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Search for Long-lived Massive Neutrinos in Z Decays^{*}

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ABSTRACT

We search for events in the Mark II detector at SLC with the topology of a Z boson decaying into a pair of long-lived, massive particles. No events that are consistent with the search hypothesis are found. Interpreting the long-lived particle as a sequential Dirac neutrino ν_4 of the fourth generation, we exclude at the 95% confidence level a significant range of mixing matrix elements of ν_4 to other generation neutrinos for a ν_4 mass from 10 GeV/ c^2 to 43 GeV/ c^2 .

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Despite the success of the present three generation Standard Model, there is immense interest in looking for new generations of fermions. The Standard Model itself imposes no restrictions on the number of possible generations or on the masses of the new particles. The existence and masses of these new particles would provide valuable information in helping to understand the pattern of fermion masses, the presence of generations, and physics beyond the Standard Model.

Motivation to search for long-lived, massive particles comes from various theories¹ asserting their existence. Among all possible new particles, a fourth generation neutrino draws the most immediate attention. If one extends the mass structure of the known fundamental fermions, the neutrino would be the lightest member of a new fourth generation and thus the most accessible to discovery by present experiments. Recently, measurements have been made at SLC^2 and LEP^3 to determine the number of neutrino species. The results rule out at 95% confidence level (C.L.) the possibility of a fourth generation neutrino. These measurements, however, assume the neutrino to be massless and stable.

In this Letter, we present a search for long-lived, massive particles in Z boson decays. The data used in this analysis were obtained with the Mark II detector at the SLAC e^+e^- Linear Collider (SLC) operating in the e^+e^- center-of-mass energy ($E_{\rm cm}$) range from 89.2 to 93.0 GeV. Although the search can be applied to other long-lived particles,⁴ we parametrize the results in terms of an hypothesized sequential massive Dirac neutrino ν_4 . If such a neutrino exists with its charged lepton partner L^- , which we assume is heavier than the neutrino, then the neutrino weak eigenstates ν_ℓ ($\ell = e, \mu, \tau$, and L) could be a mixture of the mass eigenstates ν_i (i = 1, 4) in analogy with the quark sector:

$$\nu_{\ell} = \sum_{i=1}^{4} U_{\ell i} \nu_i$$

where $U_{\ell i}$ is a unitary mixing matrix.

Through this mixing, ν_4 would decay by the weak charged current ($\nu_4 \rightarrow \ell + W^*$; $\ell = e, \mu, \tau$). The lifetime of the ν_4 is calculable given the mixing matrix elements $U_{\ell 4}$ and the mass of the neutrino m_4 . Assuming ν_4 mixes with only one other generation of ℓ , it can be expressed in terms of the muon lifetime as

$$\tau(\nu_4 \to \ell^- X^+) = \left[\frac{m_{\mu}}{m_4}\right]^5 \frac{\tau(\mu \to e\nu\bar{\nu})Br(\nu_4 \to \ell^- e^+\nu)}{|U_{\ell 4}|^2 f}$$

where f is a phase-space suppression factor⁵ for massive final state particles which differs appreciably from unity only when one or more of the final state particles is a τ lepton or charm quark, and m_4 is relatively small. The predicted branching fraction for $\nu_4 \rightarrow \ell^- e^+ \nu$ is constant (~11% for $Br(\nu_4 \rightarrow \ell^- X^+) = 100\%$) for most of our mass search range from 10 GeV/ c^2 to 45 GeV/ c^2 .

Although the unitarity of the mixing matrix and $e - \mu$, $\mu - \tau$ weak universality restrict the allowed mixings, there is a large region where the mixing is of a value such that the decay length of the ν_4 is experimentally observable.

The expected cross section for $\nu_4 \bar{\nu}_4$ pair production at the Z peak is very large: $2.2(\beta/4)(3 + \beta^2)$ nb, where β is the velocity of the ν_4 in the e^+e^- center-of-mass frame. For a 35 GeV/ $c^2 \nu_4$, a total of 26.2 produced events would be expected in the accumulated Mark II data sample of 19.2 nb⁻¹.

