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CP VIOLATION*

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ABSTRACT

Predictions for CP violation in the three generation Standard Model are reviewed based on what is known about the Cabibbo-Kobayashi-Maskawa matrix. Application to the K and B meson systems are emphasized.

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Introduction

It is now 25 years since the initial discovery of CP violation and we are still faced with the question of its origin and its ultimate significance:

- Is it a curiosity? Could it be physics from a much higher mass scale, at which we are allowed only a peek – a tiny remnant of new physics beyond the Standard Model?

or

- Is it a cornerstone? Does it originate inside the Standard Model? Indeed, is it the signal that there are three or more generations, all quark masses unequal, and all weak mixing angles nonzero? Is it then the single statement summarizing all of this, and yielding a characteristic pattern of CP violation which is tied to quark flavor?

These are the basic questions which we seek to answer experimentally, and then to delineate the details of whatever is the mechanism of CP violation. To do so, we need to know how CP violation is manifested in the Standard Model.

CP Violation in the Three Generation Standard Model

The matrix’ that describes the mixing of three generations of quarks has three real angles and one non-trivial phase. Any difference of rates between a given process and its CP conjugate process (or of a CP violating amplitude) always has the form:

$$\Gamma - \bar{\Gamma} \propto s_1^2\ s_2\ s_3\ c_1\ c_2\ c_3\ \sin\delta_{KM} = s_{12}\ s_{23}\ s_{13}\ c_{12}\ c_{23}\ c_{13}^2\ \sin\delta_{13}\ , \quad (1)$$

where we express things first in the original parametrization of the quark mixing matrix’ and then in the “preferred” parametrization adopted by the Particle Data Group,’ using the shorthand that $s_i = \sin\theta_i$ and $c_i = \cos\theta_i$. Our present experimental knowledge assures us that the approximation of setting the cosines to unity, which we often adopt in the following, induces errors of at most a few percent. In that case the combination of angle-dependent factors in Eq. (1), involving the invariant measure of CP violation,[3] becomes the approximate combination,

$$s_1^2\, s_2\, s_3 \sin\delta_{KM} = \quad s_{12}\, s_{23}\, s_{13} \sin\delta_{13}\ , \tag{2}$$

which was recognized earlier as characteristic of CP violating effects in the three-generation standard model.[4] Eq. (1) shows us immediately that all three generations of quarks are necessary for CP violation; in particular, none of the angles can be zero, nor can any of the Cabibbo-Kobayashi-Maskawa (C-K-M) matrix elements.

The Cabibbo-Kobayashi-Maskawa factors in Eq. (1) define the "price of CP violation" in the Standard Model. This "price" must be paid somewhere. It could be paid in a specific process- by having many of these factors in both Γ and $\bar{\Gamma}$, corresponding to a very small branching ratio-for that process: Then when we form the asymmetry,

$$A_{CP \text{ violation}} = \frac{r - r}{\Gamma + \bar{\Gamma}}\ , \tag{3}$$

the smallness of the denominator results in a large asymmetry. On the other hand, the price could be paid by having few of these factors in Γ and $\bar{\Gamma}$ separately (and hence in their sum), but only in their difference; the asymmetry is correspondingly small. There is therefore a very rough correspondence between rarer decays and bigger asymmetries. This rule-of-thumb is only that; it can be mitigated or exacerbated by other factors: hadronic matrix elements, dependence of one-loop amplitudes upon internal quark masses, and the possible presence of C-K-M factors in addition to those demanded by Eq. (1). A prime example of luck in this regard is provided by CP violating effects which depend on $B - \bar{B}$ mixing, where the large top quark mass allows fairly big asymmetries between B and $\bar{B}$ decays to occur in modes which are themselves not suppressed in rate by C-K-M factors.

The Unitarity Triangle

In principle, measurement of just the magnitudes of the C-K-M matrix elements could tell us about the phase, δ_{13}, as well as the "rotation angles" θ_{12}, θ_{23}, and θ_{13} in Eq. (1). This is most easily seen for the case at hand, where the "rotation angles" are small, by using the unitarity of the matrix as applied to the first and third-columns to derive that (c_{ij} have been set to unity):

$$1 \cdot V_{ub}^* - s_{12} \cdot V_{cb}^* + V_{td} \cdot 1 \approx 0 \ . \tag{4}$$

This equation is represented graphically in Figure 1 in terms of a triangle in the complex plane, the lengths of whose sides are $|V_{ub}^*|$, $|s_{12} \cdot V_{cb}^*|$, and $|V_{td}|$, and the non-trivial phase in different parametrizations is the indicated interior or exterior angle. This triangle appears explicitly in Ref. 4, and has been commented on by many people,[5] but has been particularly emphasized by Bjorken.[6]

According to an ancient theorem, perfect measurements of the lengths of all three sides could determine a non-trivial triangle, thereby completely fixing the mixing matrix, including the phase. Alternately, a set of measurements of the lengths could show that the triangle can not exist, forcing us beyond three generations. As a special case, the triangle could collapse to a line, and we must go beyond the three generation Standard Model for an explanation of CP violation. Unfortunately, given our present experimental knowledge and our limited theoretical ability to compute hadronic matrix elements, the three sides are not known with sufficient accuracy to discriminate between these situations, let alone determine the value of δ_{13}. For now, to get information on the phase we are forced to consider a CP violating quantity and assume it can be understood within the three generation Standard Model.

Note that twice the area of the triangle is:

$$s_1^2 s_2 s_3 \sin \delta_{KM} \approx s_{12} s_{23} s_{13} \sin \delta_{13} \, . \tag{5}$$

This is "the price of CP violation," and reaffirms that if the triangle degenerates to a line, then CP is conserved.

With this representation of the ill-determined parameters of the Cabibbo-Kobayashi-Maskawa matrix, it is possible to see more directly the interplay of various pieces of experimental information. In Figures 2 to 6 we have placed[7] the side $s_{12}\, V_{cb}^*$ along the horizontal and taken $|V_{cb}|$ at its central value[2] of 0.046, so that-one vertex is at the origin and a second vertex is very near the point (0.010, 0). Constraints on the position of the third vertex follow from[8]

- $|V_{ub}|$ – An upper limit on this quantity forces the third vertex to lie inside a circle about the origin. A lower limit, taken here to be $|V_{ub}| > 0.04|V_{cb}|$, is implied by data indicating $b \rightarrow$ u transitions presented to this conference.[9]
- $B - \bar{B}$ Mixing – The combination of the experimental value of $\Delta M/\Gamma$ and an upper and lower limit on the hadronic matrix element' forces the third vertex to lie outside and inside, respectively, circles drawn with the second vertex as an origin.
- ϵ – Imposing the constraint of obtaining the experimental value of $|\epsilon|$ along with upper and lower limits on the hadronic matrix element forces the third vertex to lie between hyperbolas.

Figure 2 shows the situation for m_t = 60 GeV, where the position of the third vertex is quite limited by the solid curves indicating the various constraints. The dotted circle represents the lower limit on $|V_{ub}|$ from the observation of $b \rightarrow$ u transitions. A sample unitarity triangle is indicated by the dashed lines. For still lower values of m_t, the inner limiting circle due to $B - \bar{B}$ mixing moves outward and eventually becomes incompatible with the other constraints – this is precisely how a lower limit of around 50 GeV for m_t came about after the observation of large $B_d - \bar{B}_d$ mixing.

As we move to a top quark mass of 80 GeV in Figure 3, the region permitted for the third vertex opens up. Values of m_t = 120,160, and 200 GeV in Figures 4, 5, and 6, respectively, show a progressively longer and lower allowed region, as both the upper and lower limits from $B - \bar{B}$ mixing and from $|\epsilon|$ enter the picture.

Note in addition that the base of the triangle, $s_{12}V_{cb}^*$, is itself only moderately well determined: Figures 7 and 8 show what happens for m_t = 200 GeV when values of 0.036 and 0.056 are used for $|V_{cb}|$

The new lower limit we are using for $|V_{ub}|$ plays little role, except for the heaviest top masses, once the "ϵ constraint" is imposed. Of course, the latter assumes that CP violation originates in the Cabibbo-Kobayashi-Maskawa matrix; it is very important to ascertain without any such assumption that $|V_{ub}|$ is nonzero, and eventually, to pin down its value.

Of more import for high top masses is the constraint that follows from comparing recent experimental data[10,11] on $B(K \to \mu^+\mu^-)$ with the value expected from unitarity, *i.e.,* $K \to \gamma\gamma \to \mu^+\mu^-$, alone. The average of the two recent experiments is $B(K \to \mu^+\mu^-)$ = 7.0 ± 0.6 x 10^{-9}, while the unitarity limit is 6.8 ± 0.3 x 10^{-9}. If it is assumed that the short-distance contribution to the real part of the amplitude is not cancelled by long-distance contributions, then one obtains a bound on the charm and top quark short-distance contributions to the branching ratio of 2 x 10^{-9} at the 3σ level. While there is no fundamental reason that a cancellation between the short-distance and long-distance contributions cannot take place, any major cancellation would have to be "accidental." In any case, for large top masses this constraint becomes important, and in particular restricts ReV_{td}. After due account of QCD corrections (relevant to the small, but non-neligible, and constructively interfering charm contribution), the effect of this constraint[12] is shown for m_t = 160 and 200 GeV in Figures 5 and 6 by the dashed-dot line. As is seen in the Figures the third vertex of the unitarity triangle is forced to the right from the resulting upper bound[13] on $|ReV_{td}|$.

