SLAC–PUB–5185
January 1990
(T,Noyes)

A FINITE, RATIONAL MODEL FOR THE EPR-BOHM EXPERIMENT*

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ABSTRACT

We construct 3+1 space-time and particles from bit-strings of finite length. At an early stage in the construction, we find that we can identify single particle Dirac wave functions. Considering a space-like trajectory for a single particle we find the usual spin correlations which violate Bell's inequalities. The only "non-local" parameter is our discrete version of the global time ordering of special relativity.

Contributed to the Symposium on the Foundations of Modern Physics 1990, Joensuu, Finland, August 13–17, 1990

⋆ Work supported by the Department of Energy, contract DE–AC03–76SF00515.

1. INTRODUCTION

Bell[1] reduced the Bohm[2] version of the EPR[3] *Gedanken*-experiment to the problem of making a physically acceptable model for the calculation of $< \vec{a} \cdot \vec{b} >$. Here $\vec{a}$ and $\vec{b}$ are the spin directions of two space-like separated (massive) spin-$\frac{1}{2}$ particles in a singlet state. Under his (and Einstein's) understanding of "physically acceptable", Bell proved that it is impossible to find such a model compatible with standard quantum mechanics. To my knowledge no model acceptable to Bell (and retrospectively to Einstein) exists which explains the laboratory realizations of many EPR experiments. Our aim in this paper is to model the global correlations predicted by quantum mechanics while preserving the space-like separation and quasi-local character of the two spin-state detections. That such a model exists analogically has been claimed previously[4]. Here the general argument is replaced by a specific physical model which, although initially developed abstractly, eventually is interpreted as attributing the necessarily global correlations to the vacuum fluctuations of our discrete version of relativistic quantum mechanics.

The basic idea which we use comes from Feynman's image of an electron moving "backward in time" as a positron. From this point of view, the production of an electron-positron pair in a singlet state followed by the subsequent detection of their spins in two space-like separated regions — obviously a Bell-Bohm-EPR situation — can be thought of as a electron which originates in one region, travels backward in time to the pair production event and then forward in time to the end of its space-like trajectory. That its spin is conserved in the process, and hence produces the standard singlet spin correlations when measured is then no surprise.

We set this calculation in a new context by constructing the space-time and the wave function together starting from a finite and discrete collection of bit-strings[5]. The wave function represents a discrete *Zitterbewegung* with steps of length h/mc executed in a time h/mc^2 and hence always with velocity $\pm c$. What is novel in our construction is that these steps can be kept finite, and are Lorentz invariant under finite and discrete Lorentz transformations even though for a large number of steps the wave functions we construct can be approximated by the solutions of the conventional one-particle Dirac equation. In a more general theory, which we can only sketch here, the *Zitterbewegung* is a concommitent of the finite "background radiation" which is observed via the Casimir effect.

2. CONSTRUCTION OF COMPLEX WAVE FUNCTIONS FROM BIT-STRINGS

2.1. WHY COMPLEX AMPLITUDES?

Examination of the foundational ideas[6] needed to construct a finite and discrete relativistic quantum mechanics from bit-strings[7] led McGoveran to the conclusion that the non-classical statistics in quantum mechanics (eg. complex probability amplitudes rather than real probabilities) can be modeled in any system whose multiple paths between two "events" share indistinguishable elements. Consider first a situation with two alternatives, with P_1 or P_2 paths characterized by one or the other alternative. If the total number of paths is P, the elementary treatment takes $P_1 + P_2 = P$; this cannot always represent the situation when the paths are *independently* generated and hence define a joint probability space with $P_1 P_2$ elements. In order to satisfy both constraints, we form $P_1^2 + P_2^2 = P^2 - 2P_1 P_2 \equiv R_{12}^2$, which is identically satisfied if the two are not independent. If, due to indistinguishable paths which we do not know how to assign to either P_1 or P_2, we have indeed made the two independent in the sense that the product $P_1 P_2$ is no longer constrained other than by the inequality $2P_1 P_2 < P^2$, we can adopt R_{12}^2 as the measure of the square of the number of paths in this new space. Taking the product $2P_1 P_2 = f^2 P^2$ where f is some rational fraction less than unity, we thus arrive at the general result

$$P_1^2 + P_2^2 = R_{12}^2 = P^2(1 - f)(1 + f) \tag{2.1}$$

which has been derived by McGoveran[8] by considering case counts including indistinguishables. We can now *define*

$$\psi = P_1 + iP_2 \tag{2.2}$$

with the normalization condition

$$\psi^*\psi = R^2 \tag{2.3}$$

Clearly we can divide ψ by R to get the normalization condition $\psi^*\psi = 1$ when we are modeling the situation in which a single system engages in the two events at the two endpoints with certainty. Once we have this general result, it is simply a matter of mathematical convenience whether we use real or complex amplitudes to model this constraint, and norm it to unity when the probability of the system traversing the "space" between the two events is unity.

