SLAC–PUB–5036
September 1989
(T/E)

POLARIZED AND UNPOLARIZED INTRINSIC GLUON DISTRIBUTIONS*

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ABSTRACT

Theoretical constraints on both the polarized and unpolarized intrinsic gluon distribution functions are developed. In particular, we study the behavior of the gluon polarization asymmetry in the nucleon in the small and large x regions, and relate the intrinsic distribution to the retarded part of the spin-dependent bound-state potential. A simple model for the polarized and unpolarized intrinsic distributions is proposed which incorporates the QCD constraints. The model predicts that the spin carried by intrinsic gluons in the nucleon is approximately 0.5.

Submitted to *Physics Letters*

* Work supported in part by Department of Energy contract DE–AC03–76SF00515 (SLAC), and by Fundación Andes, Chile.

1. INTRODUCTION

The *intrinsic* gluon distribution $G_{g/H}(x, Q_0^2)$ describes the fractional light-cone momentum distribution of gluons associated with the bound-state dynamics of the hadron H, in distinction to the *extrinsic* distribution, which is derived from radiative processes or evolution. Given the intrinsic distribution, one can obtain the extrinsic distribution by applying the QCD evolution equations starting at the bound-state scale Q_0.

In principle, one must solve the non-perturbative bound state equation of motion to compute the intrinsic gluon distribution. In the case of positronium in quantum electrodynamics one can readily calculate the photon distribution, at least to first order in the fine structure constant α. The analysis requires coherence between amplitudes in which the electron and positron couple to the photons. In the infrared limit this coherence in the neutral atom ensures a finite photon distribution.

In the QCD case, the analysis of the intrinsic gluon distribution of a hadron is essentially non-perturbative. However, there are several theoretical constraints which limit its form:

1. In order to insure positivity of fragmentation functions, distribution functions $G_{a/b}(x)$ must behave as an odd or even power of $(1-x)$ at $x \to 1$ according to the relative statistics of a and b.[1] Thus the gluon distribution of a nucleon must have the behavior: $G_{g/N}(x) \sim (1-x)^{2k}$ at $x \to 1$ to ensure correct crossing to the fragmentation function $D_{N/g}(z)$. This result holds individually for each helicity of the gluon and the nucleon.

2. The coupling of quarks to gluons tends to match the sign of the quark helicity to the gluon helicity in the large x limit.[2] We define the helicity-aligned and

anti-aligned gluon distributions: $G^+(x) = G_{g\uparrow/N\uparrow}(x)$ and $G^-(x) = G_{g\downarrow/N\uparrow}(x)$. The gauge theory couplings imply

$$\lim_{x\to 1} G^-(x)/G^+(x) \to (1-x)^2. \quad (1)$$

3. In the low x domain the quarks in the hadron radiate gluons coherently, and one must compute emission of gluons from the quark lines taking into account interference between amplitudes. We define $\Delta G(x) = G^+(x) - G^-(x)$ and $G(x) = G^+(x) + G^-(x)$. We shall show that the asymmetry ratio $\Delta G(x)/G(x)$ vanishes linearly with x; perhaps coincidentally, this is also the prediction from Reggeon exchange.[3] The coefficient at $x \to 0$ depends on the hadronic wave-functions; however, for equal partition of the hadron's momentum among its constituents, we will show that

$$\lim_{x\to 0} \Delta G(x)/G(x) \to N_q\ x, \quad (2)$$

where N_q is the number of valence quarks.

4. In the $x \to 1$ limit, the stuck quark is far off-shell so that one can use perturbation theory to characterize the threshold dependence of the structure functions. We find for three-quark bound states

$$\lim_{x\to 1} G^+(x) \to C(1-x)^{2N_q-2} = C(1-x)^4, \quad (3)$$

Thus $G^-(x) \to C(1-x)^6$ at $x \sim 1$. This is equivalent to the spectator-counting rule developed in Ref. 4.

