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# Analysis of Semileptonic Decays of Mesons Containing Heavy Quarks*

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## ABSTRACT

We analyze semileptonic decays of mesons containing heavy quarks and indicate how to extract the signs and magnitudes of the helicity amplitudes for the exclusive decay mode involving a vector meson, lepton and neutrino by using the joint distribution of the decay products of the vector meson (into a pair of pseudoscalar mesons) and of the virtual $W$ (into a charged lepton and its neutrino). We apply this to specific cases of charm and bottom meson decays with form factors calculated from quark models.

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## § 1: Introduction

Weak semileptonic decays of hadrons provide a well-defined arena for testing the structure of the weak currents, for probing the quark structure of hadrons, and for extracting fundamental information on the quark mixing matrix. In particular, $K_{\ell 3}$ decays provide the most accurate value[1] of the Kobayashi-Maskawa matrix element $V_{us}$, since the hadronic matrix element of the weak vector-current deviates from unity only at second order in comparatively small SU(3) symmetry breaking and can be calculated to high accuracy. Ultimately one would like to use[2] $D_{\ell 3}$ and $B_{\ell 3}$ decays in a similar manner, even though the deviation from any symmetry limit is much larger, to provide information on the Kobayashi-Maskawa matrix elements $V_{cd}$, $V_{cs}$, $V_{ub}$, and $V_{cb}$.

The quark model has been generally regarded as giving a successful description of the semileptonic decays of heavy quarks, both inclusively[3] and exclusively[4–7] These calculations seemed to agree quite well both with the rates for decays like $D \to K\ell^+\nu_\ell$ and with the polarization of the final vector meson for decays like $B \to D^*\ell^+\nu_\ell$. They then were candidates for use in calculating the form factors so as to extract the values of the Kobayashi-Maskawa matrix elements from semileptonic decays of heavy quarks.

In the last year, however, a serious discrepancy between the quark model and experiment has apparently arisen in the decays of charmed mesons, $D \to K^*e^+\nu_e$, in that both the decay rate and the polarization of the $K^*$ do not agree with the predictions of the previous quark model calculations.[4,5] The quark model predictions would have comparable $D \to K^*e^+\nu_e$ and $D \to Ke^+\nu_e$ decay rates and comparable populations of the transverse and longitudinal polarization states of the final $K^*$; experiment,[8] however, shows that the rate for $D \to K^*e^+\nu_e$ is about half that for $D \to Ke^+\nu_e$, and that the $K^*$ is dominantly in a longitudinal state.

This has inspired several attempts to reexamine the quark model and the underlying assumptions involved in the calculations.[9–11] Up to this point there does not seem to be any theoretically well-motivated reason to modify the predictions in a significant way, although *ad hoc* adjustments can be made to fit the data.

In this paper we first derive the kinematic formulas which are necessary to make full use of the data and to extract the form factors for the semileptonic decay of a pseudoscalar meson into a vector meson, such as $D \to K^* e^+ \nu_e$, by using the joint angular distribution[12,13] of the $K^*$ decay (into $K\pi$) and of the virtual $W$ decay (into $e^+\nu_e$). This will permit obtaining the relative signs and magnitudes of the form factors and to isolate the discrepancy with the quark model. We indicate what reasonable variation of the form factors and the explicit kinematic factors do in $D$ and $B$ decays as one moves across the Dalitz plot. Then we use the information presently available (on the decay rate and just the $K^*$ polarization) to indicate how we might modify the quark-model-inspired form factors, particularly if we rescale the magnitudes of the form factors contributing to the transverse polarization state. Finally, we indicate what will happen if we apply similar ideas to Kobayashi-Maskawa suppressed $B$ decays.

## § 2: Kinematics

We are interested in the exclusive semileptonic decays of pseudoscalar mesons into pseudoscalar and vector mesons, and in particular in the information contained in the joint angular distribution of the final vector meson and the virtual $W$. Much of this is a standard exercise and has been derived elsewhere.[12,13] We repeat it here for the sake of writing a self-contained exposition and to clarify the results.

The process at hand for a parent pseudoscalar meson, containing a generic heavy quark, Q, is shown in Figure 1. We take the lepton to be an electron, and neglect its mass in what follows.[14] For the process $M \to m\ e\ \bar{\nu}$, in the parent rest frame the decay rate is given by

$$d\Gamma(M \to m\ e\ \bar{\nu}) \ = \ \frac{1}{2M}\ |A(M \to m\ e\ \bar{\nu})|^2\ d\Pi_3 \tag{2.1}$$

where

$$d\Pi_3 = (2\pi)^4 \delta^{(4)}(P - p - p' - k) \prod_f \frac{d^3 k_f}{(2\pi)^3 2E_f} \tag{2.2}$$

and

$$A(M \to m\ e\ \bar{\nu}) = \frac{G_F}{\sqrt{2}} V_{Qq}\ L^\mu\ H_\mu\ , \tag{2.3}$$

with $V_{Qq}$ being the Kobayashi-Maskawa matrix element appropriate to $Q \to q$ transitions, and the product is over all final state momenta. We are using a redundant (but hopefully not confusing notation) where $M$ ($m$) refers to the parent (daughter) meson as well as to its mass. The parent has four-momentum $P$, the daughter $k$, and the $e$ and $\nu$ have $p$ and $p'$, respectively. The virtual $W$ carries four-momentum $q = p + p'$.

The leptonic and hadronic currents are given by

$$L^\mu = \bar{u}_e \gamma^\mu (1 - \gamma_5) v_\nu \tag{2.4a}$$

$$H^\mu = < k|J_{had}(0)|P > \tag{2.4b}$$

If we have instead the final lepton states $e^+\nu$, as will be the case for a charmed quark (and not anti-quark) decaying, we must change the ordering of the spinors in the lepton current. The matrix element of the hadronic current must be constructed from Lorentz-invariant form factors and the four-vectors in the problem. Writing $J^\mu = V^\mu - A^\mu$, and with standard form factor conventions we have for a pseudoscalar meson in the final state:

$$< k|V_\mu(0)|P > = f_+(q^2)(P + k)_\mu + f_-(q^2)(P - k)_\mu\ , \tag{2.5}$$

and for a vector meson in the final state:

$$< k, \epsilon|V_\mu(0)|P > = ig(q^2)\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}(P + k)^\rho (P - k)^\sigma \tag{2.6a}$$

$$< k, \epsilon | A_\mu(0) | P > = f(q^2)\ \epsilon^*_\mu + a_+(q^2)\ (\epsilon^* \cdot P)(P+k)_\mu + a_-(q^2)\ (\epsilon^* \cdot P)(P-k)_\mu, \tag{2.6b}$$

where $\epsilon$ is the vector meson polarization and $q^2 = (P-k)^2$. It is convenient to define the dimensionless kinematic variables $y = q^2/M^2$ and $x = P{\cdot}p/M^2$ by scaling to the parent meson mass. Neglecting the mass of the electron, the kinematically allowed limits of $y$ are from a minimum of 0 to a maximum of $(1-m/M)^2$. The range allowed for $x$ depends on the value of $y$ and can be derived from Eq. (2.10) below.

In the parent rest frame we denote quantities by a '$\sim$'. We reserve $E_m$, $E_e$, *etc.*. without the tilde to denote quantities in the $e\nu$ center of mass frame ($e\nu$-frame) where the amplitude will turn out to have a simple angular dependence. Let $-\hat{\mathbf{k}}$ define the direction of the positive $z$-axis and let $\theta_e$ be the angle of the electron relative to this axis in the $e\nu$-frame, with the $y$-axis oriented perpendicular to the plane defined by the $K$, $e$, and $\nu$ momenta, as shown in Figure 2. In this frame, the natural variables are

$$E_e = E_\nu = \frac{M}{2}\sqrt{y}\ , \tag{2.7a}$$

and $\cos\theta_e$. On the other hand, in the parent rest frame the natural variables are

$$\tilde{E}_e = M\ x \tag{2.7b}$$

and $y = q^2/M^2$.

