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Phenomenology of the CKM Matrix^{*}

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ABSTRACT

- The way in which an exact determination of the CKM matrix elements tests the Standard Model is demonstrated by a two generation example. The determination of matrix elements from meson semi-leptonic decays is explained, with an emphasis on the respective reliability of quark level and meson level calculations. The assumptions involved in the use of loop processes are described. Finally, the state of the art of our knowledge of the CKM matrix is presented.

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1. INTRODUCTION

The free parameters of the quark sector in the Standard Model (SM) are the quark masses and the mixing parameters. In the interaction basis, the charged gauge interactions are, by definition, diagonal:

$$\bar{d}_L^i M_d^I d_R^i + \bar{u}_L^i M_u^I u_R^i + \frac{g}{\sqrt{2}} W_\mu^+ \bar{u}_L^i \mathbf{1} \gamma^\mu d_L^i. \quad (1)$$

For n generations, the mass matrices M_d^I and M_u^I are general $n \times n$ matrices, while $\mathbf{1}$ stands for the unit matrix. In the mass basis, the mass matrices are, by definition, diagonal:

$$\bar{d}_L^m M_d^m d_R^m + \bar{u}_L^m M_u^m u_R^m + \frac{g}{\sqrt{2}} W_\mu^+ \bar{u}_L^m V \gamma^\mu d_L^m. \quad (2)$$

The charged gauge interactions, however, are no longer diagonal: the mixings are given by the unitary matrix V . The independent parameters are n eigenvalues of each mass matrix and $(n - 1)^2$ parameters of the matrix V . At present we know of three quark generations, in which case V is the Cabibbo - Kobayashi - Maskawa (CKM) mixing matrix of *four* free parameters: three mixing angles and one phase.

If we have several independent measurements for a given CKM matrix element, or if we find the values of the nine entries, we will have the four mixing parameters *overdetermined*. Therefore, an *exact* determination of the CKM matrix elements provides us with a stringent test of the SM and with possible clues to physics beyond it. We explain this by showing what we can tell about the third generation from our present knowledge of the 2×2 Cabibbo matrix.

We survey the determination of different matrix elements from semi-leptonic meson decays. We explain the shortcomings of calculations at either the quark level or the meson level. We concentrate on the three above-diagonal elements: $|V_{us}|$, $|V_{cb}|$ and $|V_{ub}|$.

Additional information can be derived from loop processes. The assumptions made are stronger. We explain these assumptions and show the constraints from $B - \bar{B}$ mixing and from the ϵ parameter.

2. THE FIRST TWO GENERATIONS

The Cabibbo mixing matrix for the first two generations is

$$V_C = \begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix}. \quad (3)$$

The value of $|V_{ud}|$ is calculated from the comparison of $0^+ \rightarrow 0^+$ superallowed Fermi β transitions:^{1,2}

$$|V_{ud}| = 0.9747 \pm 0.0011 \quad (4)$$

This is the most accurately determined of all CKM matrix elements. The calculation of the above value follows a continuous refinement of radiative corrections. The most recent one² takes into account $O(Z\alpha^2)$ corrections, and brings the eight accurately studied Ft values to agree within less than 1σ .

The value of $|V_{us}|$ is best determined from the measured rates of $K^+ \rightarrow \pi^0 e^+ \nu_e$ and $K_L^0 \rightarrow \pi^- e^+ \nu_e$ which give³

$$|V_{us}| = 0.220 \pm 0.002. \quad (5)$$

The calculation cannot be carried out within the spectator quark model, because:

- a. The final spectrum is completely dominated by the single pion state, so that duality is not expected to hold.
- b. There are large QCD corrections as the relevant scale for $\alpha_s(\mu)$ is $\mu = O(m_s)$, but $m_s \sim \Lambda_{QCD}$ (the scale at which, by definition, $\alpha_s \sim 1$).
- c. There are large uncertainties in m_s : first, it is a running mass and we do not know the relevant scale and second, even if we knew the scale, the uncertainty in m_s is still about 30%.⁴ This is significant, as the phase space for the decay depends on $(m_s)^5$.

