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BIT-STRING SCATTERING THEORY *

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ABSTRACT

We construct discrete space-time coordinates separated by the Lorentz-invariant intervals h/mc in space and h/mc^2 in time using ***discrimination*** (XOR) between pairs of independently generated bit-strings; we prove that if this space is homogeneous and isotropic, it can have only 1, 2 or 3 *spacial* dimensions once we have related time to a global ordering operator. On this space we construct exact combinatorial expressions for free particle ***wave functions*** taking proper account of the interference between indistinguishable alternative paths created by the construction. Because the end-points of the paths are fixed, they specify ***completed*** processes; our wave functions are "born collapsed". A convenient way to represent this model is in terms of complex amplitudes whose squares give the probability for a particular set of observable processes to be completed. For distances much greater than h/mc and times much greater than h/mc^2 our wave functions can be approximated by solutions of the free particle Dirac and Klein-Gordon equations. Using an eight-counter paradigm we relate this construction to scattering experiments involving four distinguishable particles, and indicate how this can be used to calculate electromagnetic and weak scattering processes. We derive a non-perturbative formula relating relativistic bound and resonant state energies to mass ratios and coupling constants, equivalent to our earlier derivation of the Bohr relativistic formula for hydrogen. Using the Fermi-Yang model of the pion as a relativistic bound state containing a nucleon-antinucleon pair, we find that $(G^2_{\pi N})^2 = (2m_N/m_\pi)^2 - 1$.

1. INTRODUCTION

At ANPA 9, 10 and again at ANPA 11 I attempted to explain how to go from discrimination between bit-strings to a relativistic quantum scattering theory. On each occasion objections were raised by the audience which could not be successfully met while I was on my feet. Part of the problem at ANPA 11 was that I made an incorrect connection between McGoveran's Theorem (FDP Section 3.4, p p 30-34)[1]:

Theorem 13. The upper bound on the global d-dimensionality of a d-space of cardinality N with a discrete., finite and homogeneous metric is 3 for sufficiently large N.

and the definition of "event" in my scattering theory. This theorem establishes our right to claim that ***we*** have ***explained*** the 3+1 structure of "space-time" in our finite and discrete context. The theorem was not shaken. However, I admit that my attempt at ANPA 11 to tie it directly to the scattering theory was flawed. At ANPA 11 I distributed a few sheets with the same title as this paper.and a heading "SLAC-PUB-formulae". These are now only of historical interest; the flaw occurs at Eq. 1.8, p.3 in that manuscript. A correct derivation based on the construction provided in this paper is given at the end of Sec. 2.4 below.

My intent at ANPA 11 was to present "a systematic discussion of the kinematics and dynamics of the bit-string scattering theory which has been developing within the framework of discrete physics." Subsequently I have come to realize my attempt failed for a deeper reason than the technical flaw noted in the first paragraph. The problem was that I was trying to use a discretized random walk in space-time as the basic paradigm without paying sufficient attention to the broader context within which the "random walk" is constructed. The random walk model restricted to causal space-time trajectories captures much of the essence of relativistic quantum mechanics, as Stein taught us long ago[2-4], but does not lead to specific characteristics of the Schroedinger equation without an additional quantum postulate[5]. Karmanov[6] has suggested that a "Stein-like" random walk in

Feynman's sum over paths using a ***fixed*** imaginary step length of $i\,h/mc$ in space and $i\,h/mc^2$ in time can still lead to the Dirac equation in 1+1 dimensions for $x >> h/mc$ ***and*** $t >> h/mc^2$. Work by Karmanov, McGoveran, HPN and Stein[7] has not only provided a rigorous proof of this contention but led-to what I believe to be a resolution of the problems raised by my earlier attempts to construct a bit-string scattering theory.

It is characteristic of the ***ordering operator calculus*** that when there are paths with interfering alternatives due to the sharing of indistinguishable possibilities that we obtain ***at least*** two alternative types of path; further, we do not have sufficient information to choose between them without extending the context. However, in our construction we find that the sum of the squares of the number of alternative paths in each type provides the correct normalization condition for our calculated probabilities. This is a specific example of the "adding in quadrature" which David McGoveran discusses in his contribution to this conference"'. It is then a matter of convenience to introduce the complex amplitudes of quantum mechanics; no mysticism is involved. That is, we can go from our construction of rational, real probabilities for real "interfering alternatives", to the complex interfering alternatives which Feynman finds characteristic of quantum mechanics[9].

Once we have seen how these alternative paths are generated, anchored to the space-time trajectories and restricted by them, but not confined to the lattice of space-time points in the way a "classical" random walk would require, my earlier work on scattering theory falls into place. I hope that with this paper available prior to ANPA 12, I will for once be able to confront at least a fraction of the (for me) more exciting applications which are now hull up and moving rapidly toward us.

2. QUANTUM WAVE FUNCTIONS AS A CONTINUUM APPROXIMATION

2.1. BIT-STRINGS

We specify a ***bit-string***

$$\boldsymbol{X(S)} = (\ldots, b_i^x, \ldots\ldots)_S \tag{2.1}$$

by its S ordered elements

$$b_i^x \in 0,1; \quad \boldsymbol{i} \in 1,2,\ldots\ldots \boldsymbol{S}; \quad \boldsymbol{0}, 1, \ldots\ldots \boldsymbol{S} \in \textbf{\textit{ordinal integers}} \tag{2.2}$$

and its norm by

$$|\mathbf{X}(S)| = \Sigma_{i=1}^{S} b_i^x = \boldsymbol{X} \tag{2.3}$$

Define the ***null string*** by O(S), $b_i^0 = \boldsymbol{0}$ for all i and the ***anti-null*** string by $\mathbf{1}(S)$, $b_i^1 = 1$ for all ***i.*** Define ***discrimination*** (XOR) by

$$\mathbf{X} \oplus \boldsymbol{Y} = (\ldots, b_i^{xy}, \ldots)_S = \boldsymbol{Y} \oplus \boldsymbol{X}; \; b_i^{xy} = (b_i^x - b_i^y)^2 \tag{2.4}$$

from which it follows that

$$\mathbf{A} \oplus \mathbf{A} = \mathbf{0}; \; \mathbf{A} \oplus \mathbf{0} = \mathbf{A} \tag{2.5}$$