When viewed from the point of view of the "price of CP violation," **i.e.,** twice the area of the unitarity triangle, it is the altitude times the base that matters. This quantity clearly has a large range, especially once we have allowed m_t to vary all the way up to 200 GeV. A ballpark figure for $s_1^2 s_2 s_3 s_\delta$ is several times 10^{-5}, which means that $s_2 s_3 s_\delta$ is of order 10^{-3}.

Status of CP Violation in the Standard Model

Given this "price of CP violation," we can "naturally" understand why

$$|\epsilon| \approx 2.28 \times 10^{-3} . \tag{6}$$

is so small and CP seems to come so close to being a symmetry in K decays. When all the factors are put in, the size of $|\epsilon|$ is roughly governed by that of $s_2 s_3 s_\delta$. This is "naturally" of the right size in the technical sense that to have $s_2 s_3 s_\delta$ of order 10^{-3} does not require any angle to be fine-tuned to be either especially small or especially large.

This same factor of $s_2 s_3 s_\delta$ pervades all CP violation observables in the K system, so it is then not so surprising that after 25 years the total evidence for CP violation in Nature consists of a nonzero value of ϵ, and one statistically significant measurement[14] of a nonzero value of the parameter $\epsilon'/\epsilon = 3.3 \pm 1.1 \times 10^{-3}$, representing CP violation in the $K \to \pi\pi$ decay amplitude itself. Experiments at Fermilab[15] and at CERN[14] are continuing with the aim of reducing the statistical and systematic errors. The value of ϵ' from Ref. 14 is consistent[16–18] with the three-generation Standard Model. Unfortunately, this is not a very strong statement. Other values of ϵ' would be consistent as well because of our lack of knowledge both on the experimental and theoretical fronts:

- The hadronic matrix elements of the penguin operators, upon which the prediction of ϵ' depends, are fairly uncertain. Definitive results will presumably come from lattice QCD calculations which still seem several years away.
- The predictions depend on the value of $s_2 s_3 s_\delta$, which in turn depends (aside from another hadronic matrix element) on m_t through imposing the constraint of obtaining the experimental value of ϵ. Very roughly, as m_t goes up, the range allowed for $s_2 s_3 s_\delta$ goes down, and so does the prediction for ϵ'.
- Also as m_t rises, the contributions from "Z penguin" and "W box" diagrams begin to be significant. For sufficiently large m_t, a recent calculation[19]

contends that most of the usual (strong) penguin contribution to ϵ' can be cancelled in this way.

Experimental and theoretical progress over the next few years should clarify these points. But even if the situation becomes that the value of ϵ' is in significant accord with the three-generation Standard Model, this single number is unlikely to be regarded as conclusively establishing that the origin of CP violation lies in the Cabibbo-Kobayashi-Maskawa matrix. We would demand additional evidence: A single set of C-K-M angles (including the phase) must be able to fit several different processes which exhibit CP violating effects, providing a redundant check on the theory.

There are two main avenues being pursued in order to get this additional evidence. One is to look for CP violating effects in the B meson system. Here the CP violating asymmetries potentially can be very large – of order 10^{-1} or more. The second way is to consider other K decays where CP violating effects, although very small, may occur with a different weighting (from that in $K \to \pi\pi$) between effects originating in the mass matrix and in the decay amplitude. Possible K decays which come to mind include $K \to 3\pi$, $K \to \gamma\gamma$, and $K \to \pi\pi\gamma$,[20–22] and especially $K_L \to \pi^0 \ell^+ \ell^-$ and $K_L \to \pi^0 \nu\bar{\nu}$. We take up K decays in the next Section, saving the B system for last.

CP Violation in Rare K Decays

The late 1960s and early 1970s marked a peak in experiments on K decays, sparked by the discovery of CP violation.[23] This effort tailed off as many important measurements were completed and new areas of physics opened up in the 1970s at electron-positron and hadron machines.

Then in the late 1970s and early 1980s, both theoretical and experimental developments led to a "rebirth" of K physics. On the experimental side, great strides were made to create high flux beams, handle high data rates, incorporate "smart triggers," improve detectors (especially for photons), and be able to analyze enormous data samples. These matched, at least to some degree, the requirements in precision and rarity being demanded by the theory for incisive tests of the Standard Model. The last few years have seen the beginning of a parade of results which are the culmination of a decade of work in perfecting and performing the needed experiments. Much more is yet to come, and one can see the opportunity to make use of the beams and detectors which are already in existence, or are being developed, to attack the rare K decays which will give additional insight into CP violation.

On the theoretical side, the establishment of gauge theories for the strong and electroweak interactions provided a well-defined basis for calculations. The three-generation Standard Model could be used to make predictions of what, by definition, was inside, and, by its complement, outside the Standard Model. The question of "who ordered the muon" was generalized to "who ordered three generations with particular values of masses and mixing angles," and attention was directed at interactions which would connect quarks and leptons of different generations, producing flavor-changing neutral currents. It was realized that not only did the three-generation model provide an origin for CP violation in the nontrivial phase in the quark mixing matrix, but that CP violation should affect the K° decay amplitude as well as the $K^\circ - \bar{K}^\circ$ mass matrix, resulting in values of ϵ'/ϵ in the 10^{-3} to 10^{-2} range.[24] There were also predictions for short-distance contributions to a number of other rare K decay amplitudes induced at one-loop, both CP conserving and CP violating.[25]

There has also been an associated experimental development which has important theoretical consequences: The rise of the top quark. Over the past decade, the "typical" or "best" value of the top quark mass used in theoretical papers has risen monotonically, somehow always remaining one step, or maybe one and a half steps, ahead of the experimental, then-current, lower bound. Values of 15, 25, 30, 45, . . . GeV have been used in various papers (some of them mine), and subsequently fallen by the wayside as experiments have been able to search at higher and higher masses. The present lower limit is around 60 GeV, below which a top quark is said[26] to be "unlikely." It seems that limits even higher than this will be quoted at high confidence within a month, as the analysis of the present round of collider data is completed. An upper limit of around 200 GeV follows from analysis of neutral and charged current data and the measured W and Z masses (i.e., consistency of the ρ parameter with unity).[27]

The rise of the top quark mass has important consequences when we go to calculate one-loop contributions. For the penguin diagrams in Figure 9 involving a top and charm quark and a virtual photon (the "electromagnetic penguin"); the conserved nature of the current demands a factor of q^2, the square of the four-momentum carried by the virtual photon, be present in the numerator of the amplitude. This cancels the $1/q^2$ from the photon propagator; the leading term for small (compared to M_W^2) top mass in the coefficient of the appropriate operator behaves as $ln(m_t^2/m_c^2)$. By contrast, the "Z penguin" or "W box" involve nonconserved currents: the factor q^2 in the numerator is replaced by the square of the quark mass in the loop and the propagator by $1/(q^2 + M_Z^2) \approx 1/M_Z^2$ or $1/M_W^2$. The corresponding coefficient behaves like $[(m_t^2/M_W^2)ln(m_t^2/M_W^2) - (m_c^2/M_W^2)ln(m_c^2/M_W^2)]$ when the top mass is small. In the days when $m_t^2 << M_W^2$, it was completely justified to throw away the Z penguin and W box contributions to such amplitudes in comparison to that of the electromagnetic penguin. Not so any more. The various graphs give comparable contributions, as we will see below in a specific example. Moreover, the contributions from the top quark become the dominant ones to various rare K decays when $m_t^2 >> M_W^2$. In the three-generation Standard Model, as m_t rises farther and farther above M_W, more and more of one-loop K physics is top physics and we are in the interesting situation where those working at the highest energy hadron colliders are pursuing another aspect of the same physics as those working on the rarest of K decays at low energies.

Let us illustrate a number of the above remarks by looking in more detail at one particular rare K decay in which it is possible to observe CP violation and which has emerged as the object of concentrated theoretical and experimental study:

$\underline{K_L \to \pi^0 e^+ e^-}$.

If we define K_1 and K_2 to be the even and odd CP eigenstates, respectively, of the neutral K system, then $K_L \to \pi^0 e^+ e^-$ has three contributions:

(1) Through a two-photon intermediate state:

$$K_2 \to \pi^0 \, \gamma\gamma \to \pi^0 e^+ e^- \, .$$

This is higher order in α, but is CP conserving. With two real photons there are two possible Lorentz invariant amplitudes for $K_L \to \pi^0 \, \gamma\gamma$. One is the coefficient of $F^{(1)}_{\mu\nu} \, F^{(2)}_{\mu\nu}$, which corresponds to the two photons being in a state with total angular momentum zero. Consequently, it picks up a factor of m_e when contracted with the QED amplitude for $\gamma\gamma \to e^+ e^-$, as the interactions are all chirality conserving, and its contribution to the $K_L \to \pi^0 e^+ e^-$ decay rate is totally negligible.[28] The other invariant amplitude is the coefficient of a tensor which contains two more powers of momentum and one might hope for its contribution to be suppressed by angular momentum barrier factors. In chiral perturbation theory, an order of magnitude estimate[29] for the resulting branching ratio of $K_2 \to \pi^0 \, e^+ e^-$ is 10^{-14}. However, a vector dominance, pole model predicts[30] a much bigger result: a branching ratio of order 10^{-11}, roughly at the level as that arising from the CP violating amplitudes (see below). The experimental upper limit on the branching ratio for $K_L \to \pi^0 \gamma\gamma$ has very recently been considerably improved,[31] and now is only a few times larger than some of the predictions.[30,29] In the future we might have not only a measurement of the branching ratio, but a Dalitz plot distribution which could help distinguish between models. The final answer for this contribution remains to be seen both theoretically and experimentally.

