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# PROSPECTS IN K PHYSICS*

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Abstract

Prospects for future experiments involving rare $K$ decays are reviewed.

## Introduction

The possibilities for pushing the frontier of high energy physics in $K$ decays center on looking for processes forbidden in the Standard Model and looking for rare processes which are sensitive to the effects of virtual, heavy particles; especially those forbidden at lowest order in electroweak interactions, but allowed at one loop. It is possible now to envisage experiments with sufficient sensitivity to probe such processes at a level which will critically test the Standard Model predictions, including those that depend on the CP violating phase inherent in the three-generation quark mixing matrix.

While there is a concentration on looking for physics beyond the Standard Model, there are also interesting questions arising from the interplay of strong and electroweak interactions, and important information in pinning down the parameters inside the Standard Model from $K$ decay experiments. As we gain knowledge about QCD corrections, hadronic matrix elements, and parameters, we can use this information to make predictions of increasing accuracy for various processes. The measurement of their rates then becomes a more sensitive test of the Standard Model or, equivalently, a search for physics outside it.

On the theoretical side, either old or new physics in $K$ decays necessitates being acquainted with relatively few generic Feynman diagrams. There are some processes which

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On the theoretical side, either old or new physics in $K$ decays necessitates being acquainted with relatively few generic Feynman diagrams. There are some processes which

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are forbidden in the Standard Model to any order. An example is leptonic flavor-changing neutral-currents. They might occur at tree-level as shown by the diagram in Fig. 1, which could represent the exchange of a flavor-changing "horizontal" gauge boson. There are also processes, which while forbidden at tree-level in the Standard Model, can occur at one-loop, as indicated by the penguin and box diagrams shown in Fig. 2. There is not much of a theoretical entry fee to understanding the basic processes, and even if you can't do the one-loop calculations yourself, you can look them up.[1]

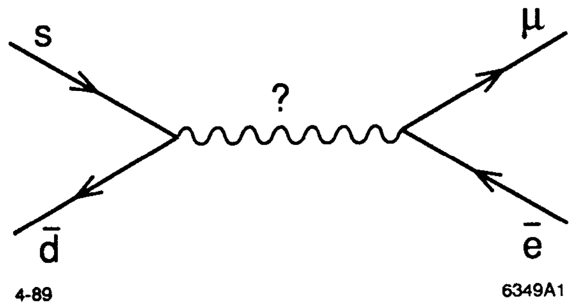


Fig. 1. Tree-level diagram involving a flavor-changing gauge boson.

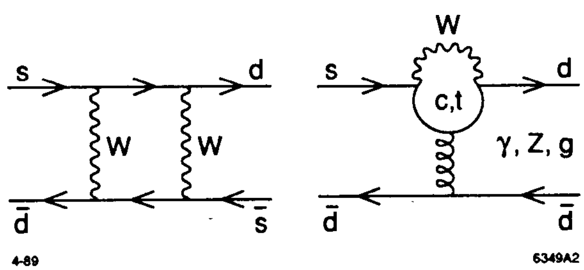


Fig. 2. One-loop diagrams giving rise to flavor-changing processes.

## The "Rebirth" of $K$ Physics

The late 1960s and early 1970s marked a peak in experiments on $K$ decays, sparked by the discovery of CP violation.[2] This effort tailed off as many important measurements were completed and new areas of physics opened up in the 1970s at electron-positron and hadron machines.

Then, in the late 1970s and early 1980s, both theoretical and experimental developments led to a "rebirth" of $K$ physics. On the theoretical side, the establishment of gauge theories for the strong and electroweak interactions provided a well-defined basis for calculations. The three-generation Standard Model could be used to make predictions of what, by definition, was inside, and by its complement, outside the Standard Model. The question of "who ordered the muon" was generalized to "who ordered three-generations with particular values of masses and mixing angles," and attention was directed at interactions which would connect quarks and leptons of different generations, producing flavor-changing neutral currents. It was realized that not only did the three-generation model provide an origin for CP violation in the nontrivial phase in the quark mixing matrix, but that CP violation should affect the $K^\circ$ decay amplitude as well as the $K^\circ - \bar{K}^\circ$ mass matrix, resulting in values of $\epsilon'/\epsilon$ in the $10^{-3}$ to $10^{-2}$ range.[3] There were also predictions for short-distance contributions to a number of other rare $K$ decay amplitudes induced at one-loop, both CP conserving and CP violating.[4]

