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LONG-RANGE WAKE POTENTIALS IN DISK-LOADED ACCELERATING STRUCTURES*

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Abstract To achieve a reasonable efficiency for the conversion of RF power to beam power, and consequently, to attain a higher luminosity, most designs for future high energy linear colliders require the acceleration of ten or so bunches per RF pulse. In such bunch trains, however, the long-range longitudinal wake potential produces a cumulative energy deviation from the first to the last bunch, while the transverse wake potential leads to cumulative growth in the transverse emittance. We have investigated the dependence of these long-range potentials on the disk aperture radius for typical $2\pi/3$-mode disk-loaded structures. We find that, excluding the lowest frequency mode, the rms long-range longitudinal and transverse wake potentials scale approximately as $(a/\lambda)^{-2}$ and $(a/\lambda)^{-3}$, respectively, where a is the aperture radius and λ is the wavelength. Furthermore, by using a structure with two different disk apertures, it is possible to produce beats in the long-range transverse wake potential. By placing the following bunch at one of the relatively broad nulls in this wake, it is possible to substantially reduce the transverse bunch-to-bunch coupling and hence, the emittance growth in a multibunch train.

INTRODUCTION

Wake potentials can be computed in two ways: directly in the time domain by a computer code which solves Maxwell's equations on a mesh, or by a sum over synchronous modes computed by a frequency domain code. In the next section, representative frequency and time domain calculations are compared to show that they give equivalent results. Since it involves less computation time, the modal sum method is then used to investigate the long-range wake potentials for several values of a/λ, where a is the disk aperture radius and λ is the free space wavelength.

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It is found that both the longitudinal and transverse wake potentials due to the higher modes only (the lowest frequency mode having been subtracted out) follow a simple scaling as a function of a/λ.

An important feature of the dipole wake potential for large a/λ is the strong dominance of the lowest frequency mode compared to the sum of all higher modes. If an accelerating structure is composed of alternating sections having different disk apertures, then the lowest frequency mode in the two types of structure will also occur at slightly different frequencies. These frequencies will then beat together to form a wake with periodic nulls. The nulls will be relatively clean if the amplitude of the higher mode wake is small compared to the wake due to the lowest frequency mode. An example of such a "beat-wave" structure is given in the final section.

COMPARISON OF FREQUENCY AND TIME DOMAIN CALCULATIONS

In a cylindrically symmetric structure, the wake potentials can be expressed as a multipole expansion with terms that vary with azimuthal angle ϕ according to cos mϕ, where $m = 0$ gives the azimuthally symmetric (longitudinal) modes, $m = 1$ gives the dipole modes, etc. The longitudinal wake potential behind a Gaussian bunch of rms bunch length σ_z and unit charge, moving in the beam aperture region of such a cylindrically symmetric structure, is given by[3]

$$W_z(s) = \sum_n 2k_{0n} \exp\left(-\omega_{0n}^2\sigma_z^2/2c^2\right) \cos\left(\omega_{0n}s/c\right) \quad . \tag{1a}$$

Here, s is the distance behind the center of the bunch, ω_{0n} is the angular frequency of the nth synchronous mode and k_{0n} is the loss factor for a point bunch, given by

$$k_{0n} = E_{zn}^2/4u_{0n} \quad , \tag{1b}$$

where u_{0n} is the stored energy per unit length in the nth mode and E_{zn} is the synchronous component of the axial electric field. The dipole wake potential is given by[3]

$$\vec{W}_d(s) = \frac{2\vec{r}_0}{a}\sum_n \frac{k_{1n}}{(\omega_{1n}a/c)} \exp\left(-\omega_{1n}^2\sigma_z^2/2c^2\right) \sin\left(\omega_{1n}s/c\right) \quad , \tag{2a}$$

where r_0 is the offset of the driving charge (r_0 is set equal to a in the examples to follow) and

$$k_{1n} = \frac{[E_{zn}(r=a)]^2}{4u_{1n}} \quad . \tag{2b}$$

Here, $E_{zn}(r=a)$ is the synchronous longitudinal electric field component evaluated at the disk hole radius.