2.2. Bit-Strings

We specify a *bit-string*

$$\mathbf{X}(S) = (..., b_i^x, ...)_S \tag{2.4}$$

by its S ordered elements

$$b_i^x \in 0,1; \quad i \in 1,2,....S; \quad 0,1,...,S \in \ \textit{ordinal integers} \tag{2.5}$$

and its norm by

$$|\mathbf{X}(S)| = \Sigma_{i=1}^S b_i^x = X \tag{2.6}$$

Define the *null string* by $\mathbf{0}(S)$, $b_i^0 = 0$ for all i and the *anti-null* string by $\mathbf{1}(S)$,

$b_i^1 = 1$ for all i. Define *discrimination* (XOR) by

$$\mathbf{X} \oplus \mathbf{Y} = (..., b_i^{xy}, ...)_S = \mathbf{Y} \oplus \mathbf{X};\ b_i^{xy} = (b_i^x - b_i^y)^2 \tag{2.7}$$

from which it follows that

$$\mathbf{A} \oplus \mathbf{A} = \mathbf{0};\ \mathbf{A} \oplus \mathbf{0} = \mathbf{A} \tag{2.8}$$

We will also find it useful to define

$$\bar{\mathbf{A}} = \mathbf{A} \oplus \mathbf{1};\ \textit{hence}\ \mathbf{A} \oplus \bar{\mathbf{A}} \oplus \mathbf{1} = \mathbf{0} \tag{2.9}$$

2.3. ONE DIMENSIONAL AMPLITUDES

Consider two independently generated strings $\mathbf{A}(S), \mathbf{B}(S)$ restricted by $|\mathbf{A} \oplus \mathbf{B}|=n$ and $A - B = c$. We call these the *boundary conditions.* We now construct two substrings $\mathbf{a}(n), \mathbf{b}(n)$ by the following recursive algorithm starting from $i, j = 0$ and ending at $i = S,\ j = n$.

$$i := i + 1$$

$$if\ b_i^A = 1\ and\ b_i^B = 0\ then\ j := j + 1\ and\ b_j^a := 1\ and\ b_j^b := 0$$

$$if\ b_i^A = 0\ and\ b_i^B = 1\ then\ j := j + 1\ and\ b_j^a := 0\ and\ b_j^b := 1$$

$$if\ (b_i^A - b_i^B)^2 = 0\ then\ j,\ b_j^a\ and\ b_j^b\ do\ not\ change$$

Once we have made this construction,

$$\mathbf{a}(n) \oplus \mathbf{b}(n) \oplus \mathbf{1}(n) = \mathbf{0}(n) \tag{2.10}$$

and we can interpret the string $\mathbf{a}$ as representing a "random walk" in which a "1" represents a step forward and a "0" represents a step backward, as in the Stein[9–12] paradigm. Define

$$a_j = \Sigma_{k=1}^{j} b_k^a; \; b_j = \Sigma_{k=1}^{j} b_k^b \tag{2.11}$$

We call the "points" $(a_j - b_j, j)$ connecting $(0,0)$ to (c,n) a *trajectory*; the new ordering parameter j then represents "causal" time order along the trajectory. Note that $a + b = n$ and $a - b = A - B = c$ for any trajectory because of our boundary conditions.