To search for these particles, we utilize the following characteristics of the event topology. In contrast to the hadronic background from the fragmentation of u, d, s, c, and b (udscb) quarks, $\nu_4 \bar{\nu}_4$ events with long-lived ν_4 would have detached

vertices, and consequently many tracks with large impact parameter with respect to the primary vertex. In addition, since a majority of $\nu_4 \bar{\nu}_4$ events are expected to contain one or two hadronic jets, the average charged particle multiplicity and total visible energy would then be significantly larger than that of the dominant background of beam-gas and beam-beampipe interaction events.

A detailed description of the Mark II detector can be found elsewhere.⁶ The tracking system of the detector is particularly important for this analysis. The Mark II central drift chamber, based on a six sense-wire cell of jet-chamber configuration, has 12 concentric cylindrical layers at radii between 19.2 cm and 151.9 cm. There are six axial layers and six stereo layers with stereo angles of approximately $\pm 3.8^{\circ}$ relative to the beam axis. The chamber is immersed in a 4.75 kG solenoidal magnetic field and has an intrinsic position resolution of less than 200 μ m, resulting in a charged particle momentum resolution of $\sigma(p)/p^2 = 0.0046$ (GeV/c)⁻¹. A double hit separation efficiency of about 80% is measured for hits 3.8 mm apart. The Mark II data acquisition system is triggered either by two or more charged tracks within $|\cos \theta| < 0.76$, or by an electromagnetic shower with localized energy deposition greater than 3.3 GeV in the barrel or greater than 2.2 GeV in the endcap electromagnetic calorimeters.

We apply the following procedure to select candidate events for $\nu_4 \bar{\nu}_4$ decays. First, in order to ensure good track reconstruction, we require charged tracks to be within the angular region $|\cos \theta| < 0.82$, where θ is the angle with respect to the beam axis, and to have transverse momentum with respect to the beam axis of at least 150 MeV/c. We also require electromagnetic showers to have shower energy greater than 500 MeV and to be within $|\cos \theta| < 0.68$ for the barrel calorimeter and within 0.74 $< |\cos \theta| < 0.95$ for the endcap calorimeters. To extract the signal events, it is necessary to eliminate the dominant background of beam-gas and beam-beampipe interaction events from the data sample without losing efficiency for the signal events. Since the beam-gas and beambeampipe interaction events usually have low multiplicity, low energy, and are forward-scattered, we apply the following selection criteria to eliminate them. We require an event to have at least eight charged tracks satisfying the track selection criteria and to have total energy greater than 35% of $E_{\rm cm}$. In addition, the minimum of the forward and backward charged energy must be greater than 7% of $E_{\rm cm}$.

To examine the effect of these cuts and to ensure that the remaining events are free of beam-gas and beam-beampipe background, we determine the most probable primary vertex of each event in the data sample. The requirement of at least eight good charged tracks essentially eliminates most events with primary vertices which are not at the known SLC beam-beam interaction point (IP) or at the beam pipe region. In contrast, the combination of the total energy requirement and the minimum forward-backward charged energy requirement eliminates most of the beam-beampipe interaction events. When all requirements are applied, there remain only events with primary vertices near the IP.

We therefore estimate that there is no contamination of beam-gas and beambeampipe interaction events in our final data sample of 350 events. When the same procedure is applied to Monte Carlo generated $\nu_4 \bar{\nu}_4$ events, no obvious change in the primary vertex distribution is observed.