We can write down a simple analytic model for the intrinsic gluon distribution in the nucleon which incorporates all of the above constraints:

$$\Delta G(x) = \frac{N}{x}[(1-x)^4 + (1-x)^5 - 2(1-x)^6] \tag{4}$$

and

$$G(x) = \frac{N}{x}[(1-x)^4 + (1-x)^5 + 2(1-x)^6] \tag{5}$$

In this model the momentum fraction carried by intrinsic gluons in the nucleon is $< x_g >= \int_0^1 dx x G(x) = (137/210)N$, and the helicity carried by the intrinsic gluons is $\Delta G \equiv \int_0^1 dx \Delta G(x) = 8/15N$. The ratio $\Delta G / < x_g >= 112/137$ for the intrinsic gluon distribution is independent of the normalization N. Phenomenological analyses imply that the gluons carry approximately one-half of the proton's momentum: $< x_{g/N} >\simeq 0.5$. We shall assume that this is a good characterization of the intrinsic gluon distribution. The momentum sum rule then implies $N \sim 0.9$ and $\Delta G \sim 0.5$. A review of the present experimental and theoretical limits on gluon and quark spin in the nucleon is given in ref.5.

In the following sections we will analyze both the polarized and unpolarized intrinsic gluon distribution functions using both perturbative and non-perturbative methods. First we study the behavior of the gluon asymmetry (unpolarized over polarized distributions) in the small x region where it turns out to be approximately independent on the details of the bound-state wavefunction. The logarithmic ultraviolet cut-off dependence of the intrinsic distribution matches with the lower cut-off of the extrinsic distribution; the Q^2 evolution of the extrinsic distribution is studied in detail in Ref. 6.

In section 3 we shall show that the intrinsic gluon distribution is related to the retarded part of the spin-dependent bound-state potential $-\left\langle \frac{\Delta \partial V}{\partial M_B^2} \right\rangle_{hfs}$. This allows us to derive sum rules for the difference of gluon distribution (and fragmentation) functions for hadrons with different spin in terms of the spin-dependent part of the bound-state potential.

2. INTRINSIC GAUGE FIELD DISTRIBUTIONS

A general bound-state wavefunction can be expanded in terms of (Fock) states of definite number (n) of elementary free fields. We define the Fock expansion at equal "time" $\tau = t + z$ in the light cone gauge $A^+ = A^0 + A^3 = 0$. Labelling the corresponding renormalized amplitudes as $\psi^{(Q)}_{n/B}(x_i, \vec{k}_{\perp_i}, \lambda_i)$, the distribution function for a constituent $\underline{a}$ in the bound state $\underline{B}$ (see Ref. 7 for details and definitions) is given by:

$$G_{a/B}(x, Q^2) = \sum_{n,\lambda_i} \int \overline{\prod_i} \frac{dx_i d^2 k_{\perp_i}}{16\pi^3} \, |\psi^{(Q)}_{n/B}\,(\psi_i, \vec{k}_{\perp_i}, \lambda_i)|^2 \sum_b \delta(x_b - x) \quad . \qquad (6)$$

We first consider positronium as an example, and calculate the intrinsic distribution function of photons $G_{\gamma/positronium}$. To leading order in the binding energy we can neglect pair annihilation, pair production, and higher particle number Fock states.

The distribution function for positive helicity photons G^+ is calculated from the diagrams of Fig. 1(a) for the case of $J_z = +1$ ortho-positronium ($u_\uparrow \bar{v}_\uparrow$). Similarly, the corresponding diagrams for negative helicity photons are shown in Fig. 1(b), where an arrow up $\uparrow$ (down $\downarrow$) indicates positive (negative) helicity. In the diagrams, the upper fermion line corresponds to a particle (electron), and the lower to an antiparticle (positron). We have also indicated the light-cone parameterization of momenta that we will use when the photon couples to an electron or positron. With this choice, the photon is always parameterized by $(x, \vec{k}_\perp)$ and the final state has the same form in all cases. The appropriate matrix elements for the various helicity transitions are listed in Table I.[8]

