{5/7} , \qquad (4.13)$$

in the vertical dimension for flat beams. For an emittance $\epsilon_n = \epsilon_c/10$, we find that $\sigma_{ad} \gg 2.68 \times 10^{-9}$ m, whereas the Oide limit for the same emittance gives $\sigma \gtrsim 1.25 \times 10^{-8}$ m.

Numerical examples have been studied by Chen et al.[8] The first is a proof-of-principle case using the beam in the SLAC End Station. The second involves the use of a focuser on the SLC, and the third is a focuser on a TeV Linear Collider (TLC). Parameters of the beam, the focuser, and the expected performance are displayed in Table II. In the first two cases, round beams, i.e., $\sigma_y = \sigma_x$, are assumed, whereas in the third case for the TLC, the beam is assumed to be flat ($\sigma_y \ll$ a,). In the End Station Focuser, the device is rather long and one may employ differential pumping to form the variation in plasma density, ramping from the initial value, n_0, to the final value, n^*, over a length L. The other two focusers require higher densities (ranging up to solid density), with variation over shorter distances.

Table II Three examples of the adiabatic focuser

	SLAC End Station	SLC	TLC
Initial Beam Properties			
$\mathcal{E}$ [GeV]	15	50	500
ϵ_n [m]	1×10^{-4}	3x10-5	1 x10-8
σ_0 [μm]	20	3	5×10^{-3}
PO [cm]	12	3	0.25
Focuser Properties			
α_0	5 x10-2	$1/\sqrt{3}$	$\sqrt{3}$
L [cm]	119	2.6	0.07
n_0 [cm^{-3}]	1.2×10^{14}	8.4×10^{15}	1.8×10^{19}
n^* [cm^{-3}]	1.2×10^{18}	8.4×10^{19}	1.8×10^{23}
Final Beam Properties			
δ	Negligible	3%	1%
σ^* [μm]	2	0.3	0.5×10^{-3}

5. BEAM SIZE MONITOR

For the purpose of monitoring the beam size, one again envisions an underdense plasma intercepting the focused beam. In principle, both the beam and the ions in the plasma carry useful signals on the beam size due to their mutual interaction. The plasma electrons, however, are too severely perturbed to provide any information. Regarding the plasma formation, either preformed plasma by external means, or self-ionization by the incoming beam can be conceived, depending on practical considerations on the device.

In the concept suggested by Chen et al.,[11] a heavy element gas jet with density n_g is ionized by a high energy beam of N particles. The number of ions created in the beam channel is

$$N_i = \frac{N\sigma_i}{4\pi\sigma_x\sigma_y} N_g \quad , \tag{5.1}$$

where a; is the ionization cross section and N_g is the total number of gas atoms in the channel:

$$N_g = 4\pi\sigma_x\sigma_y l n_g \ . \tag{5.2}$$

Here l is the thickness of the gas target. The ionization is considered to be non-saturated if $N_i < N_g$, or $N\sigma_i/4\pi\sigma_x\sigma_y < 1$, from Eq. (5.1). In this regime one can easily see that N_i is independent of the beam size. However, if the ionization cross section is-so large that $N\sigma_i/4\pi\sigma_x\sigma_y > 1$, then the ionization will be saturated, as there cannot be more ions than the available number of atoms in the channel. In this regime $N_i = N_g$, and N_i is now a function of the beam size as in Eq. (5.2).

As we have seen in Sec. 3, the focusing strength due to an ion column is proportional to the number of ions encompassed in the beam volume. The synchrotron radiation triggered by the ion focusing can again be described by the parameter Υ, in this case

$$\Upsilon = \frac{\sqrt{3}}{2} \frac{r_e \lambda_c \gamma N_i}{l\sigma_y(1+R)} \quad . \tag{5.3}$$

Inserting the expression in Eq. (5.2) for N_i, we find, to the accuracy of the order $1/R$, that

$$\Upsilon = 2\sqrt{3}\pi r_e \lambda_c \gamma n_g \sigma_y \ . \tag{5.4}$$

Since the critical energy of synchrotron radiation is $\omega_c = (3/2)\Upsilon\mathcal{E}$, where $\mathcal{E}$ is the beam particle energy, one should in principle be able to measure σ_y by analyzing the spectrum of such a radiation.