We can also define a *path* in the larger space s_i, A_i, B_i where

$$s_i = \Sigma_{k=1}^{i} s_k = \Sigma_{k=1}^{i} b_k^A b_k^B \tag{2.12}$$

$$A_i = \Sigma_{k=1}^{i} b_k^A (b_k^A - b_k^B)^2 + s_k; \; B_i = \Sigma_{k=1}^{i} b_k^B (b_k^A - b_k^B)^2 + s_k$$

Note that by construction $A_i - B_i = a_j - b_j$ and hence A_i, B_i is tied to the same trajectory in the $(a_j - b_j, j)$ plane; it acquires a third "orthogonal" coordinate due to those cases when both A_i and B_i are incremented by 1. Note also that there is no way from our boundary conditions or from the trajectory to tell those cases from those where i advances but neither A_i nor B_i nor s_i is incremented. All we know is that $s_{AB} = \Sigma_{k=1}^{S} b_k^A b_k^B$, lies in the range $0 \leq s_{AB} \leq S - n$. It is these *indistinguishable* paths which create the interfering alternatives in our model.

We now ask how many paths characterized by some ordering parameter $s = 0, 1, 2, ..., S - n$ satisfy our boundary conditions. By construction each path is tied to the n points which compose a trajectory, and can be chosen in n^s ways. Note that we have broken the causal connection between path and trajectory. Of the total number of ways of choosing a path characterized by s from the $S!/(S-s)!$ possibilities, only $S!/s!(S-s)!$ are distinct. Consequently, the probability of having a path characterized by s is

$$\frac{S!/s!(S-s)!}{S!/(S-s)!} = \frac{1}{s!} \tag{2.13}$$

Thus the total number of paths is

$$P(n;S) = \Sigma_{s=0}^{S-n} \frac{n^s}{s!} = \Sigma_{s=0}^{S-n} p_s(n) \equiv exp_{S-n}(n) \tag{2.14}$$

where $exp_{S-n}(n)$ is the *finite exponential.* This is a general result for the *transport operator* referring to *attribute distance* as has been proved by McGoveran in FDP, Theorems 36-40, pp 55-58.

Although Eq. 2.14 specifies the total number of paths, given S and n, it conceals a four-fold ambiguity arising from the construction. However the sequence of paths is generated, the order adopted in the sum implies a recursive generation of the terms $p_s(n) = n^s/s!$ given by

$$p_{s+1}(n) = np_s(n)/(s+1); \; p_0(n) = 1 \tag{2.15}$$

The first ambiguity is the fact that we do not know whether $S - n$ is even or odd outside of the uninteresting case $S = n$ when paths and trajectories coincide; hence we do not know whether the sum terminates in an even or an odd term. The second ambiguity arises because, however s is ordered, we do not know how many cases arise because *both* A_i and B_i are incremented, or *neither.* To include this dichotomy we split the even and odd sequences themselves into two sequences corresponding to these alternatives which we call 11 and 00, giving four recursion relations:

$$p_{s+4}^{e,11}(n) = \frac{n^4}{(s+4)(s+3)(s+2)(s+1)} p_s^{e,11}(n); \; p_0^{e,11}(n) = 1$$

$$p_{s+4}^{o,11}(n) = \frac{n^4}{(s+4)(s+3)(s+2)(s+1)} p_s^{o,11}(n); \; p_1^{o,11}(n) = n$$

$$p_{s+4}^{e,00}(n) = \frac{n^4}{(s+4)(s+3)(s+2)(s+1)} p_s^{e,00}(n); \; p_2^{e,00}(n) = \frac{1}{2}n^2$$

$$p_{s+4}^{o,00}(n) = \frac{n^4}{(s+4)(s+3)(s+2)(s+1)} p_s^{o,00}(n); \; p_3^{o,00}(n) = \frac{1}{6}n^3 \tag{2.16}$$

At some point which depends on whether (a) $S - n$ is even or odd and/or $2s_{AB}$

is greater or less than $S-n$, this four-fold ordering of the terms in the sum over s has to stop, and may or may not leave some terms unaccounted for. Calling the contribution of these terms to the sum ΔP, we find that our construction allows us to decompose the sum over paths as follows:

$$P(n;S) = \Sigma_{s=0}^{S-n}[p_s^{e,11} + p_s^{o,11} + p_s^{e,00} + p_s^{o,00}] + \Delta P \tag{2.17}$$

