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Moduli Spaces and Topological Quantum Field Theories.

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ABSTRACT

We show how to construct a topological quantum field theory which corresponds to a given moduli space. This method is applied to several cases. In particular we discuss the moduli space of flat gauge connections over a Riemann surface which is related to the phase space of the Chern-Simons theory. The observables of these theories are derived. Geometrical properties are invoked to prove that the global invariants are not trivial.

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Generally covariant field theories have observables which are metric independent. Hence they are global invariants. Recently, a new class of such theories, the so called topological quantum field theories (TQFTs'), were introduced by E.Witten. Originally they were affiliated with Yang-Mills instantons (TYM),[1] sigma models (TSM)[2], and gravity (TG)[3]. Later on they enveloped other domains of physical systems[5–8]. The main question is obviously whether the TQFT's probe some physical phenomena or are they merely mathematical tools to study topological properties of certain bundles? The answer to this question is two-fold: (i) The observables of the TQFT span the cohomology ring on certain moduli spaces. These moduli spaces may be intimately related to physics. An example familiar to string theorists is the moduli space of Riemann surfaces. Another example is the moduli space of instantons. (ii) The possibility that the TQFT's describe a generally covariant phase of some physical systems[1,8]. In this work we follow the first direction.

The main feature of the TQFT's is the "topological symmetry" which is the largest local symmetry possible for the fields that describe the system. This symmetry is responsible for gauging away any dependence on local properties. The classical action does not play any role and can be taken to be zero or a topological number. The quantum Lagrangian is derived via BRST gauge fixing of the topological symmetry and related "ghost symmetries"[10][11]. The observables of the theory, which are expectation values mainly of the ghost fields, can be expressed as an integral of closed forms on some moduli space. Can one write down a TQFT which corresponds to any given moduli space? In this work we present a general prescription for the building of such a TQFT. Several examples of TQFT's which correspond to interesting moduli spaces are presented. We show how the observables of the theory correspond to cohomologies on the moduli space. The perception that those global invariants are trivial is shown to be incorrect.

The connection between moduli spaces and physical systems can be described simply in the following way. Assume that a physical system is defined by a set of fields Φ_i on a d dimensional space-time manifold M, and a certain local symmetry

G under which the fields Φ_i transform in some representation of the group G . Mathematically, a certain bundle is defined over M. Very often we are interested not in the whole space of possible Φ_i configurations but in a particular subset which can be characterized by

$$\{\Phi_i^0 | F(\Phi_i^0) = 0\}, \tag{1}$$

where $F(\Phi_i)$ is a given functional of the fields Φ_i. The condition (1) can be, for example, the Euler-Lagrange equations of the action describing the physical system. We now perturb a given configuration in this subspace and demand that the perturbation does not take Φ_i out of this subspace, namely:

$$F(\Phi_i^0 + \delta\Phi_i) = 0 \qquad \rightarrow \qquad \frac{\delta F}{\delta \Phi_i}\delta\Phi_i = 0. \tag{2}$$

We want further to mod out from the possible variations, $\delta\Phi_i$, those redundant ones which are the transformation of Φ_i under G. We, therefore, choose a gauge slice by imposing a gauge condition

$$G_{GF}(\Phi_i^0 + \delta\Phi_i) = 0 \qquad \rightarrow \qquad \frac{\delta G_{GF}}{\delta \Phi_i}\delta\Phi_i = 0. \tag{3}$$

As for solutions to the equations (2-3) there are two possibilities (i) No non-trivial solutions, then the Φ_i^0 configurations are isolated. (ii) There are solutions and then these solutions span the moduli space, $\mathcal{M}$, of configurations fulfilling (1) modulo gauge transformations. For finite deformations we want to integrate $\delta\Phi_i$ but there may be obstructions[1,2] to the integration of the infinitesimal deformations. Regarding the solutions of (2-3) as the kernel of an operator $\bar{D}$ acting on $\delta\Phi_i$, then the obstructions are given by the cokernel of this operator. Therefore, the dimension of the moduli space is the number of solutions minus the number of obstructions which is :

$$dim\mathcal{M} = dim(Kernel\bar{D}) - dim(coKernel\bar{D}) = index\bar{D} \tag{4}$$

We demonstrate the statements made above in table 1 † for the moduli spaces which are related to various physical systems: (i) Yang Mills instantons in four dimensions[1,11], (ii) flat connections in two dimensions (which is equivalent to the phase-space of the correspondig three dimensional Chern-Simons theory), (iii) flat SO(2,1) connections[12] (which is equivalent to the space of Riemann surfaces with $g > 1$), (iv) World sheet instantons in two dimensions[2], (v) two-torus[4] and (vi) (1,1) forms on Calabi-Yau manifolds.