The mass shell relation $P^2 = (q+k)^2 = M^2$ may be used to obtain expressions for the energy and momentum of the final state meson:

$$E_m = \frac{M}{2\sqrt{y}}\left(1 - \frac{m^2}{M^2} - y\right) \tag{2.8a}$$

and

$$|\mathbf{k}| = K/\sqrt{y} \tag{2.8b}$$

in the $e\nu$-frame, whereas in the parent rest frame

$$\tilde{E}_m = \frac{M}{2}\left(1 + \frac{m^2}{M^2} - y\right) \qquad (2.9a)$$

and

$$|\tilde{\mathbf{k}}| = K \qquad (2.9b)$$

with

$$K \equiv \frac{M}{2}\left[\left(1 - \frac{m^2}{M^2} - y\right)^2 - 4\frac{m^2}{M^2}y\right]^{1/2}.$$

The connection of the natural variables in the two frames is made complete by expressing the angular variable in the $e\nu$-frame, $\cos\theta_e = -\cos\theta_\nu$, in terms of variables in the parent rest frame by evaluating $P \cdot p$ in the two frames:

$$M\,x = \tilde{E}_e = \frac{K}{2}\cos\theta_e + \frac{M}{4}\left(1 - \frac{m^2}{M^2} + y\right). \qquad (2.10)$$

The Feynman amplitude is Lorentz invariant and we split the phase space into Lorentz invariant pieces so that it takes on a particularly simple form:

$$d\Pi_3 = \frac{M}{(4\pi)^5}\,K\,dy\,d\Omega_e\,d\tilde{\Omega}_m \qquad (2.10)$$

where $d\Omega_e$ is the solid angle of the electron in the $e\nu$-frame and $d\tilde{\Omega}_m$ is the solid angle of the final meson in the parent rest frame. This gives the differential decay rate:

$$\frac{d\Gamma}{dy\,d\Omega_e\,d\tilde{\Omega}_m} = \frac{1}{2}\,\frac{K}{(4\pi)^5}\,|A|^2. \qquad (2.11)$$

We now calculate the amplitude squared in the $e\nu$-frame. After summing over the electron and neutrino spins one finds the usual result:

$$|A(M \to m\,e\,\bar{\nu})|^2 = \frac{G_F^2}{2}\,|V_{Qq}|^2\,L^{\mu\nu}\,H_\mu H_\nu^\dagger, \qquad (2.12)$$

and when we neglect the mass of the lepton only the spatial components of the lepton tensor are non-zero in the $e\nu$-frame:

$$L^{ij} = 4M^2\, y\left[(\delta^{ij} - \hat{e}^i\hat{e}^j) - i\eta\epsilon^{ijl}\hat{e}^l\right] . \tag{2.13}$$

where $\eta = +1$ for $e\bar{\nu}$ and $\eta = -1$ for $e^+\nu$ final lepton states and $\hat{e}$ is a unit vector along the charged lepton direction in the $e\nu$-frame.

Consequently, we only need the spatial components of $H_\mu$ in the $e\nu$-frame. It is then useful to expand $\mathbf{H}$ in terms of a helicity basis (effectively of the virtual $W$) in that frame:

$$\mathbf{H} = H_+\,\hat{\mathbf{e}}_+ + H_-\,\hat{\mathbf{e}}_- + H_0\,\hat{\mathbf{e}}_0 , \tag{2.14}$$

where

$$\begin{aligned} \hat{\mathbf{e}}_\pm &= \frac{1}{\sqrt{2}}\left[\mp\hat{\mathbf{x}} - i\hat{\mathbf{y}}\right] , \\ \hat{\mathbf{e}}_0 &= \hat{\mathbf{z}} . \end{aligned}$$

Putting Eq. (2.13) and (2.14) into (2.12), we then have

$$\begin{aligned} |A|^2 = \frac{G_F^2}{2}|V_{Qq}|^2\, 4M^2\, y \Bigg[ &\frac{1}{2}(1-\eta\cos\theta_e)^2\,|H_+|^2 \\ &+\frac{1}{2}(1+\eta\cos\theta_e)^2\,|H_-|^2 \\ &+\sin^2\theta_e\,|H_0|^2 + \frac{1}{2}\sin^2\theta_e\,(H_+H_-^* + H_+^*H_-) \\ &-\frac{\eta}{\sqrt{2}}\sin\theta_e(1-\eta\cos\theta_e)(H_+H_0^* + H_+^*H_0) \\ &-\frac{\eta}{\sqrt{2}}\sin\theta_e(1+\eta\cos\theta_e)(H_-H_0^* + H_-^*H_0) .\Bigg] \end{aligned} \tag{2.15}$$

The angular dependence in this equation is entirely a reflection of the $V-A$ character of the $W\to e\nu$ amplitude.

What remains is to relate the helicity amplitudes $H_{+,-,0}$ to the invariant amplitudes defined in Eqs. (2.5) and (2.6). For the case of a pseudoscalar final state, we have simply

$$
\begin{aligned}
H_{\pm} &= 0 \\
H_0 &= -2\,\frac{K}{\sqrt{y}}\, f_+(q^2)\ ,
\end{aligned} \tag{2.16}
$$

and $f_-(q^2)$ makes no contribution, as $(P-q)_\mu$ has no spatial component in the $e\nu$-frame.

For vector meson final states, such as the $K^*$, Eq.(2.6) gives the spatial part of this hadronic current in the $e\nu$-frame as

$$
\mathbf{H} \;=\; 2i\sqrt{y}\; M\; g(q^2)\; \hat{\boldsymbol{\epsilon}}^* \times \mathbf{k} \;-\; f(q^2)\; \hat{\boldsymbol{\epsilon}}^* \;-\; 2\,(\epsilon^* \cdot P)\; a_+(q^2)\; \mathbf{k}\ . \tag{2.17}
$$

The $a_-$ form factor, like $f_-$, does not contribute to $\mathbf{H}$. To proceed further, we express $\hat{\boldsymbol{\epsilon}}$ in terms of the polar and azimuthal angles in the vector-meson's helicity frame, $\theta^*$ and $\phi^*$, and Lorentz transform it into the $e\nu$-frame (only the z-component changes by a factor $E_m/m$), where we express it in the $\hat{\mathbf{e}}_{+,\mathbf{0},-}$ basis:[15]

$$
\hat{\boldsymbol{\epsilon}} = \frac{1}{\sqrt{2}}\sin\theta^* e^{i\varphi^*}\ \hat{\mathbf{e}}_+ \;-\; \frac{1}{\sqrt{2}}\sin\theta^* e^{-i\varphi^*}\ \hat{\mathbf{e}}_- \;-\; \frac{E_m}{m}\cos\theta^*\ \hat{\mathbf{e}}_0\ . \tag{2.18}
$$

Then we have that

$$
\begin{aligned}
H_{\pm} &\equiv \mp\,\frac{1}{\sqrt{2}}\sin\theta^*\ e^{\pm i\varphi^*}\ \bar{H}_{\pm} \\
&= \mp\,\frac{1}{\sqrt{2}}\sin\theta^*\ e^{\pm i\varphi^*}\Big(f(q^2)\ \mp\ 2MK\ g(q^2)\Big)
\end{aligned} \tag{2.19$a$}
$$

and

$$
\begin{aligned}
H_0 &\equiv \cos\theta^*\ \bar{H}_0 \\
&= \cos\theta^*\ \left(\frac{M}{2m\sqrt{y}}\right)\left[\left(1-\frac{m^2}{M^2}-y\right)f(q^2)\ +\ 4K^2\ a_+(q^2)\right]\ .
\end{aligned} \tag{2.19$b$}
$$

Note that $g(q^2)$ only occurs in $H_{\pm}$ and $a_+(q^2)$ only in $H_0$.

In the decay of the final vector-meson, the momentum direction of either of the resulting pseudoscalar particles in the vector-meson rest frame follows the direction of $\hat{\epsilon}$ and acts as a polarization analyzer. The distribution in $\theta^*, \phi^*$ is dictated by the vector-mesons helicity, which must be the same as that of the $W$: Specific $\theta_e$ dependence is tied to particular $\theta^*, \phi^*$ dependence. The relative magnitudes of the helicity amplitudes $H_{+,-,0}$ are in turn related to the sign and magnitude of the invariant amplitudes. Note especially that the relative sign of $f(q^2)$ and $g(q^2)$ yields the relative magnitudes of $H_+$ and $H_-$: It is in this way that the $V - A$ character at the quark level is translated into a positive relative sign and revealed at the hadron level as $|H_-| > |H_+|$.