We are now in the general situation discussed at the start of this chapter, except that our construction has provided us with *four* types of path rather than two. Now that we have recognized that the amplitudes — whose square gives a quantity which can be normed to form a probability — can be complex, we have no conceptual barrier to forming real combinations which can be negative as well as positive. The obvious choice is to form those which lead to the finite sines and cosines, i.e. by subtracting the two components of the odd or even series from each other:

$$R\ cos_{S-n}(n) = R\ \Sigma_{k=0}^{\frac{1}{2}(S-n)}(-1)^k \frac{n^{2k}}{(2k)!} = \Sigma_{s=0}^{(S-n)}[p_s^{e,11} - p_s^{e,00}] \tag{2.18}$$

$$R\ sin_{S-n}(n) = R\ \Sigma_{k=1}^{\frac{1}{2}(S-n)}(-1)^{k+1} \frac{n^{2k-1}}{(2k-1)!} = \Sigma_{s=1}^{(S-n)}[p_s^{o,11} - p_s^{o,00}] \tag{2.19}$$

The two constructions can now be combined by taking the normalized wave function to be

$$\psi_{S-n}(n) = exp_{S-n}(in) = \Sigma_{s=0}^{\frac{1}{4}(S-n)} \frac{(in)^s}{s!} \tag{2.20}$$

Thus, by taking proper account of the interference between *independently generated* paths which share *indistinguishable* elements, we claim to have *derived* Feynman's prescription[13]. for calculating the quantum mechanical wave function as a "sum over paths" with imaginary *finite* and *discrete* steps.

3. DISCRETE FREE-PARTICLE WAVE FUNCTIONS

3.1. Construction of Space-time Coordinates

1+1 dimensions

In any universe of bit strings of length S, all quadruples such that

$$\mathbf{A} \oplus \mathbf{B} \oplus \mathbf{C} \oplus \mathbf{D} = \mathbf{0} \tag{3.1}$$

are called *events*. Note that this implies that

$$\mathbf{A} \oplus \mathbf{B} = \mathbf{C} \oplus \mathbf{D};\ \mathbf{A} \oplus \mathbf{C} = \mathbf{B} \oplus \mathbf{D};\ \mathbf{A} \oplus \mathbf{D} = \mathbf{B} \oplus \mathbf{C} \tag{3.2}$$

$$\mathbf{A} = \mathbf{B} \oplus \mathbf{C} \oplus \mathbf{D};\ \mathbf{B} = \mathbf{C} \oplus \mathbf{D} \oplus \mathbf{A};\ \mathbf{C} = \mathbf{D} \oplus \mathbf{A} \oplus \mathbf{B};\ \mathbf{D} = \mathbf{A} \oplus \mathbf{B} \oplus \mathbf{C} \tag{3.3}$$

Consider an event defined by four independently generated strings $\mathbf{F}, \mathbf{B}, \mathbf{R}, \mathbf{L}$ whose norms are F, B, R, L; all must be less than or equal to $|\mathbf{1}| = S$. For the moment we need only define a fifth integer n by

$$|\mathbf{F} \oplus \mathbf{B}| = n = |\mathbf{R} \oplus \mathbf{L}| \tag{3.4}$$

Our intent is to construct a discrete square coordinate mesh (z_i, t_j) with $(2n+1)^2$ points within which we can model piecewise continuous ordered trajectories (z_k, t_k) which connect the "endpoint" $(0,0)$ to some "endpoint" (z,t) lying on the boundary of the square

$$t = \pm n,\ -n \le z \le n;\ z = \pm n,\ -n \le t \le n \tag{3.5}$$

The order parameter $0 \le k \le n$ traverses any space-time point along the trajectory only once; in addition we require that

$$z_{k+1} - z_k = \pm 1;\ t_{k+1} - t_k = \pm 1;\ (\textit{four choices}) \tag{3.6}$$

The description is *static* in the sense that it can be read either from 0 to n or from n to 0 and still describe the *same* trajectory. Note that in contrast to previous discussions, (a) we consider space-like as well as time-like trajectories, and (b) that the length of the strings $S \geq n$ is not specified; it is some finite integer named in advance of the construction. Note further that since we specify both endpoints, we are describing a *completed* process. The "wave functions" we will eventually construct on this mesh will be "born collapsed". All our results will belong to the "fixed past"; whether we should or should not use our theory to *predict* the future, either in a deterministic or a statistically deterministic sense, is a separate issue we will not discuss in this paper. We have picked our boundary conditions $(0,0) - - - (z,t)$ in the process of specifying the problem.