(2) Through the small (proportional to ϵ) part of the K_L which is K_1 due to CP violation in the mass matrix:

$$K_L \approx K_2 + \epsilon K_1$$

$$K_1 \to \pi^0 \, \gamma_{virtual} \to \pi^0 e^+ e^- \, .$$

We call this "indirect" CP violation and may calculate its contribution to the decay rate once we know the width for the CP conserving process $K_1 \to \pi^0 e^+ e^-$. Eventually, there will presumably be an experimental measurement of $\Gamma(K_S \to \pi^0 e^+ e^-)$, -which will take all the present theoretical model dependence away. For now, equating this width to the measured one for $K^+ \to \pi^+ e^+ e^-$ gives the estimate:

$$B(K_L \to \pi^0 e^+ e^-)_{\text{indirect}} = 0.58 \times 10^{-11} \, . \tag{7}$$

(3) Through the large part of the K_L, i.e., K_2, due to CP violation in the decay- amplitude:

$$K_2 \to \pi^0 \, \gamma_{virtual} \to \pi^0 e^+ e^- \, .$$

We call this "direct" CP violation, and the amplitude for it arises from the diagrams shown in Figure 9. For values of $m_t << M_W$, it is the "electromagnetic penguin" that gives the dominant short-distance contribution to the amplitude, which is summarized in the Wilson coefficient, C_{7V}, of the appropriate operator,

$$Q_{7V} = \alpha \left(\bar{s}\gamma_\mu(1 - \gamma_5)d\right) \left(\bar{e}\gamma^\mu e\right) \, .$$

Values of $m_t \sim M_W$ allow the "Z penguin" and "W box" contributions to become comparable to that of the "electromagnetic penguin," and bring in another operator,

$$Q_{7A} = \alpha \left(\bar{s}\gamma_\mu(1 - \gamma_5)d\right)\left(\bar{e}\gamma^\mu\gamma_5 e\right) \, .$$

The QCD corrections are substantial for the "electromagnetic penguin" contribution and have been redone for the case[32,33] when $m_t \sim M_W$. In contrast, the top quark contributions from the "Z penguin" and "W box" live up at the weak scale and get only small QCD corrections. Still, the coefficient C_{7V} comes largely from the "electromagnetic penguin," even after its reduction from QCD corrections. On the other hand, the "electromagnetic penguin" cannot contribute to C_{7A}, and here it is the "Z penguin" which gives the dominant contribution. The overall decay rate due to the "direct" CP violating amplitude can be obtained by relating the hadronic matrix elements of the operators Q_{7V} and Q_{7A} to that which occurs in K_{e3} decay. Then we find that

$$B(K_L \to \pi^0 e^+ e^-)_{direct} \approx 1 \times 10^{-5} (s_2 s_3 s_\delta)^2 [|\widetilde{C}_7|^2 + |\widetilde{C}_{7A}|^2] . \qquad (8)$$

The last factor, shown in Figure 10, ranges[32] between about 0.1 and 1.0. As $s_2 s_3 s_\delta$ is typically of order 10^{-3}, the corresponding branching ratio induced by this amplitude alone for $K_L \to \pi^0 e^+ e^-$ is around 10^{-11}. Note that when $m_t \gtrsim 150$ GeV, the contribution from C_{7A} overtakes that from C_{7V}, and it is the "Z penguin" and "W box," coming from the top quark with small QCD corrections, which dominate the decay rate.

Thus it appears at this point that the three contributions from (1) CP conserving, (2) "indirect" CP violating, and (3) "direct" CP violating amplitudes could all be comparable. The weighting of the different pieces in $K_L \to \pi^0 e^+ e^-$ is entirely different from that in $K \to \pi\pi$. The present experimental upper limit[34,35] is 4×10^{-8}, with prospects of getting to the Standard Model level of around 10^{-11} in the next several years. Hopefully, the CP conserving and "indirect" CP violating amplitudes will be pinned down much better by then, permitting an experimental measurement of this decay to be interpreted in terms of the magnitude of the "direct" CP violating amplitude.

CP Violation in B Decay

The possibilities for observation of CP violation in B decays are much richer than for the neutral K system. The situation is even reversed, in that for the B system the variety and size of CP violating asymmetries in decay amplitudes far overshadows that in the mass matrix.[36]

- To start with the familiar, however, consider the phenomenon of CP violation in the mass matrix of the neutral B system.

Here, in analogy with the neutral K system, one defines a parameter ϵ_B. It is related to p and q, the coefficients of the B° and $\bar{B}^\circ$, respectively, in the combination which is a mass matrix eigenstate by

$$\frac{q}{p} = \frac{1-\epsilon_B}{1+\epsilon_B} \tag{9}$$

The charge asymmetry in $B^\circ\bar{B}^\circ \to \ell^\pm\ell^\pm + X$ is given by[37]

$$\frac{\sigma(B^\circ\bar{B}^\circ \to \ell^+\ell^+ + X) - \sigma(B^\circ\bar{B}^\circ \to \text{e-e-} + X)}{\sigma(B^\circ\bar{B}^\circ \to \ell^+\ell^+ + X) + \sigma(B^\circ\bar{B}^\circ \to \ell^-\ell^- + X)} = \frac{|p/q|^2 - |q/p|^2}{|p/q|^2 + |q/p|^2} = \frac{\mathrm{Im}(\Gamma_{12}/M_{12})}{1 + \frac{1}{4}|\Gamma_{12}/M_{12}|^2}, \tag{10}$$

where we define $< B^\circ|H|\bar{B}^\circ >= M_{12} - \frac{i}{2}\Gamma_{12}$. The quantity $|M_{12}|$ is measured in $B - \bar{B}$ mixing to be comparable in magnitude to the total width, while Γ_{12} gets contributions only from channels which are common to both B° and $\bar{B}^\circ$, i.e., K-M suppressed decay modes. This causes the charge asymmetry for dileptons most likely to be in the ballpark of a few times 10^{-3}, and at best 10^{-2}. For the foreseeable future, it is inaccessible experimentally.

- Now we turn to where the excitement is: CP violation in decay amplitudes.

In principle, this can occur whenever there is more than one path, with different C-K-M factors, to a common final state. For example, let us consider the all-time favorite and paradigm: decay of a neutral B to a CP eigenstate, f, such as ψK_s° or D^+D^-. Since there is substantial $B^\circ - \bar{B}^\circ$ mixing, one can consider two decay chains of an initial B° meson:

$$\begin{array}{lcc} B^\circ \to B^\circ & \searrow & \\ & & f \\ B^\circ \to \bar{B}^\circ & \nearrow & \end{array} .$$

The second path differs in its phase because of $B^\circ \to \bar{B}^\circ$ mixing, and because the decay of a $\bar{B}$ involves the complex conjugate of the K-M factors involved in

B decay. The strong interactions, being CP invariant, give the same phases for the two paths. The amplitudes for these decay chains can interfere and generate non-zero asymmetries between $\Gamma(B^\circ(t) \to f)$ and $\Gamma(\bar{B}^\circ(t) \to f)$. Specifically,

$$\Gamma(\bar{B}^\circ(t) \to f) \sim e^{-\Gamma t} \left(1 - \sin[\Delta m\ t] Im\left(\frac{p}{q}\rho\right)\right) \tag{11a}$$

and

$$\Gamma(B^\circ(t) \to f) \sim e^{-\Gamma t} \left(1 + \sin[\Delta m\ t] Im\left(\frac{p}{q}\rho\right)\right) . \tag{11b}$$

Here we have neglected any lifetime difference between the mass matrix eigenstates (thought to be very small), set $\Delta m \equiv m_1 - m_2$, the difference of the eigenstate masses, and $\rho \equiv A(B \to f)/A(\bar{B} \to f)$, the ratio of the amplitudes, and then used the fact that $|\rho| = 1$ when f is a CP eigenstate in writing Eqs. (11). From this we can form the asymmetry:

$$A_{\text{CP Violation}} = \frac{\Gamma(B) - \Gamma(\bar{B})}{\Gamma(B) + \Gamma(\bar{B})} = \sin[\Delta m\ t] Im\left(\frac{p}{q}\rho\right) . \tag{12}$$

Moreover, in the particular case of decay to a CP eigenstate with one combination of K-M factors contributing to the decay amplitude, the quantity

$$Im\left(\frac{p}{q}\rho\right) = Im\left(e^{2i\Phi}\right)$$

is given entirely by the Cabibbo-Kobayashi-Maskawa matrix and is independent of hadronic amplitudes, which cancel out in the ratio, ρ . Remarkably, the angles Φ turn out to be nothing but those of the unitarity triangle, as shown in Figure 11, where the angles are labelled by examples of the neutral B decays to CP eigenstates whose asymmetries they govern.[38] Figure 12 shows the potential size of the time dependent differences[39] between B_d and $\bar{B}_d$ decaying[40] to the same (CP self-conjugate) final state, ψK_s°. The likely situation for B_s mixing is shown[41] in Figure

13c. The oscillations are so rapid that even with a very favorable difference in the time dependence for an initial B_s *versus* an initial $\bar{B}_s$, the time-integrated asymmetry is quite small. Measurement of the time dependence becomes a necessity for CP violation studies in this case.

We can also form asymmetries where the final state f is not a CP eigenstate. Examples are $B_d \rightarrow D\pi$ compared to $\bar{B}_d \rightarrow \bar{D}\bar{\pi}$; $B_d \rightarrow \bar{D}\pi$ compared to $\bar{B}_d \rightarrow D\bar{\pi}$; or $B_s \rightarrow D_s^+ K^-$ compared to $\bar{B}_s \rightarrow \bar{D}_s^- K^+$. There is a decided disadvantage here in theoretical interpretation, in that the quantity $Im\left(\frac{p}{q}\rho\right)$ is now dependent on hadron dynamics.

