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Conformal Field Theories for the Green–Schwarz Superstring *

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ABSTRACT

The energy-momentum tensor of the covariantly quantized GS heterotic superstring in D dimensions is found to be equivalent to that of a $\widehat{so}(2(D-2))_{k=2}$ theory plus a system of three bosons which carry background charges and the $c = -2$ system of world-sheet fermions familiar from the super-conformal ghost system. Both the R-symmetry and the κ-ghost number symmetry are found to be anomalous.

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The Green-Schwarz (GS) superstring was recently quantized in a Lorentz covariant gauge [1-4]. Its quantum lagrangian includes an infinite number of space-time spinor fields. In this note, we will study the conformal field theory of the gauge fixed action for this manifestly space-time supersymmetric string theory. There are many motivations for such an investigation. Among these is the possibility that there might be a simplier theory which is equivalent to the GS superstring. Additionally, such a study should shed light on the relationship between the spinning (NSR) and super (GS) strings. Whatsmore, it should be possible to see the origin (or lack) of R-symmetry in the GS theory.

After covariant gauge fixing of the κ-supersymmetry and in the conformal gauge for the reparametrization symmetry, the GS superstring lagrangian reads [1]:$\mathcal{L} = \mathcal{L}_B + \mathcal{L}_S$ with

$$\mathcal{L}_B = \partial_z x \cdot \partial_{\bar{z}} x \quad , \qquad \mathcal{L}_S = \sum_{n=0}^{\infty} (-)^{n+1} \sum_{m=0}^{n} \pi_{n+1}^{n-m+1} \partial_z \bar{c}_n^m \quad . \tag{1}$$

The reparametrization ghosts have been excluded here. Futhermore, only the anti-holomophic sector will be of concern here. It has been pointed out [5] that the change of variables [1-4] which diagonalizes the original lagrangian to this form, is not invertible. Nevertheless, charges which generate space-time supersymmetry tranformations may be derived from eqn. (1). Thus it is a valid starting point for a conformal field theory discussion of a space-time super-translationally invariant theory.

The dimensions of the N_S component, space-time spinors π and $\bar{c}$ are $[\bar{c}_n^m] = (2m-n)$ and $[\pi_{n+1}^{n-m+1}] = 1-(2m-n)$. Their Grassmann statistics assignments are the same: $(-)^{n+1}$. Consequently, the energy-momentum tensor from $\mathcal{L}_S$ is

$$T^S(\bar{z}) = -\sum_{n=0}^{\infty} (-)^{n+1} \sum_{m=0}^{n} [(2m-n-1)\pi_{n+1}^{n-m+1} \partial_{\bar{z}} \bar{c}_n^m + (2m-n)\partial_{\bar{z}} \pi_{n+1}^{n-m+1} \bar{c}_n^m] \quad . \tag{2}$$

The operator product of the spinors in eqn. (1) is

$$\bar{c}_n^m(\bar{z}) \pi_{q+1}^{q-p+1}(\bar{w}) \sim -(-)^{n+1} \frac{1}{(\bar{z}-\bar{w})} \delta^{mp} \delta_{nq} \quad . \tag{3}$$

With this, the central charge in the $T^S(\bar{z})T^S(\bar{w})$ OPE is [1]

$$c^S = -2N_S \sum_{n=0}^{\infty} \sum_{m=0}^{n} (-)^n [6(2m-n)(2m-n-1)+1] \ . \tag{4}$$

Numerically, N_S is given in terms of the space-time dimension, D, as $N_S = 2(D-2)$. Performing the 'm' summation leads to $c^S = -2N_S \sum_{n=0}^{\infty} (-)^n (n+1)[2n(n+2)+1]$. The first term in the square brackets may be regularized as $\lim_{y\to 1_-}[\frac{24N_S y}{(1+y)^4}] = \frac{3}{2}N_S$ and the last is $\lim_{y\to 1_-}[2N_S(\frac{y}{(1+y)^2} - \frac{1}{1+y})] = -\frac{1}{2}N_S$. (Alternatively, one could use ζ-function regularization.) Thus c^S evaluates to $c^S = N_S$ so that the central charge from eqn. (1) is $c = D + N_S = 26$. For example, in D=10 space-time dimensions, the spinors are indeed **16**'s of $SO(9,1)$.