We are now in a position to put everything together: Eq. (2.16) or (2.19) into the square of the amplitude, Eq. (2.15), and that into the differential decay rate in Eq. (2.11):

$$
\begin{aligned}
\frac{d\Gamma}{dy\, d\Omega_e\, d\tilde{\Omega}_m} = \frac{G_F^2 |V_{Qq}|^2\ K\ M^2\ y}{(4\pi)^5} \Big[ & \Big(\frac{1}{2}(1-\eta\cos\theta_e)^2\Big)\Big(\frac{1}{2}\sin^2\theta^*\Big)\ |\bar{H}_+|^2 \\
& + \Big(\frac{1}{2}(1+\eta\cos\theta_e)^2\Big)\Big(\frac{1}{2}\sin^2\theta^*\Big)\ |\bar{H}_-|^2 \\
& + \sin^2\theta_e\ \cos^2\theta^*\ |\bar{H}_0|^2 \\
& - \Big(\frac{1}{2}\sin^2\theta_e\Big)\Big(\frac{1}{2}\sin^2\theta^*\cos 2\phi^*\Big) 2\ \bar{H}_+\ \bar{H}_- \\
& + \frac{\eta}{\sqrt{2}}\sin\theta_e(1-\eta\cos\theta_e)\Big(\frac{1}{\sqrt{2}}\sin\theta^*\cos\theta^*\cos\phi^*\Big)2\ \bar{H}_+\ \bar{H}_0 \\
& - \frac{\eta}{\sqrt{2}}\sin\theta_e(1+\eta\cos\theta_e)\Big(\frac{1}{\sqrt{2}}\sin\theta^*\cos\theta^*\cos\phi^*\Big)2\ \bar{H}_-\ \bar{H}_0\Big)\ ,\Big]
\end{aligned}
\tag{2.20}
$$

where we have assumed that the amplitudes $H_{+,0,-}$ are relatively real in writing the last three interference terms. At $y = 0$, the overall factor of $y$ causes all the contributions to the rate to vanish except that from $|H_0|^2$, since $H_0$ contains a factor of $1/\sqrt{y}$. At $y = y_{max}$, the overall factor of $K$ vanishes, causing the rate to do likewise.

Before turning back to the differential decay rate and examining the magnitude of the different contributions, let us integrate over all angles and sum over final polarizations to obtain:

$$\frac{d\Gamma}{dy} = \frac{G_F^2 |V_{Qq}|^2 K M^2 y}{96\pi^3} \left[ |\bar{H}_+|^2 \; + \; |\bar{H}_-|^2 \; + \; |\bar{H}_0|^2 \right] \tag{2.21}$$

The helicity amplitudes $\bar{H}_{+,0,-}$, which are related to the form factors by Eq. (2.19) for a final vector-meson, are functions of $y = q^2/M^2$. For a pseudoscalar final state we have in particular from (2.16):

$$\frac{d\Gamma}{dy} = \frac{G_F^2 |V_{Qq}|^2 K^3 M^2}{24\pi^3} |f_+(M^2 \; y)|^2 \; . \tag{2.22}$$

The transverse and longitudinal widths are obtained from the parts of Eq. (2.21) involving $|\bar{H}_+|^2 + |\bar{H}_-|^2$ and $|\bar{H}_0|^2$, respectively, by integrating over $y$. It is the ratio of these quantities which has been obtained by E691 up to now.[8]

## § 3 Some Examples in the Quark Model

A glance at Eq. (2.20) tells us that much more can be learned from studying the full joint decay distribution over the Dalitz plot than from the integrated longitudinal and transverse widths. To understand in detail what can actually be learned, one needs to look at the $y$ distribution for each of the terms in Eq. (2.20) in some typical cases.

We adopt the viewpoint of Ref. 4, in that we use the quark model to calculate the values of the form factors at $y = y_{max}$ where the final meson is at rest, and then we extrapolate them over the full $y$, *i.e.*, $q^2$, range down to $y = 0$. Unlike Ref. 4, we use a pole model to do this extrapolation. We will discuss this more in the next Section.

As a first example, we consider Cabibbo-allowed $D$ decays, with the form factors taken from a specific quark model[16] applied at the point $y = y_{max}$, and the values continued to other values of $y$ using a monopole for each of the form factors with a mass $M_{pole} = 2.11$ GeV. This agrees with the measured[17] form factor in $D \to \bar{K} e^+ \nu_e$, where only the vector current is relevant, and is consistent with the mass of the appropriate meson, the $D_s^*$, as well. It is even more heartening that the decay rate for this processes from the quark model (from integrating Eq. (2.22) over $y$ with $f_+ = 1.20$ at $y = y_{max}$) is $7.1 \times 10^{10}$ sec$^{-1}$, consistent with the experimental value[2] from combining Mark III and E691 data of $\Gamma(D \to K e^+ \nu_e) = 7.8 \pm 1.1 \times 10^{10}$ sec$^{-1}$ .

With these assumptions, the values of the form factors and helicity amplitudes at $y_{min} = 0$ and $y_{max}$ for $D \to \bar{K}^*\ e^+ \nu_e$ are given in Table I. The form factors themselves change by a factor of 1.27 from $y = 0$ to $y = y_{max}$, and none of the helicity amplitudes changes sign over the physical range of $y$, although $\bar{H}_+$ and $\bar{H}_0$ both get contributions from two form factors which enter with opposite signs. The combinations of helicity amplitudes which form the coefficients of the various angular factors in Eq. (2.20) are shown in Figure 3. We note for later reference that it makes little difference in the decay rates whether one use poles, double poles, exponentials, or even a linear form when the form factors change so little over the Dalitz plot, so long as the first derivative is approximately right.

First, $\bar{H}_-$ is generally considerably bigger than $\bar{H}_+$, bearing out our previous comments on how the $V - A$ structure at the quark level is manifested at the hadron level. While the condition $\bar{H}_+ = \bar{H}_-$ is forced by kinematics at $y = y_{max}$, these two amplitudes quickly go their own ways and differ by more than a factor of five at $y = 0$.

Second, and less intuitive, is the large size of $\bar{H}_0$, which leads to an integrated longitudinal width, $4.7 \times 10^{10}$ sec$^{-1}$, comparable to the integrated transverse width, $4.0 \times 10^{10}$ sec$^{-1}$. (Figure 3 is normalized so that the width into a given helicity state is just the area under the $|\bar{H}_\lambda|^2$ curves.) Again, kinematics forces $\bar{H}_0 = \bar{H}_+ = \bar{H}_-$ at $y = y_{max}$, but the amplitudes soon separate so that in the region near $y = 0$ the amplitude $\bar{H}_0$, which contains a factor of $1/\sqrt{y}$, very much dominates the others. This feature is independent of modest changes in the form factors employed:

- Using 2.11 GeV in the vector current form factor and 2.53 GeV for the mass of the pole in the axial-vector current form factors gives $\Gamma_L = 5.2 \times 10^{10}$ sec$^{-1}$ and $\Gamma_T = 4.3 \times 10^{10}$ sec$^{-1}$, about 10% larger than the numbers above for a single value of $M_{pole} = 2.11$ GeV.
- While the negative value of the form factor $a_+$ means that it interferes destructively with $f$ in the helicity amplitude $\bar{H}_0$, even setting $a_+$ to zero, as was done in early versions of Ref. 4, just increases $\Gamma_L$ from $5.2 \times 10^{10}$ sec$^{-1}$ to $7.7 \times 10^{10}$ sec$^{-1}$ (with the axial-vector pole fixed at 2.53 GeV).
- On the other hand, using the form factors of Ref. 5, which almost doubles the magnitude of $a_+$, only results in a decrease in $\Gamma_L$ to $4.5 \times 10^{10}$ sec$^{-1}$. At the same time, shifts in other form factors make $\Gamma_T$ increase to $4.9 \times 10^{10}$ sec$^{-1}$, so that the total width is almost unchanged.

Since the experimental value[8] of the full $K^*$ semileptonic width is $4.1 \pm 0.7 \pm 0.5 \times 10^{10}$ sec$^{-1}$, we see that we are already in some trouble; the quark model form factors give about twice the correct width (as contrasted to $D \to \bar{K} e^+ \nu_e$). It is this, and the ratio of the longitudinal to transverse widths of[8] $2.4^{+1.7}_{-0.9} \pm 0.2$ (obtained solely from the $K^*$ decay angular distribution) to which we alluded in the Introduction as surprisingly at odds with the quark model.

Quite striking in Figure 3 are the large transverse-longitudinal interference terms, which can be picked off through their characteristic $\cos\phi^*$ angular behavior. They peak in the middle of the $y$ range, and have signs which reflect back on the underlying dynamics. They should provide a redundant determination of the relative sizes of the amplitudes, as well as a unique handle on their relative signs.

