# Effects of the mass and magnetic moment of the neutrinos in $\nu e \rightarrow \nu e \gamma^{\star}$ 

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#### Abstract

We study the inelastic $\nu e$ scattering and consider possible deviations from the Standard Model. We perform a numerical analysis of the cross section as a function of the angle and energy of the bremsstrahlung photon and show that the degree of circular polarization of the photon can be used to obtain new limits on the masses and/or magnetic moments of the ncutrinos.


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[^0]About 60 years after the Pauli hypotesis, the neutrino remains a mystcrious particle. Moreover the search for the neutrino mass and magnetic moment is of great significance for the choice of the gauge theory of particle interactions and to understand phenomena like stellar evolution, supernovae explosions and the solar neutrino problem. The importance of the reaction $\nu e \rightarrow \nu e \gamma$ for the study of neutrino properties such as mass and electro magnetic interactions was pointed out several years ago [1-6]. In [1] the cross section of that reaction was calculated including electromagnetic form-factors of the neutrino. However, it is very difficult to detect deviations when the cross section is itself very small.

It has been shown [2-6] that the study of polarization effects could provide new restrictions on the parameters of the theoretical models describing neutrino-electron interactions. In [6] it was suggested that the degree of circular polarization of the photon in the radiative $\nu e$ scattering should depend on the contribution to this process coming from the direct interaction of the neutrino with the electromagnetic field. Our aim is to find what deviations from the Clashow-Weinberg-Salam model (GWS) in the cross section for radiative $\nu e$ scattering would arise from non-standard mass and magnetic moment of the neutrino.

The limits on the electron-neutrino mass from different experiments are

$$
\begin{array}{rll}
17< & \mathrm{m}_{\nu_{e}}<40 \mathrm{eV}, & \mathrm{~m}_{\nu_{e}} \simeq 30 \mathrm{eV}[7], \\
& \mathrm{m}_{\nu_{c}}<27 \mathrm{eV}[8], & \mathrm{m}_{\nu_{e}}<18 \mathrm{eV}[9] .
\end{array}
$$

For the muon- and tan-nentrinos,

$$
\mathrm{m}_{\nu_{\mu}}<0.25 \mathrm{MeV}[10], \mathrm{m}_{\nu_{\tau}}<35 \mathrm{MeV}[11] .
$$

The limits obtained from laboratory experiments on the magnetic moments of $\nu_{e}$ and $\nu_{\mu}$ are

$$
\mu_{\nu_{e}}<1.4 \times 10^{-9} \mu_{\mathrm{B}}[12], \quad \mu_{\nu_{\mu}}<0.95 \times 10^{-9} \mu_{\mathrm{B}}[13]
$$

where $\mu_{\mathrm{B}}=e / 2 m_{e}$ is the Bohr magneton. From cosmological considerations, the upper bound, $\mu_{\nu}<(1-2) \times 10^{-11} \mu_{B}$, was derived [14] for the magnetic moment of the three neutrino species.

Let us consider the inelastic neutrino-electron scattering

$$
\begin{equation*}
\nu_{l} e \xrightarrow{z, \gamma} \nu_{l} e \gamma \tag{1}
\end{equation*}
$$

where $l=e, \mu, \tau, \ldots$ The Feynman diagrams of the $t$ channels arc shown in fig. 1. The first two diagrams, 1.a-b), correspond to the neutral weak interaction through the Z-boson exchange and the diagrams $1 . c-d$ ) to the exchange of a virtual photon coupling to a neutrino with non-zero magnetic moment.