Repeated use of the sums: $SUM[I] \equiv \sum_{n=0}^{\infty} (-)^{n+1} \sum_{m=0}^{n} (2m-n)^I$ for $I = 0$, 1 and 2, will be made in the calculations below. They evaluate to $SUM[0] = -\frac{1}{4}$, $SUM[1] = 0$ and $SUM[2] = \frac{1}{8}$.

As is well known, the GKO construction [6] offers a prescription for finding equivalent CFT's. In order to follow that procedure, one must find the currents from eqn. (1) which form a closed Kač-Moody (KM) super-algebra.

There are many independent global transformations under which eqn. (1) is invariant. It is pedagogical to first study the simplier action given by the $n = m = 0$ term in the latter equation. Thus take

$$\mathcal{L}'_S = -p\partial_z\theta \ , \tag{5}$$

where $p \equiv \pi_1^1$ and $\theta \equiv \bar{c}_0^0$ are both Grassmann odd with $[p] = 1$ and $[\theta] = 0$. $\mathcal{L}'_S$ falls into the class of first order Lagrangians studied in ref. [7]. Either by direct calculation or by regurgitating their results, one finds that the algebra of the energy-momentum tensor

$$T'_S = p\partial_z\theta \ , \tag{6}$$

has a central charge: $c'_S = -2N_S$.

Associated with $\mathcal{L}'_S$ are fermion number and $SO(N_S)$ vector symmetries. The respective currents are $j = -p\theta$ which generates a R-symmetry and $J^I = -pt^I\theta$. In addition to satisfying the respective KM algebras, these currents may be used to construct the Sugawara energy momentum tensor $T'_{Sug} \equiv T_j + T_J$ which satisfies (at level k for $\widehat{so}(N_S)$)

$$\begin{aligned} T_j &= \frac{1}{2N_S}[: jj : + N_S \partial_{\bar{z}} j] \ , \qquad T_j(\bar{z})j(\bar{w}) \sim \frac{j(\bar{z})}{(\bar{z}-\bar{w})^2} - \frac{N_S}{(\bar{z}-\bar{w})^3} \ , \\ T_J &= \frac{1}{2(k+N_S-2)} \sum_{I=1}^{N_S} : J^I J^I : \ , \qquad T_J(\bar{z})J^I(\bar{w}) \sim \frac{J^I(\bar{z})}{(\bar{z}-\bar{w})^2} \ , \end{aligned} \tag{7}$$

along with $J^I(\bar{z})j(\bar{w}) \sim 0$. Note that j is anomalous with background charge $Q_j = -N_S$. The central charge in the $T'_{Sug}T'_{Sug}$ operator product is easily computed and found to be $c'_{Sug} = c_j + c_J$ with $c_j = (1 - 3N_S)$ and $c_J^{k=2} = (N_S - 1)$ so that $c_{Sug} = -2N_S$. By the quantum equivalence theorem [6], the Sugawara construction (7) is equivalent to the Virasoro energy-momentum tensor (6): $T'_{Sug} = T'_S$ with $k = 2$.

Some time ago, a superstring algebra was suggested in ref. [8]. It reads

$$\begin{aligned} [Q^\alpha_{(m)}, Q^\beta_{(n)}\} &= 2(\Gamma^a)^{\alpha\beta} P_{a(m+n)} \ , \\ [Q^\alpha_{(m)}, P_{a(n)}\} &= 2(\Gamma_a)^{\alpha\beta} \Omega_{\beta(m+n)} \ , \\ [Q^\alpha_{(m)}, \Omega_{\beta(n)}\} &= m\delta_{m,-n}\delta_\beta{}^\alpha \ , \\ [P_{a(m)}, P_{b(n)}\} &= m\delta_{m,-n}\eta_{ab} \ , \\ [P_{a(m)}, \Omega_{\alpha(n)}\} &= [\Omega_{\alpha(m)}, \Omega_{\beta(n)}\} = 0 \ , \end{aligned} \tag{8}$$