† The notations in the table follow references:[1,11], [2,4], and[12]

Configuration	G-Symmetry	Conditions on $\delta\Phi_i$	Moduli Space
A_μ : non-abelian gauge fields in four dim.	non-abelian gauge symmetry	$D_{[\mu}\delta A_{\nu]} + \epsilon_{\mu\nu\rho\sigma}D^{[\rho}\delta A^{\sigma]} = 0$ $D_\mu \delta A^\mu = 0$	Yang Mills instantons
A_α : non-abelian gauge fields in two dim.	non-abelian gauge symmetry	$D_{[\alpha}\delta A_{\beta]} = 0$ $D_\alpha \delta A^\alpha = 0$	non-abelian flat connections
$(e_{\alpha a},\ \omega_\alpha)$: world sheet SO(2,1) connections	SO(2,1) gauge symmetry	$\partial_{[\alpha}\delta\omega_{\beta]} + \epsilon_{ab}e^a_{[\alpha}\delta e^b_{\beta]} = 0$ $(\tilde{D}_{[\alpha}\delta e_{\beta]})^a + \epsilon^{ab}e_{b[\alpha}\delta\omega_{\beta]} = 0$ $\tilde{D}^{ab}_\alpha = \delta^{ab}\partial_\alpha + \epsilon^{ab}\omega_\alpha$	Riemann surfaces of $g > 1$
x^i : coordinates on symplectic manifold	world-sheet reparametrization	$D_\alpha \delta x^i + \epsilon_{\alpha\beta}J^i_j D^\beta \delta x^j = 0$ J: complex structure	world sheet instantons
$g_{\alpha\beta}$: metric on torus	world sheet reparametrization	$\partial_z\partial_{\bar z}(g^{z\bar z}\delta g_{z\bar z}) = 0$ $g_{\bar z\bar z} = g_{zz} = 0$	torus
$g_{i\bar j}$: metric on Kahler Manifold	diffeomorphism on Khaler manifold	$\partial_i\partial_{\bar j}(\frac{\delta g}{g}) = 0$ $g = det(g_{i\bar j})$	(1,1) forms Calabi-Yau Manifold

Table 1- *Examples of moduli spaces.*

The basic idea of the use of TQFT to explore the moduli spaces is to formulate a field theory which is invariant under an additional local "topological symmetry"[9-11] of the form $\delta\Phi_i = \Theta_i(x)$ where Θ_i has the same properties as Φ_i under the Lorentz and G transformations but may differ in boundary conditions. The form of the original action is not important as long as it is invariant under the topological symmetry. In general, the Lagrangian is taken to be zero up to topological terms and up to eliminating auxiliary fields. In the case that the configurations, Φ_i^0, are characterized by a topological number which can be expressed as a d dimensional integral, it makes sense to take the later as the action. This will imply some boundary condition on the local parameter of the topological symmetry[11]. Quantization of the TQFT is performed by using the BRST method. $\epsilon\Psi_i$ is now replacing Θ_i where ϵ is an anti-commuting global parameter and Ψ_i is an anti-commuting ghost. The gauge-fixing and Faddev-Popov Lagrangians are derived by BRST variation of a "gauge condition" $\mathcal{Z}^{(1)}$:

$$\mathcal{L}^{(1)}_{(GF+FP)} = \hat{\delta}^{(1)}\mathcal{Z}^{(1)} = \hat{\delta}^{(1)}[\bar{\Psi}F(\Phi_i)] = BF(\Phi_i) - \bar{\Psi}\hat{\delta}[F(\Phi_i)]. \tag{5}$$