We will also find it useful to define

$$\bar{\mathbf{A}} = \mathbf{A} \oplus \mathbf{1}; \; \textit{hence } \mathbf{A} \oplus \bar{\mathbf{A}} \oplus \mathbf{1} = \mathbf{0} \tag{2.6}$$

2.2. ONE DIMENSIONAL AMPLITUDES

Consider two independently generated strings A(S), B(S) restricted by $|\mathbf{A} \oplus \mathrm{B}| = n$ and $\boldsymbol{A} - \boldsymbol{B}$ = c. We call these the ***boundary conditions.*** We now construct two substrings a(n),b(n) by the following recursive algorithm starting from i, j = 0 and ending at i = S, j = ***n.***

$$\mathrm{i} := \mathrm{i} + 1$$

$$\textit{if } b_i^A = 1 \textit{ and } b_i^B = 0 \textit{ then } j := j + 1 \textit{ and } b_j^a := 1 \textit{ and } b_j^b := 0$$

$$\textit{if } b_i^A = 0 \textit{ and } b_i^B = 1 \textit{ then } j := j + 1 \textit{ and } b_j^a := 0 \textit{ and } b_j^b := 1$$

$$\textit{if } (b_i^A - b_i^B)^2 = 0 \textit{ then } j,\ b_j^a \textit{ and } b_j^b \textit{ do not change}$$

Once we have made this construction,

$$\mathrm{a(n)} \oplus \mathrm{b(n)} \oplus \mathbf{1}(n) = \mathbf{0}(n) \tag{2.7}$$

and we can interpret the string a as representing a "random walk" in which a "1" represents a step forward and a "0" represents a step backward, as in the Stein paradigm. Define

$$a_j = \Sigma_{k=1}^{j} b_k^a;\ b_j = \Sigma_{k=1}^{j} b_k^b \tag{2.8}$$

We call the "points" $(a_j - b_j$, j) connecting $(0,0)$ to ***(c, n)*** a ***trajectory;*** the new ordering parameter j then represents "causal" time order along the trajectory. Note that a + ***b*** = ***n*** and a − ***b*** = ***A*** − ***B*** = c for any trajectory because of our boundary conditions.

We can also define a ***path*** in the larger space s_i, A_i, B_i where

$$s_i = \Sigma_{k=1}^{i} s_k = \Sigma_{k=1}^{i} b_k^A b_k^B \tag{2.9}$$

$$A_i = \Sigma_{k=1}^{i} b_k^A (b_k^A - b_k^B)^2 + s_k;\ B_i = \Sigma_{k=1}^{i} b_k^B (b_k^A - b_k^B)^2 + s_k$$

Note that by construction $A_i - B_i = a_j - b_j$ and hence A_i, B_i is tied to the same trajectory in the $(a_j - b_j, j)$ plane; it acquires a third "orthogonal" coordinate due to those cases when both A_i and B_i are incremented by 1. Note also that there is no way from our boundary conditions or from the trajectory to tell those cases from those where i advances but neither A_i nor B_i nor s_i is incremented. All we know is that $s_{AB} = \Sigma_{k=1}^{S} b_k^A b_k^B$, lies in the range $0 \leq s_{AB} \leq S - n$. ***It*** is these *indistinguishable* paths which create the interfering alternatives in our model.

We now ask how many paths characterized by the unknown parameter s = $0, 1, 2, \ldots)$ S − n satisfy our boundary conditions. By construction each path is tied to the n points which compose a trajectory, and can be chosen in n^s ways. Note that we have broken the causal connection between path and trajectory. Of the total number of ways of choosing a path characterized by s from the $S!/(S -$ s)! possibilities, only $S!/s!(S-s)!$ are distinct. Consequently, the probability of having a path characterized by s is

$$\frac{S!/s!(S - s)!}{S!/(S - s)!} = \frac{1}{s!} \tag{2.10}$$

Thus the total number of paths is

$$P(n; S) = \Sigma_{s=0}^{S-n} \frac{n^s}{s!} = \Sigma_{s=0}^{S-n} p_s(n) \equiv exp_{S-n}(n) \tag{2.11}$$

where $exp_{S-n}(n)$ is the ***finite exponential.*** This is a general result for the ***transport operator*** referring to ***attribute distance as*** has been proved by McGoveran in FDP, Theorems 36-40, pp 55-58.

2.3. ADDING IN QUADRATURE

Although Eq. 2.11 specifies the total number of paths, given S, n, it conceals a four-fold ambiguity arising from the construction. However the sequence of paths is generated, the order adopted in the sum implies a recursive generation of the terms $p_s(n) = n^s/s!$ given by

$$p_{s+1}(n) = np_s(n)/(s + 1);\ p_0(n) = 1 \tag{2.12}$$

The first ambiguity is the fact that we do not know whether $S - \boldsymbol{n}$ is even or odd outside of the uninteresting case $S = \boldsymbol{n}$ when paths and trajectories coincide; hence we do not know whether the sum terminates in an even or an odd term, which affects the statistical calculation in an interesting way. The second ambiguity arises because, however s is ordered, we do not know how many cases arise because ***both A;*** and B_i are incremented, or ***neither.*** To include this dichotomy we split the even and odd sequences themselves into two sequences corresponding to these alternatives which we call 11 and 00, giving four recursion relations:

$$p^{e,11}_{s+4}(n) = \frac{n^4}{(s+4)(s+3)(s+2)(s+1)} p^{e,11}_s(n);\ p^{e,11}_0(n) = 1$$

$$p^{o,11}_{s+4}(n) = \frac{n^4}{(s+4)(s+3)(s+2)(s+1)} p^{o,11}_s(n);\ p^{o,11}_1(n) = n$$

$$p^{e,00}_{s+4}(n) = \frac{n^4}{(s+4)(s+3)(s+2)(s+1)} p^{e,00}_s(n);\ p^{e,00}_2(n) = n^2\frac{1}{2}$$

$$p^{o,00}_{s+4}(n) = \frac{n^4}{(s+4)(s+3)(s+2)(s+1)} p^{o,00}_s(n);\ p^{o,00}_3(n) = n^3\frac{1}{6} \tag{2.13}$$

At some point which depends on whether (a) $S -$ n is even or odd and/or $2s_{AB}$ is greater or less than $S - \boldsymbol{n}$, this four-fold ordering of the terms in the sum over s has to stop, and may or may not leave some terms unaccounted for. Calling the

contribution of these terms to the sum AP, we find that our construction allows us to decompose the sum over paths as follows:

$$P(n;\, s) = \Sigma_{s=0}^{S-n}[p_s^{e,11} + p_s^{o,11} + p_s^{e,00} + p_s^{o,00}] + \Delta P \tag{2.14}$$