In the local approximation, $-\left(q-q^{\prime}\right)^{2} \ll M_{2}^{2}$, the matrix element of the Z channel is

$$
\begin{equation*}
M_{\mathrm{z}}=M_{\mathrm{z}}^{a}+M_{\mathrm{z}}^{b}, \tag{2}
\end{equation*}
$$

where

$$
\begin{align*}
& M_{\mathrm{z}}^{a}=-C_{\mathrm{z}} \overline{u_{\nu}^{\prime}} \gamma^{\alpha}\left(1-\gamma_{5}\right) u_{\nu} \overline{u_{e}^{\prime}} \gamma_{\alpha}\left(g_{\mathrm{v}}-g_{\mathrm{A}} \gamma_{5}\right) \frac{2 \varepsilon^{*} \cdot p-\not k 申^{*}}{2 p \cdot k} u_{e},  \tag{3}\\
& M_{\mathrm{z}}^{b}=C_{\mathrm{z}} \overline{u_{\nu}^{\prime}} \gamma^{\alpha}\left(1-\gamma_{5}\right) u_{\nu} \overline{u_{e}^{\prime}} \frac{2 \varepsilon^{*} \cdot p^{\prime}-\not k \ddagger^{*}}{2 p^{\prime} \cdot k} \gamma_{\alpha}\left(g_{\mathrm{v}}-g_{\mathrm{A}} \gamma_{5}\right) u_{e} \tag{4}
\end{align*}
$$

correspond to the diagrams 1.a) and 1.b) respectively. For the scattering via virtual photon exchange the matrix element is given by

$$
\begin{gather*}
M_{\gamma}=M_{\gamma}^{c}+M_{\gamma}^{d}  \tag{5}\\
M_{\gamma}^{c}=C_{\gamma} \overline{u_{\nu}^{\prime}} \frac{\sigma^{\alpha \beta}\left(q-q^{\prime}\right)_{\beta}}{\left(q-q^{\prime}\right)^{2}} u_{\nu} \overline{u_{e}^{\prime}} \gamma_{\alpha} \frac{2 \varepsilon^{*} \cdot p-\not k \dot{\chi}^{*}}{2 p \cdot k} u_{e}  \tag{6}\\
M_{\gamma}^{d}=-C_{\gamma} \overline{u_{\nu}^{\prime}} \frac{\sigma^{\alpha \beta}\left(q-q^{\prime}\right)_{\beta}}{\left(q-q^{\prime}\right)^{2}} u_{\nu} \overline{u_{e}^{\prime}} \frac{2 \varepsilon^{*} \cdot p^{\prime}-\not k \cdot \dot{\xi}^{*}}{2 p^{\prime} \cdot k} \gamma_{\alpha} u_{e} . \tag{7}
\end{gather*}
$$

Here $C_{z}=G_{F} e / \sqrt{2}, C_{\gamma}=4 \pi \alpha \mu_{\nu}, G_{F}$ is the Fermi coupling constant, $\mu_{\nu}$ is the neutrino magnetic moment, $g_{\mathrm{v}}=-1 / 2+2 \sin ^{2} \theta_{W}$ and $g_{\mathrm{A}}=-1 / 2$
as in the GWS model; $u_{e}$ and $u_{\nu}\left(u_{e}^{\prime}\right.$ and $\left.u_{\nu}^{\prime}\right)$ are the Dirac spinors of the incoming (outgoing) clectron and neutrino with 4 -momenta $p$ and $q$ ( $p^{\prime}$ and $q^{\prime}$ ) respectively; $k$ and $\varepsilon$ are the 4 -momentum and polarization vectors of the bremsstrahlung photon. We follow the metric convention and notations of Bjorken and Drell [15]. In the case of $\nu_{e} e$ scattering we should also consider the charged weak current. However, by applying Fierz transformations to the matrix element one obtains that the contribution from the $W$ exchange is equivalent to a change in the above coupling constants $g_{V}$ and $g_{A}$. From the eqs. (6-7) we see that the contribution of the magnetic moment interaction becomes more important at low neutrino energies. This point is also discussed in the refs. [5], [16], [17].

