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COHERENT PAIR CREATION FROM BEAM-BEAM INTERACTION*

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Abstract It has recently been recognized that in future linear colliders, there is a finite probability that the beamstrahlung photons will turn into e^+e^- pairs induced by the same beam-beam field, and this would potentially cause background problems. In this paper, we first review the probability of such a coherent pair creation process. It is seen that the constraint on the beamstrahlung parameter, Υ, is tight if these coherent pairs are to be totally suppressed. We then point out that there exists a minimum energy for the pair-created particles, which scales as $\sim 1/5\Upsilon$. When combining this condition with the deflection angle for the low-energy particles, the contraint on the allowable Υ value is much relaxed. Finally, we calculate the effective cross section for producing the weak bosons by the low-energy e^+e^- pairs. It is shown that these cross sections are substantial for $\Upsilon > 1$. We suggest that this effect can help to autoscan the particle spectrum in the high energy frontier.

INTRODUCTION

Recently, it was recognized[1] that the generally high-energy beamstrahlung photons, which have to travel through the same collective field in the remainder of the oncoming beam, have a finite probability of turning into e^+e^- pairs. Being lower in energies, these e^+e^- pairs will be deflected more severely than the high-energy primary particles, and will potentially cause background problems for high energy physics experiments. This phenomenon, which we shall call coherent pair creation, has since been a subject of intense study.[2,3]

In view of the possibile impact to high energy experimentations, it is essential to look into the problem in more detail. In this paper, we first review the coherent pair creation process, and then discuss the probability of creating such pairs during beam-beam interaction. We show that to entirely suppress these events requires that

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the beamstrahlung parameter, Υ, be less than $\sim 1/4$ for typical beam parameters discussed in next-generation linear colliders. We then calculate the energy spectrum of the pair-created particles. It is shown that a minimum energy exists, below which the probability is exponentially small. When combining this condition with the deflection angle for the low-energy particles due to the same beam-beam field, we show that the constraint on Υ is much less stringent.

With a larger allowable value of Υ in mind, we calculate the effective cross sections of weak boson production by these lower-energy coherent pair particles. It is shown that they are nonnegligible compared to that of the true high energy events in the TeV range. We therefore suggest that this effect can be exploited as a means in helping to autoscan the unknown particle spectrum in the high energy frontier.

COHERENT PAIR CREATION

The probability of pair creation per unit time by a photon with energy ω in a transverse magnetic field B can be obtained analytically in the asymptotic limits[4]:

$$\frac{dn}{dt} = \begin{cases} 0.23\,\frac{\alpha m^2}{\omega}\,\chi\exp(-8/3\chi) \quad , & \chi \ll 1 \quad , \\ 0.38\,\frac{\alpha m^2}{\omega}\,\chi^{2/3} \quad , & \chi \gg 1 \quad , \end{cases} \tag{1}$$

where $\chi \equiv (\omega/m)B/B_c$, and $B_c \equiv m^2/e \sim 4.4 \times 10^{13}$ Gauss is the Schwinger critical field. Here we adopt the convention of natural units: $\hbar = c = 1$.

For the entire range of χ, dn/dt can be well approximated by the following expression[5]:

$$\frac{dn}{dt} \simeq \frac{4}{25}\,\frac{\alpha m^2}{\omega}\,K^2_{1/3}(4/3\chi) \quad . \tag{2}$$

We see that $\chi \sim 1$ corresponds to the threshold condition for finite probability, below which the pair-creation rate is exponentially suppressed. This condition can be appreciated by the following intuitive arguments: Consider the boosted frame where the e^+e^- pair is created at rest. In this frame, there is an electric field which is $E' = (\omega/2m)B$. At the threshold, the created particle with unit charge e should acquire enough energy within one Compton wavelength to provide for its rest mass. Thus, the threshold condition is $eE'\lambda_c \sim m$, or $\chi \sim 1$.

Consider the beam particles as distributing uniformly within an elliptical cylinder with dimensions $2\sigma_x$, $2\sigma_y$, and $2\sqrt{3}\sigma_z$. In association with a constant effective field, $\overline{B}$, that represents the entire beam, the beamstrahlung parameter for a particle with energy $\mathcal{E}$ is defined as[6]

$$\Upsilon \equiv \gamma \frac{\overline{B}}{B_c} = \frac{5}{6} \frac{\gamma r_e^2 N}{\alpha \sigma_z \sigma_y (1+R)} \quad , \tag{3}$$

where $\gamma = \mathcal{E}/m$, $R \equiv \sigma_x/\sigma_y$ is the beam aspect ratio, and N is the total number of particles in the bunch. The coefficient 5/6 is empirical.