On the experimental side, great strides were made to create high flux beams, handle high data rates, incorporate "smart triggers," improve detectors (especially for photons), and be able to analyze enormous data samples. These matched, at least to some degree, the requirements in precision and rarity being demanded by the theory for incisive tests of the Standard Model. The last few years have seen the beginning of a parade of results which are the culmination of a decade of work in perfecting and performing the needed experiments. Much more is yet to come, as one can now see the possibilities for improvements which will take us to the next generation of experiments. This indeed is the point of much of this workshop.

## The Rise of the Top Quark

Over the past decade, the "typical" or "best" value of the top quark mass used in theoretical papers has risen monotonically, somehow always remaining one step, or maybe one-and-a-half steps, ahead of the experimental then-current lower bound. Values of 15, 25, 30, 45, ... GeV have been used in various papers (some of them mine), but all of which have fallen by the wayside as experiments have been able to search at higher and higher masses. The present lower limit is around 60 GeV, below which a top quark is said[5] to be "unlikely." It seems that lower limits even higher than this will be quoted at high confidence within a few months, as the analysis of the present round of collider data (which is still being taken as I speak) is completed. An upper limit of around 200 GeV follows from analysis of neutral and charged current data and the measured $W$ and $Z$ masses (*i.e.*, consistency of the $\rho$ parameter with unity).[6] Here again, we will know much more in a few months when we have a much more accurate $Z$ mass from electron-positron colliders. I suspect that we are headed for a lower limit (or a top mass value?!) in the neighborhood of 100 GeV later this year.

The rise of the top quark mass has important consequences when we go to calculate one-loop contributions. For the penguin diagrams in Fig. 2 involving a top and charm quark and a virtual photon (the "electromagnetic penguin"); the conserved nature of the current demands a factor of $q^2$, the square of the four-momentum carried by the virtual photon, be present in the numerator of the amplitude. This cancels the $1/q^2$ from the photon propagator; the leading term for small (compared to $M_W^2$) top mass in the coefficient of the appropriate operator behaves as $\ln(m_t^2/m_c^2)$. By contrast, the "$Z$ penguin" or "$W$ box" involve nonconserved currents: the factor $q^2$ in the numerator is replaced by the square of the quark mass in the loop, and the propagator by $1/(q^2 + M_Z^2) \approx 1/M_Z^2$ or $1/M_W^2$. The corresponding coefficient behaves like $[(m_t^2/M_W^2)\ln(m_t^2/M_W^2) - (m_c^2/M_W^2)\ln(m_c^2/M_W^2)]$ when the top mass is small. In days when $m_t^2 \ll M_W^2$, it was completely justified to throw away the $Z$ penguin and $W$ box contributions to such amplitudes in comparison to that of the electromagnetic penguin. Not so any more. The various graphs give comparable contributions, as we will see later in specific examples. Moreover, the contributions from the top quark become the dominant ones to various rare $K$ decays when $m_t^2 \gg M_W^2$. In the three-generation Standard Model, as $m_t$ rises farther and farther above $M_W$, more and more of one-loop $K$ physics is top physics and we are in the interesting situation where those working at the highest energy hadron colliders are pursuing another aspect of the same physics as those working on the rarest of $K$ decays at low energies.

## 25 Years After the Discovery of CP Violation

It may be disappointing that 25 years after the discovery of CP violation[2] we have not progressed to a full understanding of its origin. Nevertheless, we have made significant theoretical progress. With the advent of the three-generation Standard Model, the question after all is not any more "why is CP violated"—it would be a surprise if CP were not violated, as it would take very special choices of the mixing angles or phase to keep CP conserved.