Any space-time point (z_k, t_k) not on the axes $(z_k, 0), (0, t_k)$ lies in one of the four quadrants $(+,+) \leftrightarrow R > L,\ F > B$, $(-,+) \leftrightarrow R < L,\ F > B$, $(+,-) \leftrightarrow R > L,\ F < B$, $(-,-) \leftrightarrow R < L,\ F < B$. We define our bounding endpoints in terms of our basic parameters, and four new parameters r, l, f, b by $|t| > z \leftrightarrow z = R - L = r - l; t = n = r + l$, $|t| < -z \leftrightarrow z = R - L = r - l; t = -n$, $|z| < t \leftrightarrow z = n = f + b; t = F - B = f - b$, $|z| < -t \leftrightarrow z = -n; t = F - B = f - b$.

The advantage of introducing the new parameters r, l, f, b is that they make it easy to define what will become Lorentz invariants. Explicitly

$$t^2 - z^2 = \tau^2 = 4rl = n^2(1 - \beta^2)\ with\ \beta = \frac{2r}{n} - 1$$

$$z^2 - t^2 = -\tau^2 = 4fb = n^2(1 - \omega^2)\ with\ \omega = \frac{2f}{n} - 1 \tag{3.7}$$

As we have shown many times[6, 7] it is easy to give meaning to the concept of Lorentz invariance in our discrete context. Defining $r' = \rho r$, $l' = \rho^{-1} l$, τ^2 is obviously invariant, and if we define $\gamma_\rho = \frac{1}{2}(\rho + \rho^{-1})$, $\beta_\rho^2 = 1 - \frac{1}{\gamma_\rho^2}$ we have immediately that

$$z' = \gamma_\rho(z + \beta_\rho t);\ t' = \gamma_\rho(t + \beta_\rho z) \tag{3.8}$$

3+1 dimensions

To distinguish space from time in the model, we include additional *spacial* dimensions which we require to be *homogeneous* and *isotropic* in the sense that none of the symmetry properties depend on the choice of the labels $x, y, z, ...$ One of the great conceptual advantages of our constructive approach is that McGoveran has *proved* that in our theory the extension from 1+1 space-time to 2+1 and 3+1 has to stop there (FDP Section 3.4, pp 30-34). To see how this applies in our context, fix the F, B pair as defining the universal ordering parameter j for causal space-time events, and try to construct not only the z coordinate from the R,L pair as above but three additional independently generated pairs $W_+, W_-; X_+, X_-, Y_+, Y_-$ to construct the coordinates $w = W_+ - W_-, x = X_+ - X_-, y = Y_+ - Y_-$, and for consistency in the notation replace L,R by Z_-, Z_+ with z = $Z_+ - Z_-$.

Following the same procedure as above, we generate four substrings $\mathbf{w}_+(n)$, $\mathbf{x}_+(n)$, $\mathbf{y}_+(n)$, $\mathbf{z}_+(n)$. Since these four strings are *independent* by hypothesis, they cannot discriminate to the null string, so we need a definition of *event* appropriate to this situation. We take this to be those values of j for which all four strings have accumulated the same number of "1" 's, i.e.

$$\Sigma_{k=1}^{j} b_k^{w+} = \Sigma_{k=1}^{j} b_k^{x+} = \Sigma_{k=1}^{j} b_k^{y+} = \Sigma_{k=1}^{j} b_k^{z+} \tag{3.9}$$

The extension to D rather than 4 spacial dimensions is obvious. This reduces the probability of events occurring after j space-time steps in D dimensions to the probability of obtaining the same number of "1" 's in D independent Bernoulli sequences after j trials[14] ,

$$p(j) = \frac{1}{2^{jD}} \Sigma_{k=0}^{j} \binom{j}{k}^D < j^{-\frac{D-1}{2}} \tag{3.10}$$

Clearly this definition of events defines a "homogeneous and isotropic" d-space, but the probability of being able to *continue* to find events for large values of j vanishes for $D > 3$. Consequently we need only consider three spacial dimensions. Thus, provided we have some clear way to label independent bit strings, we can extend our construction of 1+1 space-time to 3+1 space-time, but no further.