It is useful to express the pair creation probability in terms of the primary particles, instead of the intermediate photons. For pair creation through the real beamstrahlung photons, the number of pairs per primary particle after collision, calculated recently by Chen and Telnov,[2] is

$$n = \frac{4\sqrt{3}}{25\pi} \left(\frac{\alpha \sigma_z}{\gamma \lambda_c} \Upsilon \right)^2 \Xi(\Upsilon) \quad , \tag{4}$$

where

$$\begin{aligned} \Xi(\Upsilon) &\equiv \frac{1}{2\Upsilon^2} \int_0^1 \left[\int_q^\infty K_{5/3}(p)\, dp + \frac{y^2}{1-y} K_{2/3}(q) \right] K_{1/3}^2(4/3y\Upsilon)\, \frac{dy}{y} \\ &\simeq \begin{cases} 0.5 \exp(-16/3\Upsilon) \quad , & \Upsilon \ll 1 \quad ; \\ 2.6\, \Upsilon^{-2/3} \ln \Upsilon \quad , & \Upsilon \gg 1 \quad , \end{cases} \end{aligned} \tag{5}$$

where $y \equiv \omega/\mathcal{E}$. Here, the synchrotron radiation spectrum a la Sokolov–Ternov is used with the beamstrahlung parameter defined in Eq. (3) and $q \equiv (2/3\Upsilon)y/(1-y)$. This treatment ignores the fact that the beamstrahlung photons are preferentially polarized. As a result, our expression over-estimates the probability by roughly a factor of two.[7] A numerical plot of the auxiliary function Ξ is given in Fig. 1. Coherent pair creation can also occur through virtual photons. This channel is relatively unimportant for $\Upsilon < \mathcal{O}(10^4)$,[2] so we shall not discuss it here.

It turns out that in e^+e^- linear colliders, the quantity $(\alpha\sigma_z/\gamma\lambda_c)\Upsilon$ cannot be arbitrary. For $0.1 \lesssim \Upsilon \lesssim 100$, it can be shown that the average energy loss of the entire beam is $\delta \simeq (3/10\sqrt{\pi})(\alpha\sigma_z/\alpha\lambda_c)\Upsilon$. In designing linear colliders, one usually chooses a reasonable value of δ as a constraint to the choices of other beam parameters, such that the energy resolution of the colliding beam is adequate for meaningful high energy experiments. For $\delta \sim 0.2$, then the quantity $(\alpha\sigma_z/\gamma\lambda_c)\Upsilon$ is of order unity.

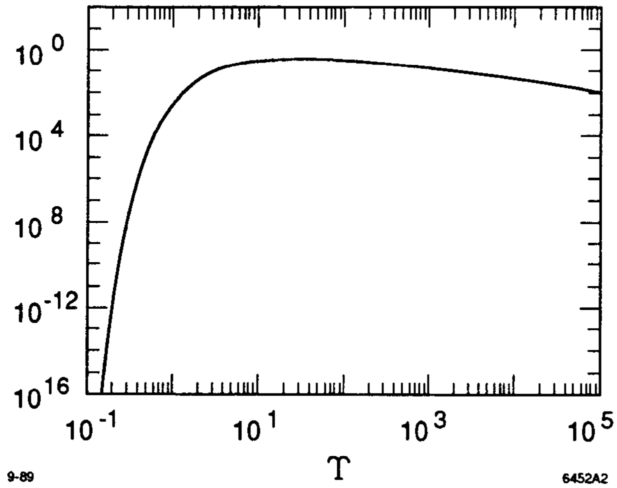

FIGURE 1 Auxiliary function Ξ of coherent pair creation probability as a function of Υ.

It is possible to suppress the coherent e^+e^- pairs entirely if the following condition is satisfied:

$$nN \lesssim 1 \quad . \tag{6}$$

The typical value for N is $\mathcal{O}(10^{10})$. Assuming that $\delta \sim 0.2$, then from Fig. 1 we find it necessary that $\Upsilon \lesssim 1/4$. This constraint seems attainable for next-generation colliders with multibunch operations; but it is generally desirable to relax design constraints wherever possible. In the section on deflection angles, we will see that the above contraint is indeed overly stringent.