The following method is used to extract the $\nu_4 \bar{\nu}_4$ signal events from the final data set. The impact parameter b in the plane perpendicular to the beam axis is defined as the distance of closest approach of a charged track to the average

beam position. The average beam position for events in a particular time period is defined as the average of the fitted primary vertex positions of the hadronic events in that period. The data is divided into blocks in which there is no evidence of a beam position movement of more than 200 μ m. Each data block contains more than 50 hadronic events, and the typical error in the average beam position is less than 40 μ m in both horizontal and vertical directions. The day-to-day variation in the position of the SLC beam-beam interaction point is estimated to be less than 200 μ m with the SLC Final Focus beam position monitoring systems. Since the SLC beam size is very small (typically 5 μ m in the plane perpendicular to the beam axis), its contribution to the uncertainty in the average beam position is negligible. We therefore assign an error of 200 μ m to the measured beam position in each event.

Next, the significance of a charged track's impact parameter is defined as the impact parameter divided by its error σ_b , which is the sum in quadrature of the track position error perpendicular to the track trajectory (about 300 μ m for high momentum tracks) and the error in the average beam position. An event search parameter χ_{imp} is defined as the fraction of charged tracks with significance b/σ_b greater than 5.0. This parameter is a powerful tool to distinguish the longlived $\nu_4 \bar{\nu}_4$ signal events from the *udscb* hadronic background events. The hadronic background events containing charm, bottom or strange quark decays rarely yield χ_{imp} greater than 0.5 as seen in Fig. 1, since there are many other tracks in the event which project to the primary vertex. Many $\nu_4 \bar{\nu}_4$ events with a reasonable lifetime would, however, yield χ_{imp} greater than 0.5, as can also be seen in Fig. 1. We therefore demand χ_{imp} of an event to be greater than 0.6 for it to be tagged as a long-lived $\nu_4 \bar{\nu}_4$ signal event.

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No events in the final data set are tagged as $\nu_4 \bar{\nu}_4$ signal events. The $\chi_{\rm imp}$ distribution of the final data set agrees reasonably with Monte Carlo *udscb* simulations (Lund 6.3 shower⁷ and Webber 4.1⁸) which predict less than 0.2 events to pass all cuts. The remaining discrepancy between the data and Monte Carlo is mainly due to nuclear interactions of particles which are not well simulated in the Monte Carlo.⁹

--In order to interpret the null search result for $\nu_4 \bar{\nu}_4$ events as an excluded region in the $m_4 - |U_{\ell 4}|^2$ plane, we apply the following procedure: $\nu_4 \bar{\nu}_4$ events are generated with various lifetimes and masses with a Monte Carlo program¹⁰ that fully simulates the response of the Mark II detector. The number of $\nu_4 \bar{\nu}_4$ events expected to be produced $N_{\nu 4}$ is normalized to the total number of hadronic events N_h that fulfill the hadronic event selection criteria described in a previous Letter:¹¹

$$N_{\nu 4} = \frac{N_h \Gamma_{\nu 4}}{\epsilon_q \Gamma_q + \epsilon_{\nu 4} \Gamma_{\nu 4}}$$

where Γ_q is the partial decay width of the Z to udscb quarks, $\epsilon_q = 0.953$ is the efficiency for udscb quarks to pass the hadronic event criteria, $\Gamma_{\nu 4}$ is the partial width of the Z to $\nu_4 \bar{\nu}_4$, and $\epsilon_{\nu 4}$ is the efficiency for the $\nu_4 \bar{\nu}_4$ events to pass the hadronic event criteria. The normalizing hadronic data sample consists of $N_h = 450$ events.

Efficiencies for tagging produced $\nu_4 \bar{\nu}_4$ events are calculated in a two-stage procedure. Since a long-lived ν_4 might not decay within the detector fiducial volume defined for triggering, charged and neutral energy trigger emulator programs are applied to simulated $\nu_4 \bar{\nu}_4$ events to determine trigger efficiencies. Secondly, event selection cuts are applied only to those events that would trigger our data acquisition system and final detection efficiencies $\epsilon_{\nu 4}$ are determined. For a 35 GeV/c² ν_4 with a mean decay length of 1 m, trigger efficiencies are 76% for the charged track trigger and 78% for the neutral energy trigger, giving an overall trigger efficiency of 90%; the final detection efficiency including the trigger efficiency is 15%. For a 35 GeV/c² ν_4 with mean decay length of 5 cm, the final detection efficiency is 45%. Detection efficiencies are equal to one another within errors for $\nu_4 \rightarrow e+W^*$ and $\nu_4 \rightarrow \mu+W^*$ but are about 10% lower for $\nu_4 \rightarrow \tau+W^*$.