The calculation is now straightforward. If we denote by $\psi(y, \vec{\ell}_\perp)$ the two-body bound-state valence wavefunction (lowest Fock state amplitude), the results are:

$$\begin{aligned} G^{+}_{\gamma/\mathrm{ortho}\uparrow}(x, \vec{k}_\perp) &= \frac{\alpha}{2\pi^2} \int \frac{d^2\ell_\perp}{2(2\pi)^3} \int_x^1 dy \\ &\times \Bigg\{ \left[\psi(y, \vec{\ell}_\perp) \frac{y\vec{k}_\perp - x\vec{\ell}_\perp}{y - x} - \psi(y - x, \vec{\ell}_\perp - \vec{k}_\perp) \frac{(1-y)\,\vec{k}_\perp + x\,\vec{\ell}_\perp}{1 - y} \right]^2 \frac{1}{x^3} \\ &+ \left[\left| \psi(y, \vec{\ell}_\perp) \right|^2 \frac{1}{(y-x)^2\, y^2} \right. \\ &+ \left. \left| \psi(y - x, \vec{\ell}_\perp - \vec{k}_\perp) \right|^2 \frac{1}{(1-y)^2\,(1 - [y - x])^2} \right] x\, m^2 \Bigg\} \frac{1}{D^2} \ , \end{aligned} \tag{7}$$

$$\begin{aligned} G^{-}_{\gamma/\mathrm{ortho}\uparrow}(x, \vec{k}_\perp) &= \frac{\alpha}{2\pi^2} \int \frac{d^2\ell_\perp}{2(2\pi)^3} \int_x^1 dy \\ &\times \left[\psi(y, \vec{\ell}_\perp) \frac{y\vec{k}_\perp - x\vec{\ell}_\perp}{y} - \psi(y - x, \vec{\ell}_\perp - \vec{k}_\perp) \frac{(1-y)\,\vec{k}_\perp + x\,\vec{\ell}_\perp}{1 - (y - x)} \right]^2 \frac{1}{x^3 D^2} \ , \end{aligned}$$

where

$$D = M_B^2 - \frac{(\vec{\ell}_\perp - \vec{k}_\perp)^2 + m^2}{y - x} - \frac{\vec{\ell}_\perp^{\,2} + m^2}{1 - y} - \frac{\vec{k}_\perp^{\,2}}{x} \ . \tag{8}$$

Here m and M_B are the electron and bound state masses, respectively. The intrinsic gluon distribution defined in Eq. 6 is obtained by integrating these expressions over the transverse momentum up to the cut-off Q_0^2. The same approach gives

$$
\begin{aligned}
G^+_{\gamma/\mathrm{para}} &= G^-_{\gamma/\mathrm{para}} \quad , \\
G^+_{\gamma/\mathrm{ortho}\ J_z=0} &= G^-_{\gamma/\mathrm{ortho}\ J_z=0} \quad .
\end{aligned}
\tag{9}
$$

The polarized and unpolarized photon distribution functions are given by:

$$
\begin{aligned}
\Delta G(x,\vec{k}_\perp) &\equiv G^+(x,\vec{k}_\perp) - G^-(x,\vec{k}_\perp) \quad , \\
G(x,\vec{k}_\perp) &\equiv G^+(x,\vec{k}_\perp) + G^-(x,\vec{k}_\perp) \quad .
\end{aligned}
\tag{10}
$$

Let us now consider the small x limit for these functions. Expanding around $x = 0$, we readily obtain:

$$
\begin{aligned}
\Delta G(x \sim 0, \vec{k}_\perp) = \frac{\alpha}{\pi^2 \vec{k}_\perp^2} \int \frac{d^2\ell_\perp}{2(2\pi)^3} \int_0^1 dy \ \left[\psi(y,\vec{\ell}_\perp) - \psi(y,\vec{\ell}_\perp - \vec{k}_\perp) \right] \\
\times \left[\frac{\psi(y,\vec{\ell}_\perp)}{y} - \frac{\psi(y,\vec{\ell}_\perp - \vec{k}_\perp)}{1-y} \right]
\end{aligned}
\tag{11}
$$

and

$$
G(x \sim 0, \vec{k}_\perp) = \frac{\alpha}{\pi^2 \vec{k}_\perp^2 x} \int \frac{d^2\ell_\perp}{2(2\pi)^3} \int_0^1 dy \ \left[\psi(y,\vec{\ell}_\perp) - \psi(y,\vec{\ell}_\perp - \vec{k}_\perp) \right]^2 .
$$

The infrared singularity at $\vec{k}_\perp^2 \to 0$ is eliminated because of the neutrality of the atom.