This can be seen very explicitly by noting that the computation of any difference of rates between a given process and its CP conjugate process (or of a CP violating amplitude) always has the form (in the three-generation case):

$$\Gamma - \overline{\Gamma} \;\propto\; s_1^2 s_2 s_3 c_1 c_2 c_3 \sin\delta_{KM} \;=\; s_{12} s_{23} s_{13} c_{12} c_{23} c_{13}^2 \sin\delta_{13} \quad , \tag{1}$$

where we express things first in the original parametrization of the quark mixing matrix[7] and then in the "preferred" parametrization adopted by the Particle Data Group,[8] using the shorthand that $s_i = \sin\theta_i$ and $c_i = \cos\theta_i$. Our present experimental knowledge assures us that the approximation of setting the cosines to unity induces errors of at most a few percent. In that case the combination of factors in Eq. (1), involving the invariant measure of CP violation,[9] becomes the approximate combination,

$$s_1^2 s_2 s_3 \sin\delta_{KM} \;=\; s_{12} s_{23} s_{13} \sin\delta_{13} \quad , \tag{2}$$

which was recognized earlier as characteristic of CP violating effects in the three-generation

Standard Model.[10] This combination of factors is (after removing $s_1^2$, whose value is accurately known)

$$s_2 s_3 \sin \delta_{KM} \equiv s_2 s_3 s_\delta \quad ,$$

where we have used the "old" parametrization.

The Kobayashi–Maskawa factors in the difference of rates in Eq. (1) defines the "price of CP violation" in the Standard Model. This "price" must be paid somewhere. It could be that it is paid in terms of these factors being found primarily in the decay rate for the process itself, which results in a very small branching ratio, but possibly then in a large asymmetry between particle and antiparticle. On the other hand, the price could be paid by having these factors mostly in the asymmetry between particle and antiparticle decays.

The latter situation is characteristic of $K$ decays. The smallness of CP violation, *i.e.*, that[8]

$$|\epsilon| \approx 2.28 \times 10^{-3} \quad , \tag{3}$$

can be "naturally" understood in the three-generation Standard Model, since $s_2 s_3 s_\delta$ is of order $10^{-3}$. No angle has to be fine tuned to be especially small or especially large in order to get a number of this magnitude.

This same factor of $s_2 s_3 s_\delta$ pervades all CP violation observables in the $K$ system, so it is then not so surprising that after 25 years the total evidence for CP violation in Nature consists of a nonzero value of $\epsilon$, and one statistically significant measurement[11] of a nonzero value of the parameter $\epsilon'$, representing CP violation in the $K \to \pi\pi$ decay amplitude itself. Experiments at Fermilab[12] and at CERN[11] are continuing with the aim of reducing the statistical and systematic errors.

Such a value[11] of $\epsilon'$ is consistent[13–15] with the three-generation Standard Model. Unfortunately, this is not a very strong statement because of our lack of knowledge both on the experimental and theoretical fronts:

- The hadronic matrix elements of the penguin operators, upon which the prediction of $\epsilon'$ depends, are fairly uncertain. Definitive results will presumably come from lattice QCD calculations which still seem several years away.
- The predictions depend on the value of $s_2 s_3 s_\delta$, which in turn depends (aside from another hadronic matrix element) on $m_t$ through imposing the constraint of obtaining the experimental value of $\epsilon$. Very roughly, as $m_t$ goes up, the range allowed for $s_2 s_3 s_\delta$ goes down, and so does the prediction for $\epsilon'$.
- Also as $m_t$ rises, the contributions from "$Z$ penguin" and "$W$ box" diagrams begin to be significant. For sufficiently large $m_t$, a recent calculation[16] contends that most of the usual (strong) penguin contribution to $\epsilon'$ can be cancelled in this way.

There is good reason to hope that experimental and theoretical progress over the next few years will clarify these points. But even if the situation at that time is that the measured value of $\epsilon'$ is consistent with the three-generation Standard Model, it is unlikely to be regarded as conclusive. We would demand additional evidence: A <u>single</u> set of Kobayashi–Maskawa angles (including the phase) must be able to fit several different processes which exhibit CP violating effects, providing a redundant check on the theory.