We are now in the general situation discussed by McGoveran in this conference[8] where we know that for any specific generation of the paths which meets our boundary we could compute the decomposition, but because of interfering indistinguishable alternatives the actual probability calculation eludes traditional approaches. Consider first a situation with two alternatives, with P_1 and P_2 paths characterized by each separately. Since the total number of paths is P, the elementary treatment takes $P_1 + P_2 = P$, but this cannot represent the situation when they are ***independently*** generated and hence define a joint probability space with $P_1\, P_2$ elements. In order to satisfy both constraints, we form $P_1^2 + P_2^2 = P^2 - 2P_1P_2 \equiv R_{12}^2$, which is identically satisfied if the two are not independent. If, due to indistinguishable paths which we do not know how- to assign to either P_1 or $P - 2$, we have indeed made the two independent in the sense that the product P_1P_2 is no longer constrained other than by the inequality $2P_1P_2 < P^2$, we can adopt R_12^2 as the measure of the square of the number of paths in this new space. Taking the product $2P_1P_2 = f^2P^2$ where f is some rational fraction less than unity, we thus arrive at the general result

$$P_1^2 + P_2^2 = R_{12}^2 = P^2(1 - f)(1 + f) \tag{2.15}$$

which has been derived by McGoveran[8] by considering case counts including indistinguishables.

Returning to the case at hand, and considering even and odd paths, this restricts P_e and P_o to the circle defined by

$$P_e^2 + P_o^2 = R^2 \tag{2.16}$$

independent of how $\boldsymbol{P}$ is partitioned between P_e and P_o. We can now ***define***

$$\psi_{S-n} = P_e + iP_o \tag{2.17}$$

with the normalization condition

$$\psi^*\psi = R^2 \tag{2.18}$$

Clearly we can divide ψ by $\boldsymbol{R}$ to get the normalization condition $\psi^*\psi = 1$ when we are modeling the case that a single system traverses the trajectory with certainty.

Our next step is to exploit the remaining ambiguity arising from the bisection of the interval between 0 and $S - n$ due to the indistinguishable parameter s_{AB}. We have already taken the basic step in Eq. 2.14 by splitting the sum into four rather than two independent collections of paths. Now that we have recognized that the amplitudes — whose square gives a quantity which can be normed to form a probability —, can be complex, we have no conceptual barrier to forming real combinations which can be negative as well as positive. The obvious choice is to form those which lead to the finite sines and cosines, i.e. by subtracting the two components of the odd or even series from each other:

$$\boldsymbol{R}\ cos_{S-n}(n) = \boldsymbol{R}\ \Sigma_{k=0}^{\frac{1}{2}(S-n)}(-1)^k \frac{n^{2k}}{(2k)!} = \Sigma_{s=0}^{(S-n)}[p_s^{e,11} - p_s^{e,00}] \tag{2.19}$$

$$R\ sin_{S-n}(n) = R\ \Sigma_{k=1}^{\frac{1}{2}(S-n)}(-1)^{k+1} \frac{n^{2k-1}}{(2k-1)!} = \Sigma_{s=1}^{(S-n)}[p_s^{o,11} - p_s^{o,00}] \tag{2.20}$$

The two constructions can now be combined by taking the normalized wave function to be

$$\psi_{S-n}(n) = exp_{S-n}(in) = \Sigma_{s=0}^{\frac{1}{4}(S-n)} \frac{(in)^s}{s!} \tag{2.21}$$

Thus, by taking proper account of the interference between ***independently generated*** paths which share ***indistinguishable*** elements, we claim to have ***derived*** Feynman's prescription for quantum mechanics as a "sum over paths" with imaginary

steps. In other words, we have shown that by using the ***ordering operator*** *calculus* to count the paths we can construct a completely finite version of "wave functions" with finite step lengths and real probabilities. The i has been introduced simply for mathematical convenience and carries no deeper significance. The ***significant*** aspect of the system that eventually leads to observable quantum interference is the fact that our construction of space-time from bit-strings includes interfering alternatives due to paths which share ***indistinguishable*** elements.

2.4. CONSTRUCTION OF SPACE-TIME COORDINATES

1+ 1 dimensions

In any universe of bit strings of length S, all quadruples such that

$$\mathbf{A} \oplus \mathbf{B} \oplus \mathbf{C} \oplus \mathbf{D} = \mathbf{0} \tag{2.22}$$

are called ***events.*** Note that this implies that

$$\mathbf{A} \oplus \mathbf{B} = \mathbf{C} \oplus \mathbf{D};\ \mathbf{A} \oplus \mathbf{C} = \mathbf{B} \oplus \mathbf{D};\ \mathbf{A} \oplus \mathbf{D} = \mathbf{B} \oplus \mathbf{C} \tag{2.23}$$

$$\mathbf{A} = \mathbf{B} \oplus \mathbf{C} \oplus \mathbf{D};\ \mathbf{B} = \mathbf{C} \oplus \mathbf{D} \oplus \mathbf{A};\ \mathbf{C} = \mathbf{D} \oplus \mathbf{A} \oplus \mathbf{B};\ \mathbf{D} = \mathbf{A} \oplus \mathbf{B} \oplus \mathbf{C} \tag{2.24}$$

Consider an event defined by four independently generated strings **F, B, R, L** whose norms are F, ***B, R, L;*** all must be less than or equal to $|\mathbf{1}| = \mathbf{S}$. For the moment we need only define a fifth integer n by

$$|\mathbf{F} \oplus \mathbf{B}| = \mathrm{n} = |\mathbf{R} \oplus \mathbf{L}| \tag{2.25}$$

We will return below to the additional flexibility provided by the remaining two equalities in Eq. 2.17. Our intent is to construct a discrete square coordinate mesh (z_i, t_j) with $(2n+1)^2$ points within which we can model piecewise continuous

ordered trajectories (z_k, t_k) which connect the "endpoint" $(0,0)$ to some "endpoint" (z, t) lying on the boundary of the square

$$t = \pm n, \quad -n \leq z \leq n; \quad z = \pm n, \quad -n \leq t \leq n \tag{2.26}$$