Note that other than the gas density, the signal is a function of σ_y only, and is independent of σ_x, σ_z, and N of the beam, and the thickness l of the target. This is important because in the case of focusing flat beams the challenge is generally in the vertical dimension, where σ_y in the range of nanometers is conceived for the next generation linear colliders. Therefore, one expects that in the final focusing process the variation in σ_x is relatively mild, and the major -uncertainty comes from the minor dimension, which is what we measure. We should also emphasize that in case saturation of self-ionization turns out to be unattainable, a preformed, fully ionized underdense plasma by external means would suffice the same purpose for radiation signals.

Buon[12] considers the case of light ions. For electron beams, the ions would experience an attractive potential well. So when the ions are light enough, they will oscillate during the beam passage. The nature of the beam collective field is such that the maximum field strength along the vertical direction is very close to that in the horizontal direction. (The actual maximum values in the two dimensions are slightly different, which will be discussed below.) In addition, the locations of these maximum fields are roughly at σ_x and σ_y, respectively. Therefore, the focusing strengths in x and y directions differ by a factor R : $K_y = RK_x$. From conservation of total energy, we have

$$\begin{aligned} \frac{1}{2}K_x\sigma_x^2 &= \frac{1}{2}Mv_{x_m}^2 \quad , \\ \frac{1}{2}K_y\sigma_y^2 &= \frac{1}{2}Mv_{y_m}^2 \quad , \end{aligned} \tag{5.5}$$

respectively. It is easy to see that the maximum horizontal and vertical velocities differ by a factor $\sqrt{R}$:

$$v_{y_m} = \sqrt{R}v_{x_m} \quad . \tag{5.6}$$

By measuring the angular distribution of the out-coming ions, one can then infer the beam aspect ratio.

Let us now introduce yet one more mechanism, using heavy ions, for monitoring flat beam size. Ionization saturation is not necessary in this approach. To a large extent, this is an extension from the ideas of Rees[9] and Prescott.[10] Consider a plasma density which is orders of magnitude-lower than that of the beam. Under this situation the ion focusing on the beam, and therefore the change of the field strength in the beam, can be ignored. For heavy ions which are initially quiescent and immobile within the time scale of the transit of the beam, the impulse received by the ions are predominantly from the local electric component of the collective beam field. Since the beam field reaches a maximum on the "boundary" of the beam and decreases outside the beam, the ion momenta will be, in general, double-valued. This makes reconstruction of the entire transverse beam field ambiguous. The maximum fields, however, are single-valued. More importantly, the maximum fields in x and y directions do not scale the same in R. As we shall see below, by comparing the maximum momenta in the x and y directions one can in principle determine both σ_x and σ_y.

The strength of the electric field in a flat beam can be described by the Basseti-Erskine formula?' To the accuracy of the order 1/R, we find

$$\begin{aligned} E_x(x,y,z) &\simeq \frac{eN}{\sigma_x\sigma_z}\, e^{-z^2/2\sigma_z^2}\ \mathcal{I}m\, F(x,y) \quad , \\ E_y(x,y,z) &\simeq \frac{eN}{\sigma_x\sigma_z}\, e^{-z^2/2\sigma_z^2}\ \mathcal{R}e\, F(x,y) \quad , \end{aligned} \tag{5.7}$$

-where

$$F(x,y) = \mathcal{W}\left(\frac{x+iy}{\sqrt{2}\sigma_x}\right) - e^{-(x^2/2\sigma_x^2+y^2/2\sigma_y^2)}\,\mathcal{W}\left(\frac{x/R+iRy}{\sqrt{2}\sigma_x}\right) \quad , \quad R \gg 1$$

Here $\mathcal{W}$ is the complex error function. The maxima of E_x along z-axis and of E_y along y-axis can be found by solving the equations $\partial \mathcal{I}m\, F(x,0)/\partial x = 0$ and $\partial\, \mathrm{Re}\, F(0,y)/\partial y = 0$, respectively. We find that the maximum E_x and E_y locates at

$$\frac{x_m}{\sigma_x} = 1.307 + \frac{0.393}{R} \quad , \tag{5.8}$$

and

$$\frac{y_m}{\sigma_y} = \sqrt{2\ln R} + \frac{1}{R}\sqrt{\frac{\pi}{2}} \quad . \tag{5.9}$$