Values of the ω_{0n}'s and k_{0n}'s for the longitudinal modes can be obtained using the program KN7C.[4] Values of the ω_{1n}'s and k_{1n}'s for the deflecting (dipole) modes can be obtained from the program TRANSVRS.[5] Both codes model a disk-loaded structures by an infinite periodic sequence of alternating irises and cylinders. The structure geometry is entirely specified by only four parameters: the radius of the iris aperture a, the radius of the cylinder between the irises b, the length of one period p, and the cylinder length g (period length minus the iris thickness). No allowance is made for rounding at the edges of the iris or for other details of the structure geometry. Thus, the high-frequency details of the wake potentials calculated using this simplified model are not meaningful. Alternatively, the meaningful features of the wake can be obtained by summing a limited number of modes (we take typically the lowest 25–30 modes). A bunch length is used such that $\omega_n \sigma_z / \sqrt{2}\, c \approx 1$ for the highest frequency mode in the sum. The damping provided by the exponential factor then helps prevent the nonphysical ringing that can occur when such a series is abruptly terminated.

The plots at the top of Figs. 1 and 2 show the longitudinal and dipole wake potentials behind a driving bunch with $\sigma_z/\lambda = 0.025$ (longitudinal), and $\sigma_z/\lambda = 0.050$ (dipole), calculated from a sum over modes for the average cell in the SLAC disk loaded structure ($a/\lambda = 0.111$, $p/\lambda = 1/3$, $g/\lambda = 0.278$, $b/\lambda = 0.391$, and $\lambda = 10.5$ cm). At the bottom of Figs. 1 and 2 are the wake potentials computed directly by the time domain code TBCI[6] for the same bunch length and structure. The qualitative features of the wake potentials computed in these two independent ways in general agree quite well, although there are some differences in detail for the dipole wake. For example, there is less detail in the long-range TBCI dipole wake potential than in the modal sum dipole potential. The reason for these differences is not well understood. Since the modal sum method uses less computer time and probably represents the long-range wake potentials more accurately, we use this method in the following section to compute the scaling of the potentials vs. a/λ.

SCALING OF WAKE POTENTIALS WITH BEAM APERTURE

The modal sums for the longitudinal and dipole wake potentials, as given by Eqs. (1a) and (2a), have been carried out for the disk-loaded structure geometry discussed in the previous section for $a/\lambda = 0.100, 0.111, 0.150, 0.200$, and 0.276. For each value of a/λ, the cylinder radius b was adjusted to keep the accelerating mode frequency fixed at 2856 MHz ($\lambda_0 = 10.5$ cm). The group velocities for the synchronous $2\pi/3$ accelerating mode corresponding to these apertures are $v_g/c = 0.0095, 0.0135, 0.038, 0.085$, and 0.185, respectively. The wake potentials for the last four cases are shown in Figs. 3 and 4. The wake potentials for the lowest frequency mode and for the sum of all higher mode are plotted separately. Note the rapid decrease in the amplitudes of the higher mode wake potentials as the aperture is increased, especially in the

