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# Electroweak Theory with spontaneous breaking of Parity and $CP^\star$

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## ABSTRACT

We consider the SM in terms of Majorana fermions and show that: the electroweak interactions conserve Parity; the Poincaré and internal symmetries cannot be factorized because both do not commute with $P$ and $T$; the fermions should be in the fundamental representations of SU(2) and SU(3); in general, the gauge groups should be U(1) or SU(n) with n-dimensional matter multiplets; the minimal Higgs mechanism has two iso-doublets and NFC follows from $P$ and $T$ invariance. $P$ is spontaneously broken but $CP$-breaking requires two more Higgs doublets.

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One of the most striking features of the standard model[1,2] (SM) of the electroweak interactions is the fact that the chiral components of the elementary fermions come in completely different representations (reps) of the symmetry group SU(2)xU(1): the left-hand ones form isospin doublets and the right chiral fields are singlets. However, this description is itself arbitrary because the degrees of freedom of a left chiral field, $\chi$, can also be represented by a right-hand one, $\chi^c$, related to it by the charge conjugation operation, defined[3] as:

$$\chi^c = \mathcal{C}\,\bar{\chi}^T . \tag{1}$$

Moreover, $\chi$ can also be replaced by a Majorana spinor, $\psi = \chi + \chi^c$, in terms of which the kinetic Lagrangian,

$$\mathcal{L}_k = 1/2\,\bar{\psi}\, i\partial\!\!\!/\, \psi = \bar{\chi}\, i\partial\!\!\!/\, \chi\,, \tag{2}$$

does not exhibit any chiral structure.

It is well known[4] that $\mathcal{L}_k$ is invariant under the parity ($P$) and $CP$ transformations (trfs) of $\psi$, defined as the ones[3] for Dirac fields, but with the intrinsic $P$ and $CP$ phases restricted to the values $\pm i$ in order to comply with the Majorana condition, $\psi = \psi^c$. This assumes a charge conjugation ($C$) trf as $\psi \rightarrow \pm\psi$. However, $\mathcal{L}_k$ is also invariant under an axial phase trf, $exp(i\alpha\gamma_5)$, so we define $C$ as:

$$C: \qquad \psi \rightarrow (\epsilon L + \epsilon^* R)\psi\,, \tag{3}$$

where $L$ ($R$) is the left (right) chirality projector ($\gamma_5 = R - L$) and $\epsilon$ denotes the intrinsic $C$ phase. Likewise, the $P$ and time-reversal ($T$) trfs are given by (ommitted the space-time coordinates):

$$P: \quad \psi \rightarrow (\pi L - \pi^* R)\mathcal{P}\,\psi\,, \tag{4}$$

$$T: \quad \psi \rightarrow (\tau L - \tau^* R)\mathcal{T}\,\psi^*\,, \tag{5}$$

where $\mathcal{P}$ and $\mathcal{T}$ are the standard[3] unitary matrices for Dirac spinors and $\pi$ ($\tau$) denotes the intrinsic $P$ ($T$) phase.

Let us consider a gauge symmetry group and a n-dimensional rep by matrices $D = exp(i\alpha^a X_a)$ with hermitian generators $X_a$. A realization on a multiplet $\psi$ of Majorana spinors is given by the matrices

$$U = exp(i\alpha^a T_a) = DL + D^* R\,, \tag{6}$$

where $T_a{=}X_a L{-}X_a^* R$. The generators $T_a$ are hermitian too and obey the same Lie algebra as the $X_a$. Clearly, the trfs (6) conserve the Lagrangian (1). In addition, the local gauge symmetry is implemented, as usual,[5] by means of the covariant derivatives

$$\mathcal{D}_\mu = \partial_\mu - igA^a_\mu T_a\,, \tag{7}$$

where the gauge fields $A^a_\mu$ undergo the canonical gauge trfs of the Yang-Mills fields. In contrast to the kinetic terms, the gauge couplings have a non-trivial chiral structure and one has to investigate whether or not they conserve parity.