3.2. SPACE-TIME WAVE FUNCTIONS

Time dependence of the Schroedinger wave function

To relate our wave function construction the time *Zitterbewegung* of an isolated system we invoke our "counter paradigm" [15,16]. If two counters a distance $|z|$ apart fire sequentially with a time interval $|t|$ as measured with clocks synchronized by the Einstein convention, we assume that they are connected by a space-time trajectory with Lorentz-invariant step parameters h/mc in space and h/mc^2 in time. As noted in the Introduction, this implies a *Zitterbewegung* with velocity steps $\pm c$ connecting "points" between the two bounding events. We have shown above (Eq. 2.20) that to finite accuracy and for n large our exact combinatorial result for the wave function can be approximated by e^{in} for n discrete steps. Putting this together with the counter paradigm (our *rule of correspondence*[6,7]) this tells us that for an isolated system with $E = mc^2$, the period is $T = 1/\nu = h/E = h/mc^2$ and the angular frequency is $E/\hbar$. Consequently any isolated system has a combinatorial wave function which can be approximated by a solution of the equation

$$\pm i\hbar \partial\psi/\partial t = E\psi \tag{3.11}$$

We emphasize that our solutions are derived only when restricted by the space-time boundary conditions which represent *completed* processes. For us it would be a *serious error* to try to interpret this or any other Schroedinger-type equation as describing the *causal* evolution of a complex amplitude.

The Klein-Gordon Equation

We have seen that this time "evolution" can be transformed from the rest system with $\tau^2 = n_0^2(h/mc)^2$ to an arbitrary system with $\tau^2 = c^2t^2 - z^2$ in which the velocity between the endpoints of the trajectory is $\beta = z/ct$. Consequently we have already constructed the discrete solutions which can be approximated by continuum solutions of the equation

$$\partial^2\psi/\partial z^2 - \partial^2\psi/c^2\partial t^2 = (mc/\hbar)^2\psi \tag{3.12}$$

Extension to 3+1 dimensions is immediate.

The Dirac Equation

Our general treatment[7] has already shown how spin enters the theory as a label for bit-strings representing event intervals. The Dirac case differs from the Klein-Gordon case because a step to the left or to the right can have either left or right helicity, and spin-conservation adds a second conservation law to the particle-antiparticle conservation implied by our boundary conditions and reflected in our use of complex amplitudes. Consequently, in addition to the two independent time sequences $t_{\pm}(s)$ we must have two independent space sequences $z_{\pm}(s)$ ordered by the *same* global ordering parameter i and characterized by the *same* path parameter s. We can take over the same space-time boundary condition used above with $Z_{\mp} = R$ the steps to the right and $Z_{-} = L$ the steps to the left, and use a imaginary step length for the $\pm c$ *Zitterbewegung*, but the wave function now has two initial states α depending on whether the initial step (or helicity) is positive or negative, and two final states β. If $\Phi_{\beta\alpha}(B)$ are the number of *trajectories* with B bends, the extension of our prescription derived above, which is equivalent to Feynman's[17,18] except that our step length is kept *fixed* at $i\ h/mc$ (ih/mc^2), amounts to calculating[?]

$$K_{\beta\alpha}(b,t_b;a,t_a) = \Sigma_{B\geq 0}\Phi_{\beta\alpha}(B)(i)^B \tag{3.13}$$

As we have shown elsewhere[19], the *exact* combinatorial result is

$$K_{-+} = (i)\Sigma_s(-)^s\frac{r^s}{s!}\frac{l^s}{s!} = (i)\Sigma_s(-)^s(\frac{\tau}{2})^{2s}\frac{1}{(s!)^2} \rightarrow iJ_0(\tau) \tag{3.14}$$

where we have used the fact that

$$4rl = [(r+l)^2-(r-l)^2] = [c^2(t_b-t_a)^2-(b-a)^2](\frac{mc}{h})^2 = \tau^2(\frac{mc}{h})^2 \tag{3.15}$$

is the square of the invariant interval. Applying the same reasoning to calculate the other three components, our final result is

$$K(z,t;0,0) = \frac{1}{2}\begin{pmatrix} -\frac{(ct+z)}{\tau})J_1(\tau) & iJ_0(\tau) \\ iJ_0(\tau) & -\frac{(ct-z)}{\tau}J_1(\tau) \end{pmatrix} \tag{3.16}$$

which for our boundary conditions is the solution of the Dirac equation

$$-i\sigma_z \partial\psi/\partial z - m\sigma_x\psi = i\partial\psi/\partial t \tag{3.17}$$

where $\hbar = 1 = c$, σ_x and σ_z are Pauli spin matrices and ψ has two components. Again, extension to 3+1 dimensions appears to be immediate.