It should be noted that the singularity in $G(x,\vec{k}_\perp)$ at $x \to 0$ is actually an ultraviolet singularity for any non-zero value of $\vec{k}_\perp$ since $x = (k^0 + k^z)/(p^0 + p^z)$ can only be zero if $k_z \to -\infty$. By definition, the intrinsic distribution $G(x, Q_0^2)$ refers to Fock states with limited parton invariant mass $\mathcal{M}$: $\mathcal{M}^2 = \sum_i \left[\frac{\vec{k}_\perp^2 + m^2}{x}\right]_i < Q_0^2$. This restriction regularizes the $x \to 0$, $\vec{k}_\perp \neq 0$ region. On the other hand, the extrinsic contribution is derived from Fock states exceeding this cut-off, $Q_0^2 < \mathcal{M}^2 < Q^2$. Physical quantities are independent of the intermediate cut-off Q_0; the logarithmic dependence on Q_0 cancels in the sum of intrinsic and extrinsic structure functions.

Note that the integral of xG, the momentum fraction carried by intrinsic photons, is always well-defined. In order to proceed further, we shall assume that the wave function $\psi(y, \vec{\ell}_\perp)$ is peaked at $y \simeq 1/2$. We then obtain

$$\frac{\Delta G(x, \vec{k}_\perp)}{G(x, \vec{k}_\perp)} \simeq x \left\langle \frac{1}{y} \right\rangle \simeq 2x \qquad (x \to 0) \quad , \tag{12}$$

for the polarization asymmetry. We have found that this result is numerically accurate for a large range of positronium wavefunctions.

The opposite region $(x \to 1)$, where the fermions emit hard photons, can be also readily studied. After changing variables $(1-y) = (1-x)(1-\tau)$, and expanding around $(1-x) \to 0$, we obtain:

$$\begin{aligned} G^+(x, \vec{k}_\perp) &= \frac{(1-x)}{2\pi^2\, 2(2\pi)^3} \int_0^1 d\tau \\ &\left\{ \left[\psi(y, \vec{\ell}_\perp) \frac{\vec{k}_\perp - \vec{\ell}_\perp}{\tau} - \psi(y-x, \vec{\ell}_\perp - \vec{k}_\perp) \frac{\vec{k}_\perp}{1-\tau} \right]^2 \right. \\ &\left. + m^2 \left[\frac{\left| \psi(y, \vec{\ell}_\perp) \right|^2}{\tau^2} + \frac{\left| \psi(y-x, \vec{\ell}_\perp - \vec{k}_\perp) \right|^2}{(1-\tau)^2} \right] \right\} \\ &\times \frac{1}{\left[\frac{(\vec{\ell}_\perp - \vec{k}_\perp)^2 + m^2}{\tau} + \frac{\vec{\ell}_\perp^2 + m^2}{1-\tau} \right]^2} \quad , \end{aligned} \tag{13}$$

$$G^-(x, \vec{k}_\perp) = \frac{(1-x)^3}{2\pi^2\, 2(2\pi)^3} \int_0^1 d\tau$$

$$\left[\ \psi(y,\vec{\ell}_\perp)\,(\vec{k}_\perp-\vec{\ell}_\perp)\ -\ \psi(y-x,\,\vec{\ell}_\perp-\vec{k}_\perp)\,\vec{\ell}_\perp\ \right]^2$$

$$\times\ \frac{1}{\left[\frac{(\vec{\ell}_\perp-\vec{k}_\perp)^2+m^2}{\tau}+\frac{\vec{\ell}_\perp^2+m^2}{1-\tau}\right]^2}\quad .$$

Thus the $x \to 1$ behavior depends on the endpoint behavior of the wavefunction $\psi(y,\vec{\ell}_\perp)$. Let us assume that $\psi(y,\vec{\ell}_\perp) \sim y^p$ for $y \to 0$, and $\sim (1-y)^q$ for $y \to 1$. If $p > q$, then the terms that contain $\left|\ \psi(y,\vec{\ell}_\perp)\ \right|^2$ dominate at $x \to 1$ since $y > x$. This regime corresponds to the photon taking most of the longitudinal momentum of the bound state from the electron. If $p < q$, the terms that contain $\left|\ \psi(y-x,\vec{\ell}_\perp-\vec{k}_\perp)\ \right|^2$ will dominate, which corresponds to the photon taking its large momentum from the positron. Then

