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RENORMALIZATION SCHEMES: WHERE DO WE STAND?*

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ABSTRACT

We consider the status of the current approaches to the application of the renormalization program to the standard $SU_{2L} \times U_1$ theory from the standpoint of the interplay of the scheme chosen for such an application and the attendant high-precision tests of the respective loop effects. We thus review the available schemes and discuss their theoretical relationships. We also show how such schemes stand in numerical relation to one another in the context of high-precision Z^0 physics, as an illustration.

1. INTRODUCTION

As it is by now well-known,[1] a primary objective of the SLC/LEP Z^0 physics program is to test the standard $SU_{2L} \times U_1$ theory at a level below 1%. When one reaches the precision of 0.1%, one is indeed at the level of the so-called electroweak quantum loop effects, so that these loop effects must be calculated in a precise way if this aspect of the SLC/LEP physics program is to be successful. We are aware also, of course, that the large so-called QED effects[1,2] in the theory must be controlled at the 0.1% level, but these effects are indeed so understood, even on an event-by-event basis[3]; thus, we do not focus on them here.

Regarding then the pure electroweak loop effects, we emphasize that several authors[4] have calculated such effects with various data sets in mind. Indeed, one can distinguish those calculations which have looked at corrections to processes involving space-like four-momentum transfers and those which have looked at corrections to processes involving only time-like four-momentum transfers. In all cases, it is necessary to choose a so-called scheme for the interpretation of the ultraviolet divergences in the loop corrections to the $SU_{2L} \times U_1$ theory via the renormalization program. If we knew how to sum up over all orders in this resultant renormalized theory, the question of scheme dependence (or independence) would be moot: if one sums to all orders, all schemes lead to the same physical observable values, so that they lead to the same uncertainty in extracting these observables from the data; assuming infinite numerical precision on the evaluation of the renormalized theory, this uncertainty is entirely determined by the experimental errors. In the $SU_{2L} \times U_1$ theory, while it is possible to sum over the infrared singularities to all orders in α, it is as yet not practical to do this in a complete way for the infrared finite, ultraviolet finite parts of the respective S-matrix elements; so far, one has been able only to compute these finite parts to a

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finite order in α. To any finite order, what one compares with experiment can greatly affect how accurate the parameters of the $SU_{2L} \times U_1$ theory are determined. But, a choice of what one compares with experiment is precisely correlated with one's scheme for implementing the renormalization at a finite order in α. Hence, it is for this reason that we may speak about the appropriateness or lack of appropriateness of a given scheme: the errors on the defining physical parameters of the theory, as determined from experiment, may be reduced if an appropriate scheme is used, for any given finite order in α calculation of the various physical processes. Hence, in the context of high-precision Z^0 physics, the issue of the scheme is indeed important for the comparison of one- (and two-) loop predictions with Z^0 production and decay phenomena; for example, in $e^+e^- \to Z^0 \to X$. Hence, in what follows, we consider the current status of the implementation of renormalization schemes in the context of one- (and two-) loop calculations of high-precision $SU_{2L} \times U_1$ physics.

Our discussion is organized as follows. In the next Section, we define precisely what actually constitutes a renormalization scheme. In Sec. 3, we then discuss the existent schemes which have been used to compute complete one (or two) loop pure electroweak corrections to $SU_{2L} \times U_1$ processes for high precision tests of the theory. In Sec. 4, we turn to the status of the use of these schemes and their possible extensions. Section 5 contains some concluding remarks.

2. DEFINITION OF A RENORMALIZATION SCHEME

In this section, we wish to define precisely what we intend by a renormalization scheme for a theory like the $SU_{2L} \times U_1$ theory. We begin by recalling the definition of renormalization itself.

Specifically, given a Lagrangian $\mathcal{L}$ dependent on a collection of fields $\{\phi_i\}$ (we suppress possible Lorentz and internal symmetry labels into the single label i), we separate $\mathcal{L}$ into a free part $\mathcal{L}_0$ and its interaction part $\mathcal{L}_{\rm int}$:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\rm int} \quad . \tag{1}$$

Focusing on the case that $\{\phi_i\}$ contains a single scalar field ϕ of rest mass m, we write