In all the above cases, to measure an asymmetry one must know if one starts with an initial B° or $\bar{B}^\circ$, i.e., one must "tag." This is one of the main difficulties experimentally, as the tagging efficiency is generally fairly low.[36]

A second path to the same final state could arise in several other ways besides through mixing. For example, one could have two cascade decays that end up with the same final state, such as:

$$B_u^- \rightarrow D^\circ K^- \rightarrow K_s^\circ \pi^\circ K^-$$

and

$$B_u^- \rightarrow \bar{D}^\circ K^- \rightarrow K_s^\circ \pi^\circ K^- \ .$$

Another possibility is to have spectator and annihilation graphs contribute to the same process.[42] Still another is to have spectator and "penguin" diagrams interfere.[43] These routes to obtaining a CP violating asymmetry have the advantage that they do not require one to know whether one started with a B or $\bar{B}$, i.e., they do not require "tagging." These decay modes are in fact "self-tagging" in that the properties of the decay products (through their electric charges or flavors) themselves fix the nature of the parent B or $\bar{B}$. Their disadvantage, which is theoretical, is that they generally bring poorly known hadronic matrix elements into the interpretation of an asymmetry, and so the association with specific combinations of K-M angles is not clean.

Conclusion

In a sense, after 25 years we are still at the beginning of the study of CP violation-;-most CP violating phenomena have yet to be explored, even those predicted by the Standard Model. The main thrusts in high energy physics are:

- K Decays: A strong effort is already underway at BNL, CERN, Fermilab, and KEK to pursue rare K decays. It includes measurements of ϵ'/ϵ and CP violating effects in $K_L \to \pi^0 e^+ e^-$ and other K decays. With a number of groups proposing to get to sensitivity levels corresponding to the Standard Model, we are almost guaranteed interesting results over the next few years.
- B Decays: We have seen that there are many manifestations of CP violation to look at in the B system. It appears that one needs of order 10^7 B's to begin to see the large asymmetries that are predicted by the Standard Model in some channels, but I would be the last to tell someone not to look for such effects if they had, say, 10^6 B's. Any nonzero asymmetry is important, and part of the signature of the Standard Model is the flavor dependence of the effects, with generally much larger CP violating asymmetries characteristic of B's than of K's. We want to know if this pattern is correct. Ultimately, CP violation in the B system is <u>the</u> way to measure the Cabibbo-Kobayashi-Maskawa angles in a redundant way. However, unlike the situation in K decays, we do not have the likelihood of significant results in the next few years. The prospects are longer term, but it seems clear what we must do: <u>Learn how to detect B's that are produced at hadron machines</u>, and <u>build electron-positron B factories</u>.

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FIGURE CAPTIONS

1) Representation in the complex plane of the triangle formed by the K-M matrix elements V_{ub}^*, s_{12} . V_{cb}^*, and V_{td}.

2) Constraints on the "unitarity triangle" for $m_t = 60$ GeV and $|V_{cb}| = 0.046$.

3) Constraints on the "unitarity triangle" for $m_t = 80$ GeV and $|V_{cb}| = 0.046$.

4) Constraints on the "unitarity triangle" for $m_t = 120$ GeV and $|V_{cb}| = 0.046$.

5) Constraints on the "unitarity triangle" for $m_t = 160$ GeV and $|V_{cb}| = 0.046$.

6) Constraints on the "unitarity triangle" for $m_t = 200$ GeV and $|V_{cb}| = 0.046$.

7) Constraints on the "unitarity triangle" for $m_t = 200$ GeV and $|V_{cb}| = 0.036$.

8) Constraints on the "unitarity triangle" for m_t = 200 GeV and $|V_{cb}|$ = *0.056.*

9) One-loop diagrams giving short distance contributions to *K* decays, and in particular, to the process $K \rightarrow \pi\ell^+\ell^-$: (a) the "electromagnetic penguin;" (b) the "Z penguin;" (c) the "W box."

10) The quantity $|\widetilde{C}_{7V}|^2 + |\widetilde{C}_{7A}|^2$, which enters the branching ratio for the CP violating decay $K_L \rightarrow \pi^0 e^+ e^-$, as a function of m_t for Λ_{QCD} = 150 MeV, from Ref. 32 .

11) The angles Φ, where $|\sin 2\Phi|$ is the magnitude of the asymmetry for $B_d \rightarrow \psi K_s$, $B_s \rightarrow \rho K_s$, and $B_d \rightarrow \pi^+\pi^-$, associated with the angles of the "unitarity triangle."

12) The time dependence for the process $B_d \rightarrow \psi K_s^\circ$ (dashed curve) in comparison to that for $\bar{B}_d \rightarrow \psi K_s^\circ$ (solid curve) for $\Delta m/\Gamma$ = n/4, a value consistent with that measured experimentally.

13) The time dependence for the process $B_s \rightarrow \rho K_s^\circ$ (solid curve) in comparison to that for $\bar{B}_s \rightarrow \rho K_s^\circ$ (dashed curve) for values of (a) Am/I' = 1, (b) $\Delta m/\Gamma$ = 5, and (c) $\Delta m/\Gamma$ = 15, from Ref. 41.

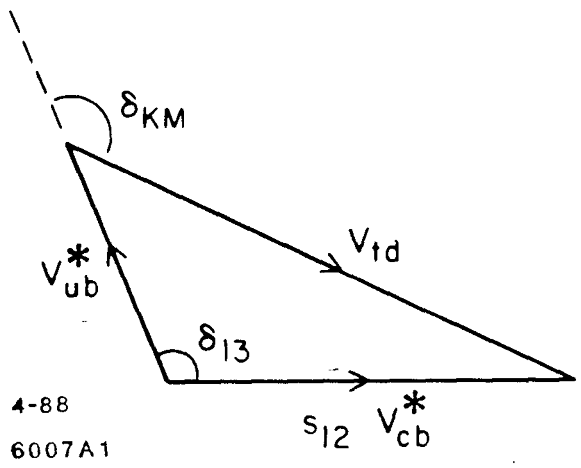
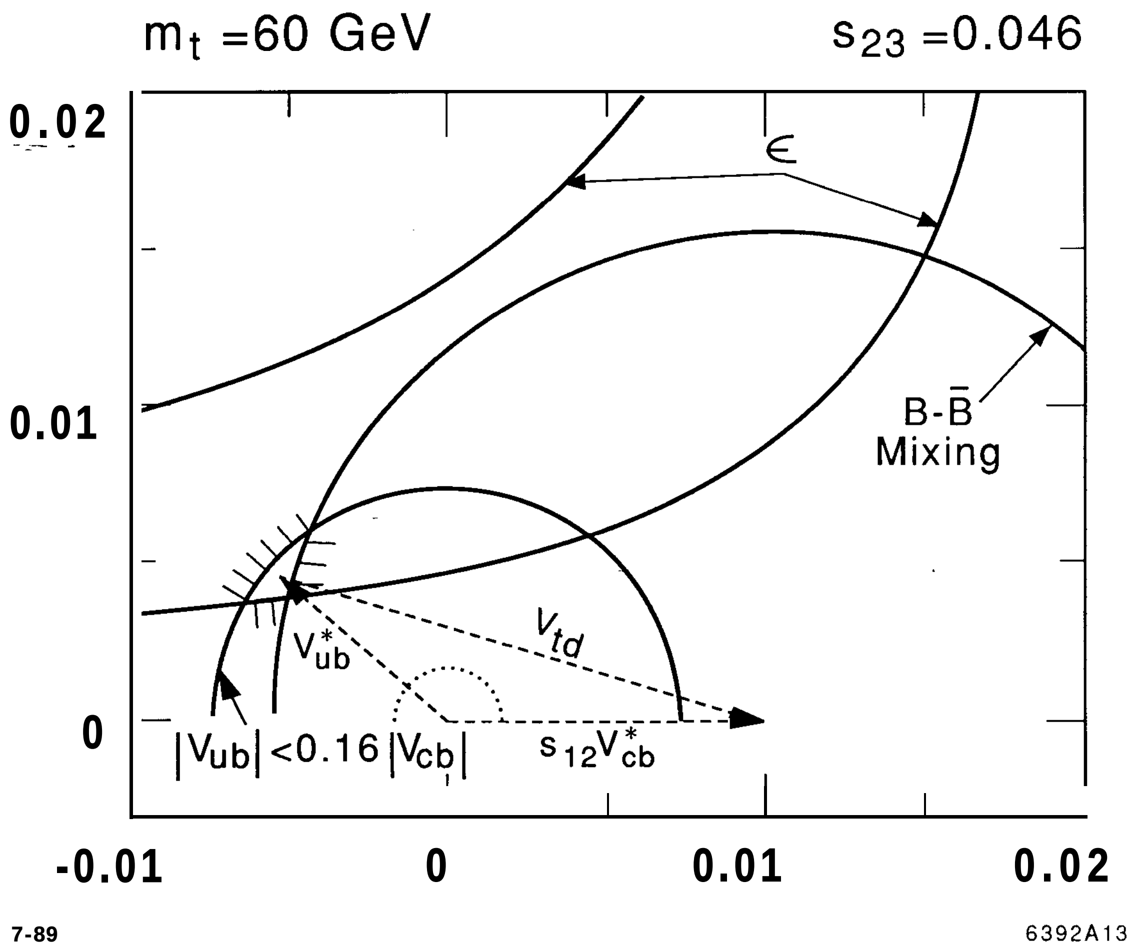
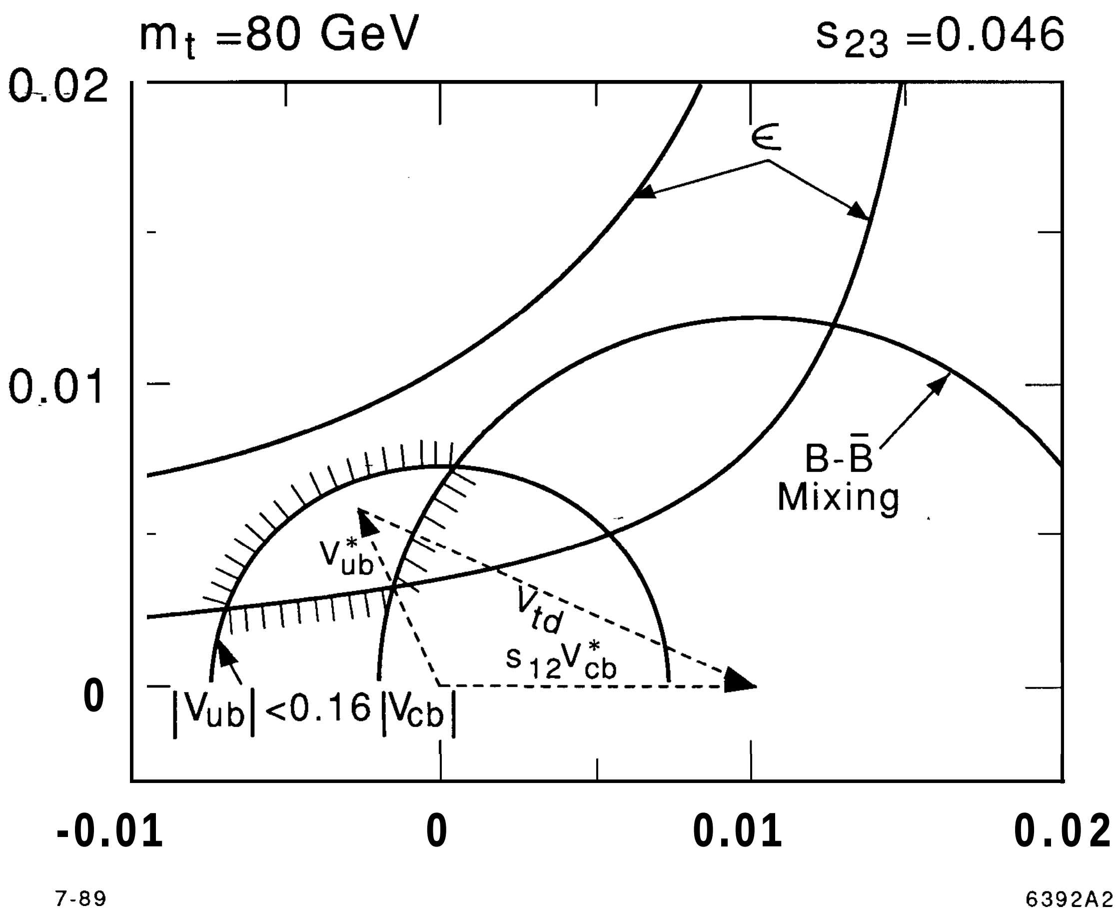