In our calculations we neglect the contribution to the differential cross section from $\left|M_{\gamma}\right|^{2}$ which is proportional to $\mu_{\nu}^{2}$. The evaluation of $|M|^{2}$ averaged over initial and summed over final electron and neutrino polarizations, and the exact analytic integration of the cross section over the neutrino

- and electron phase space were carried out with fimite electron and neutrino masses. Using a Reduce program wc obtaincd the differential cross section as a linear combination of the integrals

$$
\begin{aligned}
& I(n, m)=\int \frac{d \overrightarrow{p^{\prime}}}{E_{p^{\prime}}} \frac{d \overrightarrow{q^{\prime}}}{E_{q \prime}} \delta\left(p^{\prime}+q^{\prime}-\Delta\right) B^{n} L^{m} \\
& I\left(n, m, X_{1}, \cdots, X_{j}\right)=\int \frac{d \overrightarrow{p^{\prime}}}{E_{p \prime}} \frac{d \overrightarrow{q^{\prime}}}{E_{q \prime}} \delta\left(p^{\prime}+q^{\prime}-\Delta\right) \frac{p^{\prime} \cdot X_{1} \cdots p^{\prime} \cdot X_{j}}{B^{n} L^{m}}
\end{aligned}
$$

where: $B=-\left(q-q^{\prime}\right)^{2}, L=2 p^{\prime} \cdot k$ and $n, m=0,1,2 ; \Delta=p+q-k$; $E_{p^{\prime}}\left(E_{q^{\prime}}\right)$ is the energy of the outgoing electron (neutrino) and $X_{1}, \cdots, X_{j}$ are any of the particle 4 -momenta. We did the analytic calculation using the method of covariant integration [18] and the results presented in refs. [19] for the integration over the phase space of two identical final particles - more details on our calculations are in ref. [20]. The differential cross section of the reaction (1) in the rest frame of the electron, summed over the photon
helicity, can be written as

$$
\begin{equation*}
\frac{d \sigma^{0}}{d E_{\gamma} d \cos \theta_{\gamma}}=\frac{d \sigma_{\mathrm{z}}^{0}}{d E_{\gamma} d \cos \theta_{\gamma}}\left(1+R_{\gamma \mathrm{z}}\right) \tag{8}
\end{equation*}
$$

where $d \sigma_{\mathrm{z}}^{0}$ is the non-polarized weak contribution, $E_{\gamma}$ is the photon energy, $\cos \theta_{\gamma}=\vec{k} \cdot \vec{q} /|\vec{k}||\vec{q}|$ and $R_{\gamma z}$ denotes the relative contribution of the interference between the diagrams 1.a-b) and 1.c-d).

Since $R_{\gamma z}$ is proportional to the neutrino mass and ncutrino magnetic moment we define $R$ as

$$
\begin{equation*}
R_{\gamma \mathrm{z}}=\frac{m_{\nu}}{1 M e V} \frac{\mu_{\nu}}{\mu_{\mathrm{B}}} R \tag{9}
\end{equation*}
$$

The numerical analysis shows that for $E_{\nu} \geq 10 \mathrm{MeV},|R| \sim 10^{9}$. Hence, in the case of the electron-neutrino the present experimental limits exclude a non-negligible contribution of the interference term, so we will specialize our numerical results for $\nu_{\mu}$ and $\nu_{\tau}$. In view of the small cross sections it is likeky that it will be very difficult to get new results from unpolarized scattering. However, theoretical models which give similar results for observable quantities, when averaged over the particle spins, can give different results for polarization effects because they involve interaction terms independent of each other with different spin structures. As mentioned above, the dcpendence of the degree of circular polarization on the coupling constants for various theoretical models was discussed in refs. [2-6]. We want to sce whether the circular polarization of the photon can also be used to obtain new limits on the masses and/or magnetic moments of the neutrinos. Let us define

$$
\begin{equation*}
P_{\gamma}\left(E_{\gamma}, \cos \theta_{\gamma}\right)=\frac{d \sigma\left(s_{\gamma}=-1\right)-d \sigma\left(s_{\gamma}=+1\right)}{d \sigma\left(s_{\gamma}=-1\right)+d \sigma\left(s_{\gamma}=+1\right)}, \tag{10}
\end{equation*}
$$

where $s_{\gamma}$ is the photon helicity ( $s_{\gamma}=-1$ for left-handed and $s_{\gamma}=+1$ for right-handed photons).