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu\phi\,\partial^\mu\phi - m_0\phi^2\right) - \frac{\lambda_0\phi^4}{4!} \tag{2}$$

for purposes of illustration so that

$$\mathcal{L}_0 = \frac{1}{2}\left(\partial_\mu\phi\,\partial^\mu\phi - m_0\phi^2\right) \tag{3}$$

and

$$\mathcal{L}_{\rm int} = -\frac{\lambda_0\phi^4}{4!} \quad . \tag{4}$$

The need for renormalization is then made manifest by any effort to compute quantum loop corrections in the theory in Eq. (2): the first quantum correction to the ϕ propagator is shown in Fig. 1. (We imagine a path-space approach to the Feynman rules so that, in ϕ^4 theory, the contraction of a ϕ with itself at a single point of space-time need not be trivial.) The standard methods would give for the loop term Fig. 1 [here, we use the n–dimensional method of Ref. 5 to define the divergent integral for $n \uparrow 4$]:

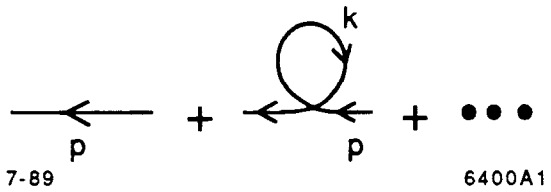

Fig. 1. Quantum corrections to the two-point connected Green function in ϕ^4 theory.

$$i\Delta_F^{(1)} = \frac{i}{p^2 - m_0^2 + i\epsilon} \frac{(-i\lambda_0)}{2} \frac{\int d^4k}{(2\pi)^4} \frac{i}{k^2 - m_0^2 + i\epsilon} \frac{i}{p^2 - m_0^2 + i\epsilon} \tag{5}$$

$$\equiv \frac{i}{p^2 - m_0^2 + i\epsilon} \frac{(\lambda_0/2)\left\{\left[-i\pi^{n/2}\,\Gamma(1-n/2)\,(m_0^2 - i\epsilon)^{n/2-1}\right]/\Gamma(1)\right\}}{(2\pi)^n} \frac{i}{p^2 - m_0^2 + i\epsilon}\,.$$

Hence, the most elementary loop graph in the theory is formally infinite. It must be cut off, as we have illustrated (this we will refer to as regularization), and then interpreted in terms of the physically observable quantities (parameters) in the theory (this we will refer to as a renormalization program). This then is all quite well known.

Hence, a renormalization scheme is a choice of definite procedures and prescriptions for implementing the interpretation of the infinities in a theory (presumed to admit this) in terms of its physically measurable parameters. It means that a regularization procedure has been chosen, a sufficient number of gauge-specifying terms and their respective compensating ghosts terms have been added to $\mathcal{L}$ to define all free contributions of the type in Eq. (3), and that a set of normalization conditions has been chosen at which one may identify the physically observable parameters in the theory in such a way that all S-matrix elements are finite order-by-order in perturbation theory when expressed in terms of these physical parameters.

In the trivial example of Eq. (2), we can use the standard power counting to identify that the two- and four-point 1PI vertices are, in fact, the only primitively divergent graphs, as it is well known. Hence, a wavefunction renormalization constant Z, a mass counter-term $\delta m^2 \equiv m_0^2 - m^2$, and a vertex renormalization constant Z_1 are sufficient to maintain the unit residue of the two-point function at its physical mass pole at $p^2 = m^2$ (Z and δm^2 are adjusted for this purpose), and to maintain the physical value λ of the four-particle coupling at the appropriate point; which may be taken, for example, at some point where $p_i^2 = \mu^2 = m^2$—for example, with $p_i = (m, \vec{o})$ [zero momentum $\phi\phi$ scattering—Z^2/Z_1 is adjusted for this purpose]. Hence, the three constants Z, Z_1 and δm^2 are uniquely fixed by the two conditions on the renormalized propagator and the one condition on the physical four-particle scattering vertex. All infinities in the theory are then absorbed into Z, Z_1 and δm^2, order-by-order in perturbation theory in the renormalized parameter λ, in a familiar way.

From this simple example, we see one very important thing:

If we sum to all orders in the renormalized coupling, we can always write every contribution to a given S-matrix element in terms of the respective all-orders series in the bare coupling λ_0. Since this bare theory does not know anything about our renormalization program, this series must be independent of this program also. Hence, in this sense, we indeed have a predictive framework in principle. More precisely, given the complete all orders in λ_0 unrenormalized 1PI vertices $\{\Gamma_u^{(n)}(\{p\}, \lambda_0, m_0)\}$ and a complete

set of all orders in λ renormalized vertices $\{\Gamma^{(n)}(\{p\};\lambda,m,\mu)\}$ (μ may not be equal to m), we know that ($\Gamma^{(n)}$ are amputated)

$$\Gamma_u^{(n)} \;=\; \left(Z^{1/2}\right)^{-n}\;\Gamma^{(n)} \quad , \tag{6}$$

so that any two different realizations of the renormalization program, $\{\Gamma_1^{(n)}\}$ and $\{\Gamma_2^{(n)}\}$, will be related by

$$\left(Z(1)^{1/2}\right)^{-n}\;\Gamma_1^{(n)} \;=\; \left(Z(2)^{1/2}\right)^{-n}\;\Gamma_2^{(n)} \quad . \tag{7}$$