Fig 1



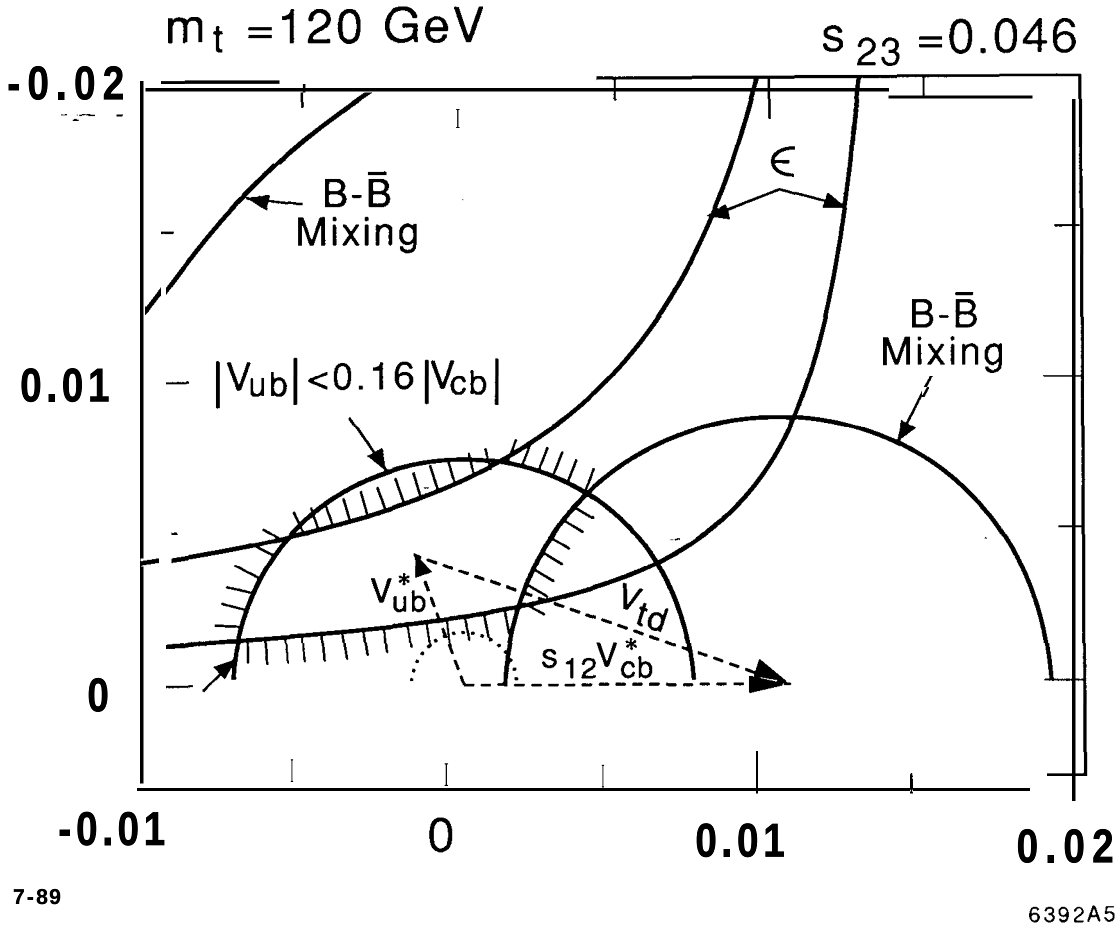
m_t =60 GeV
s_{23} =0.046
0.02
0.01
0
-0.01
0
0.01
0.02
∈
B-$\bar{B}$
Mixing
V_{td}
V^*_{ub}
$|V_{ub}|$ <0.16 $|V_{cb}|$
$s_{12}V^*_{cb}$
7-89
6392A13

Fig. 2



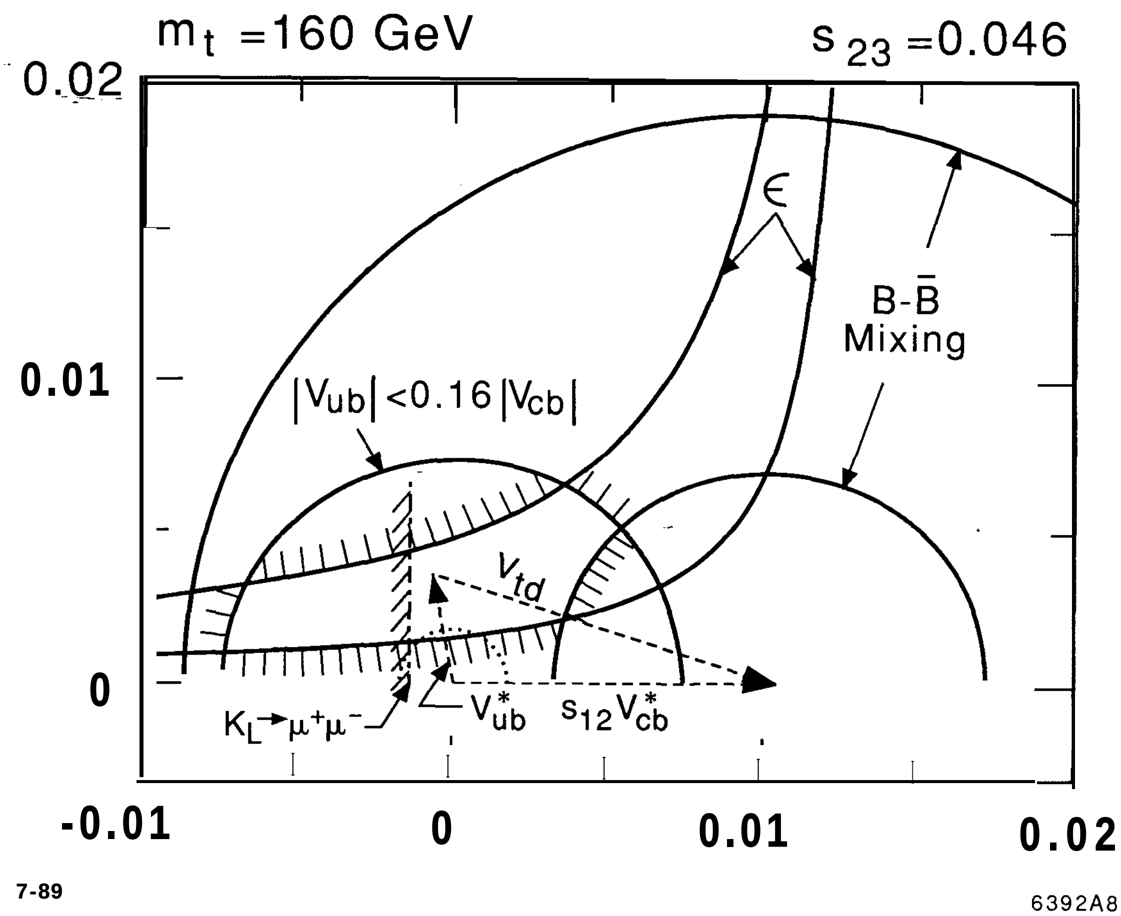
m_t =80 GeV
s_23 =0.046
0.02
0.01
0
-0.01
0
0.01
0.02
∈
B-B̄
Mixing
V*ub
Vtd
s12V*cb
|Vub|<0.16|Vcb|
7-89
6392A2