where Q^α is the super-charge, P_a is the momenta and Ω_α is a new fermionic generator whose origin will be seen below. The $m, n, \ldots$ indices here and through eqn. (12) below, are loop indices. Mathematically, Ω_α is needed for the closure of the affine algebra. The super-Jacobi Identities vanish if $(\Gamma^a)^{(\alpha\beta}(\Gamma_a)^{\gamma)\delta} = 0$. This Fierz identity is valid only in $D = 3, 4, 6$ and 10 space-time dimensions.

Following GKO [6], the Virasoro generators constructed from eqn. (8) are

$$L_m = \frac{1}{2} \sum_{p=-\infty}^{\infty} : P^a_{(m+p)} P_{a(-p)} + \Omega_{\alpha(m+p)} Q^\alpha_{(-p)} - Q^\alpha_{(m+p)} \Omega_{\alpha(-p)} : \ , \tag{9}$$

where the bosonic normal ordering is as in the latter reference and (without implied summation) the fermionic normal ordering is

$$: \Omega_{\alpha(m)} Q^{\alpha}_{(n)} : = \begin{cases} \Omega_{\alpha(m)} Q^{\alpha}_{(n)} \ , & \text{if } m < 0 \ , \\ -Q^{\alpha}_{(n)} \Omega_{\alpha(m)} \ , & \text{if } m \geq 0 \ . \end{cases} \tag{10}$$

It is straightforward to show that

$$[L_m, \mathcal{O}_n\} \ = \ -n\mathcal{O}_{m+n} \ , \tag{11}$$

where $\mathcal{O}$ is one of the operators in eqn. (8). Direct computation yields that L_m satisfies the Virasoro algebra with central charge $c = D - 2N_S = -3D + 8$. The minus sign preceding N_S is due to the fact that the spinor generators are bosonic, Grassmann odd operators on the world-sheet, *i.e.* ghost-like. More directly, recall that the central charge of a KM super-algebra [6] is proportional to its super-dimension which is [dim(bosonic) − dim(fermionic)].

Later, the KM algebra of the symmetry currents of the full lagrangian in eqn. (1) will be constructed (see eqn. (16) below). From it or by direct construction, the super-translation currents from $\mathcal{L}_B + \mathcal{L}'_S$ are:

$$\begin{aligned} Q^{\alpha}_{(m)} &= p^{\alpha}_{(m)} \ - \ (\Gamma^a)^{\alpha\beta} \sum_q \theta_{\beta(m+q)} p_{a(-q)} \ , \\ P_{a(m)} &= p_{a(m)} \ + \ (\Gamma_a)^{\alpha\beta} \sum_q q\theta_{\alpha(m+q)} \theta_{\beta(-q)} \ , \\ \Omega_{\alpha(m)} &= m\theta_{\alpha(m)} \ , \end{aligned} \tag{12}$$

with $p_a \equiv \partial_z x_a$. These currents almost satisfy eqn. (8). The exception is the super-charge commutator for which $\{Q, Q\} = 2\not{P} + \mathcal{F}$ requires the introduction of a new generator, $\mathcal{F}$.

The minimum value for D, namely $D = 3$, is precisely the smallest integer for which the central charge is negative. More importantly, as the latter quantity is necessarily negative, the theories (5-12) are not unitary. In particular, the norms of the states $[L_{-n}|0\rangle]$ are not positive definite. These ghost states are, in principle, removed by κ-supersymmetry via the full theory of eqn. (1).