Here $\delta_{BRST} = i\epsilon\hat{\delta}$, $\bar{\Psi}$ is an anti-ghost in a representation of the group G and the Lorentz group such that $\bar{\Psi}F(\Phi_i)$ is a singlet under both groups and B is the associated auxiliary field. The Euler-Lagrange equation for $\bar{\Psi}$ leads to an equation for Ψ_i which is the same as eqn. (2) for $\delta\Phi_i$. The Lagrangian (4) is further invariant under a local "ghost symmetry". The origin of this symmetry is the following: $\mathcal{Z}^{(1)}$ is obviously invariant under the G symmetry, thus transformations that leave Φ_i and $\bar{\Psi}_i$ inert and transform Ψ_i and B in the same way as Φ_i and $\bar{\Psi}$ transform under G, leave (5) invariant. In general, one can replace $\mathcal{Z}^{(1)}$ by $\mathcal{Z}^{(1)'} = \bar{\Psi}(F(\Phi_i) + \alpha B)$ where α is an arbitrary parameter. For $\alpha \neq 0$ the "ghost symmetry" mentioned above is not a symmetry. However, by adopting the "ghost symmetry" transformation, for the variation of B in $\mathcal{Z}^{(1)'}$ the resulting $\mathcal{L}^{(1)'}$ is invariant again under a "ghost symmetry"[13] . We thus use here the $\alpha = 0$ gauge.

To fix the "ghost symmetry" we introduce a commuting "ghost for ghosts" field ϕ and a its anti-ghost $\bar{\phi}$. The BRST gauge fixing Lagrangian now has the following form:

$$\mathcal{L}^{(2)}_{(GF+FP)} = \hat{\delta}^{(2)}\mathcal{Z}^{(2)} = \hat{\delta}^{(2)}[\bar{\phi}G_{GF}(\Psi_i)], \tag{6}$$

where $\hat{\delta}^{(2)}$ is the sum of the $\hat{\delta}^{(1)}$ and the BRST transformations associated with the ghost symmetry. $G_{GF}(\Phi_i)$ is the gauge condition of (3). It is now obvious that the equation of motion of the combined action will require Ψ_i to obey eqn. (3). The equations for Ψ_i are therefore identical to those which define the moduli space (2-3). The condition for having a moduli space $\mathcal{M}$, thus, translate into a condition of having ghost zero modes.

We now describe this construction for the examples of above in table 2.

Topological Symmetry	Gauge Fixing	Ghost Symmetry	Equations of Ψ
$\hat{\delta}A_\mu = \psi_\mu$	$\hat{\delta}[\bar{\psi}^{\mu\nu}(F_{\mu\nu} + \tilde{F}_{\mu\nu})]$	$\hat{\delta}\psi_\mu = iD_\mu\phi$ $\hat{\delta}B^{\mu\nu} = i[\bar{\psi}^{\mu\nu}, \phi]$	$D_{[\mu}\psi_{\nu]} + \epsilon_{\mu\nu\rho\sigma}D^{[\rho}\psi^{\sigma]} = 0$ $D_\mu\psi^\mu = 0$
$\hat{\delta}A_\alpha = \psi_\alpha$	$\hat{\delta}[\bar{\psi}^{\alpha\beta}(F_{\alpha\beta})]$	$\hat{\delta}\psi_\alpha = iD_\alpha\phi$ $\hat{\delta}B^{\alpha\beta} = i[\bar{\psi}^{\alpha\beta}, \phi]$	$D_{[\alpha}\psi_{\beta]} = 0$ $D_\alpha\psi^\alpha = 0$
$\hat{\delta}e_{\alpha a} = \psi_{\alpha a}$ $\hat{\delta}\omega_\alpha = \tilde{\psi}_\alpha$	$\hat{\delta}[\bar{\tilde{\psi}}(\epsilon^{\alpha\beta}\partial_\alpha\omega_\beta + det\ e)]$ $+\epsilon^{\alpha\beta}\bar{\psi}^a D_\alpha e_{\beta a}]$	$\hat{\delta}\psi_\alpha = iD_\alpha\phi$ $\hat{\delta}B = i[\bar{\psi}, \phi]$	$\partial_{[\alpha}\tilde{\psi}_{\beta]} + \epsilon_{ab}e^a_{[\alpha}\psi^b_{\beta]} = 0$ $(\tilde{D}_{[\alpha}\psi_{\beta]})^a + \epsilon^{ab}e_{b[\alpha}\tilde{\psi}_{\beta]} = 0$
$\hat{\delta}x^i = \psi^i$	$\hat{\delta}[\bar{\psi}^\alpha_i(D_\alpha x^i + \epsilon_{\alpha\beta}J^i_j D^\beta x^j - B^i_\alpha)]$ $\bar{\psi}^i_\alpha = \epsilon_{\alpha\beta}J^i_j\bar{\psi}^\beta_j$		$D_\alpha\psi^i + \epsilon_{\alpha\beta}J^i_j D^\beta\psi^j = 0$
$\hat{\delta}g_{\alpha\beta} = \psi_{\alpha\beta}$	$\hat{\delta}[\bar{\psi}\sqrt{g}R^{(2)}]$	$\hat{\delta}\psi_{\alpha\beta} = D_{(\alpha}\phi_{\beta)}$ $\hat{\delta}B = \phi^\alpha D_\alpha\bar{\psi}$	$D_\alpha D^\alpha\psi = 0$ $\psi = \psi^\alpha_\alpha$
$\hat{\delta}g_{i\bar{j}} = \psi_{i\bar{j}}$	$\hat{\delta}[\bar{\psi}^{i\bar{j}}\partial_i\partial_{\bar{j}}log\ g]$		$\partial_i\partial_{\bar{j}}(\psi) = 0$ $\psi = \psi_{i\bar{j}}g^{i\bar{j}}$