Fig. 3



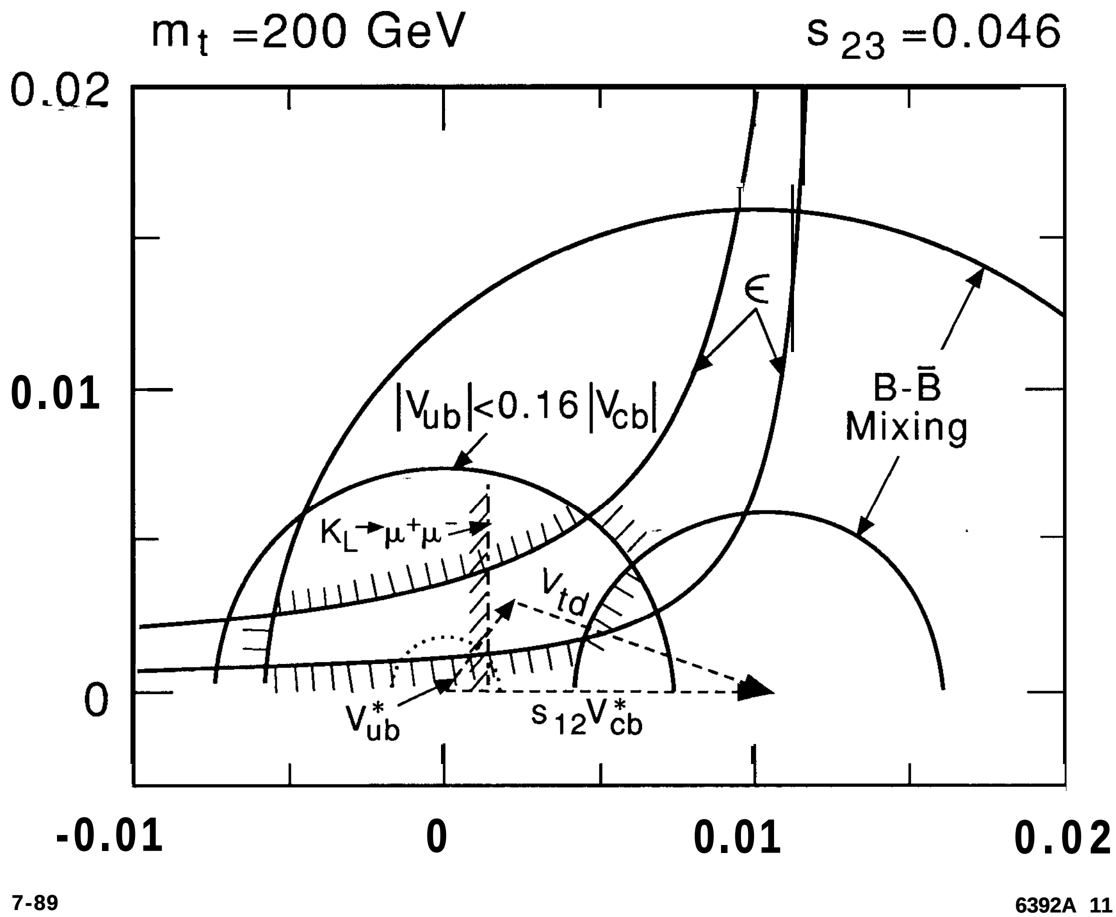
m_t =120 GeV
s_{23} =0.046
-0.02
0.01
0
-0.01
0
0.01
0.02
B-B̄ Mixing
ϵ
B-B̄ Mixing
$|V_{ub}|<0.16|V_{cb}|$
V_{ub}^*
V_{td}
$s_{12}V_{cb}^*$
7-89
6392A5

Fig. 4



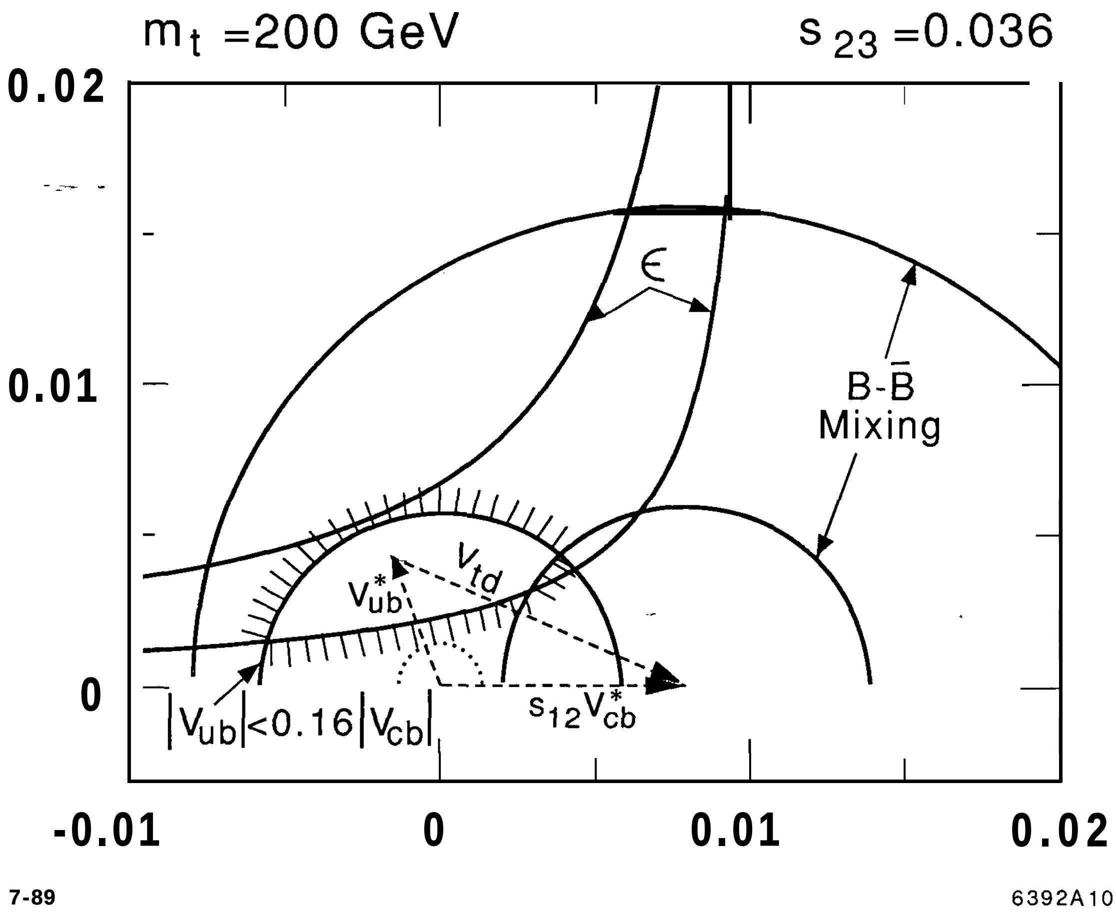
m t =160 GeV
s 23 =0.046
0.02
0.01
0
-0.01
0
0.01
0.02
ϵ
B-B̄
Mixing
|Vub| <0.16 |Vcb|
Vtd
KL→μ+μ−
V*ub
s12V*cb
7-89
6392A8