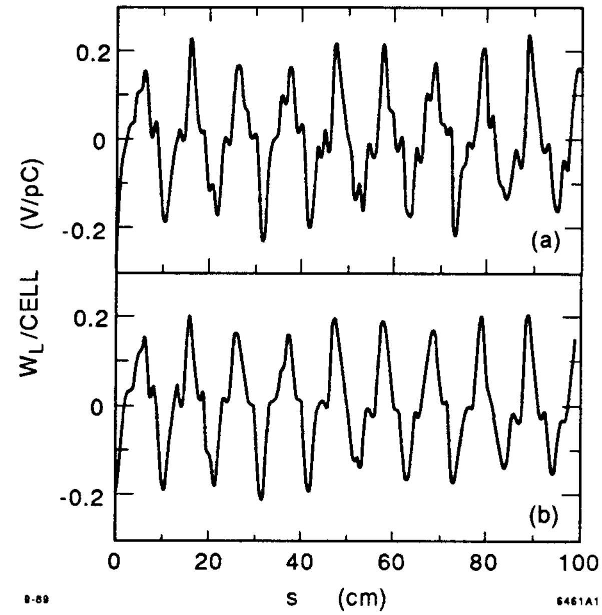

FIGURE 1 (a) Longitudinal wake potential for the SLAC disk-loaded structure ($a/\lambda = 0.111$) calculated from a sum of modes with $\sigma_z/\lambda = 0.025$. (b) Longitudinal wake potential for the SLAC structure calculated by TBCI (three cells with beam tubes, $\sigma_z/\lambda = 0.025$).

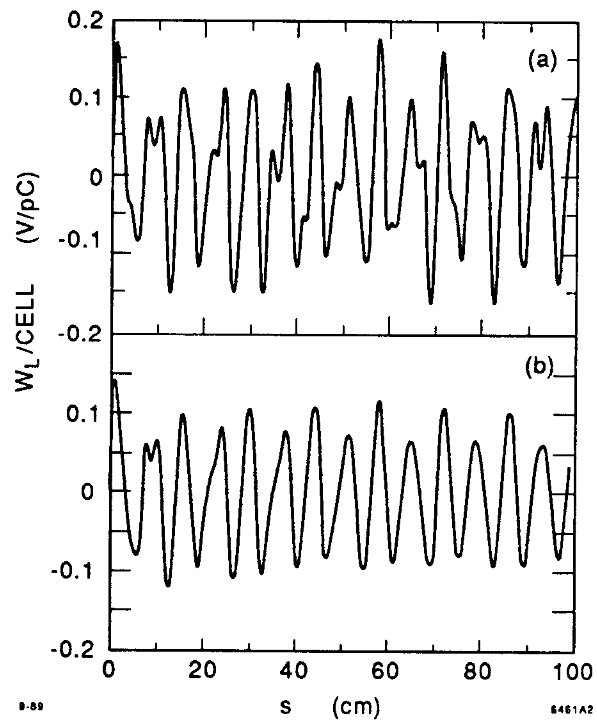

FIGURE 2 (a) Dipole wake potential for the SLAC structure calculated from a sum of modes ($\sigma_z/\lambda = 0.050$). (b) Dipole wake potential for the SLAC structure calculated by TBCI ($\sigma_z/\lambda = 0.050$).