Thus, the above value is derived from a phenomenological model:

$$\frac{BR(K \rightarrow \pi e \nu)}{\tau(K)} = C_K \left| f_+^K(0) \right|^2 |V_{us}|^2, \quad (6)$$

where C_K includes factors with small uncertainties only. In general, the major difficulty is in the calculation of the form factor $|f_+(0)|$. In this case, however, only the three light quarks take part. In the $SU(3)$ symmetry limit ($m_u = m_d = m_s$) we have $|f_+(0)| = 1$. Deviations from the symmetry limit are second order in the symmetry breaking parameter and calculable. Altogether we have a 1-1.5% error from experiment and about a 2% error in the theoretical calculations.

Our confidence in the above calculation of $|V_{us}|$ is supported by another independent measurement which gives a consistent value: a simultaneous fit to the rates of $\Lambda \rightarrow p e \nu$, $\Sigma^- \rightarrow n e \nu$ and $\Xi^- \rightarrow \Lambda e \nu$ gives⁵ $|V_{us}| = 0.220 \pm 0.001 \pm 0.003$. Consistency with the meson decay data was achieved only after recoil corrections were taken into account.

There are two methods to determine $|V_{cd}|$ and $|V_{cs}|$. The first one is using data from deep inelastic neutrino - nucleon scattering. One gets:⁶

$$\begin{aligned} |V_{cd}| &= 0.21 \pm 0.03 \\ |V_{cs}/V_{cd}| &\geq 3.3 \end{aligned} \quad (7)$$

The bound on the ratio is derived with a mild assumption on the ratio of strange sea to anti-quark sea in the nucleon, $2S \leq \bar{U} + \bar{D}$.

The second method is from D semi-leptonic decays. A reliable quark level calculation is still impossible due to the lightness of the c quark: Duality is questionable and QCD corrections may be large. However, the uncertainty in m_c at a given scale is small, so the question here is that of the relevant scale.

At the meson level we have:

$$\frac{BR(D^0 \rightarrow X_q^- e^+ \nu)}{\tau(D^0)} = C_D \left| f_+^{D \rightarrow X_q}(0) \right|^2 |V_{cq}|^2, \quad (8)$$

where C_D includes factors with small uncertainties only. The uncertainty from the

D^0 lifetime is common to both determinations,⁷ $\tau(D^0) = .422 \pm .008 \pm .010$ psec. A large uncertainty comes from the calculation of the form factor. The charm quark is too heavy to make an $SU(4)$ symmetry useful for the calculation. Various calculations of the form factors, using quark models and QCD sum rules, give:

$$|f_+(0)| = \begin{cases} .6 \pm .1 & [8] \\ .75 - .82 & [9] \\ .75 \pm .05 & [10] \end{cases} \quad (9)$$

The main difference between the determination of $|V_{cd}|$ and that of $|V_{cs}|$ comes from the experimental measurements. For $c \rightarrow s$ there are two measurements:

$$BR(D^0 \rightarrow K^- e^+ \nu) = \begin{cases} (3.8 \pm 0.5 \pm 0.6) \times 10^{-2} & [11] \\ (3.4 \pm 0.5 \pm 0.4) \times 10^{-2} & [12] \end{cases} \quad (10)$$

With enough confidence in the models for the form factor one may give a value for $|V_{cs}|$, e.g. $|V_{cs}| = 1.1 \pm 0.2$ for $|f_+(0)| = .7 \pm .1$. However, for $c \rightarrow d$ there is only one measurement and with large uncertainties:¹²

$$BR(D^0 \rightarrow \pi^- e^+ \nu) = (3.9_{-1.1}^{+2.3} \pm 0.4) \times 10^{-3}. \quad (11)$$

The ratio $|V_{cd}/V_{cs}|$ is free of the uncertainties in $\tau(D^0)$. Moreover, it depends on the ratio $|f_+^{D \rightarrow \pi}/f_+^{D \rightarrow K}|$: this ratio is 1 in the $SU(3)$ limit, which is expected to hold within 10%. Thus, we get:

$$|V_{cd}/V_{cs}| = 0.25 \pm 0.06. \quad (12)$$

With present experimental errors and theoretical uncertainties, the more restrictive bounds come from deep inelastic scattering, but the measurements of D semi-leptonic decays give further confidence in these results.