The order parameter $0 \leq \boldsymbol{k} \leq$ n traverses any space-time point along the trajectory only once; in addition we require that

$$z_{k+1} - z_k = \pm 1;\ t_{k+1} - t_k = \pm 1;\ (four\ \textbf{choices}) \tag{2.27}$$

The description is ***static*** in the sense that it can be read either from 0 to ***n*** or from ***n*** to 0 and still describe the same trajectory. Note that in contrast to previous discussions, (a) we consider space-like as well as time-like trajectories, and (b) that the length of the strings $S \geq$ n is not specified; it is some finite integer named in advance of the construction. Note further that since we specify both endpoints, we are describing a ***completed*** process. The "wave functions" we will eventually construct on this mesh will be "born collapsed". All our results will belong to the "fixed past"; whether we should or should not use our theory to ***predict*** the future, either in a deterministic or a statistically deterministic sense, is a separate issue we will not discuss in this paper. We have picked our boundary conditions $(0,0)$ — — — (z, *t)* in the process of specifying the problem.

Any space-time point (z_k, t_k) not on the axes $(z_k, 0)$, $(0, t_k)$ lies in one of the four quadrants (+, +), (-, +), (+, −), (-, −). We fix the sign convention in terms of our parameters ***R, L,*** F, ***B*** by the rule

$$(+,+) \leftrightarrow R > L,\ F > B;\ (-,+) \leftrightarrow R < L,\ F > B$$

$$(+,-) \leftrightarrow R > L,\ F < B;\ (-,-) \leftrightarrow R < L,\ F < B \tag{2.28}$$

The interiors of the light cones are specified in the usual way: Time-like $(t^2 > z^2)$: Forward $t > |z|$, Backward $t < -|z|$; Space-like $(z^2 > t^2)$: Right z > $|t|$, Left z <

-It |. We can now define our bounding endpoints in terms of our basic parameters, and four new parameters **r,** $l, f,$ **b** by

$$Time-like, Forward \leftrightarrow z = R - L = r - l; t = n = r + l$$

$$\textbf{Time-Zike, Backwurdt, z = R - L = r - Z; t = - n}$$

$$\textbf{Space — like, Right} \leftrightarrow z = n = f + b; t = F - B = f - b$$

$$\textbf{Space — like, Left} \leftrightarrow z = -n; t = F - B = f - b; \qquad \textbf{(2.29)}$$

The advantage of introducing the new parameters **r,** $l, f,$ **b** is that they make it easy to define what will become Lorentz invariants. Explicitly

$$t^2 - z^2 = \tau^2 = \textbf{4rZ} = n^2(1-\beta^2) \textbf{ with } \beta = \frac{2r}{n} - 1$$

$$z^2 - t^2 = -\tau^2 = \textbf{4fb} = n^2(1-\omega^2) \textbf{ with } w = \frac{2f}{n} - 1 \qquad (2.30)$$

Since the sign of τ^2 specifies the light cone type and hence whether we use β or w, and the sign of one or the other specifies the appropriate octant (up to an overall sign ambiguity which can only be resolved by reference to the laboratory situation which is being modeled), we can start from z, t and calculate ***n*** and β or n and w unambiguously; from these we get **r,** $l, f,$ **b.** This leaves one unknown parameter $s = S -$ ***n*** which we will use to characterize all possible trajectories which fit our boundary conditions by requiring that

$$R = r + s;\ L = l + s;\ F = f + s;\ \textbf{B = b + s} \qquad (2.31)$$

As we have shown many times[10] it is easy to give meaning to the concept of Lorentz invariance in our discrete context. Defining **r' = pr,** $l' = \rho^{-1}l,$ τ^2

is obviously invariant, and if we define $\gamma_\rho = \frac{1}{2}(\rho + \rho^{-1})$, $\beta_\rho^2 = 1 - \frac{1}{\gamma_\rho^2}$ we have immediately that

$$z' = \gamma_\rho(z + \beta_\rho t); \qquad t' = \gamma_\rho(t + \beta_\rho z) \tag{2.32}$$

Although our original derivation applied only in the forward light cone, the current context allows us to extend it to the square space-time mesh we constructed above. Then we find that ***n*** is simply the space-like or time-like interval in some rest system, and the coordinate-system dependent quantities β, w can be ignored until we start making ***physical*** use of the model for discussing laboratory experiments.

In order to relate this model to the ***single*** bit-string "random walk" we have used in the past, we construct two substrings $\mathbf{r}(n), \mathbf{l}(n)$ by the same recursive algorithm starting from i, j = 0 and ending at i = S, j = n which led to Eq. 2.7 above. Once we have made this construction,

$$r(n) \oplus \mathbf{l}(n) \oplus l(n) = \mathbf{0}(n) \tag{2.33}$$

and we can interpret the string r as representing a "random walk" in which a "1" represents a step to the right and a "0" represents a step to the left, as in the Stein paradigm; the new ordering parameter j now represents *causal* time order along the trajectory when ***F − B > 0.***

3+1 dimensions

Clearly, until we make physical application of the formalism, the distinction between space and time in this construction is only suggested by our choice of symbols (z, t); the Minkowski symmetries are maintained. That is, any set of labels R,L,F,B; r,l,f,b; n, z, t which maintain the connections defined above model the same situation. An appropriate interchange of the symbols $0, 1$ which maintains both the dichotomy and the asymmetry [i.e. 1 $1 = 0 = 0 $\oplus$ 0] used above makes the whole scheme "label invariant" within the ***combinatorial*** *hierarchy*[11] labeling scheme constructed by Kilmister; his latest version was presented at this conference [12].

To distinguish space from time in the model, we include additional *spacial* dimensions which we require to be ***homogeneous*** and ***isotropic*** in the sense that none of the symmetry properties depend on the choice of the labels x, y, z, ... One of the great conceptual advantages of our constructive approach is that McGoveran has ***proved*** that in our theory the extension from 1+1 space-time to 2+1 and 3+1 has to stop there. This is *McGoveran's* ***Theorem*** (FDP Section 3.4, pp 30-34) which we already quoted in the first paragraph of the introduction. To see how this applies in our context, fix the F, B pair as defining the universal ordering parameter j for causal space-time events, and try to construct not only the z coordinate from the R,L pair as above but three additional independently generated pairs W_+, W_-; X+,X-, Y+, Y- to construct the coordinates w = $W_+ - W_-$, x = $X_+ - X_-$, y = Y+ $- Y_-$, and for consistency in the notation replace L,R by Z-, Z_+ with z = $Z_+ - Z_-$.