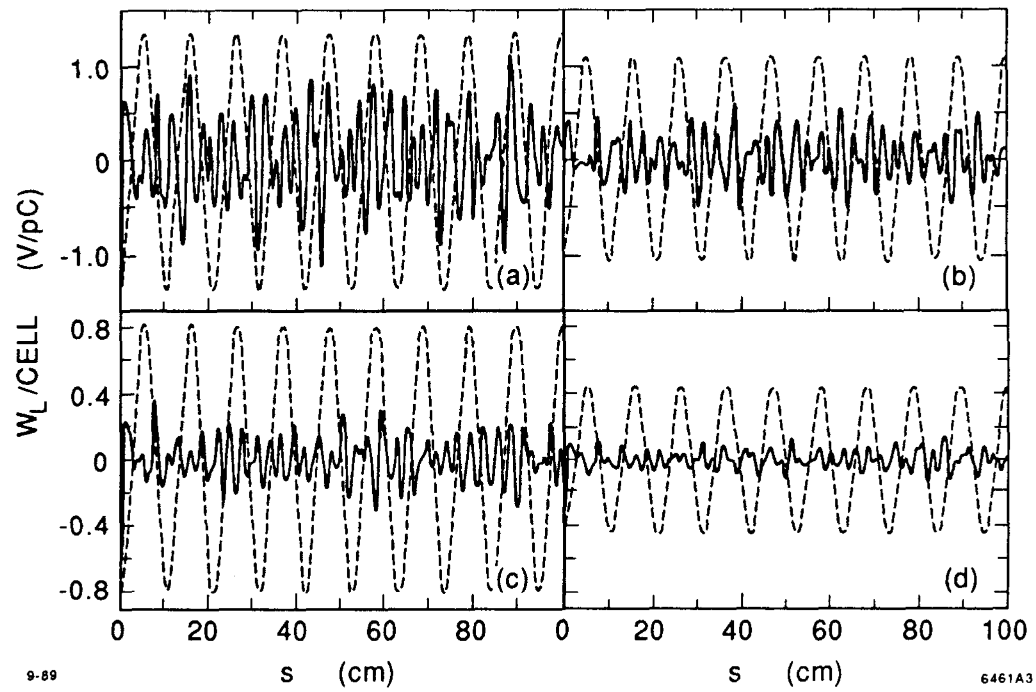

FIGURE 3 Longitudinal wake potential for a disk-loaded structure calculated from a modal sum; $\sigma_z/\lambda = 0.025$ and (a) $a/\lambda = 0.111$, (b) $a/\lambda = 0.150$, (c) $a/\lambda = 0.200$, and (d) $a/\lambda = 0.278$. Accelerating mode (dashed); higher modes (solid).

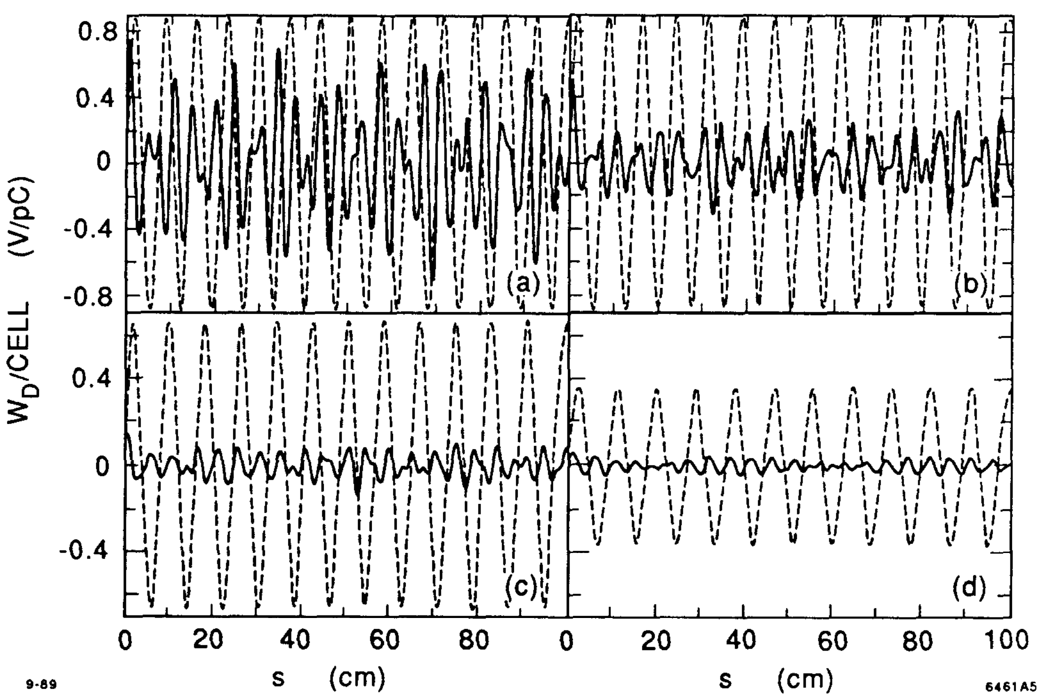

FIGURE 4 Dipole wake potential for a disk-loaded structure calculated from a modal sum; $\sigma_z/\lambda = 0.050$ and (a) $a/\lambda = 0.111$, (b) $a/\lambda = 0.150$, (c) $a/\lambda = 0.200$, and (d) $a/\lambda = 0.278$. Lowest frequency mode (dashed); higher modes (solid).

case of dipole wake. This illustrates in a graphic manner the advantage of a large beam aperture in an accelerating structure for a linear collider.