THE ENERGY SPECTRUM

For our purpose, it is important to study the energy spectrum of the pair-created secondary particles. It can be shown that in the asymptotic limits,

$$\frac{dn(y)}{dx} = \frac{1}{8\pi}\frac{\alpha\sigma_z}{\gamma\lambda_c}\begin{cases} \frac{\sqrt{\pi}}{2}(3\Upsilon)^{1/2}\,\frac{3+(1-2x/y)^2}{[xy(y-x)]^{1/2}}\,\exp\left\{-\frac{2}{3\Upsilon}\,\frac{y}{x(y-x)}\right\}, & y\Upsilon \ll 1, \\ \Gamma\left(\frac{2}{3}\right)(3\Upsilon)^{2/3}\,\frac{1+(1-2x/y)^2}{[xy^2(y-x)]^{1/3}}\ , & y\Upsilon \gg 1, \end{cases} \tag{7}$$

where $x \equiv \varepsilon/\mathcal{E}$, and the effective traversing time for the photon is $t = \sqrt{3}\sigma_z/2$, half of that for the first generation particles. For $\Upsilon \ll 1$, the pair tends to equally share the photon energy, while for $\Upsilon \gg 1$, the spectrum becomes much broader. A numerical plot of the energy spectrum using the exact form[8] is shown in Fig. 2, where the pair-created particle energy is normalized as $x\Upsilon$. For a given value of Υ, the minimum energy x_{min} is independent of the intermediate photon energy y. In addition, for different values of Υ's, we find that $x_{min} \sim 1/5\Upsilon$.

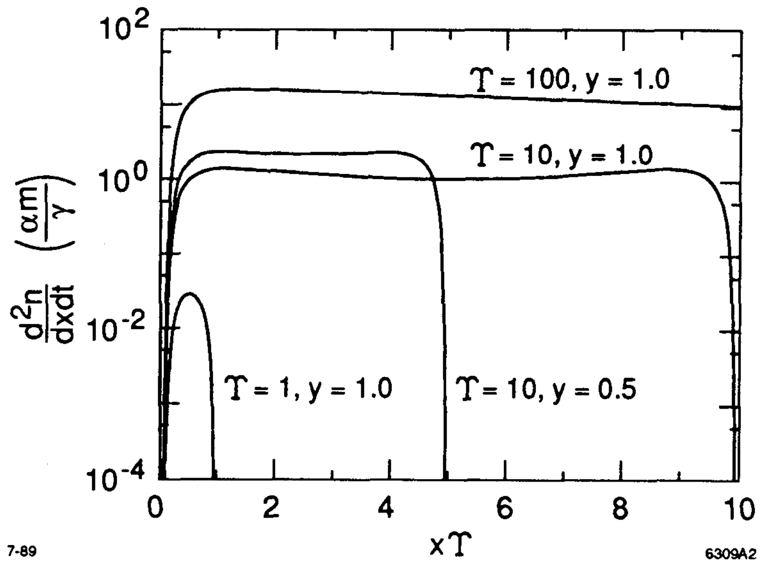

FIGURE 2 Normalized spectrum for coherent pair creation, in units of $\alpha m/\gamma$, as a function of $x\Upsilon$. One sees that $x_{min} \simeq 1/5\Upsilon$.

The mean energy spectrum from the entire beam can be calculated, again, by folding in the Sokolov–Ternov spectrum for beamstrahlung. To obtain an analytic expression for later use in this paper, we find it more convenient to approximate the beamstrahlung spectrum as

$$\psi(y) \simeq \frac{5}{2\Gamma(1/3)}\left(\frac{\alpha\sigma_z}{\gamma\lambda_c}\right)\left(\frac{3\Upsilon}{2}\right)^{2/3} y^{-2/3} \quad , \tag{8}$$

for $y \ll 1$ if $\Upsilon \ll 1$, and $y \lesssim 1$ if $\Upsilon \gtrsim 1$. Folding in the above two spectra,

$$\frac{d\overline{n}}{dx} = \int_x^1 \frac{dn}{dx}\psi(y)dy \quad , \tag{9}$$

we obtain, for $x \ll 1$,

$$\frac{d\overline{n}}{dx} = \left(\frac{\alpha\sigma_z}{\gamma\lambda_c}\right)^2 \begin{cases} 0.65\Upsilon^{7/3}\exp(-2/3\Upsilon x) \quad , & \Upsilon \ll 1 \quad , \\ 0.28\Upsilon^{4/3}x^{-1} \quad , & \Upsilon \gg 1 \quad . \end{cases} \tag{10}$$