Following the same procedure as above, we generate four substrings w+(n), $\mathbf{x}_+(n), \mathbf{y}_+(n), \mathbf{z}_+(n)$. Since these four strings are ***independent*** by hypothesis, they cannot discriminate to the null string, so we need a definition of *event* appropriate to this situation. We take this to be those values of j for which all four strings have accumulated the same number of "1" 's, i.e.

$$\Sigma_{k=1}^{j} b_k^{w+} = \Sigma_{k=1}^{j} b_k^{x+} = \Sigma_{k=1}^{j} b_k^{y+} = \Sigma_{k=1}^{j} b_k^{z+} \tag{2.34}$$

The extension to D rather than 4 spacial dimensions is obvious. This reduces the probability of events occurring after j space-time steps in D dimensions to the probability of obtaining the same number of "1" 's in D independent Bernoulli sequences after j trials,

$$p(j) = \frac{1}{2^{jD}} \Sigma_{k=0}^{j} \binom{j}{k}^D < j^{-\frac{D-1}{2}} \tag{2.35}$$

Clearly this definition of events defines a "homogeneous and isotropic" d-space, but the probability of being able to *continue* to find events for large values of j

vanishes for $D > 3$. Consequently we need only consider three spacial dimensions. Thus, provided we have some clear way to label independent bit strings, we can extend our construction of $1+1$ space-time to $3+1$ space-time, but no further. We will return to the calculation of scattering probabilities in this space in the next chapter.

2.5. DISCRETE FREE-PARTICLE WAVE FUNCTIONS

Time dependence of the Schroedinger wave function

In the past we have used the macroscopic space-time interval between two counter firings as our basic means of connecting our theory to the laboratory (the "counter paradigm"). We have now extended our model to include left-right motion in space ***and*** "forward-backward motion in time" using a single global ordering parameter. The Lorentz invariance of the system allows us, in principle, to talk about one ***or*** the other by going to the appropriate "rest system". To relate this to the time *Zitterbewegung* of an isolated system-of rest energy mc^2, which has period. $T = h/mc^2 = 1/\nu$ or angular frequency w = $mc^2/\hbar$, we require a new variant of our "counter paradigm" which touches an empirical reflection of this basic aspect of the theory. Fortunately Feynman has supplied us the clue by allowing us to think of a particle moving "backward in time" as an antiparticle moving forward in time.

This line of thought suggests that, just as we use counter telescopes to define "monochromatic velocities" which are the starting point for making measurements that exhibit deBroglie wave interference effects, we think about external devices before and after the counter firings [which in a rest system would be $t_0 = n_0(h/mc^2)$ seconds apart] that tell us whether a particle or an antiparticle enters or leaves the system. This is easy for charged particles, since all we need do is to put a region of magnetic field between the two counters in the entrance and exit counter telescopes, and see which way the macroscopic space-time trajectory bends. In this way, we find that particles of opposite charge bend in opposite directions,

and that on the macroscopic scale a single particle has the same charge in the entrance and exit telescopes. Although we cannot reverse time flow, we can reverse velocities, and find that for all the time intervals between the four counter firings kept the same, particles of opposite charge ***and*** opposite velocity follow identical trajectories, although they are obviously traversed in the opposite sense. Thus the same experimental setup does allow us to distinguish "forward" from "backward" while passing along the same macroscopic trajectory, and we accept both this fact and the conservation of charge (or more generally number of particles minus the number of antiparticle with all other quantum numbers reversed) as part of our *rules* ***of correspondence*** connecting our constructed model to laboratory experience.

With this experimental preliminary out of the way, we can now immediately apply our general "adding in quadrature" result to construct free particle wave functions. For time-like intervals, we need simply take the period to be h/mc^2 = ***h/E*** in the rest system, and find that ***any*** isolated system with rest energy $mc^2 = E$ has a combinatorial wave function which can be approximated by a solution of the equation

$$\pm i\hbar\partial\psi/\partial t = E\psi \qquad \textbf{(2.36)}$$

We emphasize that our solutions are derived only when restricted by the space-time boundary conditions which represent ***completed*** processes. For us it would be a ***serious error*** to try to interpret this or any other Schroedinger-type equation as describing the *causal* evolution of a complex amplitude.

The Klein-Gordon Equation

We have seen that this time evolution can be transformed from the rest system with $\tau^2 = n_0^2(h/mc)^2$ to an arbitrary system with $\tau^2 = c^2t^2 - z^2$ in which the velocity between the endpoints of the trajectory is $\beta = z/ct$. Consequently we have already constructed the discrete solutions which can be approximated by continuum solutions of the equation

$$\partial^2\psi/\partial z^2 - \partial^2\psi/c^2\partial t^2 = (mc/\hbar)^2\psi \qquad \textbf{(2.37)}$$

Extension to $3+1$ dimensions is immediate.

The Dirac Equation[7]

The Dirac case differs from the Klein-Gordon case because a step to the left or to the right can have either left or right helicity, and spin-conservation adds a second conservation law to the particle-antiparticle conservation implied by our boundary conditions and reflected in our use of complex amplitudes. Consequently, in addition to the two independent time sequences $t_\pm(s)$ we must have two independent space sequences $z_\pm(s)$ ordered by the ***same*** global ordering parameter i and characterized by the same path parameter s. We can take over the same space-time boundary condition used above with Z_+ = ***R*** the steps to the right and Z_- = ***L*** the steps to the left, and use a imaginary step length for the $\pm c$ *Zitterbewegung*, but the wave function now has two initial states α depending on whether the initial step (or helicity) is positive or negative, and two final states β. If $\Phi_{\beta\alpha}(B)$ are the number of ***trajectories*** with ***B*** bends, the extension of our prescription derived above, which is equivalent to Feynman's[13,14] except that our step length is kept *fixed* at $i\,h/mc$ (ih/mc^2), amounts to calculating[7]

$$K_{\beta\alpha}(b,t_b;a,t_a) = \Sigma_{B\geq 0}\Phi_{\beta\alpha}(B)(i)^B \qquad \textbf{(2.38)}$$