**Theorem.** The gauge interactions between the gauge bosons in the Lie algebra of a group G and a n-dimensional Majorana multiplet $\psi$, conserve parity, $CP$ and time-reversal, regardless of the $\psi$ basis, if and only if: the group G is SU(n) or U(1); the gauge fields are even eigenstates of $C$ and transform as:

$$P: \qquad A^\mu \to -A^*_\mu , \tag{8}$$

$$T: \qquad A^\mu \to +A^*_\mu , \tag{9}$$

where $A_\mu = A^a_\mu X_a$, and $X_a$ is a number if G is U(1).

Proof. Given the $\psi$ trfs as in the eqs. (3-5), the above $A_\mu$ trfs are the necessary ones for the gauge couplings to be invariant under $P$, $T$ and $C$. In order that they really represent trfs of the gauge fields $A^a_\mu$, the matrices $X^*_a$ have to lie on the same Lie algebra. In addition, since the $\psi$ kinetic terms do not depend on the basis, that should be true for any basis of the $X_a$, if the rep is irreducible and the group is simple or U(1).

Let $S \in$ SU(n) be a trf from a basis $\mathcal{D}$ to $\mathcal{D}'$: $X'_a = S X_a S^\dagger$. In order that $X^*_a$ lie on $\mathcal{D}$ and $X'^*_a$ on $\mathcal{D}'$, the matrix $S^T S$ has to leave the Lie algebra invariant i.e., $S^T S X_a S^\dagger S^*$ generate $\mathcal{D}$ and likewise for all equivalent reps $\mathcal{D}''$. Hence, $[Y, \mathcal{D}'']$ is contained in $\mathcal{D}''$ for some $Y \neq 0$ ($S^T S = exp(iY)$). From $\mathcal{D} = U \mathcal{D}'' U^\dagger$, $U \in$ SU(n), it follows that all $[Y_U, \mathcal{D}]$, $Y_U = U Y U^\dagger$, lie on $\mathcal{D}$. The $Y_U$ generate the complete SU(n) algebra because they form an invariant subalgebra (note that $[Y, Z]$ can be written as the derivative at $t = 0$ of $F(t) = iV(t) Y V^\dagger(t)$, with $V(t) = exp(itZ)$) of SU(n), which has no proper invariant subalgebras.[6] Therefore, apart from a U(1) subalgebra, $\mathcal{D}$ is an invariant subalgebra of SU(n) and for that very reason, is either the SU(n) or the nil algebra. Finally, the $A^a_\mu$ trfs induced by different $X_a$ basis are equivalent because they only differ for global gauge trfs.

The sufficient condition is proved by noting that, first, the two n-dimensional fermion reps of SU(n), complex conjugate of each other, induce the same trfs of the gauge fields, second, the Yang-Mills "kinetic" term[5,7] ($\mathrm{tr} F_{\mu\nu} F^{\mu\nu}$) is invariant under $P$ and $T$. q.e.d.

We remark that in the SM the gauge group and fermion reps are in agreement with this theorem: indeed, the simple groups are SU(2) and SU(3) and the non-trivial irreducible reps are iso-doublets and color-triplets. Of course, the chiral reps can be replaced by Majorana multiplets without changing the Lagrangian.

It is of some importance to understand why are the fermions in chiral and not Dirac reps. It is well known that the massless Dirac fields are not irreducible reps of the Poincaré group: they are reducible into two chiral-Weyl spinors of opposite helicity which, in addition, are not mixed by the gauge interactions. Finally, since parity reverses the helicity, its reps are realized by Majorana rather than Weyl spinors. It is worth to remark that the standard $CP$ and $T$ trfs of a Dirac multiplet induce the very trfs (8-9) and the resultant restrictions on the gauge groups and reps. In fact, the difference between Majorana and Dirac reps just lies on the $C$ (or $P$) trf.

Since the gauge fields are in the adjoint rep, their $C$, $P$ and $T$ trfs should result from the Lie algebra. Using the eqs. (4-6) one derives the algebra

$$P\,T_a\,P^{-1} = \Pi_a{}^b T_b = -T\,T_a\,T^{-1}\,, \tag{10}$$

where the matrix $\Pi$ is defined by: $X_a^* = -\Pi_a{}^b X_b$. $\Pi$ is real and $\Pi^2 = 1$. The algebra is necessarily satisfied by the gauge fields rep since they are in the adjoint rep. The $\Pi$ eigenvalues, $\pm 1$, represent their intrinsic $P$ and $T$ phases. Because $P$ and $T$ do not commute with the Poincaré group and are geometric trfs by themselves, the eqs. (10) establish the unification between the internal and space-time symmetries - they cannot be factorized.