$$\begin{aligned} G^+ &= \text{constant } (1-x)^{1+2h} \\ G^- &= \text{constant } (1-x)^{3+2h} \end{aligned} \qquad (x \to 1)\quad , \tag{14}$$

where $h = min(p,q)$ is the lowest endpoint power $(y \to 0, y \to 1)$ behavior of $\psi(y,\vec{\ell}_\perp)$. If $\psi(y,\vec{\ell}_\perp)$ is invariant under $y \to (1-y)$, then the two endpoint powers are the same. In any case:

$$\frac{\Delta G(x,\vec{k}_\perp)}{G(x,\vec{k}_\perp)} \to 1 \qquad (x \to 1)\quad ; \tag{15}$$

i.e., the helicity of the photon tends to be aligned with that of the bound state at large x. In the case of relativistic positronium $h = 1$.[9]

We now extend this analysis to QCD bound states. A perturbative analysis is certainly justified for heavy quark systems[10]. Since the general structure of the fermion $\to$ fermion plus gluon vertices given in Table I is dictated by Lorentz invariance and parity conservation, we will assume that this perturbative structure is also applicable to light-quark systems. We thus analyze the intrinsic gluon distribution retaining only first order corrections to the valence Fock state. The appropriate color factor is obtained by the replacement of (α) by $(C_F \alpha_s)$ where $C_F = 4/3$ for $N_C = 3$. We find similar endpoint behavior to that found in the abelian calculation. In particular, the gluon asymmetry at $x \to 0$ is $\Delta G(x)/G(x) \simeq < 1/y > x \simeq N_q x$ where N_q is the number of fermions in the valence Fock state. The $x \to 1$ behavior for the three-quark proton can also be determined[11]

$$\begin{aligned} G^+ &\sim (1-x)^4 \\ G^- &\sim (1-x)^6 \end{aligned} \qquad (x \to 1) \quad . \tag{16}$$

3. CONNECTION WITH THE BOUND STATE POTENTIAL

On general grounds we expect a connection between the probability for emission (distribution function of photons or gluons) and the hyperfine interaction part of the bound state potential since both depend on the exchange of transverse gauge quanta. In fact, each diagram that contributes to the transverse potential has a corresponding cut-diagram in the expression for the distribution function. In the actual calculation, these quantities differ by just a denominator D. Thus

$$\int_0^1 dx\, G_{g/B}\,(x, Q_0^2) \;=\; -\left\langle \frac{\partial V}{\partial M_B^2} \right\rangle_{Q_0^2} \quad , \tag{17}$$

where $G_{g/B}$ is the unpolarized distribution function of gauge fields g in the bound state B, V is the potential due to gluon exchange and self-energy corrections, and M_B is the bound-state mass. Note that the instantaneous (non-retarded) piece does not depend on M_B, so it does not contribute. As discussed in section 2, these quantities are regulated at $x \rightarrow 0$ by the ultraviolet cutoff Q_0^2 in the invariant mass. This singularity cancels in the hyperfine splitting:

$$\int_0^1 dx \ \left[G_{\gamma/\text{ortho}\uparrow}(x) - G_{\gamma/\text{para}}(x) \right] = -\left\langle \frac{\Delta \partial V}{\partial M_B^2} \right\rangle_{hfs} . \tag{18}$$

where $\langle \ \rangle_{hfs}$ refers to the spin-dependent part of the bound state potential.

In the case of gluons in QCD bound states, we obtain analogous results:

$$\int_0^1 dx \ \left[G_{g/\rho}(x) - G_{g/\pi}(x) \right] = -\left\langle \frac{\Delta \partial V}{\partial M_B^2} \right\rangle_{hfs} , \tag{19}$$

for mesons (ρ and π), and

$$\int_0^1 dx \ \left[G_{g/p}(x) - G_{g/\Delta}(x) \right] = -\left\langle \frac{\Delta \partial V}{\partial M_B^2} \right\rangle_{hfs} , \tag{20}$$

for baryons (p and Δ).

These expressions can be analytically continued, relating the difference of fragmentation functions of gluons $D_{H/g}(z, Q^2)$ into hadrons H of different spin to the hyperfine splitting piece of the bound state potential.