As another example, we take the decay $\bar{B} \to D^* e^- \bar{\nu}_e$ using quark model form factors[18] at $y = y_{max}$, but this time the monopole mass is chosen as 6.8 GeV (to approximately represent the mass of the relevant bottom-charm mesons), so that the form factors change by a factor of 1.3 from $y = 0$ to $y = y_{max}$ in $\bar{B} \to D^* e^- \bar{\nu}_e$. In the same model, the width for $\bar{B} \to D e^- \bar{\nu}_e$, with $f_+ = 1.16$ at $y = y_{max}$ and $V_{cb} = 0.046$, is $2.6 \times 10^{10}$ sec$^{-1}$ (corresponding to a branching ratio around 3%), and is again, within fairly big experimental errors, consistent with experiment.[19,20] The resulting form factors and helicity amplitudes for $\bar{B} \to D^* e^- \bar{\nu}_e$ are given in Table II, and the combinations of helicity amplitudes which form the coefficients of the angular factors in Eq. (2.20) are shown in Figure 4 as a function of $y$.

As in the case of $D \to \bar{K}^* e^+ \nu_e$, the amplitudes $\bar{H}_-$ and $\bar{H}_+$ differ substantially away from $y = y_{max}$, as does $\bar{H}_0$. The longitudinal and transverse integrated widths are comparable ($2.3 \times 10^{10}$ sec$^{-1}$ and $2.6 \times 10^{10}$ sec$^{-1}$, respectively), and their sum corresponds to a branching ratio of about 5.8% . Both because the experimental $\bar{B} \to D^* e^- \bar{\nu}_e$ branching ratio[21,22] and the ratio of longitudinal to transverse widths[23] is consistent with the numbers above, we are presently quite comfortable here with the predictions of the quark model.

The interference terms in Eq. (2.20) are again very dramatic. The factor of $\eta$ in front of the $\bar{H}_\pm - \bar{H}_0$ interference terms means that the sign of these effects is flipped between $D$ and $B$ decays.

The question now is whether the disagreement in $D \to \bar{K}^* e^+ \nu_e$ is simply a consequence of the inapplicability of the quark model to a situation with a low final quark mass, *i.e.*, $m_s$, or represents some more fundamental breakdown of the quark model for semileptonic decays of hadrons containing heavy quarks. If the latter is the case, we should ask what would be the effect in $\bar{B}^* \to D^* e^- \bar{\nu}_e$ and how detectable will it be using the angular correlations between leptons and hadrons. If, on the other hand it is the former situation, then we still might expect problems in the processes $\bar{B} \to \pi e^- \bar{\nu}_e$ and $\bar{B} \to \rho e^- \bar{\nu}_e$, decays which are crucial to extracting a quantitative value for the Kobayashi-Maskawa matrix element $V_{ub}$. So in either case it will be important to understand what "goes wrong" in $D \to \bar{K}^* e^+ \nu_e$, if anything indeed does disagree with the quark model. In the next Section we consider possible modifications to the quark model amplitudes, how they can be diagnosed experimentally, and what their consequences would be in the various decays we have mentioned.

## § 4: Modifying the Quark Model and its Consequences

If we take as given the disagreement between the quark model and experiment in $D \to K^* e^+ \nu_e$, both in absolute rate and in $K^*$ polarization, then we might look for how we might modify the quark model in order to accommodate these data. There are fairly strong arguments[24] in the context of the quark model that when both the initial and final quarks are heavy and the fractional energy release is small,

$$\left(\frac{\Lambda_{QCD}}{M}\right) << \frac{M^2 - m^2}{2M^2} << 1 ,$$

that the inclusive quark level calculation should be good. It has further been noted[11] that in this regime the sum of the pseudoscalar and vector meson final states, with form factors calculated in the quark model, mock up the inclusive quark level calculation, not just at $y_{max}$, but point by point in the Dalitz plot. It would seem that in this case the quark model "must" be right.

This may well be the case for $B$ decays to charmed mesons. However, in $D$ decay, even if we consider the initial charmed quark as heavy, the final strange quark is not. It is at least possible to contemplate modifications. How might we modify the quark model predictions in $D$ decay, if indeed they need "fixing?"

Proceeding in an *ad hoc* manner, we first ask whether just changing the form factor $a_+$, which had once been under suspicion as to its calculability in the quark model, could change the total $D \to K^* e^+ \nu_e$ width and the longitudinal to transverse ratio so as to better agree with the central values from experiment?[8] The answer is no – Both the total width and the longitudinal to transverse ratio are quadratic functions of $a_+$ with a common minimum, and one cannot make the total width smaller while at the same time increasing the longitudinal to transverse ratio (which is what is needed when starting from the quark model).

A second *ad hoc* procedure is to rescale the form factors that contribute to the transverse helicity states, $f$ and $g$. Viewed from the perspective of the quark model, especially when applied at $y = y_{max}$ with the final meson at rest, it makes little sense to single out a particular polarization state. It has more appeal when viewed in terms of an infinite-momentum-frame approach where the transverse polarizations correspond to so-called "bad" operators. This has been especially considered in Ref. 9 .

An example of such rescaling is to take:

$$\begin{aligned} f &\rightarrow \lambda\, f \\ g &\rightarrow \lambda\, g \;. \end{aligned} \tag{4.1}$$

This obviously reduces both the transverse helicity amplitudes by a factor $\lambda$, but since the form factor $f$ also enters the helicity zero amplitude, that changes also. We are left with the form factor $a_+$ at our potential disposal. Leaving it with a value from the quark model calculations discussed in the previous Section gives a poor fit to the data of Ref. 8.

Instead, we have made a search in the $\lambda$, $a_+$ parameter space for values which fit both the total decay rate and the longitudinal to transverse ratio in the decay $D \rightarrow K^* e^+ \nu_e$ within the experimental errors. An example of the amplitudes which are close to optimum for fitting the central values of the data[8] is shown in Figure 5. Here $\lambda = 0.5$ and $a_+ = 0.1$ GeV$^{-1}$. The resulting longitudinal and transverse decay rates are $2.2 \times 10^{10}$ sec$^{-1}$ and $1.0 \times 10^{10}$ sec$^{-1}$, respectively.

Note that $a_+$ is of opposite sign to that given by the quark model. This is because we have rescaled $f$ and $g$ in order to lower the transverse portion of the decay rate dramatically, but then the longitudinal part would decrease as well (through the form factor $f$). Since the magnitude of the longitudinal decay rate was of about the right magnitude beforehand, it is necessary to change the sign of $a_+$ so as to get constructive rather than destructive interference with $f$ in the longitudinal helicity amplitude, and thereby boost the longitudinal decay rate back up to near its central value experimentally. On comparing Figures 5 and 3, we see the suppression of the transverse amplitudes (but still the large longitudinal transverse interference terms) and the rapid rise of $|\bar{H}_0|^2$ as $y \rightarrow 0$ because of the constructive interference of $f$ and $a_+$. These results are similar to those of Ref. 9, but those authors determine $a_+$ theoretically in the course of their rescaling procedure; we obtain a value (close to theirs) from approximately fitting the experimental data.

We could have also obtained a reasonable fit to the data of Ref. 8 by leaving $g$ alone. An example of this is shown in Figure 6, where again in an *ad hoc* manner we have modified $f$ and $a_+$ compared to the quark model as in the previous example, but not $g$. Note the zero in the amplitude $\bar{H}_+$ which arises because $f$ has been reduced and the cancelling contribution of $g$ in $\bar{H}_+$ is then able to overcome it in the middle of the physical region.

Overall we see that a fairly drastic change from the quark model is required to fit the central values of the data from Ref. 8. A small modification of one form factor will not do it.

Now let us consider what happens in $B$ decay. As already noted, there seems to be good reason to trust the calculation of $\bar{B} \to De^-\bar{\nu}_e$ and $\bar{B} \to D^*e^-\bar{\nu}_e$. This is not true for $\bar{B} \to \pi e^-\bar{\nu}_e$ or $\bar{B} \to \rho e^-\bar{\nu}_e$, where light quarks appear in the final state and the $q^2$ range is large.

If we nevertheless proceed in a straightforward manner and apply the quark model of Ref. 6 at $y = y_{max}$, then we find the results[25] given in Table III and Figure 7 for $\bar{B} \to \rho e^-\bar{\nu}_e$. Here we have used a vector current form factor pole at $M_{pole} = 5.33$ and an axial-vector one at $M_{pole} = 5.75$ GeV. Note that in this case, more than any other we have considered, the poles are very close to the edge of the physical region. This means that on the one hand, they should indeed dominate the behavior of the form factors near $y = y_{max}$. On the other hand, they change by roughly a factor of three (for the $\rho e^-\bar{\nu}_e$ final state) in the $q^2$ range which is available (see Table III), and different form factors give quite distinct results.