Although the constants $\Pi_a{}^b$ depend on the basis of the $X_a$, the Lie algebra is well defined because, under a similarity trf $U$, not only the generators $T_a$ but also $P$ and $T$ are transformed. Yet, since $U$ is itself a gauge trf (see eq.(6)), $UPU^\dagger$ ($UTU^\dagger$) is equivalent to $P$ ($T$) up to a global gauge trf.

In the gauge bosons rep, $C$ and $T$ commute with $P$. Since the fermions should be just a different rep of the same symmetry group, the constraints $\epsilon = \pm 1$ and $\pi = \pm i\tau$ follow. As a result, $CPT$ reduces to: $\psi(x) \rightarrow \pm\psi(-x)$. It is well known[8] that, because $T$ is antilinear, $\tau$ can be made equal to 1 using a field redefinition by a phase factor. In the case of a Majorana spinor one can use an axial phase, so we fix $\tau = 1$ for all spinor fields.

Now, let us consider the SM[1,2,5] in detail. We denote the quark Majorana multiplets as: $\xi_i, \chi_i$ are the up, down quarks, singlets of SU(2), and $\psi_i$ are the SU(2) doublets (with $i$ as the generation indice). The SU(2) generators are $(\sigma_j L - \sigma_j^* R)/2$; the SU(3) ones are $(\lambda_a L - \lambda_a^* R)/2$ for $\psi_i$ and $(-\lambda_a^* L + \lambda_a R)/2$ for $\xi_i, \chi_i$ - here, $\sigma_j (\lambda_a)$ are the Pauli (Gell-Mann) matrices. The U(1) generator is $Y(L-R)$ where $Y$ is the hypercharge, 1/3, -4/3 and 2/3 for $\psi_i, \xi_i$ and $\chi_i$ respectively. Denoting by $W^i, B, G^a$ the gauge bosons of SU(2), U(1), SU(3) respectively, one finds that $W^2, G^2, G^5, G^7$ are vector fields and all the others are axial-vectors. Needless to say, the SM electroweak interactions conserve parity.

It has been conceived that[2,5] one scalar iso-doublet is the minimal Higgs content to break SU(2)xU(1). However, no linear rep of parity anticommutes with the hypercharge, as required by the eqs. (10). Therefore we consider a 4-dimensional rep, namely a bi-doublet $\phi \equiv (\phi_1, \phi_2)$, where $\phi_1, \phi_2$ are doublets in complex conjugate reps of each other, with hypercharges 1, -1 respectively. The symmetry group rep is defined as:

$$\text{SU(2)xU(1)}: \quad \phi \rightarrow (U\phi_1, U^*\phi_2)\,, \tag{11}$$

$$P: \quad \phi \rightarrow (\pi_\phi \phi_2, \pi_\phi^* \phi_1)\,, \tag{12}$$

$$T: \quad \phi \rightarrow \phi^*\,, \tag{13}$$

where $U$ is a unitary 2x2 matrix and the intrinsic $T$ phases of $\phi_{1,2}$ are fixed to be 1. The constraint $PT=TP$ requires the phase $\pi_\phi$ to be $\pm 1$. Since the $\pi_\phi$ sign can be absorbed in $\phi_2$ without changing the $T$ trf, we fix $\pi_\phi=1$. $\phi_1$ and $\phi_2$ are irreducible reps of $T$ and SU(2). We point out that all scalar multiplets in irreducible reps of SU(2) should be iso-doublets. Otherwise, as we showed for the fermions, the gauge couplings would not be invariant under time-reversal regardless of the basis chosen.

The specification $\pi=i\epsilon$ for all fermions and the definition of $C$ for the scalars as

$$C: \quad \phi \rightarrow (\phi_2^*, \phi_1^*)\,, \tag{14}$$

brings their $CPT$ trfs to be like $\psi(x) \rightarrow \psi(-x)$. As a result, any algebraic coupling like the mass terms is trivially invariant under $CPT$. In addition, the Yukawa couplings and scalars potential conserve $C$ provided that they are invariant under $P$ and $T$. Finally, the $\phi$ gauge interactions conserve $C$.