This expresses the equivalence of all consistent renormalization schemes when the respective theories are summed to all orders in their respective couplings, and implies that the 1PI vertices satisfy the famous renormalization group equation:

$$\left(\mu\,\frac{\partial}{\partial\mu}+\beta(\lambda)\,\frac{\partial}{\partial\lambda}-\gamma_\Theta\;m\,\frac{\partial}{\partial m}-\gamma_\Gamma\right)\;\Gamma^{(n)} \;=\; 0 \tag{8}$$

where, from Eqs. (6) and (7), we see that β, γ_Θ and γ_Γ are computable order-by-order in perturbation theory in λ. In fact, one can use Eq. (8) as a check of the consistency of a calculation in a given scheme by verifying that Eq. (8) is satisfied order-by-order in perturbation theory. For a computation to order λ^n, Eq. (8) should be respected to the same order in λ. Such checks of the various calculations in schemes[4] for the $SU_{2L} \times U_1$ are in progress[6] and have been done in part by others to some degree.

Equation (8) makes it clear why precision tests of a theory like the $SU_{2L} \times U_1$ theory need to pay attention to the issue of the respective renormalization scheme. One sees that $\Gamma^{(n)}$ at different μ are connected by solving Eq. (8). Thus, to pin down the theory most precisely, one should choose as parameters to be compared to experiment those values or functions of the $\{\Gamma^{(n)}\}$ which can be most accurately measured at the respective scales $\{\mu\}$. This will then pick out a scheme. Deviation from this may propagate unnecessary errors from the comparison of theory and experiment into the defining parameters of the theory.

With this definition and elucidation of what we mean by a scheme, we turn now to the existent schemes which have been used for the order α (and α^2) corrections to the $SU_{2L} \times U_1$ theory for precision tests of the pure electroweak effects therein. This we do in our next Section.

3. RENORMALIZATION SCHEMES FOR THE ELECTROWEAK THEORY

The effort to use the comparison between theory and precise experiments to check the one-loop (or two-loop) corrections to the $SU_{2L} \times U_1$ theory in the electroweak sector has a long history. Each effort in this history has, in fact, employed a renormalization scheme by necessity, although in some cases, this scheme is only partially implemented, due to the nature of the specific calculation under study. We then wish to identify the various schemes that have been successfully used or advanced for precision tests of loop effects in the standard $SU_{2L} \times U_1$ theory. This is the purpose of our discussion in this section.

Specifically, we consider the various schemes in Refs. 4 and 7, in turn. We consider first the formal schemes employed by Lee and Zinn-Justin and 't Hooft and Veltman[7] in their arguments for the explicit renormalizability of the $SU_{2L} \times U_1$-type theory and the equivalence of its R and U gauge formulations.

More precisely, the regularization at m–loops may be either effected with gauge-invariant Pauli–Villars type (higher-derivative) regulators or with the n–dimensional methods of 't Hooft (some questions still remain about the higher-derivative regulator in higher loops). The normalization point may be the Symanzik-type point $p_i^2 = -\mu^2$, $p_i p_j = (n-1)^{-1}\,\mu^2$ for an n-point irreducible vertex which is primitively divergent. The vacuum expectation value of any renormalized field is then maintained at its physical value order-by-order in perturbation theory. Counterterms are then used to remove the respective divergent terms in the theory's S–matrix so that this S–matrix is finite. A gauge of either the so-called renormalizable type, in which the vector boson propagator projection $P^{\alpha\beta}$ in $iD_{Fab}^{\alpha\beta} = -i\delta_{ab}\,P^{\alpha\beta}(k)/(k^2 - M_V^2 + i\epsilon)$ is bounded for $|k^2| \to \infty$ (in the Euclidean region), or of the U–type, in which all unphysical fields are removed from the theory and $P^{\alpha\beta} = -g^{\alpha\beta} + k^\alpha\,k^\beta/M_V^2$, may be chosen. This is really a whole class of schemes. Indeed, whenever the infrared singularities of the $\{\Gamma^{(n)}\}$ allow it, one may consider that some of $\{p_i\}$ are at $p_i^2 \geq 0$ (i.e., are on-shell) at the normalization point. The arguments of Lee, Lee and Zinn–Justin, and 't Hooft and Veltman then show that any two schemes of this type are equivalent. This is a powerful result for the precision tests of the loop corrections to the standard $SU_{2L} \times U_1$ theory.

