SLAC–PUB–5140
November 1989
T/E

A New Look at the Riemann–Cartan Theory*

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Submitted to *Physical Review Letters*

⋆ Work supported by the Department of Energy, contract DE–AC03–76SF00515.

ABSTRACT

The geometry of torsion in the Riemann–Cartan theory can be described by an abelian axial vector field interacting with the chiral fermion current in a purely Riemannian background. On the basis of this observation we note that the Schwinger model formulated in curved spacetime can be interpreted as the two dimensional version of the Riemann–Cartan theory. In two dimensions as well as in four dimensions there is a one parameter family of regulators that can be used to compute the chiral anomaly. In four dimensions we set the value of the arbitrary parameter equal to zero and compute the chiral anomaly, including counterterms, using Fujikawa's approach. The addition of the Wess–Zumino lagrangian changes the original RC–theory into a non-anomalous abelian gauge theory of the torsion field. Guided by the analogy with the Schwinger model, we offer several forms of $L_{GRAVITY}$ from which one can deduce the spin content of the quanta of torsion.

The Riemann–Cartan theory, hereafter referred to as the RC–theory, was proposed long ago as the natural candidate for the gauge formulation of the gravitational interaction.[1] We shall refer to the existing literature for a review of the wide range of arguments supporting the gauge formulation of gravity.[2] Our own motivation to investigate the RC–theory stems from some of the recent and current work on anomalies, especially in connection with solvable models in two dimensions.[3–6] To elaborate briefly on this point, consider the lagrangian of the (euclidean) Schwinger model in curved spacetime: $L = L_M + L_F$, where

$$L_M \equiv i\overline{\psi}\, e^{\mu}_{a}\gamma^a \left(\partial_\mu + \frac{1}{4}\,\omega^{ab}_{\mu}\Sigma_{ab} - ie\, V_\mu\right)\, \psi \ , \tag{1}$$

L_F stands for the free Maxwell lagrangian, $\Sigma_{ab} \equiv \frac{1}{2}\left[\gamma_a, \gamma_b\right]$ and ω^{ab}_{μ} represents the Lorentz spin connection defined in terms of Ricci's rotation coefficients. Since in two dimensions $\epsilon^{ab}\gamma^5\gamma_b = \gamma^a$, vector fields and axial vector fields are dual to each other and L_M is equivalent to

$$L_M = i\overline{\psi}\, e^{\mu}_{a}\gamma^a \left(\partial_\mu + \frac{1}{4}\,\omega^{ab}_{\mu}\Sigma_{ab} - ie\,\gamma^5 A_\mu\right)\, \psi \ . \tag{2}$$

Accordingly, in evaluating the quantum anomaly and the effective action, one has a choice of possible regulators

$$D = i\gamma^\mu \left(\partial_\mu + \frac{1}{4}\,\omega^{ab}_{\mu}\Sigma_{ab} + e\,aV_\mu - ie(1-a)\gamma^5 A_\mu\right) \tag{3}$$

in terms of the parameter "a" first noted by Rajaraman and Jackiw[7] in connection with the chiral Schwinger model.

Evidently, the equivalence between (1) and (2) does not hold in four dimensions. However, in four dimensions (with euclidean signature: ++++) $\epsilon^{abcd}\gamma^5\gamma_d =$

$\{\gamma^a, \Sigma^{bc}\}$ so that totally antisymmetric tensors of rank three and axial vector fields are dual to each other. In particular, the spin current

$$S^{abc} \equiv 2\,\frac{\delta L_{\rm M}}{\delta\omega_{abc}} = \frac{i}{4}\,\overline{\psi}\left\{\gamma^a, \Sigma^{bc}\right\}\psi = \frac{i}{4}\,\overline{\psi}\epsilon^{abcd}\gamma^5\gamma_d\psi \tag{4}$$

is the dual of the axial vector current $J^5_d \equiv i\overline{\psi}\gamma^5\gamma_d\psi$. The spin current arises as the response of the Riemann–Cartan action to a local Lorentz transformation; it is easy to show that S^{abc} is coupled to the totally antisymmetric part T_{abc} of the torsion tensor according to