We adopt the following principles:

1. The Lagrangian is invariant under the extended symmetry group, which includes

$C$, $P$ and $T$, the gauge groups and the Poincaré group.

2. Universality is extended to $C$, $P$ and $T$: fields in equal reps of the continuous symmetry group have the same quantum numbers $\epsilon$, $\pi$ and $\tau$.

Then, the Yukawa couplings of the down quarks for example, are determined to be of the form

$$K_{ij}\,\bar{\psi}_i\,(\phi_1 - \pi_\psi \pi_\chi\,\phi_2^*)R\,\chi_j + h.c.\,, \tag{15}$$

with real coupling constants ($T$ invariance). Remarkably enough, Natural Flavor Conservation (NFC) in the exchange of neutral Higgs particles results from parity conservation, because $\pi_\psi, \pi_\chi, \pi_\xi$ are the same for all generations (Universality Principle).

For the sake of equality we choose that both

$$\phi_\pm = (\phi_1 \pm \phi_2^*)/\sqrt{2} \tag{16}$$

couple with the quarks, not to the same singlets, of course. There is no loss of generality in specifying which one couples with what singlets because the replacement $\phi_2 \to -\phi_2$ ($\phi_+ \leftrightarrow \phi_-$) leaves the scalars potential invariant up to a redefinition of the free parameters. Then, the full expression of the Yukawa couplings is

$$\begin{aligned} \mathcal{L}_Y = &-K_{ij}\,\bar{\psi}_i\,(\phi_- R + \phi_-^* L)\,\chi_j \\ &-\tilde{K}_{ij}\,\bar{\psi}_i\,(\tilde{\phi}_+ R + \tilde{\phi}_+^* L)\,\xi_j\,, \end{aligned} \tag{17}$$

where, as usual, $\tilde{\phi}$ stands for $i\sigma_2\phi^*$. This corresponds to $\epsilon_\psi = \epsilon_\xi = -\epsilon_\chi$. For definiteness, $\epsilon_\psi = 1$. Because $C$ and $PT$ reduce to the reflection $\phi_\pm \to \pm\phi_\pm$, it is convenient to write the invariant quartic potential in terms of $\phi_\pm$ rather than $\phi_{1,2}$:

$$V = \sum \mu_a^2 \mid \phi_a \mid^2 + p_{ab} \mid \phi_a \mid^2 \mid \phi_b \mid^2 + q_{ab}\, \phi_a^\dagger \phi_b\, \phi_b^\dagger \phi_a + r_{ab}\, (\phi_a^\dagger \phi_b - \phi_b^\dagger \phi_a)^2 , \quad (18)$$

where $\phi_a$ stand for $\phi_\pm$ and the coupling constants are real. Whatever the vacuum is (consistent with a massless photon), it can be written as

$$< \phi_+ >= \begin{pmatrix} 0 \\ v_+ \end{pmatrix}, \qquad < \phi_- >= \begin{pmatrix} 0 \\ v_- \end{pmatrix} e^{i\theta}, \quad (19)$$

where $v_\pm, \theta$ are real. Since both up and down quarks are massive, $C$ and $PT$ are necessarily broken; a non-trivial phase $\theta$ results in $CP$-violation.

We assume the same number of fields $\psi_i, \xi_i$ and $\chi_i$. Then, with orthogonal trfs of $\psi, \xi, \chi$ in the flavor space one can bring the matrices $\tilde{K}$ and $K$ to be diagonal and symmetric respectively. The mass eigenstates are Dirac spinors, $u_i, d_i$, related to the weak eigenstates, in a matrix notation, by:

$$u = L\psi_u + R\xi\,, \qquad d = V^T(L\psi_d + e^{i\theta} R\,\chi)\,;$$

$\psi_{u,d}$ are the $T_3$=$\pm1/2$ components of $\psi$ and $V$ is the generalized (orthogonal) Cabibbo matrix. Under $C$ and $P$ (see eqs. (3-4)), they transform as

$$C: \quad u \to u \quad , \qquad d \to (L - R)d\,;$$

$$P: \quad u \to i\mathcal{P}\, u^c, \qquad d \to i(L - e^{2i\theta} R)\mathcal{P}\, d^c\,;$$

$$CP: \quad u \to i\mathcal{P}\, u^c, \qquad d \to i(L + e^{2i\theta} R)\mathcal{P}\, d^c\,.$$