To conclude, different direct measurements give the following range for the Cabibbo matrix elements:

$$V_C = \begin{pmatrix} .9747 \pm .0011 & .220 \pm .002 \\ .21 \pm .03 & \geq .60 \end{pmatrix}. \quad (13)$$

Now, suppose we knew about two generations only. Then unitarity would imply that the above matrix depends on one parameter only:

$$V_C = \begin{pmatrix} c_{12} & s_{12} \\ -s_{12} & c_{12} \end{pmatrix}. \quad (14)$$

With the above measurements we have certainly *overdetermined* the Cabibbo angle. The test to the two generation SM is the following: Can we find a range for the Cabibbo angle which is consistent with all measurements? The answer is positive: for $.219 \leq s_{12} \leq .222$ we get the following ranges for the matrix elements:

$$V_C = \begin{pmatrix} .9750 - .9758 & .219 - .222 \\ .219 - .222 & .9750 - .9758 \end{pmatrix}, \quad (15)$$

which is consistent with the measurements (13). Thus, the two generation picture is still consistent and we could not tell that there is a third generation if not for its direct observation (or from CP violation). From our knowledge about $|V_{cb}|$ and $|V_{ub}|$ we know that the third generation mixings would be probed only if we reached an accuracy level of 10^{-4} in the determination of $|V_{ui}|$ or 10^{-3} in the determination of $|V_{ci}|$ ($i = d, s$); this is well beyond the present level of accuracy. At present, the values in (13) imply only the following mild bounds on the possible mixings of a third generation:

$$V = \begin{pmatrix} \cdot & \cdot & \leq .07 \\ \cdot & \cdot & \leq .78 \\ \leq .14 & \leq .77 & \leq 1 \end{pmatrix} \quad (16)$$

Additional information on the parameters of the first two generations can be derived from indirect measurements, namely SM loop processes. To extract useful

information, we need to know all the significant contributions to such a process. Thus, we make two major assumptions:

- a. There are no additional generations. This assumption is unnecessary in the case of direct measurements.
- b. There are no significant “beyond standard” contributions. For direct measurements we assume that there are no beyond standard processes which compete with the tree level SM processes, which is indeed the case for most “reasonable” models (with the possible exception of models with a light charged Higgs). For indirect measurements we assume that there are no processes which compete with SM loop processes (which are suppressed by the high order in the weak interaction coupling and by the GIM mechanism). This is *not* the case in many extensions of the SM.

Finally, we note that as the GIM mechanism is in operation, the results have strong dependence on the masses of intermediate quarks.

The only loop process which does not a priori necessitate the existence of a third generation is ΔM_K , the mass difference between the two neutral K -mesons:

$$\frac{\Delta M_K (1 - D)}{N_K B_K} = \eta_1 \frac{m_c^2}{M_W^2} (s_{12})^2 \quad (17)$$

The N_K parameter is a known quantity, $N_K \equiv \frac{G_F^2 f_K^2 M_K M_W^2}{6\pi^2} = 2.1 \times 10^{-10} \text{ GeV}$. The long distance contributions are given by $D \cdot \Delta M_K$. The B_K parameter gives the ratio between the short distance contribution and its value in the vacuum insertion approximation. The η_1 parameter gives the QCD corrections, $\eta_1 = 0.7$. In the above we used unitarity for two generations by putting

$$\text{Re}[(V_{cd}^* V_{cs})^2] \approx (s_{12})^2. \quad (18)$$

We note the strong dependence on m_c . When the original study of the $K - \bar{K}$ mixing¹³ was performed, the c -quark was not yet experimentally discovered.