$$L^{\rm INT}_{\rm RC} = \frac{1}{2}\,T_{abc}\,S^{abc}\ . \tag{5}$$

Introducing now the axial torsion field $A^a \equiv \epsilon^{abcd}T_{bcd}$ and using Eq. (4), one is led to consider the linear combination

$$L^{\rm INT}_{\rm RC} = \frac{1}{2}\left[a\,T_{abc}\,S^{abc} - \frac{1}{4}(1-a)A^a J^5_a\right] \tag{6}$$

which reflects, just as in two dimensions, the ambiguity in the choice of the regulator used to compute the quantum anomaly and the effective action of the RC–theory. In this letter we shall restrict ourselves to the case $a = 0$; then the RC–lagrangian takes the form: $L = L_M + L_{\rm GRAVITY}$, with

$$L_M = i\,\overline{\psi}\,e^\mu_a\gamma^a\left(\partial_\mu + \frac{1}{4}\,\omega^{ab}_\mu\Sigma_{ab} + \frac{1}{4}\,\gamma^5 A_\mu\right)\psi \tag{7}$$

where ω^{ab}_μ is the Lorentz spin connection which depends on the vierbein field through the Ricci rotation coefficients. Thus, while the conventional interpretation of the Schwinger model is that of QED_2 (in curved spacetime), other interpretations are admissible (see for instance Ref. 8). Comparing L_M in Eq. (7) with

L_M in Eq. (2) we suggest that the matter part of the Schwinger Lagrangian be interpreted as the RC–theory in two dimensions. *In this novel interpretation the geometry of the original Riemann–Cartan spacetime is represented, in two as well as in four dimensions, by the dynamics of an abelian axial vector field in a purely Riemannian background.* Note that we have left the form of L_{GRAVITY} unspecified. The form of L_{GRAVITY} in the RC–theory is notoriously non-unique but can be ignored in the computation of quantum effects in the external field approximation. We shall come back to the possible forms of L_{GRAVITY} later.

The lagrangian (7) is formally invariant under a $U(1)$ chiral transformation and we now proceed to evaluate the effect of the chiral anomaly on the effective action for the torsion field. The same technique (Fujikawa's method[9]) can be applied to the RC–theory in either four dimensions or in two dimensions. We are interested in the generating functional

$$Z \equiv \exp[-W(\omega, A)] \equiv \int d\overline{\psi}\, d\psi \exp[-S_M(\overline{\psi}, \psi, \omega, A) + \text{source terms}] \qquad (8)$$

where $S_M(\overline{\psi}, \psi, \omega, A)$ represents, up to gauge fixing terms, the euclidean action corresponding to the lagrangian (7). The regularization procedure to calculate the chiral anomaly is by now standard and involves the following steps: i) write the RC–lagrangian (7) in the form $L_M = \overline{\psi} i \not{D} \psi$ and introduce an orthonormal set $\{\phi_n\}$ of eigenfunctions of the (hermitian) operator $i\not{D}$ with eigenvalues $\{\lambda_n\}$, ii) define the generalized ζ-function $\zeta(s, x) \equiv \Sigma_n \phi_n^+(x) \lambda_n^{-2s} \phi_n(x)$ corresponding to the operator $(i\not{D})^2$, iii) introduce the heat-kernel $K(x, y, \tau) = \Sigma_n \phi_n^+ e^{-\lambda_n^2 \tau} \phi_n(y)$ associated with the Dirac operator $i\not{D}$. The ζ-function is related to the heat kernel by a Mellin transform. In particular, the following relationship holds between $\zeta(0, x)$ and the coefficient $a_2(x)$ in the asymptotic expansion of the diagonal part $K(x, x, \tau)$ of the heat kernel, $\zeta(0,x) = \frac{1}{(4\pi)^2}\, a_2(x)$. The chiral anomaly is caused by the change in the functional measure induced by an infinitesimal (euclidean) γ^5 rotation of the fermion fields. The effect of a chiral rotation $\psi = e^{-\gamma^5\beta(x)}\chi$; $\overline{\psi} = \overline{\chi}\, e^{-\gamma^5\beta(x)}$ on the generating functional is