Table 2- *the TQFT's which correspond to the moduli spaces given in table 1.*

Several remarks on the TQFT's given in table 2 are in order: (i) The TFC case and its relation to the Chern-Simons theory and to conformal field theory were presented in ref. [5].

(ii) The TFC for the SO(2,1) group was shown to correspond to the space of Reimann surfaces[12,5]

(iii) The combination of the TSM and TG leads to a theory of topological strings[4]. The corresponding target manifold, which has to be a Kahler manifold, can have any number of dimensions. This bosonic string theory is freed from tachyons.

The BRST algebra that we have at the present stage is not nilpotent but rather it is closed up to a G transformation, $\hat{\delta}_G$, with the ghost for ghost ϕ as the parameter of transformation. For example $(\hat{\delta}^{(2)})^2\Phi_i = \hat{\delta}_G\Phi_i$.

So far we considered only configuration which minimize the action. In particular Φ_i^0 and Ψ_i configurations which are solutions to eqn.(2-3). This is justified only if the path integral is dominated by those configurations. As for Φ^0, this is obvious since this was the gauge fixing we used. As for the rest of the fields we can modify the BRST transformations $\hat{\delta} \to \hat{\delta}' = \kappa\hat{\delta}$ such that the $\mathcal{L} \to \kappa\mathcal{L}$. It is straightforward to see that correlation functions are also κ independent[5]. Now in the large κ limit it is obvious that the path integral is dominated by the minima of the action.

The correspondence between the TQFT and the related moduli spaces includes the obstruction as well. Recall that the dimension of the moduli space is equal to the index of the operator defined in (2) and (3). In the TQFT the kernel corresponds to the Ψ zero modes. The cokernel is given by the zero modes of $\bar{\Psi}$ and $\hat{\delta}\bar{\phi}$. Thus the number of obstructions is given by the number of the latter zero modes.

$$index\bar{D} = \#(\Psi\ zero\ modes) - \ \#(\bar{\Psi}, \hat{\delta}\bar{\phi}\ zero\ modes) = Dim\mathcal{M} \qquad (7)$$

We want to address now the question of the observables of the TQFT's. The correlation functions of BRST invariant operators are independent of arbitrary variations of the metric[1].

$$\delta_{g_{\alpha\beta}} < \mathcal{O} >= \delta_{g_{\alpha\beta}} \int DX\mathcal{O}e^{i\int d^dx\hat{\delta}\mathcal{Z}} = \int DXe^{i\int d^dx\hat{\delta}\mathcal{Z}}\mathcal{O}\hat{\delta}[\delta_{g_{\alpha\beta}}\int d^dx\mathcal{Z}] = 0, \quad (8)$$

where DX is the measure, and $\mathcal{O}$ is an operator which is a BRST scalar and is independent on the metric. We used here the fact that a vev of any BRST transformation is zero.