<u>Momentum-space equations</u>

A major conceptual advantage arising from our finite and discrete approach to relativistic quantum mechanics using end-point boundary conditions is that we obtain the momentum-space wave function without additional effort. We have already seen that for the interval specified $z = (r-l)(h/mc)$ and $mc^2t/h = (r+l) = n$; consequently the velocity in units of the limiting velocity c is $\beta = z/ct = \frac{2r}{n} - 1$ Since we have already established our discrete version of Lorentz invariance for the equations, we must use the implied definition of energy $E = \gamma mc^2$ and momentum $p_z = \gamma\beta mc = \beta E/c$. This gives us immediately the Klein-Gordon equation in "momentum space"

$$(p_z^2 + m^2)\phi(p_z) = E\phi(p_z) \tag{3.18}$$

where (and from now on) $\hbar = 1 = c$. Another way to see this is to recognize that our energy (or momentum) conservation law, allows us to treat the left-right *Zitterbewegung* in z as a one-dimensional problem analagous to our treatment of forward and backward movement in time. Thus we can immediately conclude that $\psi_{p_z}(z) = e^{ip_z z}$. Since the space-motion and the time-motion are generated independently in our model, we can multiply the two independent amplitudes to obtain

$$\psi(z,t) \to e^{\pm i(p_z z \pm Et)} \tag{3.19}$$

and hence provide an alternative derivation of the Klein-Gordon equation which is completely equivalent to our treatment above. Clearly, this route applied to two amplitudes which conserve helicity at the end points along the same lines will yield the 1+1 Dirac equation in momentum space, and make extension to 3+1 dimensions even easier to accomplish. For instance, instead of p_x, p_y we can use $p_\perp, j$ and the wave function $e^{i(p_\perp r_\perp + j\phi)}$ where the boundary condition on ϕ is periodic with period 2π or 4π depending on whether j is integer or half-integer.

4. THE EPR-BOHM EXPERIMENT IN THIS DISCRETE CONTEXT

Granted our discrete construction and our *rules of correspondence*[6,7] which connect this model to laboratory experience, Feynman's imaginative picture of an electron-positron pair as a single particle trajectory connecting two space-like separated spin measurements reduces the EPR "paradox" to triviality. The space-like connection, granted the Minkowski symmetries and the CPT theorem which are part and parcel of the construction, is just as "causal" as a time-like connection. In fact, once one grants our contention that quantum mechanics refers to completed processes that lie in the fixed past, there can be no "collapse of the wave function". But I am sure this formal discussion will leave many physicists unconvinced.

A slightly subtler difficulty is that we have invoked a large number of paths which lie "outside" of space time. My contention here is that in any discrete theory, this is a *necessary* consequence of our discrete version of the Lorentz transformation. By making a "boost" we can always interject any large number of points between any two points in the original mesh. That this — in general statistical — process is an alternative and completely consistent way of looking at the Lorentz transformation has been argued in detail by McGoveran (FDP, Theorem 41, pp.58-60). Of course the same thing happens in a continuum theory, but gets swept under the rug by the assumption that the space-time continuum is structureless. In our opinion, this is one of the obvious sources of the objectionable infinities in second quantized relativistic field theory. We have the conceptual advantage that for us these additional degrees of freedom represent additional external processes that for the moment we can ignore, but which will become relevant when a larger number of particles enter the system. Just as Einstein's coordinates are specified by "events" where a light ray could be reflected by a mirror, our coordinates represent potential events where a photon could be emitted or absorbed. The fact that for us the "coordinate string" for a photon is simply the null or the anti-null string[7], means that we have no way of defining its energy and momentum except by its source and sink (as in Wheeler-Feynman theories).

This brings us to a *physical* interpretation of our *Zitterbewegung* — it is simply the inevitable interaction of any particle with what would be called in second quantized field theory the vacuum fluctuations of the zero-point energy of the massless quantum fields. Casimir had the courage to take this idea seriously, and predicted an attractive force between two conducting plates which has been observed. Others predicted a *repulsive* force on a conducting sphere which has also been observed. It is easy to see that our theory predicts the right sign and geometry dependence for these two cases. In fact for us these experiments confirm our prediction that space is 3-dimensional, since they depend only on hc and are independent of the masses and coupling constants involved. A good check on our theory will be whether we can calculate the correct $1/120$ factor in the Casimir effect combinatorially.