$$
\begin{aligned}
Z = \int d\overline{\chi}\, d\chi \exp \Big[-2 \int d^4x\, (\det e^a_\mu)\beta(x)\, \frac{1}{(4\pi)^2}\, \mathrm{Tr}(\gamma^5 a_2) \\
- \int d^4x\, (\det e^a_\mu)\, \big(L_M(\overline{\chi},\chi) - \beta(x)\, \nabla_\mu J^{\mu 5}\big) \Big]
\end{aligned}
\tag{9}
$$

and the requirement $\delta Z/\delta\beta(x) = 0$ gives

$$
\begin{aligned}
\nabla_\mu\, J^{\mu 5} = \frac{2}{(4\pi)^2}\, \mathrm{Tr}(\gamma^5 a_2)\ . \\
(\nabla_\mu \equiv \underline{\text{Riemannian}} \text{ covariant derivative})\ .
\end{aligned}
\tag{10}
$$

The coefficient $a_2(x)$, which is a polynomial in the background fields, has been tabulated by various authors. There are discrepancies among the results reported in the literature.[10] Our own calculations are in agreement with those of Yajima and Kimura.[11] In our notation the expression of the anomaly is

$$
\begin{aligned}
\nabla_\mu J^{\mu 5} = \frac{2}{(4\pi)^2} \left[\frac{1}{96}\, \epsilon^{\alpha\beta\mu\nu} F_{\alpha\beta} F_{\mu\nu} + \frac{1}{48}\, \epsilon^{\alpha\beta\mu\nu} R^{\sigma\rho}{}_{\alpha\beta} R_{\sigma\rho\mu\nu} \right] \\
+ \frac{2}{(4\pi)^2}\, \nabla_\mu K^\mu \equiv G(\omega, A)
\end{aligned}
\tag{11}
$$

where

$$
K^\mu \equiv \frac{1}{6} \left[\nabla_\lambda \nabla^\lambda A^\mu + \frac{1}{4}\, A^2 A^\mu - \frac{R}{2}\, A^\mu \right]\ , \tag{12}
$$

$F_{\mu\nu} \equiv \partial_{[\mu} A_{\nu]}$ and $R^{\sigma\rho}{}_{\alpha\beta}$ and R represent the *Riemannian* curvature tensor and curvature scalar, respectively. The anomaly equation consists of two contributions:

the first term in the square bracket of Eq. (11) represents the "minimal" abelian form of the anomaly and is gauge invariant; the second term represents the Riemannian contribution from the background geometry; finally, the term $\nabla_\mu K^\mu$ is gauge variant and can be written as a gauge variation of local counterterms in the original action. In other words, the anomaly equation can be brought to the form

$$G(\omega, A) = \frac{2}{(2\pi)^4}\left[\frac{\epsilon^{\alpha\beta\mu\nu}}{96}\, F_{\alpha\beta}F_{\mu\nu} + \frac{\epsilon^{\alpha\beta\mu\nu}}{48}\, R^{\sigma\rho}{}_{\alpha\beta}R_{\sigma\rho\mu\nu}\right] + \nabla_\mu\left(\frac{1}{\sqrt{g}}\,\frac{\delta S_c}{\delta A_\mu}\right) \tag{13}$$

and one readily finds that

$$\begin{aligned} S_c &\equiv \int d^4x\sqrt{g}\, L_c(\omega, A) \\ &= \frac{1}{2(2\pi)^4}\int d^4x\sqrt{g}\left(\frac{1}{12}\, A_\mu\nabla_\nu\nabla^\nu A^\mu + \frac{1}{96}\, A^4 - \frac{1}{24}\, A^2 R\right)\ . \end{aligned} \tag{14}$$

Evidently the original lagrangian (7) does not fully define the RC–theory. The complete lagrangian must incorporate the *full* anomaly equation. This can be achieved by introducing the Wess-Zumino lagrangian

$$\begin{aligned} L_{WZ}(\omega, A, \theta) = &-\frac{1}{(4\pi)^2}\,\theta(x)\left(\frac{1}{48}\,\epsilon^{\alpha\beta\mu\nu}\, R^{\sigma\rho}_{\alpha\beta}R_{\sigma\rho\mu\nu} + \frac{1}{96}\,\epsilon^{\alpha\beta\mu\nu}F_{\alpha\beta}F_{\mu\nu}\right) \\ &+ L_c(\omega, A + \partial\theta) - L_c(\omega, A) \end{aligned} \tag{15}$$