In any case, the results (see Figure 7) are quite spectacular:

- There are zeroes in both the $\bar{H}_+$ and $\bar{H}_0$ amplitudes at different values of $y$. This makes for an especially interesting behavior of the interference terms between helicity amplitudes, which each have a characteristic $\phi$ dependence given in Eq. (2.20).

- The $\bar{H}_+$ ($\bar{H}_-$) amplitude gets destructive (constructive) contributions from the $f$ and $g$ form factors. The zero in $\bar{H}_+$ arises here since $g$ is much larger than for $\bar{B} \to D^* e^- \nu_e$, and its coefficient of $2M_B\ K$ in Eq. (2.19a) causes it's contribution to this helicity amplitude to quickly catch up to that of $f$ as we move away from $y = y_{max}$. Of course this increases $\bar{H}_-$ correspondingly, according to Eq. (2.19a).

- A similar situation, but less dramatic, pertains to the amplitude $\bar{H}_0$ where $f$ and $a_+$ destructively interfere. Still, with our choice of parameters a zero results near $y = 0$. This pulls the whole longitudinal portion of the decay rate down and it is far smaller than the transverse one ($0.012 \times 10^{10}$ sec$^{-1}$ compared to $0.071 \times 10^{10}$ sec$^{-1}$).

- These zeros persist, even though the magnitudes of the ampitudes change substantially, if we use all exponential form factors, all double poles, or all single poles with somewhat different masses than we took initially. However, the zeros go away if we used a single pole for $f$, but double poles for $g$ and $a_+$, as in Ref. 12, for in this case the double pole (chosen, as before, quite near the physical region) causes the latter two form factors to fall-off so fast that they are not large enough to overcome $f$ in either $\bar{H}_+$ or $\bar{H}_0$.

- The rate for $\bar{B} \to \pi e^- \bar{\nu}_e$ is extremely sensitive to what we take for the form factor, for most of the decay rate comes from the region near $y = 0$, while we are fixing the amplitude at $y = y_{max}$, very near the pole, and using a form factor to extrapolate to $y = 0$. With $M_{pole} = 5.33$ GeV$^{-1}$, there is a factor of about 14 in the form factor between these two $y$ values! The extrapolation to $y = 0$ using a single pole is unreliable, but other choices of form factors can be criiicized as well. We very much need experiment to guide us here.

- If we scale the $f$ and $g$ form factors by a factor of 0.5 and set $a_+ = 0.1$ GeV$^{-1}$, as we did for $D \to K^* e^+ \nu_e$, then we get the situation shown in Figure 8.

The zero in the $\bar{H}_-$ amplitude remains, but that in $\bar{H}_0$ is gone, for $a_+$ now interferes constructively with $f$ in that amplitude.

Thus, it would appear that serious modifications of the quark model amplitudes for exclusive semileptonic decays need to be considered, at least when there are light quarks in the final state, if the central values of the E691 experiment for $D \to K^* \, e^+ \, \nu_e$ persist with further data and analysis. These modifications have dramatic effects on the helicity amplitudes for decays with final vector mesons, which show up in the full, joint angular distribution. The behavior as a function of $q^2$ of the coefficients of the terms in the joint angular distribution is a powerful tool for untangling these amplitudes, and especially for seeing the zeros that might occur, the relative signs of amplitudes, and their magnitudes in a clean way.

## Acknowledgements

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## Table I

Variation of form factors and helicity amplitudes between $y = 0$ and $y = y_{max} = 0.27$ for the decay $D \to K^* \, e^+ \, \nu_e$ with form factors from the quark model,[16] and $M_{pole} = 2.11$ GeV.

| | Value at $y_{max}$ | Value at $y_{min} = 0$ |
|---|---|---|
| K (GeV) | 0 | 0.72 |
| f (GeV) | 2.58 | 2.03 |
| g ($GeV^{-1}$) | 0.68 | 0.53 |
| $a_+$ ($GeV^{-1}$) | -0.26 | -0.20 |
| $\bar{H}_+$ (GeV) | 2.58 | 0.60 |
| $\bar{H}_-$ (GeV) | 2.58 | 3.46 |
| $\sqrt{y}\bar{H}_0$ (GeV) | 1.35 | 1.20 |

## Table II

Variation of form factors and helicity amplitudes between $y = 0$ and $y = y_{max} = 0.38$ for the decay $B \to D^* \, e^- \, \bar{\nu}_e$ with form factors from the quark model,[18] and $M_{pole} = 6.8$ GeV.

| | Value at $y_{max}$ | Value at $y_{min} = 0$ |
|---|---|---|
| K (GeV) | 0 | 2.25 |
| f (GeV) | 6.51 | 5.01 |
| g ($GeV^{-1}$) | 0.17 | 0.13 |
| $a_+$ ($GeV^{-1}$) | -0.15 | -0.11 |
| $\bar{H}_+$ (GeV) | 6.51 | 1.88 |
| $\bar{H}_-$ (GeV) | 6.51 | 8.15 |
| $\sqrt{y}\bar{H}_0$ (GeV) | 4.03 | 2.61 |

## Table III

Variation of form factors and helicity amplitudes between $y = 0$ and $y = y_{max} = 0.73$ for the decay $\bar{B} \to \rho\ e^-\ \bar{\nu}_e$ with form factors from the quark model,[25] with $M_{pole} = 5.33$ GeV for vector current, and 5.75 GeV for axial-vector current form factors.

| | Value at $y_{max}$ | Value at $y_{min} = 0$ |
|---|---|---|
| K (GeV) | 0 | 2.58 |
| f (GeV) | 4.03 | 1.55 |
| g ($GeV^{-1}$) | 0.58 | 0.17 |
| $a_+$ ($GeV^{-1}$) | -0.20 | -0.08 |
| $\bar{H}_+$ (GeV) | 4.03 | -2.94 |
| $\bar{H}_-$ (GeV) | 4.03 | 6.05 |
| $\sqrt{y}\bar{H}_0$ (GeV) | 3.44 | -1.83 |

## REFERENCES


1. H. Leutwyler and M. Roos, Z. Phys. C25, 91 (1984).
2. Particle Data Group, Phys. Lett. 204B, 107 (1988).
3. G. Altarelli *et al.*, Nucl. Phys. B208, 365 (1982).
4. B. Grinstein, N. Isgur, and M. B. Wise. Phys. Rev. Lett. 56, 298 (1986); Caltech preprint CALT-68-1311, 1986 (unpublished); B. Grinstein, N. Isgur, D. Scora, and M. B. Wise, Phys. Rev. D39, 799 (1989).
5. M. Bauer, B. Stech and M. Wirbel, Z. Phys. C29, 637 (1985).
6. T. Altomari and L. Wolfenstein, Phys. Rev. Lett. 58, 1583 (1987) and Phys. Rev. D37, 681 (1988).
7. J. G. Korner and G. A. Schuler, Z. Phys. C38, 511 (1988).
8. J. C. Anjos *et al.*, Phys. Rev. Lett. 62, 722 (1989). The Mark III result for the decay width is consistent with E691: J. M. Izen, SLAC preprint SLAC-PUB-4753, 1988 (unpublished).
9. M. Bauer and M. Wirbel, Heidelberg preprint HD-THEP-88-22, 1988 (unpublished).
10. E. Golowich *et al.*, Phys. Lett. 213B, 521 (1988).
11. N. Isgur and D. Scora, Toronto preprint UTPT-89-12, 1989 (unpublished).
12. J. G. Korner and G. A. Schuler, Mainz preprints MZ-TH/88-14, 1988 and MZ-TH/89-01, 1989 (unpublished).
13. K. Hagiwara, A. D. Martin, and M. F. Wade, Durham preprint DTP/89/12, 1989 (unpublished).
14. There is one form factor for the pseudoscalar and one for the vector meson final state whose contributions to the respective amplitudes are proportional to the lepton mass. For muons, neither the contributions from these form

factors nor the phase space corrections are large. For taus, see J. G. Korner and G. A. Schuler, DESY preprint DESY 89/062, 1989 (unpublished).

15. The vector-meson's helicity frame has the -x, y, and -z axes as a coordinate frame, and the angles usual azimuthal angles $\theta$ and $\phi$ are related to $\theta^*$ and $\phi^*$ by $\theta = \pi + \theta^*$ and $\phi = -\phi^*$.

16. We use the formulas of Ref. 6, $f_+ = \sqrt{m/M}[1+(M-m)/2m_q]$, $f = \sqrt{4\ m\ M}$, $g = (1/2m_q)\sqrt{m/M}$, and $a_+ = -(1/\sqrt{4mM})[1+(m/M)(1-m/m_q)]$ for the values of the form factors at $y = y_{max}$, but very similar results are obtained if Ref. 4 or Ref. 5 is used instead. Note that although we use the values of the form factors at $y = y_{max}$ as a starting point, we take monopole form factors rather than the exponential form factors of Ref. 4. The values to be compared to our Table I from Ref. 5 are $f = 2.85$ GeV, $g = 0.56$ GeV$^{-1}$, and $a_+ = -0.45$ GeV$^{-1}$.