Uncertainties in detection efficiency from Monte Carlo statistics ($\approx 2\%$), detector simulation and beam backgrounds ($\approx 4\%$), tracking efficiencies for tracks with large impact parameters ($\approx 10\%$), and different fragmentation models ($\approx 1\%$) are estimated. Uncertainties in the number of produced events arise from the statistical error in N_h and from small errors in ϵ_q and $\epsilon_{\nu 4}$. The total error is calculated by summing the individual statistical and systematic errors in quadrature.

Detection efficiencies are determined for $\nu_4 \bar{\nu}_4$ events for a grid of values in the $m_4 - |U_{\ell 4}|^2$ plane. Two-dimensional polynomial interpolation is used to determine efficiencies between grid points. Finally, the expected number of $\nu_4 \bar{\nu}_4$ events after all cuts is determined, and reduced by the total error described above to obtain a conservative 95% C.L. limit contour in the $m_4 - |U_{\ell 4}|^2$ plane. The results are shown in Fig. 2. The upper and lower boundary of the contour at a mass of 35 GeV/ c^2 correspond to mean decay lengths of 0.6 cm and 83 cm, respectively. Also shown in Fig. 2 are the region excluded by an isolated track search in the same data sample,¹² and the region excluded by the Mark II experiment at PEP.¹³

In conclusion, we observe no events that are consistent with the search hypothesis of a sequential, long-lived, massive Dirac neutrino. We significantly extend the excluded region at 95% C.L. in the $m_4 - |U_{\ell 4}|^2$ plane determined by lepton universality¹⁴ and other experiments: most recently by the AMY¹⁵ collaboration, the CELLO¹⁶ collaboration, and monojet searches at PEP¹⁷. The results are summarized in Fig. 3 for $\ell = e$. The excluded regions are very similar for $\ell = \mu$. It is interesting to note that whereas most other searches are restricted to limiting $|U_{e4}|^2 + |U_{\mu4}|^2$, this search also limits $|U_{\tau4}|^2$. Finally, it should be noted that the described search procedure is also valid for any general long-lived particles decaying into many particles.

REFERENCES

- E. Witten, Phys. Lett. **91B**, 81 (1980); R.N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980); A. Davidson and K. Wali, Phys. Lett. **98B**, 183 (1981); E. Papantonopoulos and G. Zoupanos, Phys. Lett. **110B**, 465 (1982); J. Bagger and S. Dimopolous, Nucl. Phys. **244**, 247 (1984); J. Bagger *et al.*, Phys. Rev. Lett. **54**, 2199 (1985); Nucl. Phys. **B258**, 565 (1985).
- 2. Mark II Collaboration, G.S. Abrams et al., Phys. Rev. Lett. 63 2173 (1989).
- ALEPH Collaboration, D. Decamp et al., CERN preprint EP/89-132; OPAL Collaboration, M.Z. Akrawy et al., CERN preprint EP/89-133; L3 Collaboration, B. Adeva et al., CERN preprint L3-preprint-001; DELPHI Collaboration, P. Aarnio et al., CERN preprint EP/89-134
- 4. The search is applicable to long-lived charged particles as well as to long-lived neutral particles.
- R.E. Shrock, Phys. Rev. D24, 1275 (1981); Y.S. Tsai, Phys. Rev. D4, 2821 (1971) and Phys. Rev. D19, 2809 (1979).
- Mark II Collaboration, G. Abrams et al., Nucl. Instrum. Methods A 281, 55 (1989).
- T. Sjöstrand, Comput. Phys. Comm. **39**, 347 (1986); M. Bengtsson and T. Sjöstrand, Nucl. Phys. **B289**, 810 (1987).
- G. Marchesini and B.R. Webber, Nucl. Phys. B238, 1 (1984); B.R. Webber, Nucl. Phys. B238, 492 (1984).
- 9. In the Monte Carlo simulation, when a nuclear interaction occurs, the interacting particle stops at the point of the interaction and disappears without generating further fragmentation particles.