where $\theta(x)$ is an auxiliary field which under a chiral rotation transform as $\theta \to \theta - \beta$. If we now start with the new lagrangian

$$L_M^{\rm NEW} = L_M(\overline{\psi}, \psi, \omega, A) + L_{\rm WZ}(\omega, A, \theta) \tag{16}$$

then the RC–theory is defined, up to gauge fixing terms, as a non-anomalous gauge theory where the torsion field is coupled to a conserved current $\tilde{J}^{\mu 5}$ as a consequence of the equation of motion of the θ-field: indeed, the conserved current is the sum of two contributions

$$\widetilde{J}^{\mu 5} = J^{\mu 5} + J^{\mu}_{\theta} \tag{17}$$

where

$$J^{\mu}_{\theta} = \frac{1}{\sqrt{g}} \frac{\delta S_{\rm WZ}}{\delta A_{\mu}} = -\frac{1}{24(4\pi)^2} \epsilon^{\mu\nu\rho\sigma} (\partial_{\nu}\theta) F_{\rho\sigma} + \frac{1}{\sqrt{g}} \frac{\delta}{\delta A_{\mu}} \left[S_c(\omega, A + \partial\theta) - S_c(\omega, A)\right] . \tag{18}$$

The covariant divergence of J^{μ}_{θ} is given by

$$\nabla_{\mu} J^{\mu}_{\theta} = \nabla_{\mu} K^{\mu}[A + \partial\theta] - \nabla_{\mu} K^{\mu}[A] . \tag{19}$$

The field equation for $\theta(x)$ requires

$$\nabla_{\mu} \frac{\delta L^{\rm WZ}[A,\theta]}{\delta\partial_{\mu}\theta} = -\frac{1}{48(4\pi)^2} \epsilon^{\mu\nu\rho\sigma} \left(F_{\mu\nu}F_{\rho\sigma} + 2R^{\alpha\beta}{}_{\mu\nu}R_{\alpha\beta\rho\sigma}\right) , \tag{20}$$

and since

$$\frac{\delta L^{\rm WZ}[A,\theta]}{\delta\partial_{\mu}\theta} \equiv \frac{\delta L_c[A + \partial\theta]}{\delta A_{\mu}} = K^{\mu}[A + \partial\theta] \tag{21}$$

it follows that

$$\nabla_{\mu} K^{\mu}[A + \partial\theta] = -\frac{1}{48(4\pi)^2} \epsilon^{\mu\nu\rho\sigma} \left(F_{\mu\nu}F_{\rho\sigma} + 2R^{\alpha\beta}{}_{\mu\nu}R_{\alpha\beta\rho\sigma}\right) \tag{22}$$

and therefore

$$\nabla_{\mu} J^{\mu}_{\theta} = -\frac{1}{48(4\pi)^2} \epsilon^{\mu\nu\rho\sigma} \left(F_{\mu\nu}F_{\rho\sigma} + 2R^{\alpha\beta}{}_{\mu\nu}R_{\alpha\beta\rho\sigma}\right) - \nabla_{\mu} K^{\mu}[A] . \tag{23}$$

Hence, the new current $\widetilde{J}^{\mu 5}$ satisfies, on shell, the Ward identity

$$\nabla_{\mu} \widetilde{J}^{\mu 5} = \text{anomaly} + \nabla_{\mu} J^{\mu}_{\theta} = 0 \tag{24}$$

and the modified RC–theory is gauge invariant at the quantum level.

The calculations in the two dimensional case are much simpler. The inclusion of the Riemannian connection in Eq. (2) does not affect the anomaly equation in any significant way.[3] We have computed the modified lagrangian of the Schwinger model using the general regulator (3) as a Dirac operator in the RC spacetime. The Wess-Zumino lagrangian and the associated current are,

$$L_{\rm WZ}(A,\theta) = m^2(a-1)\left(\frac{1}{2}\,\partial_\mu\theta\partial^\mu\theta - \theta\nabla_\mu A^\mu\right)\ . \tag{25}$$

$$J^\mu_\theta = \frac{m^2}{e}\,(a-1)\partial^\mu\theta \qquad \left(m^2 = \frac{e^2}{\pi}\right) \tag{26}$$

and one readily finds that the modified current $\widetilde{J}^\mu = J^\mu + J^\mu_\theta$ is conserved in view of the field equation for $\theta\ :\ \partial^2\theta = -\nabla_\mu A^\mu$.