Thus one could use eq. (17) to predict the mass of the c -quark. In the original calculation, the vacuum saturation approximation was used ($B_K = 1$), and neither long-distance contributions nor QCD corrections were taken into account ($D = 0$, $\eta_1 = 1$). This led, somewhat coincidentally, to the correct prediction: $m_c = 1.5 \text{ GeV}$. With the full range of uncertainties in B_K and D one gets:

$$8 \times 10^{-6} \leq \frac{\Delta M_K (1 - D)}{N_K B_K} \leq 5 \times 10^{-5} \quad (19)$$

which gives $1.3 \text{ GeV} \leq m_c \leq 3.2 \text{ GeV}$. As we now know that $m_c \approx 1.4 \text{ GeV}$, the two generation picture is still self-consistent, even when information from the loop process ΔM_K is taken into account. Due to the very small mixings of the third generation, at present we could not find it from inconsistencies in the Cabibbo matrix.

3. THE ABOVE-DIAGONAL ELEMENTS

In this section we concentrate on the determination of the three above diagonal elements:

$$V = \begin{pmatrix} \cdot & V_{us} & V_{ub} \\ \cdot & \cdot & V_{cb} \\ \cdot & \cdot & \cdot \end{pmatrix} \quad (20)$$

from semi-leptonic meson decays. The determination of $|V_{us}|$ was explained in the previous section: the s quark is too light to allow a quark level calculation, but light enough to allow a reliable calculation of the form factor at the meson level.

The value of $|V_{cb}|$ is best determined from semi-leptonic B decays: $B \rightarrow X_c e \nu_e$. At the quark level the process is $b \rightarrow c e \nu_e$. In this case:

- a. The dominant semi-leptonic modes are those with $X_c = D, D^*$. Duality should hold for the decay rate within about 10%.
- b. The relevant scale for QCD corrections is of order m_b . As $\alpha_s(m_b) \sim 0.2$, a first-order calculation should be fine to within 4% or so.

- c. The mass of the b quark at a certain energy scale is known at the 2% accuracy level. Consequently, the crucial question is that of the relevant energy scales. We will argue that there is no ambiguity of energy scales for m_c or, more accurately, in the ratio m_c/m_b . However, the question of energy scale for m_b in the $(m_b)^5$ factor is still open and remains the main source of uncertainty in the calculation. One possible way to overcome this difficulty is by fitting m_b to the leptonic spectrum. The fit is model-dependent, but if we use several models and let their parameters vary in a reasonable range, we may learn what is the uncertainty involved[†].

Within the spectator quark model:

$$\frac{BR(b \rightarrow ce\nu)}{\tau_b} = \left[\frac{G_F^2}{192\pi^3} \right] m_b^5 F_{ps}(\rho_c) F_{QCD}(\rho_c) |V_{cb}|^2. \quad (21)$$

The experimental quantities on the left hand side are known with about 15% error, mainly from the b lifetime determination. The phase-space factor F_{ps} and the QCD correction factor F_{QCD} both depend on the mass ratio $\rho_c = m_c^2/m_b^2$. As mentioned, a priori there is an ambiguity, because quark masses are running, so that ρ depends on two scales:

$$\rho_c = \frac{[m_c(\mu_c)]^2}{[m_b(\mu_b)]^2}. \quad (22)$$

The question is what are the relevant scales μ_c and μ_b . The answer is¹⁴ that to every choice of two scales, there corresponds a specific QCD correction factor. The modification of F_{QCD} is such that the product $F_{ps}(\rho) \cdot F_{QCD}(\rho)$ is independent of the choice of scales:

$$F_{ps}(\rho_c) F_{QCD}(\rho_c) = 0.46 \pm 0.04. \quad (23)$$

Various arguments suggest that the value of m_b should be taken as

$$m_b = 4.9 \pm 0.3 \text{ GeV}. \quad (24)$$

As the decay width depends on $(m_b)^5$, this gives a 30% uncertainty. With the

[†] We thank K. Schubert and G. Altarelli for discussions on this point.