The corresponding field strengths at these locations are

$$\begin{aligned} E_{x_m} &= \frac{eN}{\sigma_x\sigma_z}\, e^{-z^2/2\sigma_z^2}\, \sqrt{\frac{2}{\pi}}\frac{\sigma_x}{x_m}\left[1 - \frac{1}{R}\, e^{-x_m^2/2\sigma_x^2}\right] \quad , \\ E_{y_m} &= \frac{eN}{\sigma_x\sigma_z}\, e^{-z^2/2\sigma_z^2}\, \sqrt{\frac{2}{\pi}}\frac{\sigma_x}{y_m}\left[1 - \left(R + \frac{1}{R}\right)\, e^{-y_m^2/2\sigma_y^2}\right] \quad . \end{aligned} \tag{5.10}$$

Inserting the explicit forms of x_m and y_m in Eq. (5.8) and Eq. (5.9), we obtain

$$\begin{aligned} E_{x_m} &= \frac{eN}{\sigma_x\sigma_z}\, e^{-z^2/2\sigma_z^2}\, \sqrt{\frac{2}{\pi}}\frac{1}{1.307}\left[1 - \frac{0.726}{R}\right] \quad , \\ E_{y_m} &= \frac{eN}{\sigma_x\sigma_z}\, e^{-z^2/2\sigma_z^2}\left\{1 - \frac{\sqrt{\pi}}{2R}\left[\sqrt{\ln R} + \frac{1}{\sqrt{\ln R}}\left(\frac{1}{2} + \frac{2}{\pi}\right)\right]\right\} \quad . \end{aligned} \tag{5.11}$$

We see that E_{x_m} and E_{y_m} are asymptotically independent of R, but the asymptotic values differ by a factor 0.610.

With the assumption that the ions are immobile during the beam transit time, the impulse received by the ions is a trivial function of the local field strength. This is obtained by integrating the fields in Eq. (5.11) over time ($t = z/c$). For singly charged ions we find

$$\begin{aligned} \Delta p_{x_m} &= \frac{2}{1.307}\,\frac{e^2N}{c\sigma_x}\left[1 - \frac{0.726}{R}\right] \quad , \\ \Delta p_{y_m} &= \sqrt{2\pi}\,\frac{e^2N}{c\sigma_x}\left\{1 - \frac{\sqrt{\pi}}{2R}\left[\sqrt{\ln R} + \frac{1}{\sqrt{\ln R}}\left(\frac{1}{2} + \frac{2}{\pi}\right)\right]\right\} \quad . \end{aligned} \tag{5.12}$$

As long as the total bunch population N, or the beam current, is known, measurements of Δp_{x_m} and Δp_{y_m} should in principle determine uniquely the values of σ_x and ***R***, or in turn σ_x and σ_y. In practice, however, this scheme may suffer from low statistics. From Eq. (5.12) we see that the relative accuracy of ***R*** diminishes roughly as 1/R:

$$\frac{\delta\Delta p_{x_m}}{\Delta p_{x_m}} = \frac{0.726}{R}\,\frac{\delta R}{R} \quad . \tag{5.13}$$

To have a 10% error in $\boldsymbol{R}$, one needs to have an accuracy of 1% in Δp_{x_m} when $\boldsymbol{R}$ is around 10, and of 0.1% when $\boldsymbol{R} \sim 100$. Evidently, a large collection of data is necessary to achieve this goal. This could be realized by increasing the gas jet density and the repetition of measurements. One other concern is that ions can never be infinitely heavy. This would result in a certain degree of smearing of the signal due to ion motions across the peak fields in the beam. The best way to assess this degradation is through computer simulations, which are still in progress.

It is also of interest to note that the angular distribution of the out-coming heavy ions should also provide useful information on the beam size. This can be appreciated by realizing first that the maximum field lies on the "boundary" of the beam. Secondly, the transverse field tends to "flip" from the horizontal direction&wards the vertical direction as soon as one departs from x_m along the boundary. This asymmetry of the field pattern should also help to diagnose the aspect ratio.[21]

6. DISCUSSION

Confirmation of the existence of strong self-focusing in plasma wakefields has been experimentally verified in tests performed at Argonne Advanced Accelerator Test Facility.[14] More recently, a dedicated plasma lens experiment has been performed in Japan,[15] which covers both the overdense and the underdense regimes. These experiments use electron beams of energies of the order 20 MeV. Although the physics of self-focusing should not be different as long as the beams are relativistic, it is of interest to test the plasma lens with beam characteristics pertinent to that in a true linear collider. To this end, such a test at the Final Focus Test Beam (FFTB) now under construction at SLAC, with the possibility of testing both round and flat 50 GeV electron and positron beams, should be meaningful.

