# COMMISSIONING EXPERIENCE WITH THE SLC ARCS* 

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#### Abstract

The SLC Arc transport line brings high energy, high intensity electron and positron bunches from the SLAC linac to the Stanford Linear Collider final focus section. In this paper we will revicw the latest techniques developed to diagnose and correct the optical properties of the SLC Arcs. These techniques could be relevant to the design of long beam transport lines in future particle colliders.


## INTRODUCTION

The SLC Arc beam lines ${ }^{1}$ deliver high energy ( 47 GeV ) $\mathrm{e}^{+}$and $\mathrm{e}^{-}$bunches from the SLAC linac exit to the entrance of the final focus section (FFs). Important parameters are summarized below ${ }^{2}$ :

Table 1

| Dipole field | $5.96976(\mathrm{kG}):$ for $50 \mathrm{GeV} / \mathrm{c}$ beam |
| :--- | :--- |
| Field gradient | $7.0189(\mathrm{kG} / \mathrm{cm})$ |
| Sextupole terms | $+1.629\left(\mathrm{kG} / \mathrm{cm}^{2}\right):$ Focussing magnets |
|  | $-2.702\left(\mathrm{kG} / \mathrm{cm}^{2}\right):$ Defocussing magnets |
| Bending radius | $279.378(\mathrm{~m})$ |
| $1 / 2$ cell length | $2.596201936(\mathrm{~m})$ |
| Phase advance | $108($ degrees $/$ cell $), 20$ cells ==> achromat |
| Total beam line length | $1350(\mathrm{~m} / \mathrm{Arc})$ |

A problem encountered in practical operations of the SLC is residual $x$-y coupling at the exit of the Arcs. This is caused by systematic positioning errors of the combined function magnets which form the Arc FoDo cells. These magnets contain design sextupole fields in addition to dipole and quadrupole fields. Horizontal errors result in focusing errors. Focusing errors can create $x$ - $y$ coupling 3,4 , because the SLC Arcs are rolled, and the $x$ - $y$ coupling generated by the rolls is cancelled at the Arc exit only if the Arc lattice tune errors are small. Vertical errors introduce skew quadrupole fields which directly generate $x$ - $y$ coupling.

[^0]To illustrate how residual $x$ - $y$ coupling at the Arc exit can degrade the overall performance of the SLC, let $R$ represent the $4 \times 4$ transfer matrix between two points $A$ and $B$. The beam matrix $\sigma$ is transferred as: $\sigma_{B}=R \quad \sigma_{A} R^{T}$. The x-y cross-term of the outgoing beam $\sigma_{B}$ is written as:

$$
\begin{align*}
\sigma_{13, B} & =R_{11}\left(\sigma_{11, A} R_{31}+\sigma_{12, A} R_{32}\right)+R_{12}\left(\sigma_{12, A} R_{31}+\sigma_{22, A} R_{32}\right) \\
& +R_{13}\left(\sigma_{33, A} R_{33}+\sigma_{34, A} R_{34}\right)+R_{14}\left(\sigma_{34, A} R_{33}+\sigma_{44, A} R_{34}\right) \tag{1}
\end{align*}
$$

The presence of off-diagonal elements in the R matrix causes a complex dependence of the output beam matrix $\sigma_{\mathrm{B}}$ on the incoming emittance, $\beta^{\prime}$ s, and $\alpha^{\prime}$ s, even when $\sigma_{\mathrm{A}}$ is uncoupled. In the SLC this effect could be contributing to detector background instabilities. The relative shadowing of collimators in the FFs assumes a nearly matched beam from the Arcs, and once set up, it is difficult to re-optimize the collimators in response to subtle changes in the beam from the linac.

In this paper we report on techniques we have developed to solve this problem. The procedure is to: (1) deduce $4 \times 4$ R-matrices from betatron oscillation datasets, (2) evaluate phase-tune errors, (3) estimate skew-field errors, (4) calculate corrective actions to take, (5) apply corrections, check results, and iterate the procedure if necessary. The result of the tune-up work is presented.