As we have shown elsewhere[7], the *exact* combinatorial result is

$$K_{-+} = (i)\Sigma_s(-)^s\frac{r^s}{s!}\frac{l^s}{s!} = (i)\Sigma_s(-)^s(\frac{\tau}{2})^{2s}\frac{1}{(s!)^2} \rightarrow iJ_0(\tau) \qquad (2.39)$$

where we have used the fact that

$$\textbf{4rZ} = \textbf{[(r + l)}^2 - \textbf{(r - l)}^2] = [c^2(t_b - t_a)^2 - \textbf{(b - a)}^2](\frac{mc}{h})^2 = \tau^2(\frac{mc}{h})^2 \qquad \textbf{(2.40)}$$

is the square of the invariant interval. Applying the same reasoning to calculate

the other three components, our final result is

$$K(z,t;0,0)=\frac{1}{2}\begin{pmatrix}-\frac{(ct+z)}{\tau}J_1(\tau) & iJ_0(\tau)\\ iJ_0(\tau) & -\frac{(ct-z)}{\tau}J_1(\tau)\end{pmatrix} \tag{2.41}$$

which for our boundary conditions is the solution of the Dirac equation

$$-i\sigma_z\partial\psi/\partial z-m\sigma_x\psi=i\partial\psi/\partial t \tag{2.42}$$

where $\hbar$ = 1 = c, σ_x and σ_z are Pauli spin matrices and ψ has two components. Again, extension to 3+1 dimensions appears to be immediate.

Momentum-space equations

A major conceptual advantage arising from our finite and discrete approach to relativistic quantum mechanics using end-point boundary conditions is that we obtain the momentum-space wave function without additional effort. We have already seen that for the interval specified $z = (r-l)(h/mc)$ and $mc^2t/h = (r+l) =$ n; consequently the velocity in units of the limiting velocity c is $\beta = z/ct = \frac{2r}{n} - 1$ Since we have already established our discrete version of Lorentz invariance for the equations, we must use the implied definition of energy $E = \gamma mc^2$ and momentum $p_z = \gamma\beta mc = \beta E/c$. This gives us immediately the Klein-Gordon equation in "moment um space"

$$(p_z^2 + m^2)\phi(p_z) = E\phi(p_z) \tag{2.43}$$

where (and from now on) $\hbar$ = 1 = c. Another way to see this is to recognize that our energy (or momentum) conservation law, allows us to treat the left-right ***Zitterbewegung*** in z as a one-dimensional problem analagous to our treatment of forward and backward movement in time. Thus we can immediately conclude that $\psi_{p_z}(z) = e^{ip_z z}$. Since the space-motion and the time-motion are generated independently in our model, we can multiply the two independent amplitudes to

obtain

$$\psi(z,t) \rightarrow e^{\pm i(p_z z \pm Et)} \tag{2.44}$$

and hence provide an alternative derivation of the Klein-Gordon equation which is completely equivalent to our treatment above. Clearly, this route applied to two amplitudes which conserve helicity at the end points along the same lines will yield the 1+1 Dirac equation in momentum space, and make extension to 3+1 dimensions even easier to accomplish. For instance, instead of p_x, p_y we can use $p_\perp$, j and the wave function $e^{i(p_\perp r_\perp + j\phi)}$ where the boundary condition on ϕ is periodic with period 2π or 4π depending on whether j is integer or half-integer.

3. A BRIEF LOOK AT INTERACTIONS

3.1. THE 8-COUNTER PARADIGM FOR 4-EVENTS

So far we have considered only single particle wave functions modeled by pairs of bit-strings of length S from which we constructed coordinate substrings c(n), c $\in$ x, y, z, t subject to the constraint

$$\mathbf{c}_+(n) \oplus \mathbf{c}_-(n) \oplus \mathbf{1}(n) = \mathbf{0}(n) \tag{3.1}$$

Now label four sets of coordinate strings by strings of length 4 and some discriminately independent choice of three of them, eg (1010) = **a,** (1001) = **b,** (1111) = 1 = $\mathbf{q_0}$. Clearly these model the seven strings and discriminately closed subsets of the second level of the ***combinatorial hierarchy***[10,11,12]. Such a choice provides a convenient way to model two types of particle, their antiparticles and three quanta. This description carries with it two additive conservation laws[10] and Feynman diagrams corresponding to the processes

$$\mathbf{a} \oplus \bar{\mathbf{a}} = \mathbf{q_0} = \mathbf{b} \oplus \bar{\mathbf{b}}$$

$$\mathbf{a} \oplus \bar{\mathbf{b}} = \mathbf{q}_+ = \bar{\mathbf{a}} \oplus \mathbf{b} \tag{3.2}$$

$$\mathbf{a} \oplus \mathbf{b} = \mathbf{q}_- = \bar{\mathbf{a}} \oplus \bar{\mathbf{b}}$$

We now put this together as an 8-counter paradigm in figure 1. If we now construct the content ***stings*** corresponding to each of these labels, following the recursive algorithm leading to 2.24, but now in 3+1 dimensions, considering all the pairs in Eq. 2.16 and all the triples in 2.17, we will find that we have the full ***crossing-symmetric*** kinematics for all 2-2 elastic and anelastic scatterings and all 3-body decays or bound states. We leave these technical details to another publication.

To get the (free particle) Bohr-Sommerfeld quantization for finite velocity, note that our definition of velocity (and coherence length) necessarily implies a periodicity N_λ specified by w or by $\beta = \frac{2k}{n} - 1 = \frac{2N_\lambda k_\beta}{N_\lambda n_\beta} - 1$ where k_β, n_β have no common factor. If we are considering boundary conditions defined by constant (average) velocity between a coherent string of possible time-like events or constant average separation along a string of possible space-like separated events — which is the usual condition for free particle quantum mechanical interference — this means that two events can be causally connected or spacially correlated only when they are some integral number N_λ of (relativistic) deBroglie wavelengths apart. Thus, in addition to the ***Zitterbewegung,*** we have a grosser quantized "random walk" with the positions where events can either occur (in time-like sequence) and/or are correlated in their space-like separations specified by their mass and some velocity parameter $0 < \beta < 1$ in terms of the deBroglie wavelength. In contrast to the unobservable ***Zitterbewegung,*** this structure can extend over macroscopic dimensions for low velocities and leads to our relativistic Bohr-Sommerfeld quantization[1,10]. The space-time, and momentum-energy conservation laws come from the deBroglie periodicities in the velocities, since events can occur only when all particles have moved an integral number of deBroglie wavelengths along their trajectories. We will spell this out in detail elsewhere.