Examples of adiabatic focuser have been shown in Table II in Section 4. When applied to the beam parameters similar to those of the SLAC End Station, where the beam energy is 15 GeV, and those of SLC, the necessary parameters for the plasma adiabatic focuser are shown to be very reasonable; and, in principle, to yield a significant increase in the luminosity for the SLC. To apply the scheme to TeV-range linear colliders, it is found necessary to invoke liquid or even solid-state materials. Although the necessary technology for the focuser is yet to be developed, such a focuser should in principle be more compact than the conventional focusing system. In particular, for a focuser relevant to the SLAC End Station-type parameters, the requirements for the system seems to be immediately realizable.

The several ideas of beam size monitoring using plasmas were reviewed and introduced. These concepts have been intensely pursued recently. Experimental effort on such a monitor is now under way through an Orsay-SLAC collaboration. The device is expected to be tested at the FFTB at SLAC.

To conclude, we have reviewed various novel plasma focusing and diagnosis concepts, and have introduced a new scheme of beam size measurement. The potential applications of plasma physics to high energy particle beam instrumentation is seen to be very rich. Most importantly, many of these concepts have been followed by experimental efforts, and are in a very healthy state of progress. We expect that in a few years, more experimental verifications will be available.

ACKNOWLEDGEMENTS

I would like to thank my collaborators in Refs. 4, 6, 8, and 11 for using the materials directly from the papers. I would also like to thank J. Buon of Orsay, France, and D. L. Burke and R. D. Ruth of SLAC, Stanford, for many helpful discussions on beam size measurements.

REFERENCES

[1]K. Oide, Phys. Rev. Lett. 61, 1713 (1988).

[2]P. Chen, Part. Accel. 20, 171 (1987).

[3]P. Chen, J. J. Su, T. Katsouleas, S. Wilks, and J. M. Dawson, IEEE Trans. Plasma Sci. PS-15, 218 (1987).

[4]J.B. Rosenzweig and P. Chen, Phys. Rev. D39, 2039 (1989).

[5]D. B. Cline, B. Cole, J. B. Rosenzweig and J. Norem Proceedings of ***the 1987*** *Washington Accelerator* Conference (IEEE, Washington, 1987) p. 241.

[6]P. Chen, S. Rajagopalan, and J. B. Rosenzweig, Phys. Rev. D40, 923 (1989).

[7]J.J.Su, T. Katsouleas, J. Dawson, and R. Fedele, UCLA Report No. PPG-1177, 1988; to appear in Phys. Rev. A.

[8]P. Chen, K. Oide, A. Sessler, and S. Yu, Phys. Rev. Lett. 64, 1231 (1989).

[9]J. Rees, SLAC Report SLAC-AP-24, 1984.

[10]C. Prescott, SLAC internal memorandum, 1984, unpublished.

[11]P. Chen, D. L. Burke, M. D. Hildreth, and R. D. Ruth, SLAC Report SLAC-AAS-41, 1988, presented at the International Workshop on the Next Generation of Linear Colliders, Stanford, 1988.

[12]J. Buon, Part. Accel. 31, 39 (1990).

[13]T. Katsouleas, Phys. Rev. A33, 2056 (1986).

[14]J. Rosenzweig, P. Schoessow, B. Cole, C. Ho, W. Gai, R. Konecny, S. Mtingwa, J. Norem, M. Rosing, and J. Simpson, Fermilab Report FERMILAB–PUB–89/213, 1989; to appear in Phys. Fluids B.

[15]A. Ogata ***et*** al., KEK Report, in preparation.

[16]P. Chen and K. Yokoya, Phys. Rev. D38, 987 (1988).

[17]E. D. Courant and H. S. Snyder, Ann. Phys. 3, 1 (1958).

[18]A. A. Sokolov and I. M. Ternov, *Synchrotron* ***Radiation*** (Pergamon, Berlin, 1968).

[19]This approximation was first suggested by P. B. Wilson, ***Proceedings of the IEEE Particle Accelerator Conference,*** IEEE Catalog No. 87CH2387–9, 53 (1987). Our expression, however, has different boundary values.

[20]M. Basseti and Erskine, CERN Report CERN-ISR-TH/80-06, 1980.

[21]P. Chen and R. D. Ruth, in preparation.