Remarkably enough, none of these can be identified with the standard trfs of Dirac masses. The mismatch in $CP$ just signals the non-commutation of $CP$ with the matrix that diagonalizes the masses. The standard $CP$ is of practical interest to study the $CP$-violating phases because it conserves the mass terms rather than the short range interactions. The discrepancy in $C$ and $P$ is more significant - it results from the fact that the fermions are in real (Majorana) reps. Hence, $P$ can only differ from $CP$ for a phase factor.

Let us find out if $CP$-violation really occurs in this model. Of course, it cannot be like in the Kobayashi-Maskawa (KM) model[9] because the Lagrangian is $CP$-invariant. But it could be as in the Weinberg model,[10] i.e. via exchange of Higgs particles and with NFC at tree level, provided that the vacuum breaks $CP$. Actually, the potential given in the eq. (18) was already investigated because the same symmetry as $\phi_\pm \rightarrow \pm\phi_\pm$, was introduced[10] in order to assure NFC. Here, NFC results from parity (or $C$) conservation. It turns out[10,11] that such a potential does not lead to spontaneous $CP$-breaking and, at least three Higgs doublets are required for that.

We enlarge the Higgs sector by one bi-doublet, $\varphi \equiv (\varphi_1, \varphi_2)$, since that is the right rep of $P$ and $C$. In order to keep NFC in the quark sector, an additional symmetry is introduced,[10] namely the reflection $\varphi \rightarrow -\varphi$ with the quarks and $\phi$ fields unchanged. Depending on the trfs of the leptons under this reflection, they couple to either $\phi_\pm$ or $\varphi_\pm = (\varphi_1 \pm \varphi_2^*)/\sqrt{2}$. In any case Universality holds, so at least one of the $C$-eigenstates $\varphi_\pm$ say $\varphi_+$, does not couple to the charged lepton singlets. The potential is still given by the eq. (18) but now $\phi_a$ stand for $\phi_\pm$, $\varphi_\pm$. The relevant terms for the phase content of the vacuum are the ones proportional to $r_{ab}$. In the case of three doublets, Branco[11] showed that $CP$-breaking occurs if all constants $r_{ab}$ are positive. Here, the same result holds in the limit that $\varphi_+$ does not couple with the other scalars. In conclusion, spontaneous $CP$-violation can be implemented within the present theory. Parity and $C$ are spontaneously broken since both $\phi_+$ and $\phi_-$ have non-trivial vacuum expectation values.

It is clear that the free parameters of the theory are to be fixed from the relevant experimental data. The models of $CP$-violation[12] use to be classified according to their predictions, if any, of the values of $\varepsilon'/\varepsilon$ in the kaon system, and the neutron electric dipole moment, $d_n$. Previous calculations[12] based on the Weinberg model with three doublets indicate as possible values, $\varepsilon'/\varepsilon \sim 10^{-3}$ and $d_n \sim 10^{-25}\, e\,cm$, i.e. of the order of their contemporary experimental limits.[12] At present the situation is not clear. The measurement of $\varepsilon'/\varepsilon$ from NA31,[13] $(3.3 \pm 1.1)\mathrm{x}10^{-3}$, has not been confirmed by other experiments. On the other hand, recent results on $d_n$, $(-1.4 \pm 0.6)\mathrm{x}10^{-25} e\,cm$[14] and $(-5 \pm 5)\mathrm{x}10^{-26} e\,cm$,[15] indicate the possibility of a non-zero $d_n$ of the order of $10^{-25} e\,cm$. Such a value would be too large to be accomodated within the KM model.[12] Hopefully, the $CP$-question will be settled soon.

We emphasize that not only the SM gauge interactions are invariant under $C$, $P$ and $T$, but also the right representation of the symmetry group requires the Higgs mechanism to have at least two Higgs doublets and consequently, charged Higgs particles. Hence, there is no reason to postulate a Lagrangian not invariant under $CP$. $CP$-violation should result from spontaneous symmetry breaking.

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