3.2. SCATTERING CROSS SECTION, RESONANCES AND BOUND STATES

Our eight counter paradigm with constant velocities for labeled trajectories connecting all pairs of counters defines a scattering volume measured by deBroglie wavelengths with a linear dimension of at least one wavelength perpendicular to some beam direction. This allows us to define a cross sectional area $4\pi(\hbar/p)^2 = 4\pi/k^2$ which can be taken out of the beam and scattered in any direction. If the total scattered intensity is a^*a compared to the beam with no scattering, this corresponds to an isotropic average differential cross section

$$P = N^2(1 + a^*a) = 1;\ d\sigma/d\Omega = t^2/k^2$$

where the complex amplitude $t = a/\sqrt{1 + a^*a}$ represents the interfering alternatives between scattering and not scattering and arises from the same type of "adding in quadrature" analysis we developed above.

For a situation where the probability of scattering is small, .and is indeed isotropic (i.e. the probable size of the scatterer is much smaller than the deBroglie wavelength defined by the beam velocity and mass), we can model this by an "interaction" with coupling constant f^2 to which t is proportional. This is the actual situation for the universal Fermi interaction between distinct spin-1/2 particles one of which is a massless neutrino. Since there are four particle types, and each has two spin states and is a particle or antiparticle there are 16 x 16 possible initial states and similarly final states. Thus the probability of any one specific type of scattering occurring is $1/(256)^2$ which is our calculation of the Fermi constant. Put in appropriate units this calculation is good to 7%, and as McGoveran reported at this meeting, a correction similar to that computed for the fine structure constant gives four figure accuracy in comparison with experiment.

Of course recognizing the coulomb interaction parametere$^2/\hbar c$ implied by the combinatorial hierarchy as 1/137 was one of the earliest successes of the program. At first sight the "short-range" approach used above for scattering due to the Fermi

interaction looks hopeless when applied to the "infinite range" coulomb interaction. Actually the fact that there is no range makes the coulomb case independent of h (for non-identical particles), which is why Rutherford was able to find a classical solution. However, once one introduces finite angular momentum conservation and recognizes that the infinite cross section which the classical (or non-relativistic quantum mechanical) theory predicts in the forward direction has to be modified to take account of finite angular resolution, a simple treatment becomes possible. It fits neatly with the McGoveran[15] calculation of the Bohr[16] -Sommerfeld[17] -Dirac[18] -Biedenharn[19] problem, and will be presented elsewhere.

One can unify these treatments by considering a system whose free constituents (eg two masses ml, m_2) have invariant four-momentum squared $s = (m_1 + m_2)^2$ and one bound ($s < s_0$) or resonant ($s > s_0$) state at s_0 due to some interaction energy $f^2\mu$. This state decays once the resonance is formed with a lifetime 1/l? where Γ is the width of the resonance. The two invariant attributes $(s - s_0)$ and Γ^2 must be added in quadrature by the now familiar argument to give

$$(f^2\mu)^4 = (s - s_0)^2 + \Gamma^4 \tag{3.3}$$

This our derivation of the relativistic Breit-Wigner probability unitarity condition for a single resonance, familiar in S-matrix theory.

Note that for $\Gamma = 0$ this formula describes bound rather than scattering states. To emphasize its importance I call it "A *handy-dandy non-perturbative formula* for bound states and resonances". It allows a uniform treatment of strong and weak interactions, and might allow us to abolish the *infrared slavery* of QCD[20]. In particular, it allows us make the following calculation. If we follow Fermi and Yang's suggestion[21] that the pion is a bound state of a nucleon-antinucleon pair, $s_0 = m_\pi^2$, $(m_1 + m_2) = 2M_N$ and

$$(G^2 m_\pi)^2 = (2m_N)^2 - m_\pi^2 \simeq (14 m_\pi)^2 \tag{3.4}$$

which is, so far as I know, the first *prediction* of the pion-nucleon coupling constant.

But this just scratches the surface of what now becomes possible for both strong (QCD), electromagnetic-weak (unified) and gravitational interactions. I wish I had time to sketch some of these applications, but must close here. The summary table give an inkling of what lies ahead.

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FIGURE CAPTIONS

1) The 8-counter paradigm: four pairs of counters on a sphere of diameter D which are uniquely sensitive to particles of type a, *b*, c, ***d*** and may be traversed in either direction. For explicitness, the single counters of each type carry the index i or *f* for initial or final firing. The same apparatus can record may different types of 4-events. Suppose that by using devices exterior to the counters we can identify particle types W, X, $Y.Z$, W_i^a means that a particle of type W fires counter a as one of the initial firings, etc. Then $(W_i^a; W_f^a)$ calibrates the counters and, together with similar calibrations for all pairs defines what we mean by "no scattering". We consider all cases when four counters fire: (1) $(W_i^a, X_i^b; W_f^b, X_f^a)$ e1ast ic scattering, illustrated in the figure. (2) $(W_i^a, X_i^b; Y_f^c, Z_f^d)$ anelastic scattering. (3) $(W_i^a; X_f^b; Y_f^c, Z_f^d)$ S-decay. (4) $(W_i^a, X_i^b, Y_i^c; Z_f^d)$ coalescence. (5) $(W_f^a, X_f^b; Y_f^c, Z_f^d)$ 4-decay (requires a source inside the scattering volume).

Summary of WHERE WE ARE in January, 1990

General structural results

- 3+1 asymptotic space-time
- combinatorial free particle Dirac wave functions
- supraluminal synchronization and correlation ***without*** supraluminal signaling
- discrete Lorentz transformations for event-based coordinates
- relativistic Bohr-Sommerfeld quantization
- non-commutativity between position and velocity
- conservation laws for Yukawa vertices and 4- events
- crossing symmetry, CPT, spin and statistics

Gravitation and Cosmology

- the equivalence principle
- electromagnetic and gravitational unification
- the three traditional tests of general relativity
- event horizon
- zero-velocity frame for the cosmic background radiation
- mass of the visible universe: $(2^{127})^2 m_p$ = 4.84 x 10^{52} ***gm***
- fireball time: $(2^{127})^2 \hbar/m_p c^2$ = 3.5 million years
- critical density: of $\Omega_{Vis} = \rho/\rho_c$ = 0.01175 $[0.005 \leq \Omega_{Vis} \leq 0.02]$
- dark matter = 12.7 times visible matter [lo??]
- baryons per photon = $1/256^4$ = 2.328... x 10^{-10} [2 *x* 10^{-10}?]