Indeed, the freedom created by the general theorems of Lee and Zinn-Justin has resulted in the specific choices of a convenient scheme by several authors in the practical calculations.[4] We list in Table 1 a compendium of such choices, together with the specific physical process analyzed. The key characteristics of these schemes are their normalization points (they are all on-shell, in that the basic parameters are defined by subtracting the ultraviolet divergences at the points in the respective external momenta where the particles involved are on their mass shell). The existent practical schemes either use the renormalizable R_ξ–type gauge or the unitary gauge. Most of the work in the R_ξ gauge has had $\xi = 1$, so that it is work in the 't Hooft-Feynman gauge. The regularization procedures are all based on the 't Hooft n-dimensional methods.[5] The key differences arise in the field renormalization.

Specifically, it is well known from Ref. 7 that the infinities in the spontaneously broken gauge theory are identical to those in the unbroken theory. In the symmetric theory, there is but one field renormalization constant for each multiplet of the symmetry group of the theory. Hence, all particles in the multiplet have the same wavefunction renormalization constant. In the broken theory, these particles may, in general, all get a different mass: m_a = rest mass of a. If the subtractions are made on-shell, then the symmetric wavefunction renormalization constant, for a totally broken multiplet, can only give one particle from the original multiplet the unit residue at its propagator pole: all others from the multiplet will have a nonunit residue if only the symmetric wavefunction renormalization constant is used. There is, however, no theoretical reason to restrict oneself to a single wavefunction renormalization constant: the only requirement is that the divergences in the respective constants for the members of a multiplet must be the same. The finite parts of these constants may differ, however, in such a way that the residue of each member's propagator pole is unity. In doing this, one gains in the simplicity of the particle wavefunction normalizations, but one has more Z_{2a}'s to calculate, where Z_{2a} is the wavefunction renormalization constant for particle a. As one can see from Table 1, some authors prefer fewer Z_{2a}'s, and some prefer simple external line normalizations. The physics does not depend on these preferences.

The issue of field renormalization is amplified in the A–Z^0 mixing phenomenon. Any one-loop calculation must treat this effect. If one only uses the symmetric wavefunction renormalization constants, mass counterterms for $W^\pm$ and Z^0, and the requirement that the A is massless and couples with the usual Thomson limit charge at zero four-momentum transfer squared, one in general cannot have $\widehat{\pi}_{AZ^0}(q^2 = 0) = 0$ and unit residues for the poles in the A, W, and Z^0 propagators, where a hat denotes a renormalized quantity. However, in the case that we treat each independent field as truly independent, we can, in fact, arrange that π_{AZ^0} vanishes for $q^2 = 0$ and the vector particles have unit residue. Again, to do this or not is a matter of taste; the physics cannot depend on such issues of taste.

Table 1. Existent Practical Schemes.

Authors	Gauge	Normalization Point	Regularization Method	$Z_L^{up} = Z_L^{down}$	$\pi_{AZ}\vert_{phys} = 0$	$\pi'_{VV}\vert_{phys} = 0$	Process
Ross & Taylor	Feynman–'t Hooft	On-shell	't Hooft	Yes	No	No	General
M. Igarashi et al.; Aoki et al.	Feynman–'t Hooft	On-shell	't Hooft	No	Yes	Yes	$e^+e^- \to X$; $\nu_\ell e \to \nu_\ell e(\ell\nu_e)$
Wetzel	Feynman–'t Hooft	On-shell ($\nu\bar{\nu}Z^0$)	't Hooft	No	Yes	Yes	$e^+e^- \to \mu^+\mu^-$
Paschos & Wirbel; Sakakibara; Brown et al.	Feynman–'t Hooft	On-shell ($f\bar{f}Z^0$)	't Hooft	Yes	Yes	No	$\nu N \to \nu(\ell) + X$; $e^+e^- \to \mu^+\mu^-$
Antonelli et al.; Consoli	Feynman–'t Hooft	On-shell ($f\bar{f}Z^0$)	't Hooft	No	Yes	No	General
Llewellyn-Smith & Wheater	Feynman–'t Hooft	$\overline{MS}$ (On-shell)	't Hooft	No	Yes	Yes	$\nu N \to \nu(\ell) + X$; $ed \to e + X$
Marciano & Sirlin (II)	Feynman–'t Hooft	On-shell	't Hooft	No	Yes	Yes	General
Bardin et al.	Unitary	On-shell	't Hooft	No	Yes	Yes	$ff' \to f''f'''$; $Z^0 \to f\bar{f}$, etc.
Cole	R_ξ	On-shell	't Hooft	Yes	No	No	Neutral Current Processes
Hollik & Timme	Feynman–'t Hooft	On-shell Off-shell	't Hooft	Yes	Yes	No	$e^+e^- \to \mu^+\mu^-$

(continued)

Table 1. Existent **Practical** Schemes (continued).