Due to the BRST symmetry, the fermionic determinant is equal to the bosonic up to a sign[1]. Therefore, in the case of no ghost zero modes ($dim\mathcal{M} = 0$) , the partition function is given by $Z = \Sigma_j (-1)^{S_j}$ where the sum is over all isolated $\Phi_i^{0(j)}$ configurations and S_j is the sign of the ratio of determinantes at the (j) configuration. In general, it was shown[1] that an expectation of an operator has the form of an integral over the moduli space of a closed form on this space.

$$< \mathcal{O} >= \int da_1....da_n \; d\psi_1....d\psi_n \; \Omega_{i_1...i_n} \; \psi_{i_1}...\psi_{i_n} = \int_{\mathcal{M}} \mathbf{\Omega}, \tag{9}$$

where $da_i, d\psi_i$ denote the bosonic and fermionic zero modes respectively and

$\Omega_{i_1...i_n} da^{i_1}...da^{i_n} = \mathbf{\Omega}$ is an n form on $\mathcal{M}$.

The global invariants $I_i^{(i,l)}$, $i = 0, ..., d$ (the pair of superscripts are the degrees of the form on M and $\mathcal{M}$ respectively) obey the following properties:

$$\begin{aligned} I_i^{(i,l)} &= \int_{\gamma_i} W_i^{(i,l)} \\ \hat{\delta}^{(2)} W_i^{(i,l-i)} &= dW_{(i-1)}^{(i-1,l+1-i)} \quad \hat{\delta}^{(2)} W_0^{(0,l)} = 0 \quad dW_d^{(d,l-d)} = 0, \end{aligned} \tag{10}$$

where γ_i is a non-trivial i^{th} homology cycle. In case that $dim\mathcal{M} \neq 0$ there are fermion zero modes for the Ψ system. Therefore the only non-trivial expectation values are of operators which can soak up those zero modes. This condition translates to a requirement on an observable I:

$$< I >=< \Pi_j I_j > \qquad \Sigma l_j = Dim\mathcal{M} \tag{11}$$

In fact, the W_i are mappings from closed forms on M to closed forms on $\mathcal{M}$ and therefore the global invarinats span the cohomology ring on $\mathcal{M}$. It is, thus, clear why they can be sensors only for topological properties on $\mathcal{M}$ but not local ones.

The dimensions[1,5,12,2,4] and global invariants of the previous examples are given in table 3.

	Dim. of moduli space $(\#\Psi^0 - \#(\bar{\Psi}, \hat{\delta}\bar{\phi})^0)$	Global Invariants $I_i = \int_{\gamma_i} W_i$
TYM	$8P - \frac{3}{2}(\chi(M) + \sigma(M))$ for G=SU(2) P-*Pontryagin#* $\chi(M)-$ *Euler #* , $\sigma(M)-$*signature* 8P- 3 *for Euclidean M*	$W_0 = \frac{1}{2}Tr(\phi^2)$ $W_1 = Tr(\phi\psi)$ $W_2 = Tr(\frac{1}{2}\psi^2 + i\phi F)$ $W_3 = iTr(\psi F)$ $W_4 = -\frac{1}{2}Tr(F^2)$
TFC	$(2g-2)DimG$ *for genus g*	$W_0 = \frac{1}{2}Tr(\phi^2)$ $W_1 = Tr(\phi\psi)$ $W_2 = \frac{1}{2}Tr(\psi^2)$
TFC $SO(2,1)$	$(6g-6)$	*same as above*
TSM	$2(2n+1)$ *for* $\Sigma^0 \to CP^n$ 6 *for* $\Sigma^1 \to$ *cubic surface in* CP^3	$W_0 = \Omega\psi^1...\psi^k$ $W_1 = \Omega dx^1\psi^2...\psi^k$ $W_2 = \Omega dx^1 dx^2\psi^3...\psi^k$ $\Omega dx^1...dx^k$ – k-form on M
TG	2	*none*