17. J. C. Anjos *et al.*, Phys. Rev. Lett. 62, 1587 (1989).

18. We apply the formulas of Ref. 6 with $M = 5.28$ GeV, $m = 2.01$ GeV, and $m_c = 1.8$ GeV. The values from Ref. 11 for the form factors are $f = 6.86$ GeV, $g = 0.16$ GeV$^{-1}$, and $a_+ = -0.15$ GeV$^{-1}$, very close to the values in Table II. There is even less variation between different versions of the quark model predictions here than in $D$ decay.

19. M. Danilov, invited talk at the XIV International Symposium on Lepton and Photon Interactions, Stanford, August 6 - 12, 1989 (unpublished) quotes an ARGUS branching ratio of $1.7 \pm 0.5 \pm 0.5$ %.

20. P. Baringer, invited talk at the 1989 SLAC Summer Institute on Particle Physics, July 10 - 21, 1989 (unpublished) quotes a preliminary CLEO branching ratio of $2.4 \pm 0.8\ ^{+0.7}_{-0.8}$ %.

21. An ARGUS value of $5.4 \pm 0.9 \pm 1.3$ % is quoted in Ref. 19 for $B^0$ decay.

22. CLEO values of $4.6 \pm 0.5 \pm 0.7$ % and $3.9 \pm 0.8\ ^{1.1}_{0.8}$ % are quoted for neutral and charged $B$ decays, respectively, in Ref. 20.

23. H. Albrecht *et al.*, Phys. Lett. B219, 121 (1989) find a value of $\Gamma_L/\Gamma_T 0.85 \pm 0.45$

24. M. A. Shifman and M. B. Voloshin, Yad. Fiz. 47, 801 (1988) [Sov. J. Nucl. Phys. 47, 511 (1988)].

25. We use the formulas of Ref. 6 with $m_u = 0.33$ GeV. For definiteness, we choose $V_{ub} = 0.005$, and our results can be scaled easily to other values.

## FIGURE CAPTIONS

1) The semileptonic decay of a heavy quark, $Q$, into a lighter quark, $q$, and a virtual $W$ which becomes a lepton and neutrino.

2) Coordinate system for semileptonic decay of a heavy meson: (a) The decaying virtual $W$, (b) The decaying final vector meson.

3) The coefficients in a quark model[16] of the angular factors in Eq. (2.20), as a function of $y$ for the decay $D \to \bar{K}^* e^+ \nu_e$, where $\eta = -1$: (++ solid curve) $\bar{H}_+^2$, (− − solid curve) $\bar{H}_-^2$, (00 solid curve) $\bar{H}_0^2$, (dashed curve) $-2\bar{H}_+ \bar{H}_-$, (dotted curve) $2\eta\ \bar{H}_+\bar{H}_0$, (dash-dotted curve) $-2\eta\ \bar{H}_-\ \bar{H}_0$, all multiplied by $G_F^2|V_{cs}|^2 K M_D^2 y/96\pi^3$ with $V_{cs} = 0.975$ .

4) The coefficients in a quark model[18] of the angular factors in Eq. (2.20), as a function of $y$ for the decay $\bar{B} \to D^* e^- \bar{\nu}_e$, where $\eta = +1$: (++ solid curve) $\bar{H}_+^2$, (− − solid curve) $\bar{H}_-^2$, (00 solid curve) $\bar{H}_0^2$, (dashed curve) $-2\bar{H}_+ \bar{H}_-$, (dotted curve) $2\eta\ \bar{H}_+\bar{H}_0$, (dash-dotted curve) $-2\eta\ \bar{H}_-\ \bar{H}_0$, all multiplied by $G_F^2|V_{cb}|^2 K M_B^2 y/96\pi^3$ with $V_{cb} = 0.046$ .

5) The coefficients, with rescaling of the $f$ and $g$ form factors by a factor $\lambda = 0.5$ from Figure 3 and changing $a_+ = +0.1$ GeV$^{-1}$ at $y = y_{max}$ (see text), of the angular factors in Eq. (2.20), as a function of $y$ for the decay $D \to \bar{K}^* e^+ \nu_e$, where $\eta = -1$: (++ solid curve) $\bar{H}_+^2$, (− − solid curve) $\bar{H}_-^2$, (00 solid curve) $\bar{H}_0^2$, (dashed curve) $-2\bar{H}_+ \bar{H}_-$, (dotted curve) $2\eta\ \bar{H}_+\bar{H}_0$, (dash-dotted curve) $-2\eta\ \bar{H}_-\ \bar{H}_0$, all multiplied by $G_F^2|V_{cs}|^2 K M_D^2 y/96\pi^3$ with $V_{cs} = 0.975$ .

6) The coefficients, with rescaling of the $f$ form factor by a factor $\lambda = 0.5$ from Figure 3 and changing $a_+ = +0.1\ \text{GeV}^{-1}$ at $y = y_{max}$ (see text), of the angular factors in Eq. (2.20), as a function of $y$ for the decay $D \to \bar{K}^* e^+ \nu_e$, where $\eta = -1$: (++ solid curve) $\bar{H}_+^2$, (– – solid curve) $\bar{H}_-^2$, (00 solid curve) $\bar{H}_0^2$, (dashed curve) $-2\bar{H}_+\ \bar{H}_-$, (dotted curve) $2\eta\ \bar{H}_+\bar{H}_0$, (dash-dotted curve) $-2\eta\ \bar{H}_-\ \bar{H}_0$, all multiplied by $G_F^2|V_{cs}|^2 K M_D^2 y/96\pi^3$ with $V_{cs} = 0.975$ .

7) The coefficients in a quark model[25] of the angular factors in Eq. (2.20), as a function of $y$ for the decay $\bar{B} \to \rho\ e^-\ \bar{\nu}_e$, where $\eta = +1$: (++ solid curve) $\bar{H}_+^2$, (– – solid curve) $\bar{H}_-^2$, (00 solid curve) $\bar{H}_0^2$, (dashed curve) $-2\bar{H}_+\ \bar{H}_-$, (dotted curve) $2\eta\ \bar{H}_+\bar{H}_0$, (dash-dotted curve) $-2\eta\ \bar{H}_-\ \bar{H}_0$, all multiplied by $G_F^2|V_{ub}|^2 K M_B^2 y/96\pi^3$ with $V_{ub} = 0.005$ .

8) The coefficients, with rescaling of the $f$ and $g$ form factors by a factor $\lambda = 0.5$ from Figure 7 and changing $a_+ = +0.1\ \text{GeV}^{-1}$ at $y = y_{max}$ (see text), of the angular factors in Eq. (2.20), as a function of $y$ for the decay $\bar{B} \to \rho\ e^-\ \bar{\nu}_e$, where $\eta = +1$ with form factors rescaled as in the text: (++ solid curve) $\bar{H}_+^2$, (– – solid curve) $\bar{H}_-^2$, (00 solid curve) $\bar{H}_0^2$, (dashed curve) $-2\bar{H}_+\ \bar{H}_-$, (dotted curve) $2\eta\ \bar{H}_+\bar{H}_0$, (dash-dotted curve) $-2\eta\ \bar{H}_-\ \bar{H}_0$, all multiplied by $G_F^2|V_{ub}|^2 K M_B^2 y/96\pi^3$ with $V_{ub} = 0.005$ .