above values we get:

$$|V_{cb}| = 0.046 \pm 0.008. \quad (25)$$

Various phenomenological models are, at present, in the stage of being tested against the experimental data. However, they all give $|V_{cb}|$ values which are somewhat higher than the spectator quark model value. To account for the model dependence of the calculation we take:

$$|V_{cb}| = 0.048 \pm 0.009. \quad (26)$$

The value of $|V_{ub}|$ can be determined from semi-leptonic charmless B decays: $B \rightarrow X_u e \nu_e$. At the quark level the process is $b \rightarrow u e \nu_e$. The calculation is subject to uncertainties similar to those of $|V_{cb}|$. It is advantageous to consider the ratio $q \equiv |V_{ub}/V_{cb}|$ rather than $|V_{ub}|$ itself:

$$\frac{BR(b \rightarrow u e \nu)}{BR(b \rightarrow c e \nu)} = \frac{F_{ps}(\rho_u) F_{QCD}(\rho_u)}{F_{ps}(\rho_c) F_{QCD}(\rho_c)} q^2. \quad (27)$$

The ratio is free of the uncertainties in $(m_b)^5$ and τ_b . Moreover, the ratio between the QCD correction factors does not depend (to $O(\alpha_s)$) on the choice of scale for α_s and, due to the lightness of the u quark, $F_{ps}(\rho_u) = 1$ with no uncertainty. We get:

$$F_{ps}(\rho_u) F_{QCD}(\rho_u) = 0.85. \quad (28)$$

The only theoretical uncertainty is then in $F_{ps}(\rho_c)$. We get:

$$q = (0.74 \pm 0.03) \left[\frac{BR(b \rightarrow u e \nu)}{BR(b \rightarrow c e \nu)} \right]^{1/2}. \quad (29)$$

Experiment does not provide us, at present, with $BR(b \rightarrow u e \nu)$ as there is no direct observation of charmless B decays. If one tried to subtract from the *measured* semi-leptonic rate the theoretically *calculated* charmed semi-leptonic decay rate, one would be left with zero and the $b \rightarrow u$ contribution “buried” within the large error bars.

Instead, $|V_{ub}|$ is determined from the electron energy spectrum. The spectator quark model is not appropriate for this analysis¹⁵, while various phenomenological models give very different results. The strongest experimental results with the weakest theoretical constraints give¹⁶:

$$q \leq 0.16. \quad (30)$$

The CLEO collaboration recently reported¹⁷ a measurement of $BR(b \rightarrow ue\nu) \neq 0$, but as the errors are still large and the result is not yet confirmed by other experiments we do not use it here.

To summarize: The above diagonal elements in the CKM matrix are best determined from semi-leptonic meson decays. For light mesons, or correspondingly light quarks, quark-meson duality does not hold because the spectrum is dominated by one final state. Moreover, even if the spectator quark model held, we would have practical difficulties in the calculation due to large QCD corrections and large uncertainties in the light quark masses. On the other hand, we are able to calculate rather accurately within phenomenological models, due to the approximate flavor symmetry. For heavier mesons, or correspondingly heavier quarks, the spectator quark model should give a reasonable description of the inclusive decay rate. QCD corrections are small and heavy quark masses are known rather well, though they remain the major source of uncertainty. In the case of heavy quarks, phenomenological models have no approximate symmetry to help control the hadronic matrix elements, and at this stage they should be tested against the experimental results rather than used to estimate the CKM matrix elements.