## DEDUCTION OF BEAM TRANSFER MATRICES (R-MATRICES)

Four horizontal and four vertical corrector dipole magnets, located at the end of the SLAC linac, are used to induce betatron oscillations of the beam centroid about the central trajectory. The difference between the orbits recorded with different settings of each corrector directly give the $\mathrm{R}_{12}$ and $\mathrm{R}_{32}$ elements (for horizontal correctors) or the $\mathrm{R}_{14}$ and $\mathrm{R}_{34}$ elements (for vertical correctors) of the transfer matrices between the corrector and the BPMs. The measurement is repeated for each corrector one at a time, and the data recorded.

We assume that (1) the transfer matrices between the eight linac correctors and (2) the transfcr matrices between four adjacent BPMs within the Arc are close to the known design. Then a least squares constrained fit is performed to measure the complete $4 \times 4$ transfer matrix between one of the eight correctors and one of the four Arc BPMs. The constraint imposed in the least squares fit is that the transfer matrix we are solving for be symplectic ${ }^{5}$.

Once one of these transfer matrices has been reconstructed, we move to the next set of four adjacent Arc BPMs and repeat the least squares fit. When the analysis is completed we end up with a set of $4 \times 4$ transfer matrices between the end of the linac and all BPMs within the Arc and FFs. The procedure for reconstructing the complete $4 \times 4$ transfer matrix is described in detail in Ref. 6 .

## IN-PLANE OPTICS DIAGNOSIS

Horizontal and vertical $\beta$ 's and phase advances are calculated using $2 \times 2$ submatrices of the measured $4 \times 4$ transfer matrix. We define the horizontal and vertical 2 x 2 submatrices $\mathrm{U}_{\mathrm{x}}, \mathrm{U}_{\mathrm{y}}$ to be:

$$
U_{x}=\frac{1}{\sqrt{R_{11} R_{22}-R_{12} R_{21}}}\left(\begin{array}{ll}
R_{11} & R_{12}  \tag{2}\\
R_{21} & R_{22}
\end{array}\right) \quad, \quad U_{y}=\frac{1}{\sqrt{R_{33} R_{44}-R_{34} R_{43}}}\left(\begin{array}{ll}
R_{33} & R_{34} \\
R_{43} & R_{44}
\end{array}\right) .
$$

The $2 \times 2$ transfer matrix between locations A and B can be written as:

$$
\mathrm{U}=\left(\begin{array}{cc}
\sqrt{\frac{\beta_{B}}{\beta_{A}}}\left(\cos \Delta \psi+\alpha_{A} \sin \Delta \psi\right) & \sqrt{\beta_{A} \beta_{B}} \sin \Delta \psi  \tag{3}\\
\frac{\left(1+\alpha_{A} \alpha_{B}\right) \sin \Delta y+\left(\alpha_{B}-\alpha_{A}\right) \cos \Delta \psi}{\sqrt{\beta_{A} \beta_{B}}} & \sqrt{\frac{\beta_{A}}{\beta_{B}}}\left(\cos \Delta \psi-\alpha_{B} \sin \Delta \psi\right)
\end{array}\right)
$$

where $\beta_{i}$ and $\alpha_{i}$ represent the values of $\beta$ and $\alpha$ at location $i(i=A, B)$, and $\Delta \psi$ is the phase shift in between. By transporting the design $2 \times 2$ beam matrix $\sigma_{\mathrm{A}}$ using $\sigma_{\mathrm{B}}=\mathrm{U} \sigma_{\mathrm{A}} \mathrm{U}^{\mathrm{T}}$, we abtain:

$$
\begin{equation*}
\beta=\frac{\sigma_{11}}{\sqrt{\sigma_{11} \sigma_{22}-\sigma_{12}^{2}}}, \quad \cos \Delta \psi=\sqrt{\frac{\beta_{\mathrm{A}}}{\beta_{\mathrm{B}}}}\left(U_{11}-\frac{\alpha_{\mathrm{A}} U_{12}}{\beta_{\mathrm{A}}}\right), \quad \sin \Delta \psi=\frac{U_{12}}{\sqrt{\beta_{\mathrm{A}} \beta_{\mathrm{B}}}} . \tag{4}
\end{equation*}
$$