A second consequence of our finite vacuum fluctuations is that we should be able to make finite combinatorial calculations of all mass ratios and all "mass renormalizations". Our original calculation of the pion mass and the ratio of the proton mass to the Planck mass[20] based on Dyson's limit[21] have both now been found by McGoveran to have well defined combinatorial corrections[8]. The construction presented in this paper allows a firm foundation to be placed under the originally heuristic argument[22] for the Parker-Rhodes calculation[23] of the proton-electron mass ratio. Other mass, mass-ratio and coupling constant calculations are already made (see Summary table) or in the offing.

Whether or not this finite approach holds up in the long run, we trust that we have made a case here for our contention that it allows a new way for us to look at the ERP experiments which avoids some of the old problems.

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Summary of **WHERE WE ARE** in January, 1990

General structural results

- 3+1 asymptotic space-time
- combinatorial free particle Dirac wave functions
- supraluminal synchronization and correlation *without* supraluminal signaling
- discrete Lorentz transformations for event-based coordinates
- relativistic Bohr-Sommerfeld quantization
- non-commutativity between position and velocity
- conservation laws for Yukawa vertices and 4- events
- crossing symmetry, CPT, spin and statistics

Gravitation and Cosmology

- the equivalence principle
- electromagnetic and gravitational unification
- the three traditional tests of general relativity
- event horizon
- zero-velocity frame for the cosmic background radiation
- mass of the visible universe: $(2^{127})^2 m_p = 4.84 \times 10^{52}\ gm$
- fireball time: $(2^{127})^2 \hbar / m_p c^2 = 3.5$ million years
- critical density: of $\Omega_{Vis} = \rho/\rho_c = 0.01175$ $[0.005 \leq \Omega_{Vis} \leq 0.02]$
- dark matter = 12.7 times visible matter [10??]
- baryons per photon $= 1/256^4 = 2.328... \times 10^{-10}$ $[2 \times 10^{-10}?]$

Unified theory of elementary particles

- quantum numbers of the standard model for quarks and leptons

with confined quarks and exactly 3 weakly coupled generations

- gravitation: $\hbar c/Gm_p^2 = 2^{127} + 136 = 1.70147...[1 - \frac{1}{3 \cdot 7 \cdot 10}] \times 10^{38}$
  $= 1.6934... \times 10^{38}$ $[1.6937(10) \times 10^{38}]$
- weak-electromagnetic unification:
  $G_F m_p^2/\hbar c = (1 - \frac{1}{3 \cdot 7})/256^2\sqrt{2} = 1.02\ 758... \times 10^{-5}$ $[1.02\ 684(2) \times 10^{-5}]$;
  $sin^2\theta_{Weak} = 0.25(1 - \frac{1}{3 \cdot 7})^2 = 0.2267...$ $[0.229(4)]$
  $M_W^2 = \pi\alpha/\sqrt{2} G_F sin^2\theta_W = (37.3\ Gev/c^2 sin\ \theta_W)^2;\ M_Z cos\ \theta_W = M_W$
- the hydrogen atom: $(E/\mu c^2)^2[1 + (1/137N_B)^2] = 1$
- the Sommerfeld formula: $(E/\mu c^2)^2[1 + a^2/(n + \sqrt{j^2 - a^2})^2] = 1$
- the fine structure constant: $\frac{1}{\alpha} = \frac{137}{1 - \frac{1}{30 \times 127}} = 137.0359\ 674...$ $[137.0359\ 895(61)]$
- $m_p/m_e = \frac{137\pi}{\frac{3}{14}\left(1 + \frac{2}{7} + \frac{4}{49}\right)\frac{4}{5}} = 1836.15\ 1497...$ $[1836.15\ 2701(37)]$
- $m_\pi^{\pm}/m_e = 275[1 - \frac{2}{2 \cdot 3 \cdot 7 \cdot 7}] = 273.1292...$ $[273.12\ 63(76)]$
- $m_{\pi^0}/m_e = 274[1 - \frac{3}{2 \cdot 3 \cdot 7 \cdot 2}] = 264.2\ 1428..$ $[264.1\ 160(76)]$
- $(G_{\pi N}^2 m_{\pi^0})^2 = (2m_p)^2 - m_{\pi^0}^2 = (13.86811 m_{\pi^0})^2$

  [( )] = empirical value (error) or range