Before returning to the study of the full theory, it is convenient to first introduce some notation. First, define† $\pi^{/1/} \equiv \pi_{n+1}^{n-m+1}$ and $\bar{c}^{/1/} \equiv \bar{c}_n^m$ with $(-)^{/1/} \equiv (-)^{n+1}$. The superscript $/1/$ denotes the κ index structure with indices (m,n). For example, $\pi^{/2/} = \pi_{q+1}^{q-p+1}$ where $/2/$ denotes the (p,q) collection of indicies and $\delta^{/1//2/} \equiv \delta^{mp}\delta^{nq}$. Next, introduce the "circle" notation as a short-hand for the sum: $\pi \circ \bar{c} \equiv \sum_{n=0}^{\infty}(-)^{n+1}\sum_{m=0}^{n}\pi_{n+1}^{n-m+1}\bar{c}_n^m$. Similarly, the "bullet" notation will be used to represent the weighted sum: $\pi \bullet \bar{c} \equiv \sum_{n=0}^{\infty}(-)^{n+1}\sum_{m=0}^{n}(2m-n)\pi_{n+1}^{n-m+1}\bar{c}_n^m$. Finally, denote the dimension of $\bar{c}^{/1/}$ as $d_{\bar{c}}^{/1/} \equiv (2m-n)$ and that of $\pi^{/1/}$ as $d_{\pi}^{/1/} = 1 - d_{\bar{c}}^{/1/}$. Henceforth, the indices $m, n \ldots$ will denote loop indices with the notations above implicitly understood.

Continuing with the symmetries of eqn. (1), it is found that they include [2,3]:

$$\delta x^a = \mathcal{E} \circ \Gamma^a \bar{c} \ , \qquad \delta\pi^{/1/} = -2\mathcal{E}^{/1/}\Gamma^a \partial_{\bar{z}} x_a \ , \tag{13a}$$

$$\delta\bar{c}^{/1/} = \bar{\mathcal{E}}^{/1/} \ , \tag{13b}$$

$$\delta\bar{c}^{/1/} = d_{\bar{c}}^{/1/}\Lambda_{\mathcal{G}}\bar{c}^{/1/} \ , \qquad \delta\pi^{/1/} = -d_{\bar{c}}^{/1/}\Lambda_{\mathcal{G}}\pi^{/1/} \ . \tag{13c}$$

The first two sets of transformations account for the space-time supersymmetry. The transformations given in eqn. (13c) are those of the ghost number symmetry. From the Noether procedure, the respective currents are:

$$\mathcal{C}^{/1/} = -2\Gamma^a \bar{c}^{/1/}\partial_{\bar{z}} x_a \ , \tag{14a}$$

$$\pi^{/1/} \ , \tag{14b}$$

$$\mathcal{G} = -\pi \bullet \bar{c} \ . \tag{14c}$$

The dimensions of the new currents are $[\mathcal{C}^{/1/}] = (d_{\bar{c}}^{/1/} + 1)$ and $[\mathcal{G}] = 1$. $\mathcal{C}$ is the current for the $\mathcal{E}$ transformation and is a member of an infinite set of currents. These particular symmetries are selected because of their relation to the KM algebras of other theories; eqn.

† Rest assured that there are no π's of the $\frac{22}{7}$ variety in the formulae to follow.

(8) for example. It is highly suggestive that the form of the graded currents in eqn. (14a) is isomorphic to the NSR super-current.

With the notations above, it is possible to unambiguously introduce loop indices so that

$$[\bar{c}_m^{/1/}, \pi_n^{/2/}\} = (-)^{/1/}\delta^{/1//2/}\delta_{m+n,0} \quad . \tag{15}$$