- 10. The four vectors from the ν_4 decay are generated for a particular mass and mean lifetime. The Lund 6.3 shower model, Ref. 7, then handles the subsequent fragmentation.
- 11. Mark II Collaboration, G.S. Abrams et al., Phys. Rev. Lett. 63, 724 (1989).
- 12. Mark II Collaboration, G.S. Abrams et al., Phys. Rev. Lett. 63, 2447 (1989).
- 13. Mark II Collaboration, C. Wendt et al., Phys. Rev. Lett. 58, 1810 (1987).
- M. Gronau, C.N. Leung, and J.L. Rosner, Phys. Rev. D 29, 2359 (1984); V.
 Barger, W.Y. Keung, and R.J. Phillips, Phys. Lett. 141B, 126 (1984).
- 15. AMY Collaboration, N.M. Shaw et al., Phys. Rev. Lett. 63, 1342 (1989).
- 16. CELLO Collaboration, H.-J. Behrend et al., Z. Phys. C41, 7 (1988).
- 17. F.J. Gilman and S.H. Rhie, Phys. Rev. D 32, 324 (1985).

FIGURE CAPTIONS

- 1) Distribution of ν_4 search parameter χ_{imp} for data (solid dots with error bars), udscb Monte Carlo (solid line), a 35 GeV/ $c^2 \nu_4$ with a lifetime of 100 ps (dotted line, normalized to data), and a 35 GeV/ $c^2 \nu_4$ with a lifetime of 1000 ps (dashed line, normalized to data).
- 2) 95% C.L. excluded regions for a hypothesized fourth generation long-lived massive Dirac neutrino ν₄ as a function of mass and mixing matrix element |U_{ℓ4}|², ℓ = e, μ, or τ. Also shown are the equivalent excluded regions for a Mark II isolated track search described in Ref. 12, and a Mark II search for detached vertices at PEP described in Ref. 13.
- 3) 95% C.L. excluded regions for a hypothesized fourth generation long-lived massive Dirac neutrino ν_4 as a function of mass and mixing matrix element

 $|U_{e4}|^2$ for the present analysis compared to those obtained by (1) Mark II Ref. 12, (2) AMY, Ref. 15, (3) CELLO, Ref. 16, (4) Mark II secondary vertex search at PEP, Ref. 13, (5) monojet searches at PEP, Ref. 17, and (6) e- μ universality, Ref. 14.



 $\sum_{i=1}^{n-1} \sum_{j=1}^{n-1} \sum_{i=1}^{n-1} \sum_{j=1}^{n-1} \sum_{j=1}^{n-1} \sum_{i=1}^{n-1} \sum_{j=1}^{n-1} \sum_{i=1}^{n-1} \sum_{j=1}^{n-1} \sum_{i=1}^{n-1} \sum_{j=1}^{n-1} \sum_{j=1}^{n-1} \sum_{i=1}^{n-1} \sum_{j=1}^{n-1} \sum_{i=1}^{n-1} \sum_{j=1}^{n-1} \sum_{i=1}^{n-1} \sum_{j=1}^{n-1} \sum_{j=1}^{n-1} \sum_{i=1}^{n-1} \sum_{j=1}^{n-1} \sum_{i=1}^{n-1} \sum_{j=1}^{n-1} \sum_{j=1}^{n-1} \sum_{i=1}^{n-1} \sum_{j=1}^{n-1} \sum_{i=1}^{n-1} \sum_{j=1}^{n-1} \sum_{j=1}^{n-1}$





Fig. 2



Fig. 3