Unified theory of elementary particles

- quantum numbers of the standard model for quarks and leptons

with confined quarks and exactly 3 weakly coupled generations

- gravitation: $\hbar c/Gm_p^2 = 2^{127} + 136 = 1.70147...[1 - \frac{1}{3\cdot7\cdot10}] \; x \; 10^{38}$
  $=1.6934...$ x 10^{38} [1.6937(10) x 10^{38}]
- weak-electromagnetic unification:
  $G_F m_p^2/\hbar c$ = $(1 - \frac{1}{3\cdot7})/256^2\sqrt{2}$ = 1.02 758... x 10^{-5} [1.02 684(2) x 10^{-5}];
  $sin^2\theta_{Weak}$ = $0.25(1 - \frac{1}{3\cdot7})^2$ = 0.2267... [0.229(4)]
  $M_W^2 = \pi\alpha/\sqrt{2}G_F sin^2\theta_W$ = ***(37.3*** $Gev/c^2 sin\ \theta_W)^2$; $M_Z cos\ \theta_W = M_W$
- the hydrogen atom: $(E/\mu c^2)^2[1 + (1/137N_B)^2] = 1$
- the Sommerfeld formula: $(E/\mu c^2)^2[1 + a^2/(n + \sqrt{j^2 - a^2})^2] = 1$
- the fine structure constant: $\frac{1}{\alpha} = \frac{137}{1-\frac{1}{30\times127}}$ = 137.0359 674...[137.0359 895(61)]
- $m_p/m_e = \frac{137\pi}{\frac{3}{14}\left(1+\frac{2}{7}+\frac{4}{49}\right)\frac{4}{5}}$ = 1836.15 1497... [1836.15 2701(37)]
- $m_\pi^\pm/m_e$ = $275[1 - \frac{2}{2\cdot3\cdot7\cdot7}]$ =273.1292... [273.12 63(76)]
- m_{π^0}/m_e = $274[1 - \frac{3}{2\cdot3\cdot7\cdot2}]$= 264.2 1428.. [264.1 160(76)]
- $(G_{\pi N}^2 m_{\pi^0})^2 = (2m_p)^2 - m_{\pi^0}^2 = (13.86811 m_{\pi^0})^2$

[( )] = empirical value (error) or range

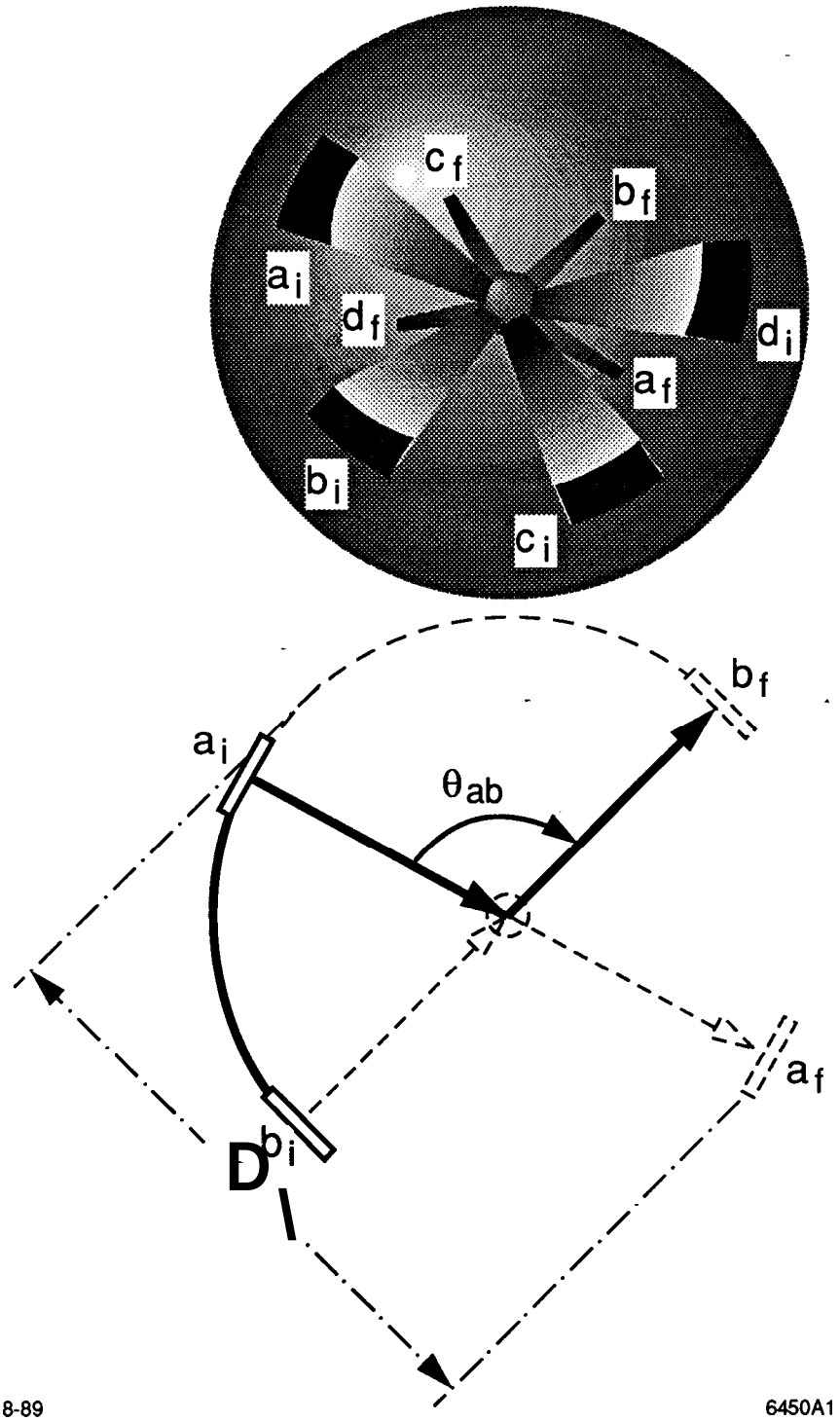

Fig. 1