The non-vanishing graded-commutators of the currents are found to be

$$\begin{aligned}
[\mathcal{G}_m, \bar{c}_n^{/1/}\} &= d_{\bar{c}}^{/1/}\bar{c}_{m+n}^{/1/} \quad , \qquad [\mathcal{G}_m, \pi_n^{/1/}\} = -d_{\bar{c}}^{/1/}\pi_{m+n}^{/1/} \quad , \\
[\mathcal{G}_m, \mathcal{C}_n^{/1/}\} &= d_{\bar{c}}^{/1/}\mathcal{C}_{m+n}^{/1/} \quad , \qquad [\mathcal{G}_m, \mathcal{G}_n\} = \frac{1}{8}N_S m\delta_{m+n,0} \quad , \\
[\mathcal{C}_m^{/1/}, \pi_n^{/2/}\} &= -(-)^{/1/}2\Gamma^a p_{a(m+n)}\delta^{/1//2/} \quad , \\
[\mathcal{C}_m^{/1/}, p_n^a\} &= 2n\Gamma^a\bar{c}_{m+n}^{/1/} \quad , \qquad [\mathcal{C}_m^{/1/}, \mathcal{C}_n^{/2/}\} = \mathcal{S}_{m+n}^{/1//2/} + m\mathcal{J}_{m+n}^{/1//2/} \quad , \\
[\mathcal{S}_m^{/1//2/}, \pi_n^{[3]}\} &= -4D[(-)^{/1/}n\bar{c}_{m+n}^{/2/}\delta^{/1/[3]} - (-)^{/1//2/}(-)^{/2/}(m+n)\bar{c}_{m+n}^{/1/}\delta^{/2/[3]}] \quad , \\
[\mathcal{G}_m, \mathcal{S}_n^{/1//2/}\} &= (d_{\bar{c}}^{/1/} + d_{\bar{c}}^{/2/})\mathcal{S}_{m+n}^{/1//2/} + md_{\bar{c}}^{/1/}\mathcal{J}_{m+n}^{/1//2/} \quad , \\
[\mathcal{J}_m^{/1//2/}, \pi_n^{[3]}\} &= -4D[(-)^{/1/}\bar{c}_{m+n}^{/2/}\delta^{/1/[3]} + (-)^{/1//2/}(-)^{/2/}\bar{c}_{m+n}^{/1/}\delta^{/2/[3]}] \quad , \\
[\mathcal{G}_m, \mathcal{J}_n^{/1//2/}\} &= (d_{\bar{c}}^{/1/} + d_{\bar{c}}^{/2/})\mathcal{J}_{m+n}^{/1//2/} \quad , \\
[p_m^a, p_n^b\} &= m\eta^{ab}\delta_{m+n,0} \quad ,
\end{aligned} \tag{16}$$

where $\mathcal{S}_m^{/1//2/} \equiv -4D\sum_{p=-\infty}^{\infty} pc_{m+p}^{/2/}\bar{c}_{-p}^{/1/}$ and $\mathcal{J}_m^{/1//2/} \equiv -4D\sum_{p=-\infty}^{\infty} c_{m+p}^{/2/}\bar{c}_{-p}^{/1/}$ are bi-spinors. The factor of $\frac{1}{8}$ in the $\mathcal{G}$-$\mathcal{G}$ commutator arises from $SUM[2]$. Turning to the computation of the semi-direct product with the Virasoro algebra, one finds

$$\begin{aligned}
L_m &= \sum_{p=-\infty}^{\infty} : [\frac{1}{2}p_{m+p}^a p_{a(-p)} - p\pi_{m+p} \circ \bar{c}_{(-p)} - m\pi_{m+p} \bullet \bar{c}_{-p}] : \quad , \\
[L_m, \bar{c}_n^{/1/}] &= -(n + md_{\pi}^{/1/})\bar{c}_{m+n}^{/1/} \quad , \\
[L_m, \pi_n^{/1/}] &= -(n + md_{\bar{c}}^{/1/})\pi_{m+n}^{/1/} \quad , \\
[L_m, \mathcal{C}_n^{/1/}] &= -(n + md_{\bar{c}}^{/1/})\mathcal{C}_{m+n}^{/1/} \quad ,
\end{aligned}$$

$$
\begin{aligned}
[L_m, \mathcal{G}_n] &= -\, n\mathcal{G}_{m+n} + \frac{1}{8} N_S m(m+1)\delta_{m+n,0} \quad , \\
[L_m, p^a_n] &= -\, n p^a_{m+n} \quad , \\
[L_m, \mathcal{S}^{/1//2/}_n] &= -\, n\mathcal{S}^{/1//2/}_{m+n} \;+\; m(d^{/1/}_{\bar{c}} + d^{/2/}_{\bar{c}})\mathcal{S}^{/1//2/}_{m+n} \;+\; m^2 d^{/1/}_{\bar{c}} \mathcal{J}^{/1//2/}_{m+n} \quad , \\
[L_m, \mathcal{J}^{/1//2/}_n] &= -\, n\mathcal{J}^{/1//2/}_{m+n} \;+\; m(d^{/1/}_{\bar{c}} + d^{/2/}_{\bar{c}} - 1)\mathcal{J}^{/1//2/}_{m+n} \quad , \qquad (17)
\end{aligned}
$$