Authors	Gauge	Normalization Point	Regularization Method	$Z_L^{up} = Z_L^{down}$	$\pi_{AZ}\vert_{phys} = 0$	$\pi'_{VV}\vert_{phys} = 0$	Process
Lynn et al.	Feynman–'t Hooft	On-shell s_*^2	't Hooft	No	Yes	No	$f\bar{f} \to f'\bar{f}'$
Fleischer & Jegerlehner	R_ξ	On-shell	't Hooft	No	Yes	Yes	$H \to \tau^+\tau^-$; $H \to W^+W^-$; $H \to Z^0Z^0$
Passarino & Veltman	Feynman–'t Hooft	On-shell	't Hooft	No	Yes	No	$e^+e^- \to \mu^+\mu^-$
Bohm et al.; Hollik	Feynman–'t Hooft	On-shell	't Hooft	Yes	Yes	No	$\mu \to \nu_\mu + e\bar{\nu}_e$, $\overset{(-)}{\nu}_\mu e \to \overset{(-)}{\ell}_\mu \ell_e$; $e^+e^- \to \mu^+\mu^-$ $e^+e^- \to W^+W^-$
Marciano & Sirlin (I)	Unitary	On-shell $W\bar{\nu}_\mu\mu$	't Hooft	No	Yes	Yes	$W \to \ell + \bar{\nu}_\ell$
Appelquist et al.	Unitary	On-shell $W\bar{\nu}_\mu\mu$	't Hooft	No	Yes	Yes	$W \to \bar{\mu}\nu_\mu$; $\mu \to \nu_\mu e\bar{\nu}_e$
Salomonson & Ueda	Unitary	On-shell $W\bar{\nu}_\mu\mu$	't Hooft	No	Yes	Yes	$\overset{(-)}{\nu}_\mu e \to \overset{(-)}{\nu}_\mu e$
Philippe	Feynman–'t Hooft	On-shell	't Hooft	No	Yes	Yes	$e^+e^- \to W^+W^-$

Finally, we re-emphasize that the choice of gauge in Table 1 is the famous R_ξ gauge such that the vector meson propagator is, for four-momentum k,

$$iD_{F_{\mu\nu}}(k) = \frac{-i\left[g_{\mu\nu} - (1-\xi)k_\mu k_\nu/(k^2 - \xi M^2)\right]}{k^2 - M^2 + i\epsilon} \quad , \tag{9}$$

where either ξ is left as a free parameter, or $\xi = 1$, or $\xi \to \infty$. Conspicuously absent are the axial and the background field gauges, with

$$D^\mu_{cl} V_\mu = 0 \quad \text{and} \quad n \cdot V = 0 \qquad V = A,\, Z^0,\, W^\pm \quad , \tag{10}$$

respectively, for some fixed classical covariant background derivative and for some fixed vector n^μ; also, the Landau gauge $\xi = 0$ is not very plural in Table 1. ($\xi \to \infty$ gives the unitary gauge.) Hence, the gauge choices are not complete in this sense. However, the independence of those calculations in the general R_ξ gauge of the choice of ξ is a strong proof of the gauge invariance of the respective calculations in practice. We emphasize here that we do not question that the $SU_{2L} \times U_1$ theory can be calculated in a gauge-invariant framework. Rather, we advocate the eventual check of the work on the theory in several types of gauges, such as axial versus R_ξ gauge, etc., as a practical check on what is done in efforts to effect the calculations in such a framework.

The regularization method is that due to 't Hooft.[5] The usual problematic issue of the meaning of γ_5 in n-dimensions gets circumvented here by the absence of anomalies in the $SU_{2L} \times U_1$ theory. Hence, any method of defining γ_5 in n–dimensions should suffice, such as a totally anticommuting γ_5 in n–dimensions.[8] Here, we see no reason to pursue alternatives. However, there are alternatives, such as the zeta function regularization[9] or the higher derivative-Pauli-Villars methods[7] (for one-loop), etc. In principle, these should be pursued as eventual checks on what one does in practice in the $SU_{2L} \times U_1$ radiative correction program.