Since the details of the higher mode wake potentials are sensitive to small perturbations in structure geometry, the present calculation cannot be used to predict the precise values of those potentials at bunch locations in a multibunch train with bunch spacing $n\lambda_0$. The probable amplitude of the higher mode potential can, however, be estimated from the rms values plotted in Fig. 5. The lowest frequency mode is also shown for comparison. Numerical values are listed in Table I for the total wake potentials, W_{rms}(all), and for the higher mode potentials, W_{rms}(hm). The longitudinal rms higher mode wake potential variation with aperture is best fitted by $(a/\lambda)^{-2.2}$, while the dipole higher mode potential varies as $(a/\lambda)^{-2.9}$. Although these fits for the higher mode scaling are quite good, the lowest frequency modes for the longitudinal and transverse cases do not follow a simple power law scaling.

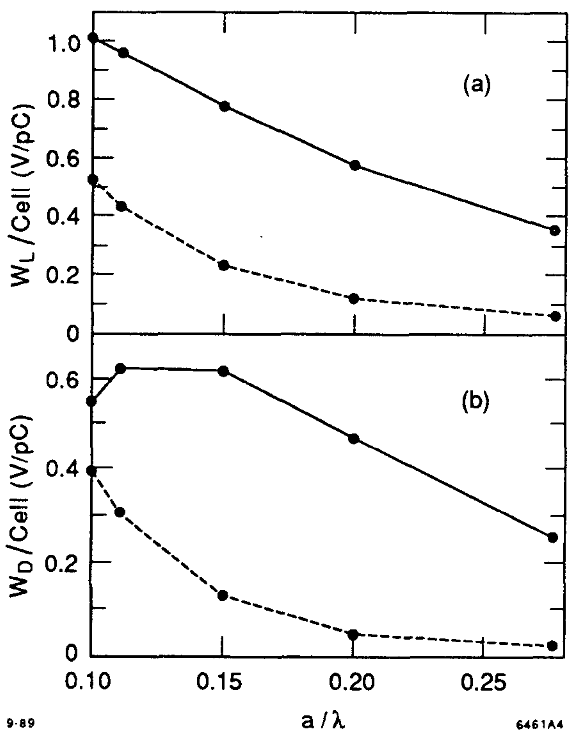

FIGURE 5 (a) Scaling of longitudinal higher mode (dashed curve) and accelerating mode (solid curve) wake potentials with beam aperture. (b) Scaling of dipole higher mode (dashed curve) and lowest frequency mode (solid curve) with beam aperture.

Also given in Table I is the ratio of the rms value of the higher mode wake potential to the peak value of the potential for the lowest frequency mode, $\widehat{W}(n=1)$. This ratio is useful for estimating the residual effects of higher modes on the multibunch energy spread (in the case of the longitudinal wake) and multibunch trans-

TABLE I Variation of rms values of wake potential with beam aperture.

a/λ	Longitudinal (V/pC per cell)			Dipole (V/pC per cell)		
	W_{rms}(all)	W_{rms}(hm)	$\frac{W_{rms}(\text{hm})}{\widehat{W}_L(n=1)}$	W_{rms}(all)	W_{rms}(hm)	$\frac{W_{rms}(\text{hm})}{\widehat{W}_D(n=1)}$
0.100	1.134	0.520	0.37	0.681	0.396	0.51
0.111	1.052	0.427	0.31	0.697	0.302	0.34
0.150	0.806	0.222	0.20	0.635	0.128	0.15
0.200	0.582	0.113	0.14	0.470	0.047	0.07
0.276	0.351	0.057	0.11	0.254	0.020	0.05

verse emittance growth (in the case of the dipole wake), after the effect due to the lowest frequency mode has been taken into account.[1,2]