DEFLECTION ANGLES

If the e^+e^- pair is created in a field-free space, their outgoing angles will be simply the final angles from the creation process. In the case of beam-beam interaction, however, these low-energy secondary particles will be strongly deflected by the same macroscopic collective field. The nature of the deflection differs between the pair. Consider, for example, a pair created by a primary electron moving against the positron beam. In this case, the secondary electron sees a focusing field, while the positron sees a defocusing field. Since both particles are generally low in energy, the electron tends to be confined by the potential and oscillate on its way out, whereas the positron would be deflected without bound. In general, for flat beams (i.e., $R \gg 1$), the most effective deflection occurs for positrons in the vertical direction. This is because for flat beams, the vertical field extends beyond the beam height $2\sigma_y$ to a distant $\sim 2\sigma_x \gg 2\sigma_y$, with a fairly uniform strength $E + B \sim 2eN/\sqrt{3}\sigma_x\sigma_z$, whereas in the horizontal direction, the field strength inceases only linearly to the same value at $2\sigma_x$.

Define the diagonal angle of the field to be $\theta_d \equiv 2\sigma_x/\sqrt{3}\sigma_z$; then the vertical deflection angle can be shown to be approximately:

$$\theta_y = \begin{cases} \frac{\sqrt{3}}{2}\frac{D_x}{x}\theta_d \ , & \theta_y \leq 2\theta_d \ ; \\ \left[\sqrt{3}\frac{D_x}{x}\right]^{1/2}\theta_d \ , & \theta_y \geq 2\theta_d \ , \end{cases} \tag{11}$$

where $D_x \equiv 2Nr_e\sigma_z/\gamma\sigma_x^2$ is the horizontal disruption parameter.

Assume that the beams are colliding at an angle θ_c. Then the maximum allowable value of Υ, which corresponds to x_{min} of the energy spectrum, is

$$\Upsilon_{max} = \frac{1}{5\sqrt{3}}\frac{1}{D_x}\left(\frac{\theta_c}{\theta_d}\right)^2 \quad . \tag{12}$$

This limit is generally much less stringent than the constraint for the total suppression of coherent pairs. As long as the Υ value of a machine is smaller than the above Υ_{max}, all secondary particles will be able to pass through the exhaust port without creating background problems. Since typically $\theta_d \sim \mathcal{O}(10^{-3})$ and $D_x \ll 1$, the needed θ_c is quite modest even when $\Upsilon > 1$.

WEAK BOSON BACKGROUNDS

As explained in the previous section, the particle in the e^+e^- pair that has the opposite sign of charge to that of the oncoming beam will be trapped due to its lower energy. These trapped secondary particles will contribute to the backgrounds through direct high energy processes. For example, they will produce weak interaction vector bosons Z^0 and W^+W^- pairs.

A. $e^+e^- \longrightarrow Z^0 \longrightarrow f\bar{f}$

First, let us calculate the effective cross section for Z^0 production. The Z^0 cross section can be represented by the relativistic Breit–Wigner resonance formula[9]:

$$\sigma_z(s) = \frac{12\pi}{m_z^2}\frac{s\Gamma_e\Gamma_f}{(s-m_z^2)^2+s^2\Gamma^2/m_z^2} \quad , \tag{13}$$

where s is the center-of-mass energy squared, m_z the mass of Z^0, Γ the total decay width of Z^0, and Γ_e and Γ_f are the Z^0 partial widths into electron pairs and into decays in the fiducial volume, respectively. From the recent SLC–Mark II results,[9] $m_z = 91.11$ GeV, $\Gamma = 2.47$ GeV, $\Gamma_e = 0.083$ GeV, and $\Gamma_f = 1.82$ GeV.