Regarding the normalization point, the on-shell prescription is overwhelmingly preferred. Again, there is nothing intrinsically lacking here. Indeed, this is the physically relevant normalization point. For some purposes, however, it may be more convenient to normalize at a Euclidean point $-\mu^2$ off the mass shell and, subsequently, to make a final finite renormalization transformation to the on-shell normalized theory. When this intermediate step is used, the parameters of the theory become $g'(\mu)$, $g(\mu)$, $M_{Z^0}(\mu)$, $m_{h,R}(\mu)$, $m_{f,R}(\mu)$; and on passing to the on-shell theory, we obtain the physical parameters α, G_F and M_{Z^0}, ..., as functions of the intermediate scale parameters. A particularly useful application of this intermediate scale device (in the textbooks[10] this is referred to as intermediate renormalization) is in conjunction with the 't Hooft–Weinberg[11] renormalization group improvement of $\mathcal{O}(\alpha)$ radiative corrections. For, when the theory is viewed at the Euclidean normalization point, its partial differential equations for the 1PI vertices $\{\Gamma\}$ take the simple form of Eq. (8):

$$\left[\mu \frac{\partial}{\partial \mu} + \beta \frac{\partial}{\partial g} + \beta' \frac{\partial}{\partial g'} - \sum_A m_{A,R}\, \gamma_{\theta_A} \frac{\partial}{\partial m_{A,R}} - \gamma_\Gamma\right] \Gamma = 0 \quad . \tag{11}$$

This equation has the well-known solution

$$\Gamma\left(\{\lambda p_j\}, g, g', \{m_{A,R}\}, \mu\right) =$$

$$\Gamma\left(\{p_j\}, g(\lambda), g'(\lambda), \{m_{A,R}(\lambda)\}, \mu\right)\ \lambda^{D_\Gamma}\ \exp\left\{-\int_1^\lambda \frac{d\lambda'}{\lambda'}\, \gamma_\Gamma\ [g'(\lambda'), g(\lambda')]\right\} \quad , \tag{12}$$

where D_Γ is the engineering dimension of Γ, $g'(\lambda)$ and $g(\lambda)$ are the famous running charges[11] for the U_1 and SU_{2_L} groups, and $\{m_{A,R}(\lambda)\}$ are the respective running mass parameters[11] of the theory. Hence, from Eq. (12), we learn immediately how to get the rigorous renormalization group improvement of an $\mathcal{O}(\alpha)$ result. We do not have to "guess" that certain parts of an $\mathcal{O}(\alpha)$ expression are a part of a geometric series and other parts are not. This device, then, can be of substantial practical use.

Finally, we remark that the processes considered in Table 1 cover the key places of experimental accessibility. The LEP/SLC physics has been amply addressed from the standpoint of the number of efforts to treat the radiative correction issues which pertain thereto. Similarly, the lepton-lepton and lepton-hadron scattering physics issues have had considerable attention from the standpoint of the attendant radiative corrections. And, indeed, Sirlin, Marciano and others[12] have shown that, with the radiative corrections taken into account, the different scattering experiments become more consistent with one another and yield the result, for example, $M_{Z^0} = 91.8 \pm 0.9$ GeV for 44 GeV $\leq m_t \leq$ 200 GeV, and 10 GeV $\leq m_H \leq$ 1 TeV. Regarding the SLC/LEP Z^0 physics, the corresponding data are only beginning to be accumulated. Therefore, it is quite fitting to address the issue of the agreement of the respective theoretical work in that area. To this we now turn in the next section.

4. RENORMALIZATION SCHEMES IN PRACTICE

Given the scenario illustrated by Table 1, the question naturally arises as to what happens in practice when different schemes are used to compute the same physical process. Hence, in this section we explore this question.

Specifically, two schemes may be said to be numerically equivalent [at some order (α^n)] for a given process if the difference between their respective predictions for that process is below one-third of the level of the respective experimental errors in the measurement of that process. Here, the reader may think of a "process" as a cross section or a decay rate, etc. Accordingly, for comparing schemes in practice, it is convenient to use the high precision Z^0 physics processes as benchmarks for the numerical equivalence of different schemes. It is known[1] that such physics has as a goal the 0.1% precision checks of the $SU_{2L} \times U_1$ theory. Thus, the $\mathcal{O}(\alpha)$ pure electroweak corrections to such physics in two different schemes are numerically equivalent if they agree to better than 0.03%, for example. Let us consider a simple quantity, such as the cross section for $e^+e^- \to \mu^+\mu^- + n(\gamma)$ near the Z^0 resonance, as our pedagogical example. We now wish to compare the results of several schemes for this process to get some idea about the practical aspects of "scheme" effects. Here, we have in mind that the number of real photons may be arbitrarily large.