For the purpose of tuning the Arcs we are interested in measuring the average phase advance per cell in each achromat. In order to make maximal use of the transfer matrix information we measure the phase shift $\Delta \psi$ across every 10 cell section of the Arc. This means we calculate $\Delta \psi$ for the beam line section consisting of the last $n$ cells of achromat $j$ and the first $10-n$ cells of achromat $j+1$, and we do this for $n=1$, $2, \ldots, 10$ and $\mathrm{j}=1,2, \ldots, 22$. For each of these calculations the phase advance per cell $\mu$ is defined to be

$$
\begin{equation*}
\mu=108^{\circ}+\frac{\Delta \psi}{10} \tag{5}
\end{equation*}
$$

The measurements of $\mu_{\mathrm{y}}$ for the north Arc on June 27, 1989 are shown in Figure 1(a).
We perform a least squares fit to solve for the tunes of each achromat assuming that the tune is constant within each achromat but not necessarily across achromats. We assume that if $\mu_{\mathrm{j}}$ is the tune of achromat $j$ and $\mu_{j+1}$ is the tune of achromat $j+1$, then the tune $\mu$ of the beamline section consisting of the last $n$ cells of achromat $j$ and the first $10-n$ cells of achromat $j+1$ is given by:

$$
\begin{equation*}
\mu=\frac{\mathrm{n} \mu_{\mathrm{j}}+(10-\mathrm{n}) \mu_{\mathrm{j}+1}}{10} \tag{6}
\end{equation*}
$$

With this assumption the fitted values of $\mu$ form straight lines between achromats and show a change in slope at each achromat boundary. The least squares fit to the data of Figure $1(a)$ is shown in Figure 1(b). The parameters of the fit are the achromat tunes $\mu_{j}, j=1,2, \ldots, 23$. We use these fit parameters as our best estimates of the achromat tunes.

## CRŌSS-PLANE OPTICS DIAGNOSIS

Let $C$ be the following $2 \times 2$ submatrix of the measured $4 \times 4$ transfer matrix $R$.

$$
C=\left(\begin{array}{ll}
R_{31} & R_{32}  \tag{7}\\
R_{41} & R_{42}
\end{array}\right)
$$

The magnitude of the elements of C indicates the strength of cross-plane coupling. It has been found ${ }^{5}$ that the determinant of this sub-matrix has several useful properties in diagnozing the cross-plane coupling problem: (a) $\operatorname{det}(C)$ is identically 0 if no $x-y$ coupling exists in the stretch described by the R-matrix, (b) $\operatorname{det}(\mathrm{C})$ remains constant if no additional x -y coupling occurs, (c) a variation of $\operatorname{det}(\mathrm{C})$ along the beam line means a presence of $x-y$ couplings. However, just because det $(C)$ is zero, it does not necessarily mean that $x-y$ coupling is totally absent.

## OPTICS TUNE-UP OF THE ARCS

After calculating the achromat tunes $\mu_{\mathrm{j}}$ we can determine which achromats need their tunes adjusted, and, using the phase-fix prescription ${ }^{3}$, we know how to correct the tunes. Figure. 1(c) shows the fitted tunes of the north Arc following one iteration of phase-fix ( 8 achromats in total were fixed).

Vertical errors in the Arc magnets generate skew quadrupole fields. These errors can arise either (1) from errors in the estimates of the vertical offsets of the BPMs, or (2) from vertical magnet alignment errors, or both ${ }^{1}$. If the effects of the former (1) are larger than the latter (2), then the BPM offset errors can be estimated by fitting the measured absolute orbit to an orbit consistent with the known positions of the vertical magnet movers. The residuals from such a fit give the BPM offset errors. This technique was applied to all BPMs in the north Arc. In some achromats vertical BPM offsets were found to be off by 400 to 800 microns. Corrections were applied and a significant reduction of cross-plane coupling was seen.