Fig. 5



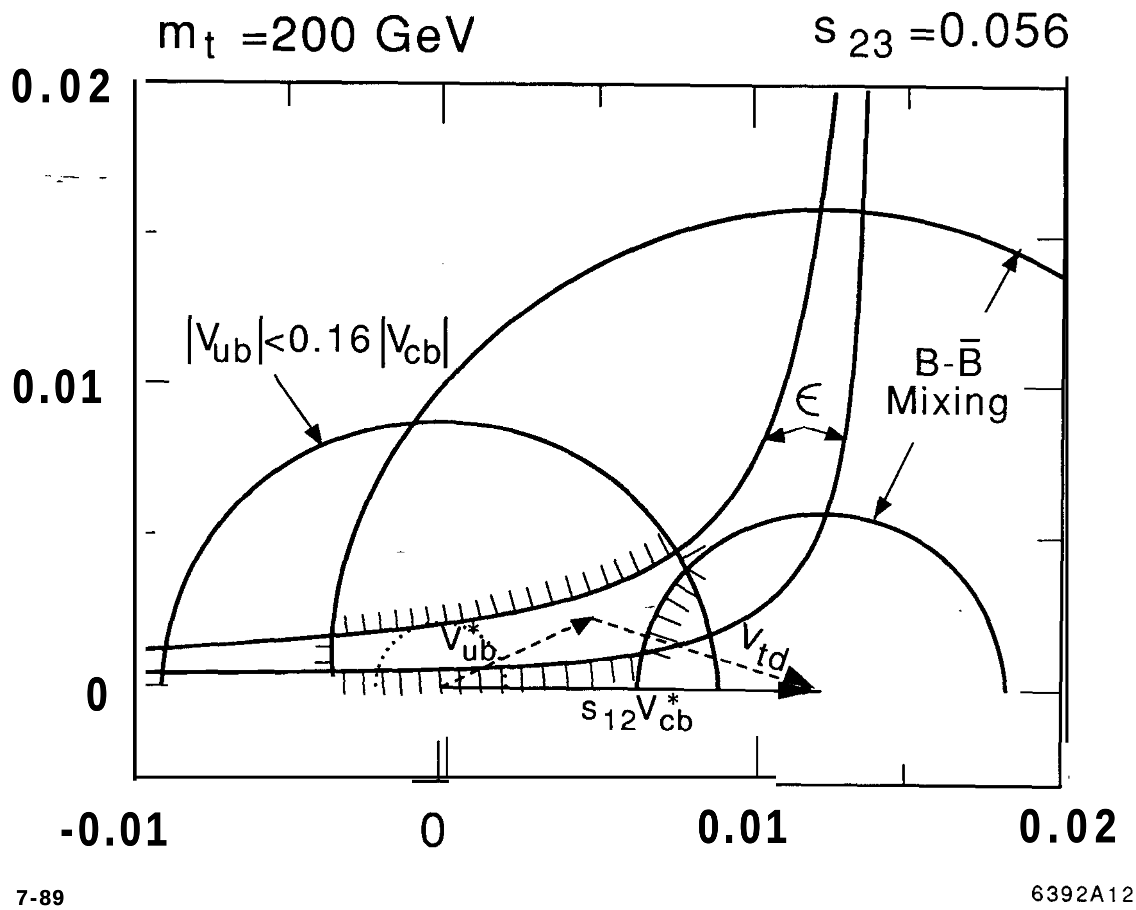
m_t =200 GeV
s_{23} =0.046
0.02
0.01
0
-0.01
0
0.01
0.02
ϵ
$|V_{ub}|<0.16\,|V_{cb}|$
B-$\bar{B}$
Mixing
$K_L \to \mu^+\mu^-$
V_{td}
V_{ub}^*
$s_{12}V_{cb}^*$
7-89
6392A 11

Fig. 6



m_t =200 GeV
s_{23} =0.036
0.02
0.01
0
-0.01
0
0.01
0.02
ϵ
B-$\bar{B}$
Mixing
V_{td}
V^*_{ub}
$s_{12}V^*_{cb}$
$|V_{ub}|<0.16|V_{cb}|$
7-89
6392A10

Fig. 7



m$_t$ =200 GeV
s$_{23}$ =0.056
0.02
0.01
0
|V$_{ub}$|<0.16 |V$_{cb}$|
∈
B-B̄
Mixing
V$^*_{ub}$
V$_{td}$
s$_{12}$V$^*_{cb}$
-0.01
0
0.01
0.02
7-89
6392A12

Fig. 8

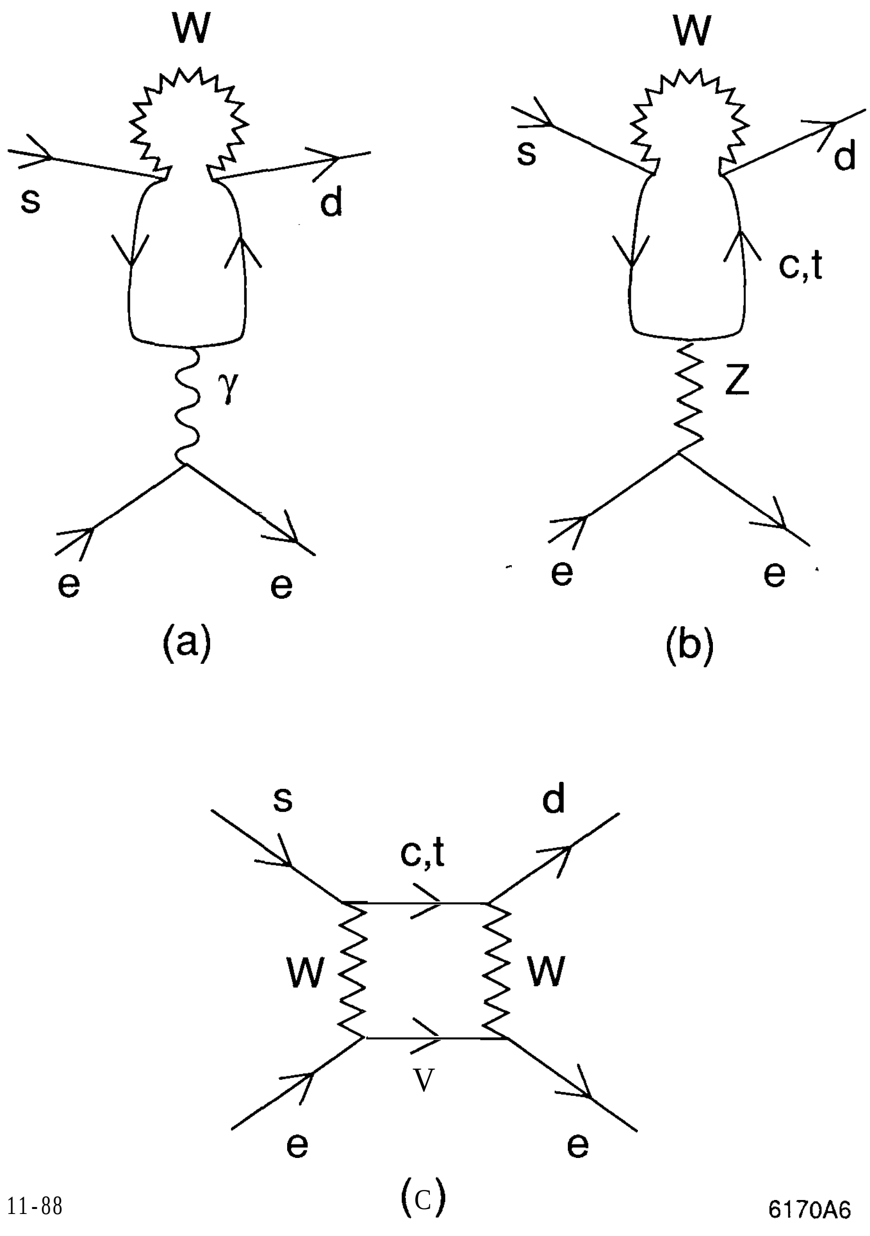
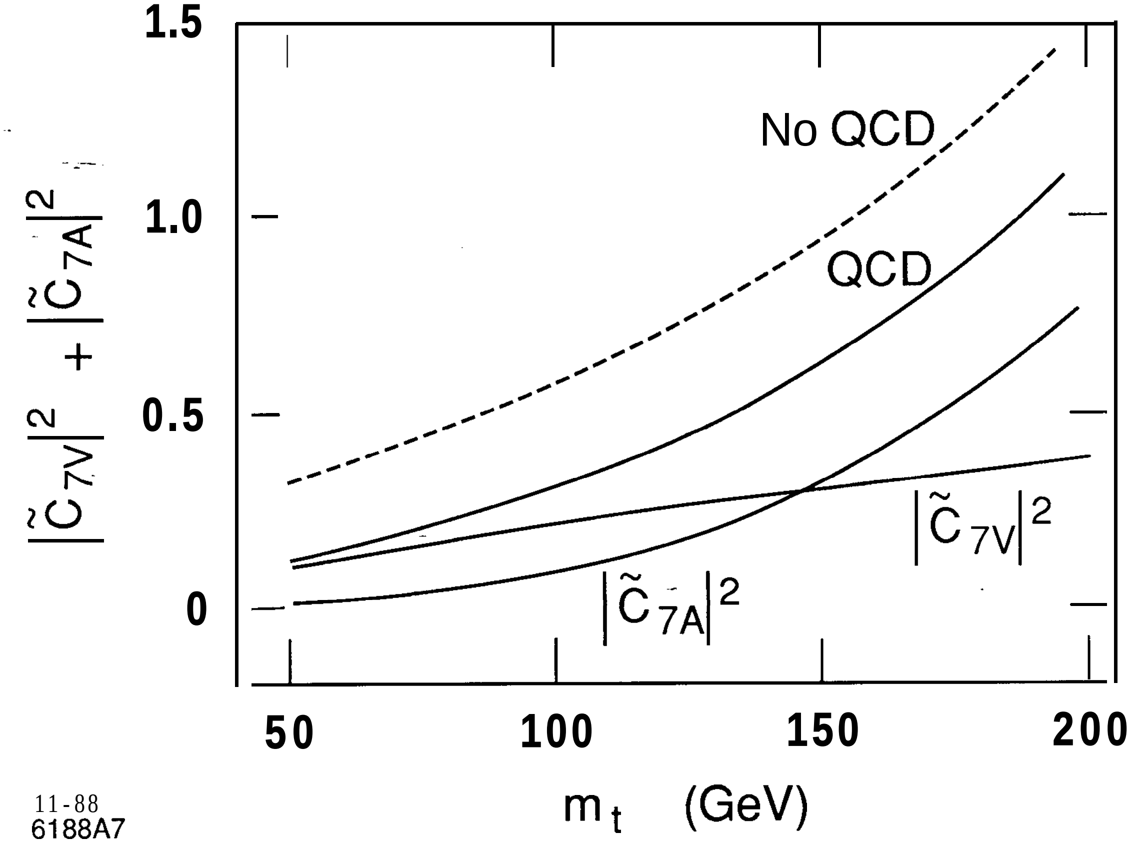
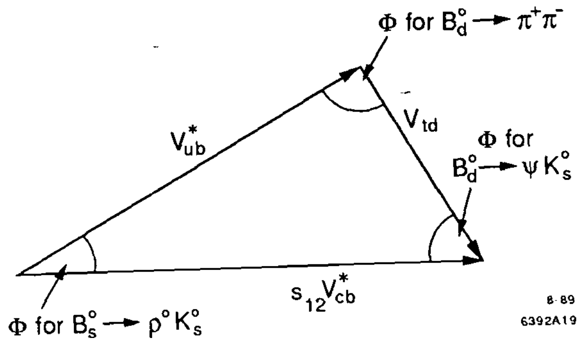