A substantial difference between the two dimensional RC–theory and the four dimensional one is that in two dimensions it is possible to determine the operator solutions corresponding to the enlarged system $\{\overline{\psi},\psi,A,\theta\}$. For instance, in two dimensions the anomaly equation can be accounted for by a *bosonic* lagrangian which admits canonical quantization.[6] In four dimensions bosonization of fermion fields does not work and one has to deal with a fully fledged quantum theory of the torsion field interacting with fermions in a (Riemannian) curvature background. It is conceivable that this problem can be consistently formulated in a spacetime with non-vanishing torsion but zero curvature and we shall investigate this question elsewhere. However, even in the limiting case of zero Riemannian curvature the kinetic term for the torsion field plays a critical role in determining the dynamical properties of the quanta of torsion. This brings us back to the problem of choice of $L_{\rm GRAVITY}$ since in the RC–theory the "free" torsion term is an integral part of $L_{\rm GRAVITY}$. Inspired by the new interpretation of the Schwinger model as a Riemann–Cartan theory, in the following we shall list four possible choices of L_{GRAVITY} in four dimensions.

a) One possible choice of L_{GRAVITY} includes the Maxwell lagrangian for the *axial* torsion field A_μ,

$$L_{\text{GRAVITY}} = +\frac{1}{2}\,\frac{1}{2!}\,F^{\mu\nu}F_{\mu\nu} - \frac{1}{2!}\,F^{\mu\nu}\left(\partial_{[\mu}A_{\nu]}\right) - \frac{1}{16\pi G_N}\,R\,. \tag{27}$$

($R \equiv$ Riemannian curvature scalar).

Interestingly enough, in the limit of zero Riemannian curvature, the lagrangian system (7), (27) corresponds to the model of dynamical symmetry breaking proposed long ago by Jackiw and Johnson.[12] Here again we see the analogy with the Schwinger model at work: under the assumption that a chiral symmetry breaking solution exists, Jackiw and Johnson show that the vacuum polarization tensor acquires a pole at zero momentum transfer, precisely as in the Schwinger model, so that fermion and axial vector meson masses are spontaneously generated. In the light of our present discussion, the above mechanism of mass generation can now be incorporated in the dynamics of the Riemann–Cartan theory.

b) A second choice of L_{GRAVITY} involves a simple extension of the Maxwell lagrangian in the Schwinger model. Recall that in our code of correspondence the vector V_μ in two dimensions corresponds to the antisymmetric tensor $T_{\mu\nu\rho}$ in four dimensions. Hence,

$$L_{\text{GRAVITY}} = \frac{1}{2\cdot 4!}\,F_{\mu\nu\rho\sigma}F^{\mu\nu\rho\sigma} - \frac{1}{4!}\,F^{\mu\nu\rho\sigma}\left(\partial_{[\mu}T_{\nu\rho\sigma]}\right) - \frac{1}{16\pi G_N}\,R\,. \tag{28}$$

In this case, the torsion tensor $T_{\nu\rho\sigma}$ plays the role of a gauge potential; the field strength $F_{\mu\nu\rho\sigma}$ does not propagate torsion quanta[8] just as $F_{\mu\nu}$ in two dimensions does not propagate "photon" quanta. Rather, $F_{\mu\nu\rho\sigma}$ possesses only one independent component and represents a background field *constant* over the Riemannian manifold. The additional coupling of $F_{\mu\nu\rho\sigma}$ to gravity via the energy momentum tensor induces a cosmological constant into the Einstein equations.

c) An alternative but equally exotic choice of kinetic terms for the torsion field is given by the lagrangian,

$$L_{\text{GRAVITY}} = \frac{1}{2}\,\frac{1}{3!}\,T^{\mu\nu\rho}T_{\mu\nu\rho} - \frac{1}{3!}\,T^{\mu\nu\rho}\left(\partial_{[\mu}A_{\nu\rho]}\right) - \frac{1}{16\pi G_N}\,R\,. \tag{29}$$