Direct measurements give:

$$|V_{us}| = 0.220 \pm 0.002, \quad |V_{cb}| = 0.048 \pm 0.009, \quad q \equiv \frac{|V_{ub}|}{|V_{cb}|} \leq 0.16. \quad (31)$$

4. INDIRECT MEASUREMENTS

We now proceed in the same manner as in the two generation case. We assume that there are only three generations. Unitarity implies that the following values for the CKM matrix elements:

$$V_{CKM} = \begin{pmatrix} .9747 \pm .0011 & .220 \pm .002 & \leq .009 \\ .21 \pm .03 & \geq .60 & .048 \pm .009 \\ \leq .14 & \leq .77 & \leq .9992 \end{pmatrix}, \quad (32)$$

should be consistent with a parametrization of four free parameters only:

$$V_{CKM} = \begin{pmatrix} c_{12} & s_{12} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23} \\ s_{12}s_{23} - c_{12}s_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23} \end{pmatrix}. \quad (33)$$

The above parametrization, recently adopted by the Particle Data Group,¹⁸ is given here with the only approximation $c_{13} = 1$, which is good to $O(10^{-4})$, better than any of the experimental determinations.

Indeed, there is a range for the mixing parameters consistent with all data. It is simple to find it, as the values of the three mixing angles are equal to the absolute values of the above diagonal elements, which were derived in the previous section. Thus, the allowed ranges for the parameters is:

$$s_{12} = .220 \pm .002, \quad s_{23} = .048 \pm .009, \quad q \equiv \frac{s_{13}}{s_{23}} \leq .16. \quad (34)$$

Direct measurements do not constrain δ : $0^\circ \leq \delta \leq 360^\circ$.

Additional information on the matrix elements is derived from indirect measurements, namely loop processes. At present, we have no direct information on

the mixings of the top quark:

$$V = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \quad (35)$$

The values of V_{ts} and V_{tb} are determined from unitarity, but V_{td} is still poorly determined:

$$|V_{tb}| \approx 1, |V_{ts}| \approx |V_{cb}|, |V_{td}| \leq .022. \quad (36)$$

The GIM mechanism implies a strong m_t dependence in loop processes. Thus, we will use the known values of five quark masses and of s_{12} and s_{23} to get constraints in the three parameter space (m_t, q, δ) .¹⁹ To put constraints on the parameters from the indirect measurements, one assumes that

- a. There are only three quark generations.
- b. There are no significant contributions from any new physics.

The most useful measurements are those of the $B - \bar{B}$ mixing parameter x_d and the CP violating parameter ϵ . The x_d relation can be presented as follows:

$$N_B = \frac{x_d}{(\tau_b s_{23}^2)(B_B f_B^2)} \cdot F(m_t, q, \delta), \quad (37)$$

where N_B contains factors with small uncertainties only. There are large uncertainties in the quantities on the r.h.s of eq. (37). We use:

$$\begin{aligned} x_d &= .71 \pm .14 \\ \tau_b s_{23}^2 &= (4.1 \pm 1.0) \times 10^9 \text{ GeV}^{-1} \\ B_B f_B^2 &= (.15 \pm .05 \text{ GeV})^2 \end{aligned} \quad (38)$$

F is a function of the three unknown parameters (m_t, q, δ) . We show the x_d bounds for either fixed m_t values (fig. 1) or fixed q values (fig. 2). The bounds correspond

to the full range of parameters in eq. (38). The ϵ relation can be presented as follows:

$$N_\epsilon = B_K \cdot G(m_t, q, \delta), \quad (39)$$

where N_ϵ contains factors with small uncertainties only. The only large uncertainty is in the B_K parameter. We use:

$$B_K = .7 \pm .3. \quad (40)$$

G is a function of the three unknown parameters (m_t, q, δ) . We show the ϵ bounds for either fixed m_t values (fig. 1) or fixed q values (fig. 2). The bounds correspond to the full range of B_K in eq. (40) and of s_{23} in eq. (34).

The final allowed range is that which lies within the direct bounds and within both the x_d -band and the ϵ -band. As the top mass becomes smaller, the allowed range in the (q, δ) plane becomes smaller. For $m_t \leq 47 \text{ GeV}$ there is no allowed range, thus excluding this range for the top mass. For $m_t \sim 200 \text{ GeV}$ almost all of the original range is allowed. For the mixing parameters, we get the following bounds from indirect measurements:

$$q \geq 0.015, \quad 12^\circ \leq \delta \leq 178^\circ. \quad (41)$$

Within the three generation SM, and using the unitarity conditions and all measurements (direct and indirect) we have:

$$V = \begin{pmatrix} .9750 - .9758 & .219 - .222 & .0008 - .009 \\ .217 - .223 & .9734 - .9753 & .039 - .057 \\ .006 - .020 & .037 - .057 & .9985 - .9993 \end{pmatrix} \quad (42)$$

The SM with three quark generations is still consistent with all measurements of the CKM matrix elements.