From this it is seen that $\mathcal{G}$ is anomalous. None of the other currents will be used henceforth. The graded-commutators in eqn. (16) demonstrate the manner in which space-time supersymmetry is present in eqn. (1). They also illustrate how deceptively complicated the latter lagrangian is. There isn't much experience in dealing with a theory in which there are an infinite number of fields. It is important to find a simplier theory (free or interacting) which is equivalent to eqn. (1).

From the experience gained through the calculation of eqns. (5-7), it is best to first assemble the various KM currents. As before, there are the fermion number current j and the $SO(N_S)$ currents J^I. Now, there is also the ghost number current of eqn. (14c). Explicitly, these currents may be written as

$$
j = -2\pi \circ \bar{c} \quad , \qquad J^I = -\pi t^I \circ \bar{c} \quad , \qquad \mathcal{G} = -2\sqrt{2}\pi \bullet \bar{c} \quad . \qquad (18)
$$

The normalizations used here reflect the presence of the factors of $SUM[0]$ (j) and $SUM[2]$ ($\mathcal{G}$); $\mathcal{G}$ differs from eqn. (14c). The $U(1)$ currents satisfy the following OPE's:

$$
\begin{aligned}
j(\bar{z})j(\bar{w}) &\sim \frac{N_S}{(\bar{z}-\bar{w})^2} \quad , \qquad T(\bar{z})j(\bar{w}) \sim \frac{j(\bar{z})}{(\bar{z}-\bar{w})^2} + \frac{\frac{1}{2}N_S}{(\bar{z}-\bar{w})^3} \quad , \\
\mathcal{G}(\bar{z})\mathcal{G}(\bar{w}) &\sim -\frac{N_S}{(\bar{z}-\bar{w})^2} \quad , \qquad T(\bar{z})\mathcal{G}(\bar{w}) \sim \frac{\mathcal{G}(\bar{z})}{(\bar{z}-\bar{w})^2} + \frac{\frac{1}{2}\sqrt{2}N_S}{(\bar{z}-\bar{w})^3} \quad , \\
J^I(\bar{z})J^J(\bar{w}) &\sim i f^{IJ}{}_K \frac{J^K(\bar{w})}{(\bar{z}-\bar{w})} + \frac{\frac{1}{4}\tilde{k}\delta^{IJ}}{(\bar{z}-\bar{w})^2} \quad , \qquad T(\bar{z})J^I(\bar{w}) \sim \frac{J^I(\bar{z})}{(\bar{z}-\bar{w})^2} \quad , \qquad (19)
\end{aligned}
$$

along with $j(\bar{z})\mathcal{G}(\bar{w}) \sim 0$. Notice that there are background charges associated with the $U(1)$ currents: $Q_j = \frac{1}{2}N_S$ and $Q_{\mathcal{G}} = \frac{1}{2}\sqrt{2}N_S$. This means that both of these currents are anomalous and that there is a non-zero difference in the number of zero-modes of π and $\bar{c}$. Also, J^I generates a $\widehat{so}(N_S)_k$ KM algebra with central term $k = \frac{1}{4}\tilde{k}$ where $tr(t^I t^J) \equiv \tilde{k}\delta^{IJ}$. The factor of $\frac{1}{4}$ arises from $SUM[0]$.