The effective cross section from the secondary, pair-created e^+e^-'s can be calculated by folding in the pair creation energy spectrum in Eq. (10),

$$\overline{\sigma}_z = \int\int dx_1 dx_2\, \sigma_z(x_1,x_2)\frac{d\overline{n}}{dx_1}\frac{d\overline{n}}{dx_2} \quad , \tag{14}$$

where $s = 4\mathcal{E}^2 x_1 x_2$ in our case. The result is

$$\overline{\sigma}_z = \begin{cases} 19.2\left(\dfrac{\alpha\sigma_z}{\gamma\lambda_c}\right)^4 \Upsilon^{31/6}\, \exp(-8\mathcal{E}/3\Upsilon m_z)\left(\dfrac{m_z}{\mathcal{E}}\right)^{5/2}\left(\dfrac{\Gamma_e\Gamma_f}{\Gamma m_z}\right)\dfrac{1}{m_z^2} \quad , & \Upsilon \ll 1 \; ; \\ 9.3\left(\dfrac{\alpha\sigma_z}{\gamma\lambda_c}\right)^4 \Upsilon^{8/3}\, \ln 5\Upsilon\left(1+\dfrac{\Gamma^2}{m_z^2}\right)\left(\dfrac{\Gamma_e\Gamma_f}{\Gamma m_z}\right)\dfrac{1}{m_z^2} \quad , & \Upsilon \gg 1 \; . \end{cases} \tag{15}$$

Notice that the effective cross section is independent of the beam energy $\mathcal{E}$ in the large Υ regime.

For $\Upsilon = 10$, and taking $(\alpha\sigma_z/\gamma\lambda_c)\Upsilon = 1$ (i.e., $\delta \sim 17\%$), we find that $\overline{\sigma}_z \gtrsim 50$ pb. But if the energy loss is twice as large, or $(\alpha\sigma_z/\gamma\lambda_c)\Upsilon^{2/3} = 1$, then $\overline{\sigma}_z \gtrsim 1$ nb. This is to be compared with its cross section on resonance, $\sigma_z(\sqrt{s} = 91\text{ GeV}) \gtrsim 50$ nb. Since the accelerator luminosity generally increases as s, the yield of this background from a TeV collider could in principle be as high as that from a Z-factory.

B. $e^+e^- \longrightarrow W^+W^-$

Next, we investigate the effective cross section for $e^+e^- \longrightarrow W^+W^-$ pairs. The cross section from the Standard Model is

$$\sigma_W(s) = \frac{\pi\alpha^2}{2\sin^4\theta_W}\,\frac{1}{s}\,\ln\frac{s}{m_W^2} \quad , \qquad s \geq m_W^2 \quad , \tag{16}$$

where $\sin^2\theta_W = 0.23$, and the W-boson mass is $m_W \simeq 82$ GeV. We find that

$$\overline{\sigma}_W = 0.03\frac{\alpha^2}{\sin^4\theta_W}\left(\frac{\alpha\sigma_z}{\gamma\lambda_c}\right)^4 \Upsilon^{8/3}\,\ln 5\Upsilon\left[\frac{4\mathcal{E}^2}{m_W^2} - 1 - \ln\frac{4\mathcal{E}^2}{m_W^2}\right]\frac{1}{\mathcal{E}^2} \quad , \qquad \Upsilon \gg 1\,. \tag{17}$$

Here, we also see that to the leading order, the effective cross section is independent of the beam energy. Again, for $\Upsilon = 10$, we find $\overline{\sigma}_W \simeq 1.3$ pb for $(\alpha\sigma_z/\gamma\lambda_c)\Upsilon = 1$, and $\overline{\sigma}_W \simeq 28$ pb for $(\alpha\sigma_z/\gamma\lambda_c)\Upsilon^{2/3} = 1$. This is to be compared with the cross section $\sigma_W(\sqrt{s} = 200 \text{ GeV}) \simeq 20$ pb.

It should be noted that while our expressions for the effective cross section are in terms of the primary particles, the effective luminosity for these backgrounds is larger than the nominal one. This is because the lower-energy trapped particles tend to have a smaller effective beam area than that of the primary particles.

AN AUTOSCANNING MACHINE

In the uncharted ocean of the high energy frontier, it would be desirable to have a collider that is easily tunable in a sufficiently large range of the center-of-mass energy. Our calculation shows that the coherent e^+e^- pairs seem to provide such an energy autoscanning mechanism. In particular, the calculation on the Z^0 background should be directly applicable to any other new vector boson. On the other hand, the calculation on the W^+W^- pair production can be easily modified to describe the production of $t\bar{t}$ (the top and anti-top quark pair), where one does not expect to find bound states due to the very short lifetime.

This suggestion of exploiting the coherent pair creation effect through beamstrahlung to help scanning the particle spectrum reminds us of an earlier suggestion by Blankenbecler and Drell,[10] where beamstrahlung is to be optimized in order to study the photon-photon physics.

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