In this case the field strength coincides with the torsion tensor itself and is derivable from an antisymmetric tensor gauge potential $A_{\nu\rho}$. This representation of the torsion tensor corresponds to the "notoph" field of Ogievetskii and Polubarinov[13] introduced in the RC–theory by Hayashi.[14] Thus, in contrast to case b), the torsion field now propagates freely with *massless* and *spinless* quanta. Note, in passing, that the lagrangian (29) is symmetric under the gauge transformation $\delta A_{\nu\rho} = \partial_{[\nu}\Lambda_{\rho]}$ while (28) is invariant under the change $\delta T_{\mu\nu\rho} = \partial_{[\mu}\Lambda_{\nu\rho]}$. Thus the covariant quantization of the torsion field, in both cases b) and c), requires a sequence of gauge fixing terms and corresponding Faddeev-Popov ghosts.[15] Of course, it is possible to express Eqs. (28) and (29) directly in terms of the axial torsion A_μ. However, the corresponding expressions would hardly be recognized as kinetic terms for the torsion field.

d) The conventional choice of L_{GRAVITY} in the RC–theory is given by the RC–curvature scalar $\widehat{R}$,

$$L_{\text{GRAVITY}} = -\frac{1}{16\pi G_N}\,\widehat{R} \equiv -\frac{1}{16\pi G_N}\,g^{\mu\nu}\widehat{R}_{\mu\nu} = \frac{1}{16\pi G_N}\,A_\mu A^\mu - \frac{1}{16\pi G_N}\,R + \dots \tag{30}$$

where the dots represent vector and tensor torsion terms which are decoupled from spinorial matter. In this case the system (7), (30) describes Dirac fermions in the Riemannian manifold of general relativity interacting with a non-propagating torsion field having a mass of the order of Planck's mass. According to this choice of L_{GRAVITY}, the torsion field is confined within the spin density distribution and cannot propagate in the vacuum. De Sabbata and Gasperini[16] have observed that the system (7), (30) yields a propagating (massless) torsion field if one assumes that A_μ is a pure gradient: $A_\mu = \partial_\mu \phi$. However, it seems to us that in this case the fermions decouple altogether from the torsion field at least at the classical level, since the interaction term in the lagrangian (7) can be transformed away by a chiral field redefinition of the Dirac fermions. Interestingly enough, fermions always decouple from the gauge field in the lagrangian (2) of the Schwinger model since the axial vector field in two dimensions is always derivable from a potential. Of course, since the functional fermion measure is not invariant under a chiral field redefinition, the effect of the quantum anomaly, for instance the mass term in the Schwinger model, persists in spite of the classical decoupling between fermions and torsion.

Returning to the general form (30) of L_{GRAVITY}, since the torsion field enters quadratically in the lagrangian, as a non-dynamical variable, it can be integrated out of the generating functional in favor of a self-coupled fermion field. This step has no counterpart in the Schwinger model. The functional gaussian integration rule gives

$$L = i\overline{\psi} e^\mu_a \gamma^a \left(\partial_\mu + \frac{1}{4}\, \omega^{ab}_\mu \Sigma_{ab} \right) \psi + \frac{\pi}{4}\, G_N (\overline{\psi} \gamma^\mu \gamma^5 \psi)^2 \; . \qquad (31)$$

In particle physics the lagrangian (31) would be interpreted as a low energy approximation to the fundamental dynamics of the RC–theory. While "low energy" here means anywhere below the Planck mass, we cannot resist the temptation to draw a parallel with the effective lagrangian approach to QCD: at low energy, say 1 GeV and below, the Nambu-Jona Lasinio[17] model, which involves a four fermion interaction modeled on the BCS–theory of superconductivity, correctly describes the spontaneous chiral symmetry breaking of the QCD vacuum and is generally regarded as a good approximation to the QCD lagrangian. By analogy, the lagrangian (31) suggests that at an energy scale below the Planck mass, the effective degrees of freedom of the RC–theory correspond to bosonic states which are composed of fermion antifermion pairs. The existence of these "Cooper pairs" raises again, as in case a), the intriguing possibility that chiral symmetry and/or Lorentz symmetry may be spontaneously broken in the RC–theory.

Acknowledgements:

One of the authors (A.A.) wishes to thank the Associated Western Universities, Incorporated for financial support while at SLAC and the California State Polytechnic University, Pomona for a grant under the CSU Research, Scholarship and Creative Activity Program.

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