4. CONCLUSIONS

The gluon distribution of a hadron is usually assumed to be generated from QCD evolution of the quark structure functions beginning at an initial scale Q_0^2.[12] In such a model there are no gluons in the hadron at a resolution scale below Q_0. The evolution is completely incoherent; i.e., each quark in the hadron radiates independently.

In the approach presented here it is recognized that the bound state wavefunction itself generates gluons. This is clear from the relationship, Eq. 17, which connects the gluon distribution to the transverse part of the bound-state potential. To the extent that gluons generate the binding, they also must appear in the intrinsic gluon distribution. We emphasize that the diagrams in which gluons connect one quark to another are not present in the usual QCD evolution equations. Evolution contributions correspond in the bound-state equation to self-energy corrections to the quark lines at resolution scales $\mathcal{M}^2 > Q_0^2$.

Eqs. 4 and 5 give model forms for the polarized and unpolarized intrinsic gluon distributions in the nucleon which take into account coherence at low x and perturbative constraints at high x. It is expected that this should be a good characterization of the gluon distribution at the resolution scale $Q_0^2 \simeq M_p^2$.

It is well-known that the leading power at $x \sim 1$ is increased when QCD evolution is taken into account. The change in power is[1]

$$\Delta p_g(Q^2) = 4C_A\ \zeta(Q^2, Q_0^2) = \frac{1}{\pi}\int_{Q_0^2}^{Q^2} \frac{d\kappa^2}{\kappa^2}\alpha_s(\kappa^2), \tag{21}$$

where $C_A = 3$ in QCD. For typical values of $Q_0 \sim 1\ GeV$, $\Lambda_{\overline{MS}} \sim 0.2\ GeV$ the change in power is moderate: $\Delta p_g(2\ GeV^2) = 0.28$, $\Delta p_g(10\ GeV^2) = 0.78$. A recent determination of the unpolarized gluon distribution of the proton at $Q^2 = 2\ GeV^2$ using direct photon and deep inelastic data has been given in ref. 13. The best fit over the interval $0.05 \le x \le 0.75$ assuming the form $xG(x, Q^2 = 2\ GeV^2) = A(1-x)^{\eta_g}$ gives $\eta_g = 3.9 \pm 0.11(+0.8 - 0.6)$, where the errors in parenthesis allow for systematic uncertainties. This result is compatible with the prediction $\eta_g = 4$ for the intrinsic gluon distribution at the bound-state scale, allowing for the increase in the power due to evolution.

Acknowledgements

We wish to thank J. Gunion, M. Karliner and P. G. Lepage for helpful discussions. IS also thanks the kind hospitality of R. Blankenbecler and the theory group at SLAC.

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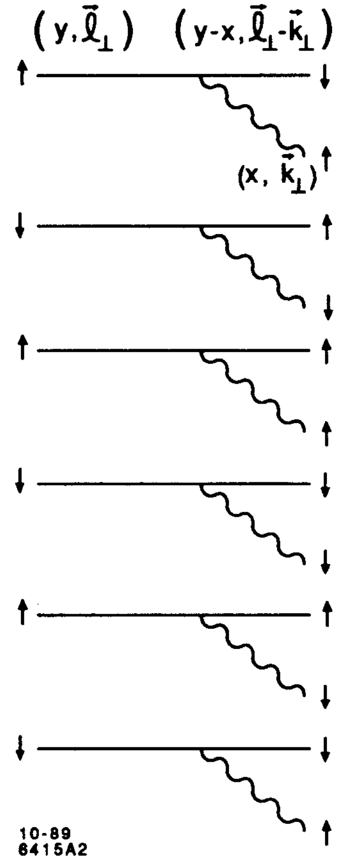

Initial	Final	Emitted	Vertex
↑	↓	↑	$\left(\frac{1}{y-x} - \frac{1}{y}\right) m$
↓	↑	↓	$\left(\frac{1}{y-x} - \frac{1}{y}\right) m$
↑	↑	↑	$\left(\frac{-k_1 + ik_2}{x} + \frac{(\ell_1 - k_1) - i(\ell_2 - k_2)}{y-x}\right)$
↓	↓	↓	$\left(\frac{k_1 + ik_2}{x} - \frac{(\ell_1 - k_1) + i(\ell_2 - k_2)}{y-x}\right)$
↑	↑	↓	$\left(\frac{k_1 + ik_2}{x} - \frac{\ell_1 + i\ell_2}{y}\right)$
↓	↓	↑	$\left(\frac{-k_1 + ik_2}{x} + \frac{\ell_1 - i\ell_2}{y}\right)$