BEAT-WAVE STRUCTURE

As noted previously, the fact that the lowest frequency dipole mode is so dominant for large beam apertures makes it possible to produce a deep null in the transverse wake potential in an accelerating structure composed of segments with two different dipole mode frequencies. Here, we choose segments with two different values of a/λ, although any perturbation in structure geometry which changes the dipole mode frequency while preserving the accelerating mode frequency would also work. The resulting dipole wake potential for such a beat-wave structure obtained from a sum of modes generated by TRANSVRS is shown in the top plot of Fig. 6. In this example, the wake potentials calculated for infinite periodic structures with $a/\lambda = 0.150$ and 0.200 were added together and the result was divided by 2. The first null occurs at $s = 48$ cm ($\approx 4.6\lambda_0$) and the second at $s = 145$ cm ($\approx 13.8\lambda_0$).

The bottom plot in Fig. 6 shows a TBCI wake potential for a structure having five cells with $a/\lambda = 0.200$ followed by five cells with $a/\lambda = 0.150$. The bunch enters through a 1-m-long beam tube with a radius equal to the larger a/λ, and exits through a 1-m-long beam tube with a radius equal to the smaller a/λ. In this example, the first null is at $s = 47$ cm ($4.5\lambda_0$) and the second at 127 cm ($12.1\lambda_0$). This is just the bunch spacing that has been proposed in one linear collider parameter set.[2] Note that at the null, the dipole wake is reduced by about an order of magnitude. In addition, the null is relatively broad. This method of wake potential reduction gives more margin for error than an alternative method[2] in which the following bunch is placed at a zero crossing of lowest dipole mode potential.

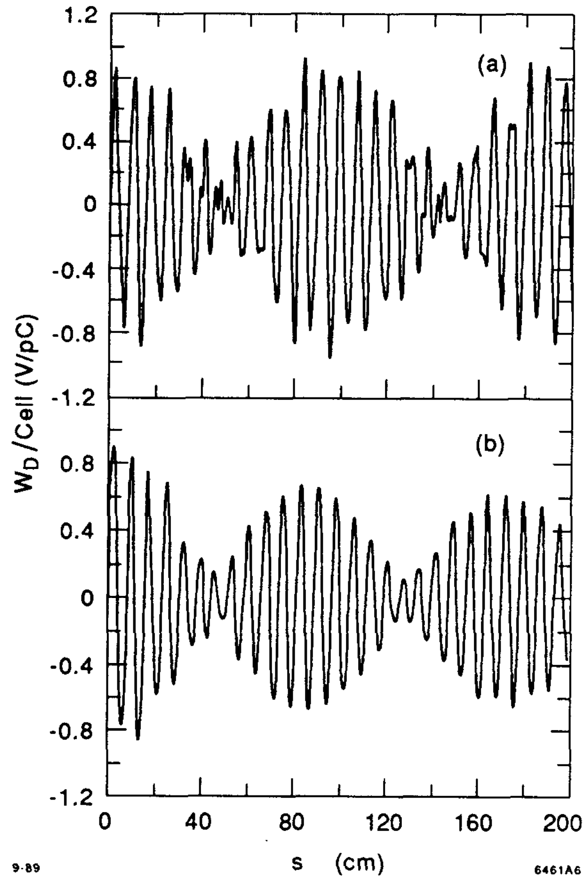

FIGURE 6 Dipole wake potential for beat-wave structure with $\sigma_z/\lambda = 0.050$ calculated from (a) sum of modes and (b) TBCI.

We have also calculated the longitudinal mode frequencies and shunt impedances for this composite structure, and found that the accelerating mode frequency is preserved and the shunt impedance is the average of the impedances of the separate structures.

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