We have shown elsewhere that the various approaches to the multiple photon effects on the line shape in $e^+e^- \to \mu^+\mu^- + n(\gamma)$ are in agreement below the level of 0.1%. Thus, it is indeed fitting to inquire as to the level of agreement of the various approaches to the pure electroweak corrections themselves at the one-loop level, for example.

Specifically, as one can see from Table 1, most of the efforts on this process have used the on-shell scheme in the strict sense of the word scheme. Where the efforts differ is in the amount of resummation or improvement of the respective perturbation series by the various techniques.

For example, one can use the Dyson–Schwinger equations to improve the expression for a given Green function (for example, a two-point function, at one-loop) to an all-orders sum of all one-particle reducible graphs built from the respective one-particle irreducible parts, where the one-particle irreducible part is computed to one loop. The classic example is the photon propagator, $iD'_{F_{\mu\nu}}(q)$, which can be represented as

$$iD'_{F_{\mu\nu}}(q) = \frac{-ig_{\mu\nu}}{[1 + e^2\hat{\pi}^{(1)}(q^2)](q^2 + i\epsilon)} + \text{gauge terms} \quad , \tag{13}$$

where $e^2\widehat{\pi}^{(1)}(q^2)$ is the renormalized one-loop vacuum polarization function. The important point is that in Eq. (13) we have summed an infinite number of graphs but that, since we only have computed $\widehat{\pi}^{(1)}$, we have also omitted the n–loop 1PI contributions for $n > 1$. Thus, we have also *omitted* an infinite number of graphs from Eq. (13). Of course, one can hope that the omitted higher-order graphs are small; and, indeed, for most applications, they are.

Hence, depending on the amount of resummation one does, one can clearly get different results in the same scheme with the identical 1PI parts. This is not a scheme dependence in the classic sense; it is a calculational procedural difference in the same scheme.

Particularly interesting in this connection is the so-called starred scheme of Lynn et al.[4] In the reduced notation, the squared unrenormalized charge e_0^2 which enters into the strength of the Thompson amplitude is replaced by the improved Schwinger–Dyson series

$$e_0^2 \rightarrow \frac{e_0^2}{1 + e_0^2\,\pi^{(1)}(q^2)} \equiv e_*(q^2)^2 \tag{14}$$

to obtain the starred charge squared. Here, $\pi^{(1)}(q^2)$ is unrenormalized. Nothing prevents one from computing a formal expression for $\pi(q^2)$ in Eq. (1) to all orders in e_0^2. The key task is to interpret systematically the respective infinities in such an expression, together with those in the analog vertex and box-type graphs for the Thomson amplitudes, in such a way that the resulting perturbation series in $e_*(q^2)$ is finite order-by-order in $e_*(q^2)$. [When $\pi^{(1)}$ is used in Eq. (14), some explicit arguments for such finiteness exist.[4]] For $q^2 \rightarrow 0$, one then imposes $e_*(q^2)^2 \rightarrow 4\pi\alpha$, where α is the fine-structure constant at $q^2 = 0$. In effect, one expands the $SU_{2L} \times U_1$ theory in terms of Dyson–Schwinger series-improved parameters. Since the resulting perturbation series still is renormalizable, one may still use the renormalization group to exploit these improved series. The key point, however, is to isolate properly what combinations of amplitudes in the starred-type series correspond to the respective $\{\Gamma\}$ that enter into Eq. (11) without double counting, for example.

Hence, the application of the various "schemes" in Table 1 can yield different results for several reasons: (1) the two compared schemes are intrinsically different; (2) the two schemes are actually the same but the calculational procedures are different; (3) the schemes and calculational procedures are both the same but there is an error in the arithmetic. The reader should keep these three possibilities in mind in what follows.