Following the Sugawara construction, introduce the energy-momemtum tensor $T'_{Sug} \equiv T_j + T_{\mathcal{G}} + T_J$:

$$
\begin{aligned}
T_j &= \frac{1}{2N_S}[: jj : \; - \; \frac{1}{2} N_S \partial_{\bar{z}} j] \ , \\
T_{\mathcal{G}} &= -\frac{1}{2N_S}[: \mathcal{G}\mathcal{G} : \; - \; \frac{1}{2}\sqrt{2} N_S \partial_{\bar{z}} \mathcal{G}] \ , \\
T_J &= \frac{1}{2(k + N_S - 2)} \sum_{I=1}^{N_S} \; : J^I J^I : \ .
\end{aligned}
\tag{20}
$$

T'_{Sug} satisfies the equations on the right-hand-side of eqn. (19). Computing the central charge for each sector yields:

$$
c_j = 1 - \frac{3}{4} N_S \ , \qquad c_{\mathcal{G}} = 1 + \frac{3}{2} N_S \ , \qquad c_J^{k=2} = N_S - 1 \ . \tag{21}
$$

Take the algebra $\hat{g} \equiv \widehat{so}(N_S)_{k=2} \oplus u_j^2(1) \oplus u_{\mathcal{G}}(1)$ where two of the anomalous $u(1)$'s are from two independent copies of j and the third is the ghost-number $U(1)$. Altogether this system has central charge $c'_{Sug} = N_S + 2$. This extra factor of two in the central charge is familiar from the bosonization of the super-conformal ghost system [7]. To cure this problem and thus obtain the correct Sugawara energy-momentum tensor, one simply introduces two world-sheet fermions χ and ρ respectively of dimension 1 and 0. Their central charge is -2. Consequently, the energy-momentum tensor

$$
T_{Sug} = T_{j^1} + T_{j^2} + T_{\mathcal{G}} + T_J + T_{(\chi,\rho)} \ , \tag{22}
$$

is quantum mechanically equivalent to eqn. (2): $T_{Sug}(\bar{z}) = T^S(\bar{z})$. A general expression for the actions of the anomalous $U(1)$ currents written in terms of bosons may be found below eqn. (78) of the first paper in ref. [7]. The well known field representations of the $\widehat{so}(N_S)_k$ KM algebras are reviewed in ref. [6]. As there are both Grassmann even and odd fields in $\mathcal{L}_S$, presumably the (χ, ρ) system plays the same statistics interpolating role as in the $N = 1$ super-conformal ghost system.

Although only the heterotic theory has been treated here, it is straightforward to carry out the same procedure for the type II GS superstrings. It is expected that separate left and right versions of the $\hat{g}$ algebra would give the quantum equivalence. Of course it is important to construct the operators and currents in eqn. (14) out of the operators which give a field representation of this algebra. These and many other important issues are left for the future.

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REFERENCES

1. W. Siegel, Talk presented at *Strings '89*, College Station, TX, March 13-18, 1989; S. J. Gates, Jr., *et al.*, *Phys. Letts.* **225B** (1989) 44; U. Lindström, *et al.*, "Construction of the Covariantly Quantized Heterotic Superstring", SUNY at Stony Brook preprint # ITP-SB-89-38 (May 1989).
2. R. Kallosh, *Phys. Letts.* **224B** (1989) 273; **225B** (1989) 49.
3. M. B. Green and C. M. Hull, *Phys. Letts.* **225B** (1989) 57.
4. M. J. Perry, "Covariant Quantization of the Green-Schwarz Superstring", DAMTP preprint # DAMTP/R-89/2 (May 1989).
5. J. M. L. Fisch and M. Henneaux, "A Note on the Covariant BRST Quantization of the Superparticle", ULB preprint # ULB-TH2/89-04 (revised version) (June 1989); F. Bastianelli, G. W. Delius and E. Laenen, "Free Quantum Action and BRST-charge for the Superparticle", SUNY at Stony Brook preprint # ITP-SB-89-51 (July 1989).
6. P. Goddard and D. Olive, *Intl. J. Mod. Phys.* **A1** (1986) 303; P. Goddard, D. Olive and G. Waterson, *Commun. Math. Phys.* **112** (1987) 591.
7. D. Friedan, E. Martinec and S. Shenker, *Nucl. Phys.* **B271** (1986) 93; *Phys. Letts.* **160B** (1985) 55; V. Knizhnik, *Phys. Letts.* **160B** (1985) 403.
8. W. Siegel, *Nucl. Phys.* **B263** (1985) 93.