Table 3- *Dimension of moduli spaces and global invariants.*

So far we ignored the necessity to gauge fix the G symmetry prior to any path integral computations. To pick a gauge slice we take the gauge fixing and Faddev-Popov actions of the form: $\mathcal{L}^{(3)}_{(GF+FP)} = \hat{\delta}_T[\bar{c}G_{GF}(\Phi)]$ where $\delta_T = \hat{\delta}^{(2)} + \hat{\delta}_G$ and $\bar{c}$ is a new anti-ghost. In general the equation of motion which corresponds to $\bar{c}$ may impose conditions on Ψ_i which are incompatible with eqns. (2-3). However, it turns out that for local symmetries like gauge symmetries and diffeomorphisms (as can be checked in the examples of above) they are compatible. Several other procedures of gauge fixing the G symmetry in the case of TYM were introduce in the past.[10,13][14] .

The question is whether the third stage of gauge fixing can alter the previous results. Since the observables in (10) are both $\hat{\delta}^{(2)}$ and gauge invariant, they are also $\hat{\delta}_T$ invariant. However, the issue of triviality[14] under the total BRST cohomology is different for the $\hat{\delta}_T$ and $\hat{\delta}^{(2)}$ operators. To discuss this question we restrict ourselves to the case where the G-Symmetry is a non-abelian gauge symmetry (TYM,TFC). The conclusion , however, will apply also to the rest of the TQFT's. It turns out that all the W_i which were in a non-trivial cohomology class of $\hat{\delta}^{(2)}$ (apart from W_1) can now be written as a sum of an exact form on M and $\mathcal{M}$ for example:

$$\begin{aligned} W_0^{(0,4)} &= \hat{\delta}_T Tr(-c\phi + \frac{i}{3}c^3) \\ W_1^{(1,3)} &= \frac{1}{2}[\hat{\delta} Tr(-A\phi - \frac{i}{3}c^2 A + c\psi) + dTr(\frac{1}{3}c^3 - ic\phi)] \\ W_0^{(2,2)} &= \hat{\delta}_T Tr(icDA + A\psi - \frac{i}{3}A^2 c + iAD\phi + iAdc) \\ &\quad + dTr(-iA\phi + ic\psi + \frac{2}{3}Ac^2 - icdc + ic\psi - cD\phi) \end{aligned} \tag{12}$$

Does it mean that the corresponding global invariants are all trivial? To get a better insight on this question we use the interesting geometrical interpretation of the BRST system that was given in[9]. Following the later reference, the BRST transformations of A, ψ, c, ϕ follow from $\tilde{d}\tilde{A} + \frac{1}{2}[\tilde{A}, \tilde{A}] = \tilde{F}$ and the associated Bianchi identity $\tilde{D}\tilde{F} = 0$ where $\tilde{d} = d + \hat{\delta}_T$, $\tilde{A} = A + ic$ and $\tilde{F} = F + \psi + i\phi$. The objects $\tilde{d}$, $\tilde{A}$ and $\tilde{F}$ are the exterior derivative, connection and curvature on the product space $P \times \mathcal{A}/\mathcal{G}$ where P is the principle bundle and $\mathcal{A}/\mathcal{G}$ is the orbit space. In this picture the set of W_i are the $(i, 4-i)$ components of the second Chern class:

$$Tr(\tilde{F} \wedge \tilde{F}) = \tilde{d}[Tr(\tilde{A} \wedge \tilde{d}\tilde{A} + \frac{2}{3}\tilde{A} \wedge \tilde{A} \wedge \tilde{A})] \tag{13}$$

Following (10) the BRST variations of the various W_i are given by exterior derivatives on M of W_{i-1} which according to (13) are derivatives of components of a second Chern class. Therefore the BRST variations of the various W_i given in eqn. (10) are also globally valid. On the other hand (12) tells us that the W_i are given by a combination of the exterior derivatives on both M and $\mathcal{A}/\mathcal{G}$ of some functional of the connections over those spaces. The moduli space, hence also the product space, are topologically non-trivial which means that they can not be covered by a single coordinate patch. Thus the statement of equation (12) is that the W_i are only locally trivial. These local properties cannot be simply extended into global properties. This is manifested in eqn. (13). An integral over the product space would not vanish, even though the integrand can locally be expressed as an exterior derivative of a generalized Chern-Simons term (a generalized instanton number). The I_i are therefore BRST invariant but not trivial.

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