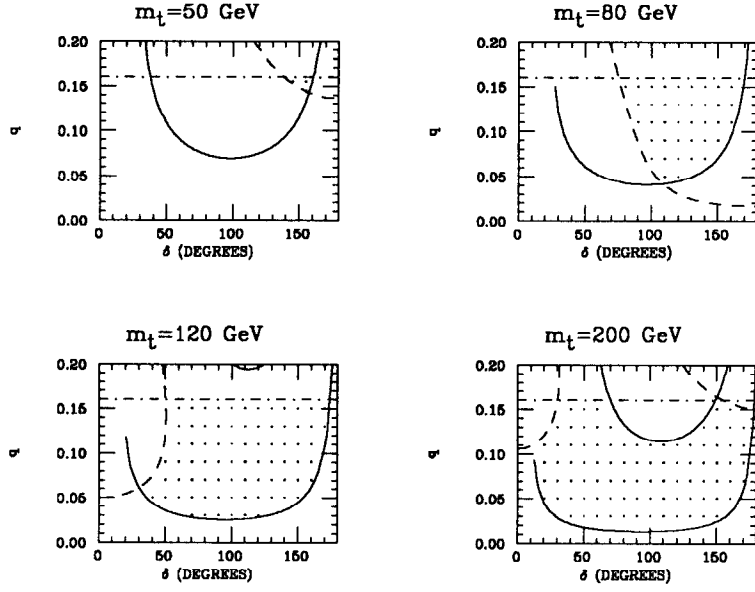


Fig. 1. Allowed range of $q = s_{13}/s_{23}$ and δ for $m_t = 50, 80, 120$ and 200 GeV. The dotdashed line gives the direct bound. The dashed lines give the x_d bounds. The solid lines give the ϵ bounds. The dotted area is the allowed range.

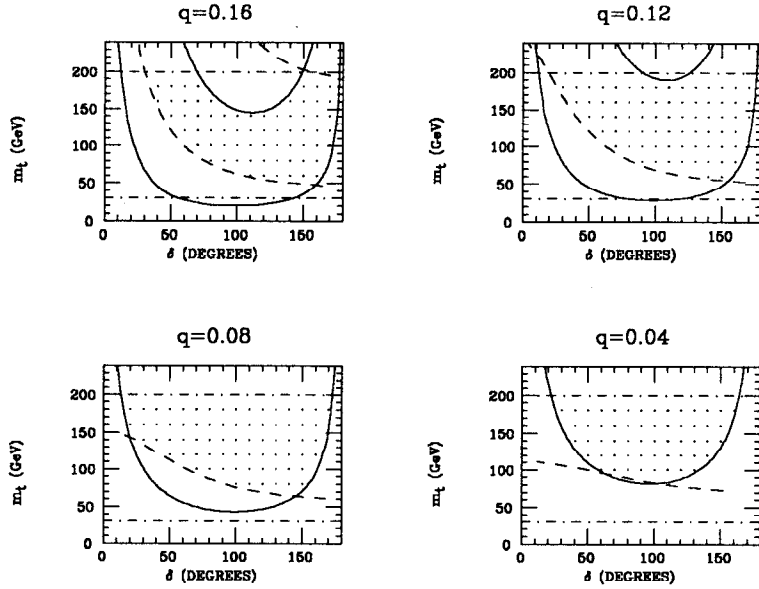


Fig. 2. Allowed range of m_t and δ for $q = 0.16, 0.12, 0.08$ and 0.04 . The dotdashed lines give the direct bounds. The dashed lines give the x_d bounds. The solid lines give the ϵ bounds. The dotted area is the allowed range.

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