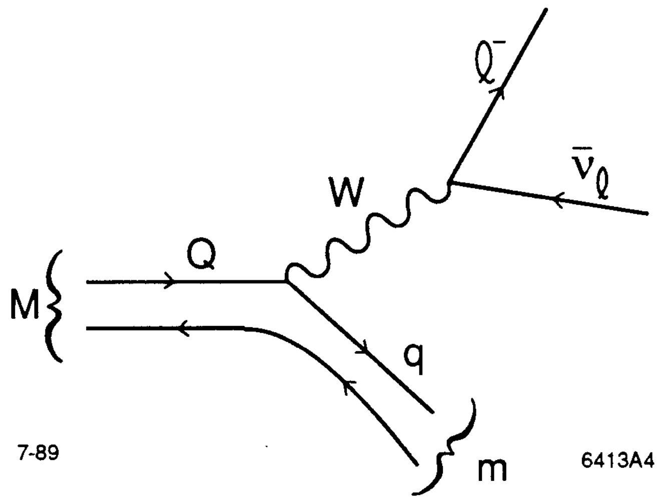


Fig. 1

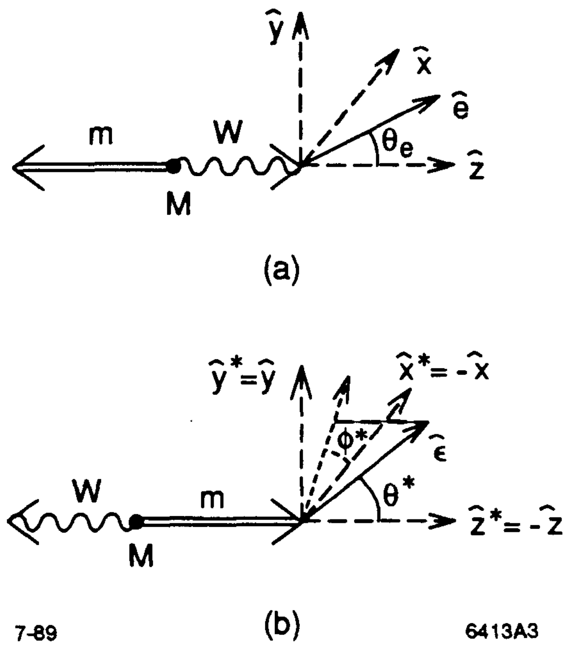


Fig. 2

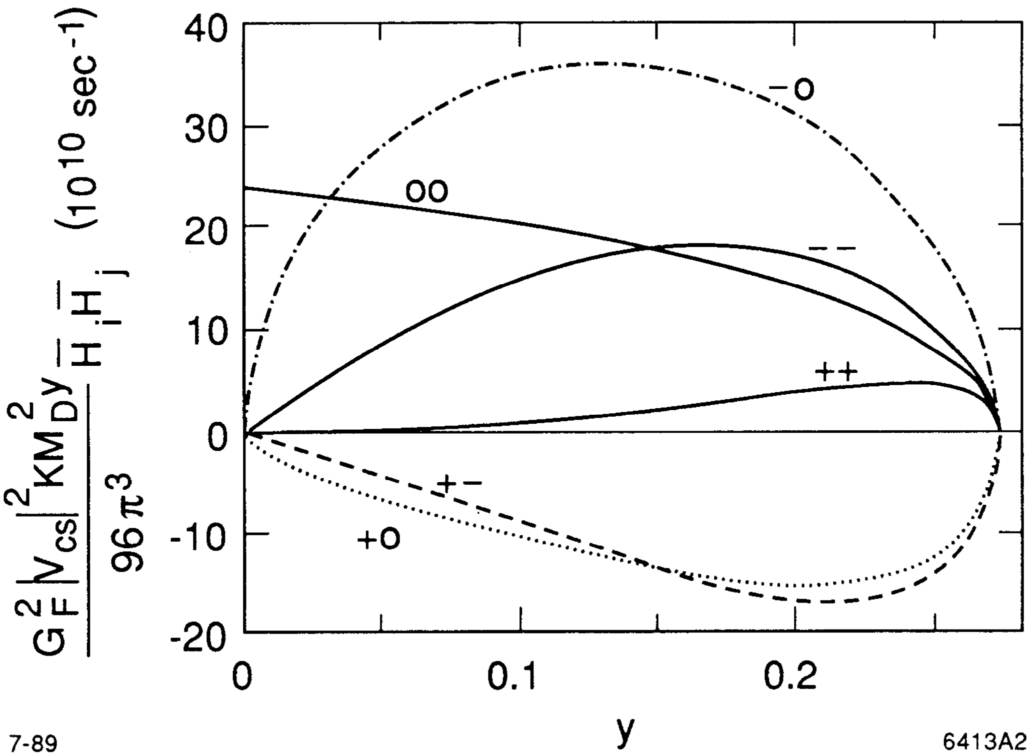
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Fig. 3

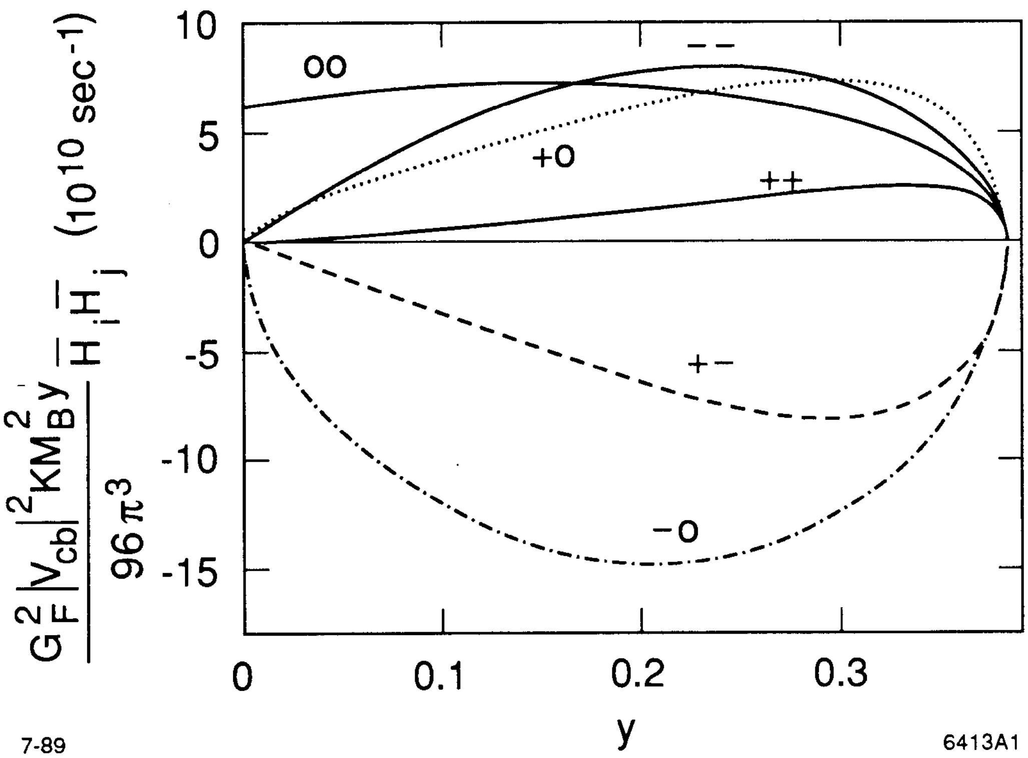

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Fig. 4

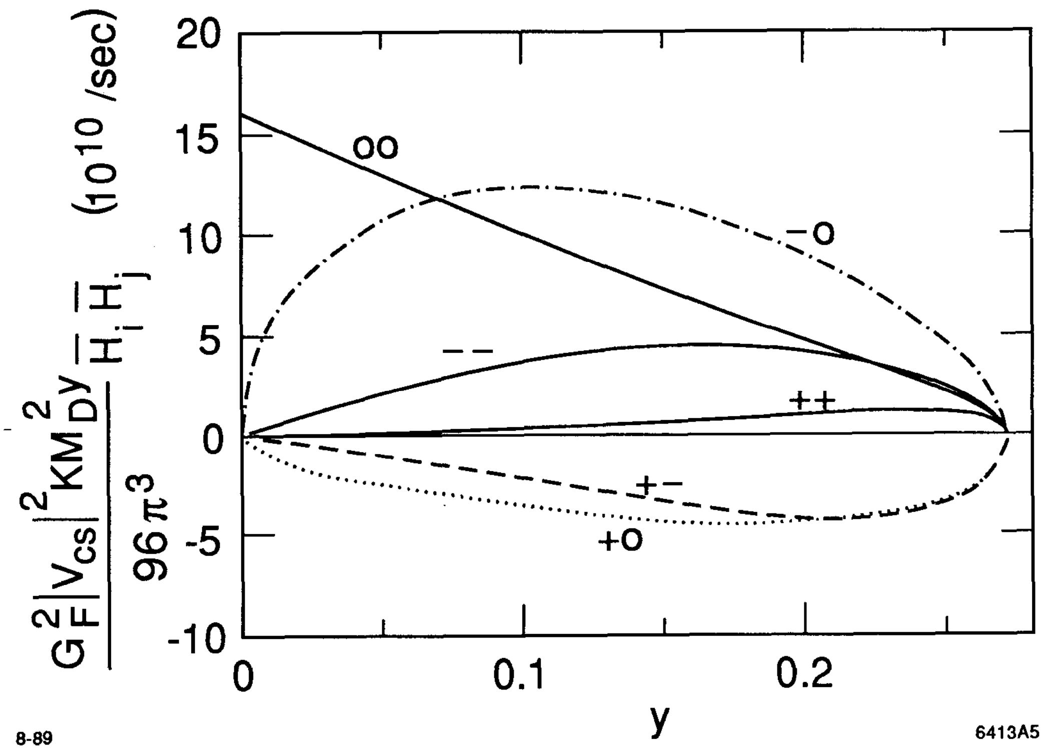
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Fig. 5