In fig. 2 we plot $P_{\gamma}$ for $F_{\nu}=50 \mathrm{MeV}, F_{\gamma}=0.1 \mathrm{MeV}$ and various values of $m_{\nu}$ and $\mu_{\nu}$. In fig. 3 we do the same for $E_{\gamma}=0.5 \mathrm{MeV}$ and $E_{\gamma}=2 \mathrm{MeV}$
with $E_{\nu}=50 \mathrm{MeV}$. It is possible to compare the behaviour of $P_{\gamma}$ in the case of the GWS model and for non-standard $m_{\nu}$ and $\mu_{\nu}$. The differential cross section for $m_{\nu}=0$ and $\mu_{\nu}=0$ is plotted in the fig. 4 as a function of $\cos \theta_{\gamma}$ for the above values of the neutrino and photon energies. Clearly, the contribution to the photon polarization due to the neutrino mass and magnetic moment is more significant for small photon energies (compared to the energy of the incident neutrino) and large angles of the photon. As far as the cross section is concerned, the fig. 4 shows that it is larger for small energies and angles of the photon.

We conclude that the study of the photon polarization in the process $\nu e \rightarrow$ $\nu e \gamma$ can be used to obtain new limits on the masses and magnetic moments of the muon- and tau-neutrinos. The development of detection methods offers hope of being able to perform experiments in the near future to study the radiative scattering of neutrinos by electrons. In particular, at LAMPF it will be possible to study the $\nu_{\mu} e$ scattering with neutrino-beam energies bellow

- 50 MeV [21]. As mentioned before, the non-standard effects are enhanced for low neutrino energies - at $E_{\nu} \leq 10 \mathrm{MeV}$ the contribution of the magnetic moment coupling to the cross section might be of the order or larger than the standard model one. It is worth to recall that for these energies the background of the $\nu e$ radiative scattering is comparatively smaller than for the elastic scattering [5].

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## FIGURE CAPTIONS

Fig. 1: Feynman diagrams for the radiative $\nu e$ scattering.

Fig. 2: $P_{\gamma}$ as a function of $\cos \theta_{\gamma}$ in the case of $E_{\nu}=50 \mathrm{MeV}, E_{\gamma}=0.1 \mathrm{MeV}$ for: $m_{\nu}=0$ and $\mu_{\nu}=0$ (dotted curve), $m_{\nu}=10 \mathrm{MeV}$ and $\mu_{\nu}=0$ (dashed curve), $m_{\nu}=0.25 \mathrm{MeV}$ and $\mu_{\nu}=2 \times 10^{-11} \mu_{B}$ (solid curve), $m_{\nu}=0.25 \mathrm{MeV}$ and $\mu_{\nu}=$ $10^{-10} \mu_{B}$ (dash-dotted curve).

Fig. 3: $P_{\gamma}$ as a function of $\cos \theta_{\gamma}$ in the case of $E_{\nu}=50 \mathrm{MeV}, E_{\gamma}=0.5 \mathrm{MeV}$ and $E_{\gamma}=2 \mathrm{MeV}$. The solid curves correspond to $m_{\nu}=0.25 \mathrm{MeV}, \mu_{\nu}=2 \times 10^{-11} \mu_{B}$, and the dash-dotted curves to $m_{\nu}=0, \mu_{\nu}=0$.

Fig. 4: Differential cross section as a function of $\cos \theta_{\gamma}$ for $E_{\gamma}=0.1 \mathrm{MeV}$ (solid curve), $E_{\gamma}=0.5 \mathrm{MeV}$ (dash-dotted curve) and $E_{\gamma}=2 \mathrm{MeV}$ (dashed curve).


Fig. 1


Fig. 2


Fig. 3


Fig. 4


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