Table I

Photon/gluon emission vertices $(e\overline{u}_{\lambda_f} \epsilon^*_{\lambda_\gamma} \cdot \gamma u_{\lambda_i})$ for particles with positive (↑) and negative (↓) helicities, in light-cone coordinates. An overall factor $A = \pm 2\sqrt{2\pi\alpha}\ \sqrt{y-x}\ \sqrt{y}$ multiplies each result, for fermions and anti-fermions respectively. For gluon emission α is replaced by $4/3\ \alpha_s$.

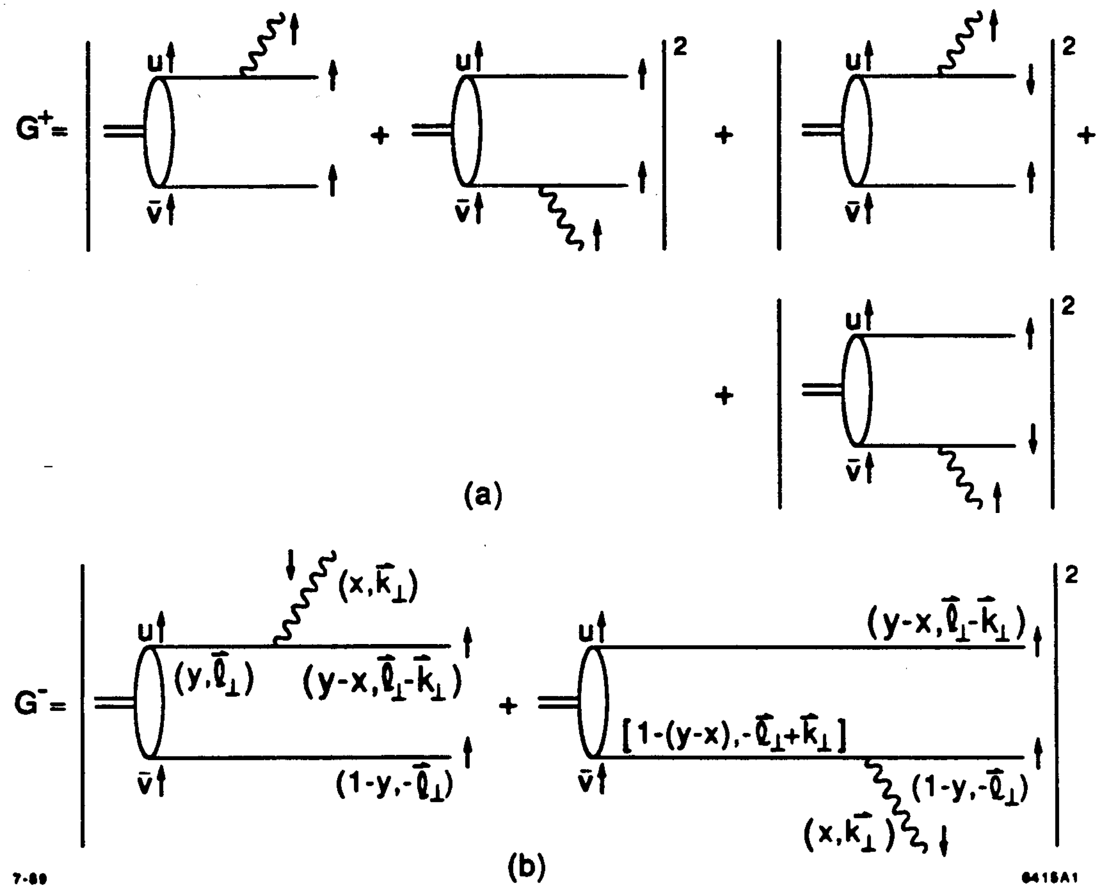

Fig. 1 Diagrams that contribute to the distribution function for positive polarized photons (a), and for negative polarized photons (b), for $J_z = +1$ ortho-positronium ($u_\uparrow \overline{v}_\uparrow$).