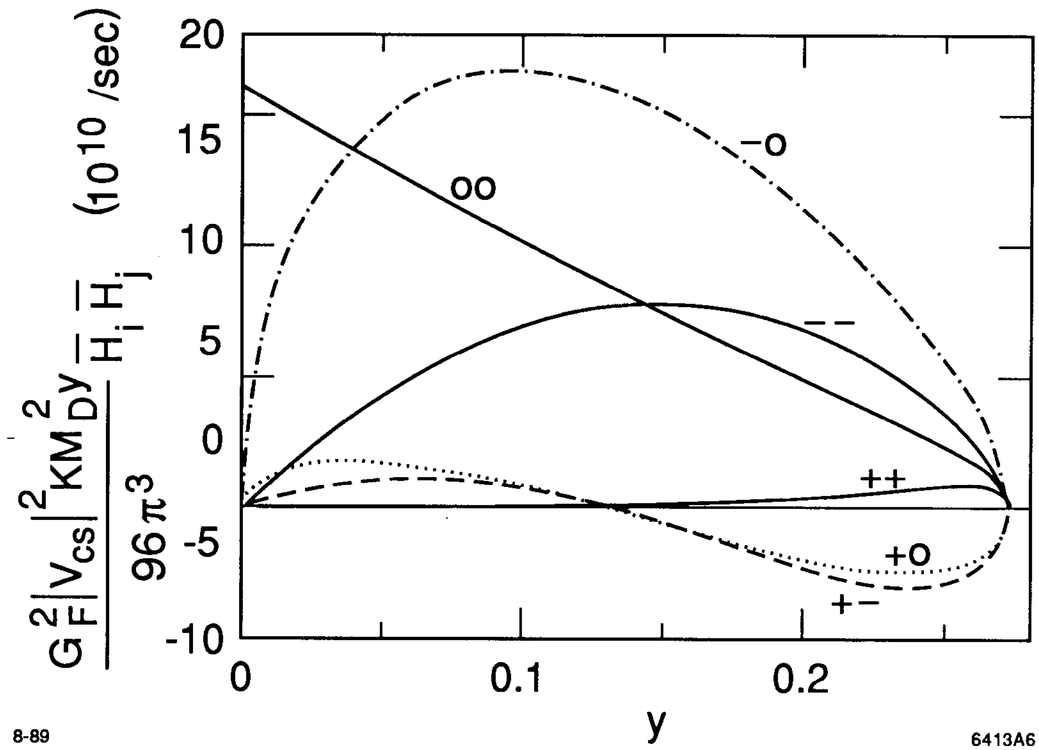


Fig. 6

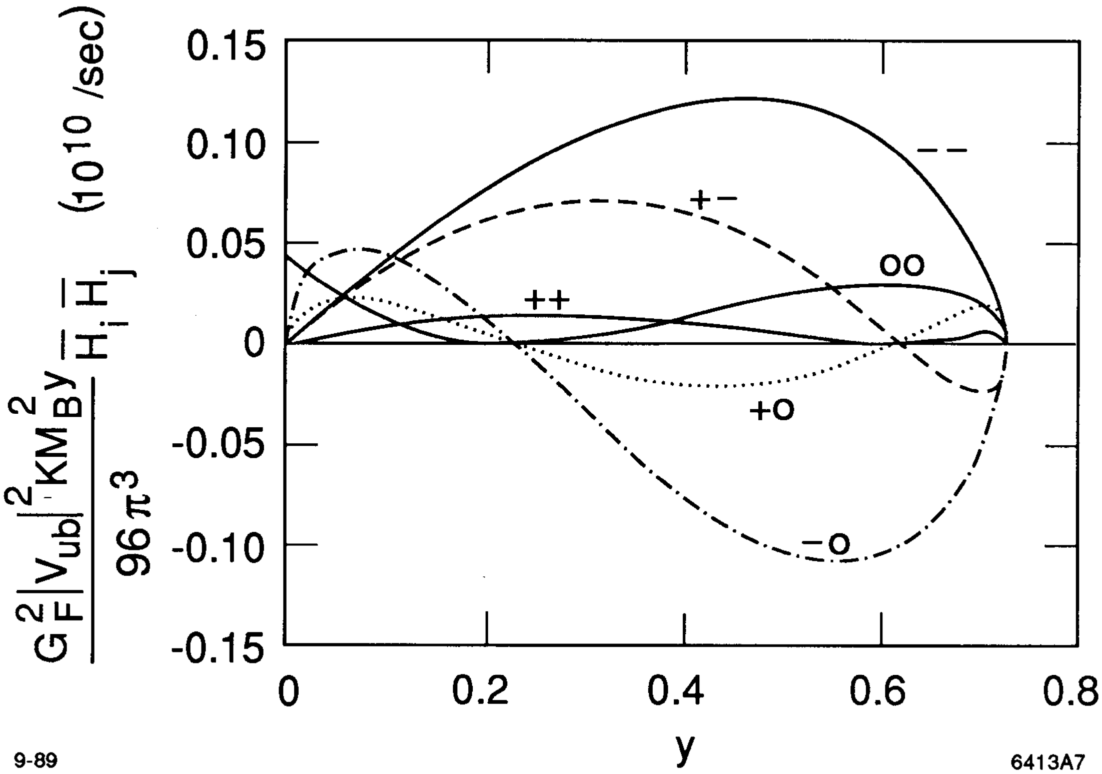


Fig. 7

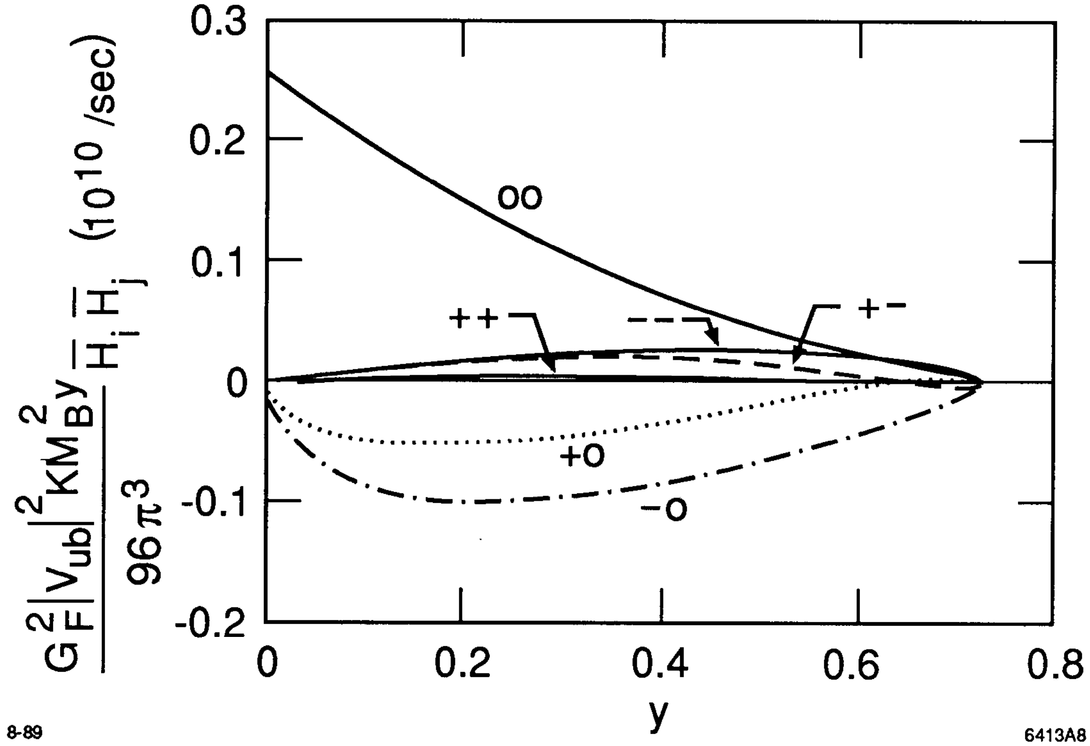


Fig. 8