Specifically, we consider the results of the Stuart et al.,[4] pure weak corrections library and the Hollik et al.,[4] pure weak corrections library in conjunction with the YFS2 multiple-photon Monte Carlo event generator,[3] as realized in the program KORALZ.[13] The result[14] for the line shape under study, for the same typical model parameters, is shown in Fig. 2, in reference to the semianalytic comparison program ZBATCH,[13] where ZBATCH corresponds to the "exponentiated exact order α^{2}" improved line shape calculation of Berends et al., Ref. 2. We see that the two pure weak libraries agree at the level of 0.1% in the line shape near the Z^0 pole and that, further, the Hollik et al., library is closer to the ZBATCH prediction at this pole, where the pole is located at $\sqrt{s} = M_{Z^0} = 92$ GeV. Clearly, this type of agreement, which is worse for $\sqrt{s} \sim 102$ GeV, would allow us to test the $SU_{2L} \times U_1$ theory below the level of 1%; it would have to be improved to explore the level below 0.1% in these $SU_{2L} \times U_1$ tests, in general. Such improvement is in progress.[14] Further, we should note that some controversy still persists regarding Fig. 2, and the resolution of this controversy is also in progress.[14]

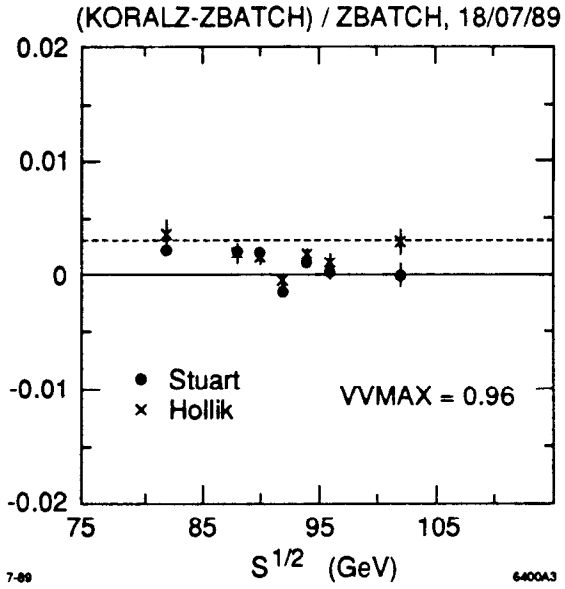

Fig. 2. Comparison of Stuart and Hollik et al., pure electroweak libraries in $e^+e^- \to \mu^+\mu^-$. Here, $M_{Z^0} = 92$ GeV, $m_t = 60$ GeV, and $m_H = 100$ GeV. In both cases, the libraries were obtained from its principal author via private communication. Note that $vvmax$ is the maximum value of $1 - s'/s \equiv v$, where $s = (p_e + p_{\bar{e}})^2$ and $s' = (p_\mu + p_{\bar{\mu}})^2$.

Turning next to the popular asymmetry A_{FB}, we illustrate the results of three different calculations in Table 2: at 90 GeV, we show the results of Bardin et al. (row one), Hollik et al. (row two), and of Lynn et al. (row three). We see a general agreement of rows one and two for low values of m_t, and a qualitatively and quantitatively different set of predictions in row three. We and others are currently investigating this latter difference, as well as the large m_t comparison of rows one and two.[15]

We emphasize that it is indeed encouraging that the various efforts on our prototypical process are all relatively close on such a complicated calculation but we, of course, feel that the high-precision Z^0 physics requires that the current residual differences be understood.

Table 2. Comparison of A_{FB} for $e^+e^- \to \mu^+\mu^-$; $M_{Z^0} = 90$ GeV, $m_H = 100$ GeV.

Authors	m_t (GeV)	50	100	150	200	
Bardin et al.		0.0036	0.0044	0.0050	0.0054	
Hollik et al.		0.0037	0.0043	0.0054	0.0071	
Lynn et al.	*	0.0038*	0.0041*	0.0035*	0.0012*	
	m_t (GeV)	60	90	130	180	230
Lynn et al.		0.0037	0.0041	0.0040	0.0028	−.0012

*Obtained by linear interpolation from the published results at $m_t = 30$, 60, 90, 130, 180 and 230 GeV.

5. CONCLUSIONS

We conclude that the current status of EWRC renormalization schemes is encouraging. The various efforts do tend to indicate that schemes have been identified and used to compute processes of physical interest to an accuracy such that the remaining higher-order corrections are indeed small. This is the basic requirement for high-precision tests of the $SU_{2L} \times U_1$ model at SLC and LEP near the Z^0 resonance. There remain, however, some interesting questions of the actual practice in this connection.

Specifically, we have shown that both in the Z^0 line shape in $e^+e^- \to \mu^+\mu^- + n(\gamma)$ and in the attendant A_{FB} for μ-pairs, there are significant deviations in the respective available results which need to be resolved for the high-precision Z^0 physics. Such a resolution is then the objective of various researchers in the field; we and others hope to report its attainment in the not-too-distant future.

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