If the remaining skew fields are well described by systematic vertical magnet misalignments, their effect may be corrected by using vertical magnet movers. The technique for determining the optimum vertical magnet mover motion is mathematically equivalent to the technique used to estimate the tunes $\mu_{j}$ of the achromats. For each 10 -cell section of the Arc we calculate the common vertical magnet mover motion $\Delta Y$ which best cancels the anomalous terms in the $2 \times 2$ submatrix $C$. We then plot the values of $\Delta Y$ as we do the measured tunes $\mu$ and perform the same type of least squares fit. The result of the fit is a set of parameters $\Delta Y_{j}, j=1,2, \ldots, 23$, which represent our best estimates of the vertical magnet mover motions required to cancel anomalous skew terms due to vertically misaligned magnets.

While the achromat tune errors can always in principle be corrected with the phase-fix procedure, it is not true that arbitrary vertical magnet misalignments can be satisfactorily corrected with uniform vertical magnet mover motions. Judgement is therefore used when deciding whether or not to implement an
achromat vertical mover motion $\Delta \mathrm{Y}_{\mathrm{j}}$. Our experience has been that when $\Delta \mathrm{Y}_{\mathrm{j}}$ is large, the implementation of the correction $\Delta Y_{j}$ produces beneficial results.

After the Arc has been phase-fixed and skew-fixed on an achromat by achromat basis, the global properties, especially the cross-plane behavior, may not be as good as we would like. In a downstream part of the north Arc we found a cross-plane coupling anomaly which could not be ascribed to phase errors or BPM offset errors, and which could not be modelled with uniform vertical mover motions. We removed the anomaly by empirically moving some vertical movers between achromats 14 and 20 until $\operatorname{det}(\mathrm{C})$ at the end of the Arc became 0 . The relationship between $\operatorname{det}(\mathrm{C})$ at the end of the $\operatorname{Arc}$ and the magnitude of these vertical mover motions was quite linear. This suggests that we can probably force, not only $\operatorname{det}(\mathrm{C})=0$, but also $\mathrm{C}=0$ at the end of the Arc through a judicious choice of independent sets of vertical movers. Such sets of movers would in effect be used as independent skew quads to remove residual coupling at the end of the Arcs.

## CONCLUSIONS

Our procedure worked. Figures 2(a) and 2(b) show the behavior of $\operatorname{det}(\mathrm{C})$ along the north $\operatorname{Arc}$ before and after applying all the tune-up corrections. The backgrounds from the north (electron) side have certainly become more stable. The SLC now delivers luminosity with $2 \times 10^{10}$ electrons per pulse whereas before the north Arc tune-up the maximum current for luminosity was limited to $1.4 \times 10^{10}$. The south Arc (positron) remains to be fixed, pending the allocation of beam time for the work. In the long run this tune-up will be important when asymmetric emittances are transmitted to the FFs. Similar techniques should be applicable to other long FoDo cells such as bunch compressors in the next generation of linear colliders.

## REFERENCES

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## FIGURE CAPTIONS

Figure 1. Vertical phase advance per cell (y axis, unit $=$ degrees) measured along the north Arc, plotted as function of BPM serial unit number (x axis, unitless). The design value is 108 degrees per cell. The (a) and (b) show the behavior before the tune-up work. (a): Phase advance per cell averaged for contiguous groups of 10 cells. (b): The result of fit to (a), assuming that each achromat has a uniform phase error. The (c) shows the behavior after completion of the tune-up work.
Figure 2. Behavior of $\operatorname{det}(\mathrm{C})$ ( y axis, unitless) plotted as function of BPM serial unit number ( x axis, unitless), before (a) and after (b) applying the full tune-up corrections described in the text.





Figure 1


Figure 2


[^0]:    * Work supported by U.S. Department of Energy Contract DE-AC03-76SF00515.

    Presented at the XIV International Conference on High Energy Accelerators, Tsukuba, Japan, August 22-26, 1989