Fig. 9



1.5
1.0
0.5
0
No QCD
QCD
$|\tilde{C}_{7V}|^2$
$|\tilde{C}_{7A}|^2$
$|\tilde{C}_{7V}|^2 + |\tilde{C}_{7A}|^2$
50
100
150
200
m_t (GeV)
11-88
6188A7

Fig. 10



Φ for $B_d^\circ \rightarrow \pi^+\pi^-$
V_{ub}^*
V_{td}
Φ for
$B_d^\circ \rightarrow \psi K_s^\circ$
$s_{12}V_{cb}^*$
Φ for $B_s^\circ \rightarrow \rho^\circ K_s^\circ$
8-89
6392A19

Fig. 11

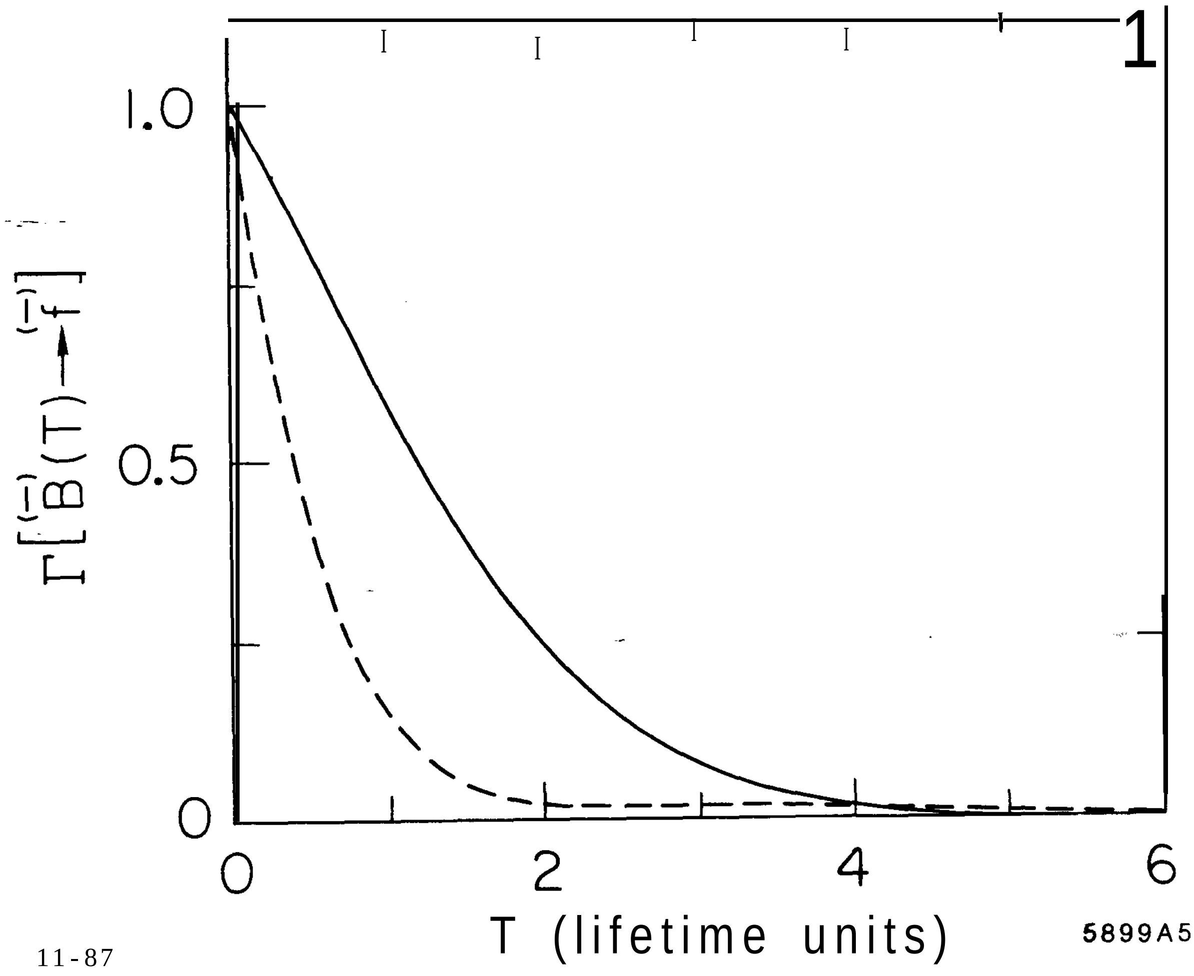
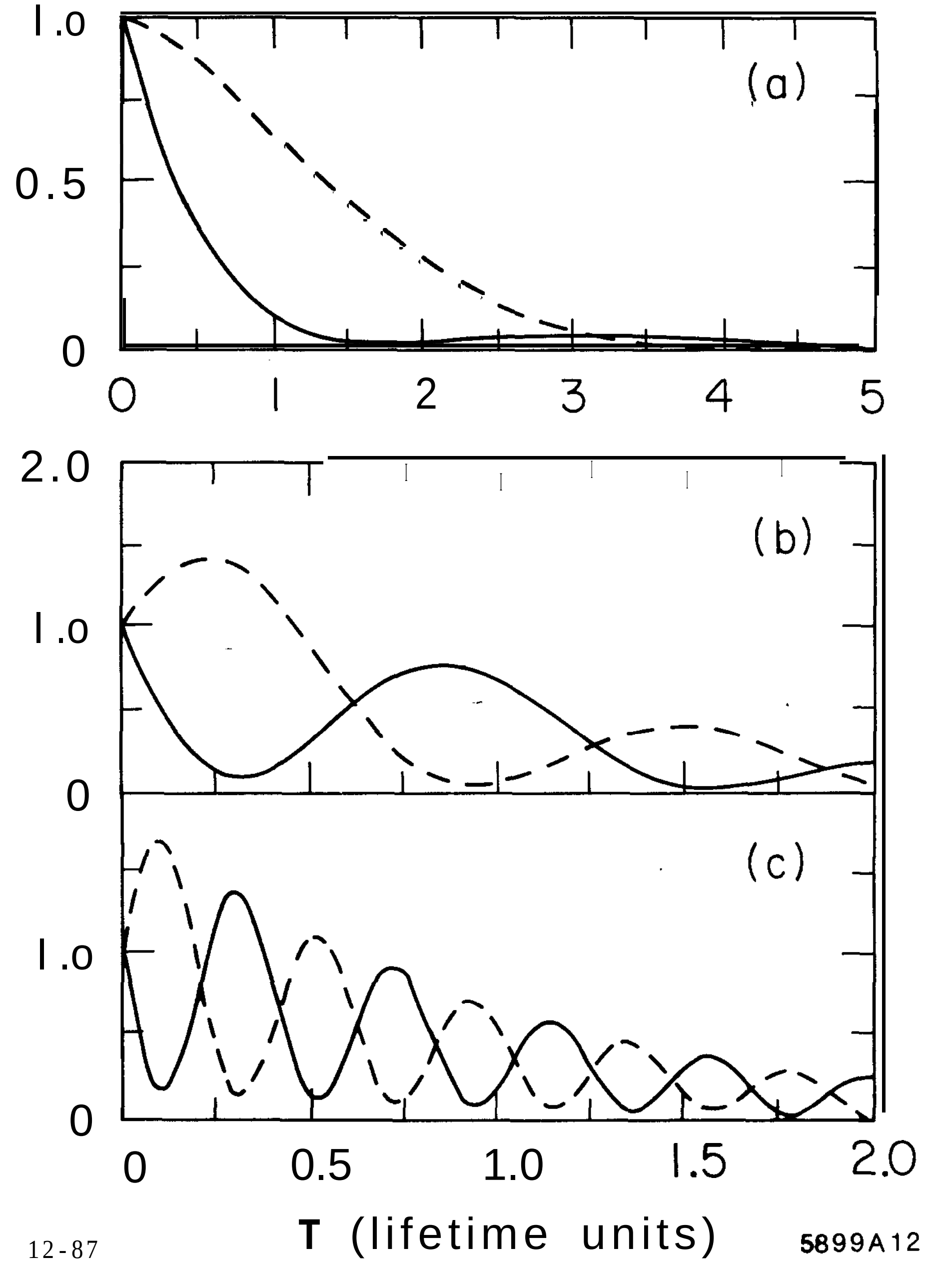


Fig. 12



1.0
0.5
0
(a)
0
1
2
3
4
5
2.0
1.0
0
(b)
(c)
1.0
0
0
0.5
1.0
1.5
2.0
T (lifetime units)
12-87
5899A12

Fig. 13