There are several ways to get this additional evidence; none of them is easy. One is to look for CP violating effects in the $B$ meson system. Here the CP violating asymmetries potentially can be very large—of order $10^{-1}$ or more in some rare modes, rather than the order $10^{-3}$ effects in the neutral $K$ mass matrix. The sheer numbers of $B$ mesons estimated to be necessary to get a statistically significant effect put this exciting possibility many years in the future.[17]

Another way is to consider other $K$ decays where CP violating effects, although very small, may occur with a different weighting (from that in $K \to \pi\pi$) between effects originating in the mass matrix and in the decay amplitude. Although these experiments are also very difficult, there is the advantage of high intensity beams and sophisticated detectors already in existence to perform the measurements of $\epsilon'$ and search for rare $K$ decays. Possible

$K$ decays which come to mind include $K \to 3\pi$, $K \to \gamma\gamma$, and $K \to \pi\pi\gamma$,[18–20] and especially $K_L \to \pi^0\ell^+\ell^-$ and $K_L \to \pi^0\nu\overline{\nu}$. It is the latter route of $K$ decays which falls within the jurisdiction of this talk and will be discussed below. If, on the contrary, the Standard Model cannot account for the results of these experiments, so much the better—we'd have evidence for physics beyond the three-generation Standard Model.

## Physics Prospects for Some Rare $K$ Decays

We will start with processes which are already measured and generally have larger rates, and move toward those with smaller branching ratios, saving the (almost?) impossible experimental measurements for the end—somehow these are also the most interesting theoretically. The decay modes discussed below are only a subset of those of interest, governed by personal prejudice and the limits of space and time. In particular, neither $K \to \mu e$ and $K \to \pi\mu e$, which involve lepton flavor-changing neutral currents and are forbidden in the Standard Model,[21] nor CP violating effects in $K \to 3\pi$, $K \to \gamma\gamma$, and $K \to \pi\pi\gamma$, are discussed here.

### $K^\circ - \bar{K}^\circ$ Mixing

The grandfather of all the calculations of amplitudes which are forbidden in lowest order of the electroweak theory is that of the off-diagonal elements of the $K^\circ - \overline{K^\circ}$ mass matrix which generate the $K_L - K_S$ mass difference and $\epsilon$. This still provides the tightest constraint on quark flavor-changing neutral currents (provided, of course, that they contribute to this process). The one-loop, short-distance contribution to $\epsilon$ has been already alluded to in our discussion of CP violation.

### $K^+ \to \pi^+\ell^+\ell^-$ and $K_S \to \pi^0\ell^+\ell^-$

Both of these processes receive short-distance contributions from the "electromagnetic penguin" with a charm quark in the loop. However, there are very large QCD corrections,[22] (so big as to change the sign of the amplitude) and the result is very untrustworthy. Not surprisingly, for the real, CP conserving, part of the amplitude which enters both these processes, it is necessary to understand significant long-distance contributions. These may be best calculable in chiral perturbation theory.[23]

The measured branching ratio for $K^+ \to \pi^+ e^+ e^-$ is[8] $2.7 \pm 0.5 \times 10^{-7}$. We may expect hundreds, if not thousands, of events from ongoing experiments, as well as some events of $K^+ \to \pi^+ \mu^+ \mu^-$. The predicted branching ratio for $K_S \to \pi^0 \ell^+ \ell^-$ is in the neighborhood of several times $10^{-9}$, and will be of importance both for a check on the chiral perturbation theory calculations[23] and for CP violation in the decay, $K_L \to \pi^0 \ell^+ \ell^-$, to be discussed later.

$\underline{K^+ \to \pi^+ \nu \bar{\nu}}$

Here the short-distance contribution from charm and especially top quarks in "$Z$ penguin" and "$W$ box" graphs provides the dominant contribution to the amplitude: all the estimates of long distance effects show them to be negligible.[24] The QCD corrections are moderate in magnitude. They particularly need to be applied to the contribution of the charm quark. The original QCD corrections,[25] have been recently updated to the case where the top mass is comparable to $M_W$.[26] The resulting branching ratio for $K^+ \to \pi^+ \nu_e \bar{\nu}_e$ is shown in Fig. 3, with the dashed lines representing upper and lower bounds (given our present freedom in choosing Kobayashi–Maskawa parameters, particularly $V_{td}$) without QCD corrections and the solid lines giving the corresponding bounds with those corrections.[26] The branching ratio ranges between about 0.2 and $2 \times 10^{-10}$ per neutrino flavor.

The upper limit on this process has recently been considerably improved to $3 \times 10^{-8}$ by a dedicated Brookhaven experiment.[27] There are prospects of getting to the $10^{-9}$ level in the next year, and eventually reaching a sensitivity where there should be a few events if the Standard Model gives the correct rate. In the meantime there is a large window still left open for new physics between where we are now and the Standard Model prediction.

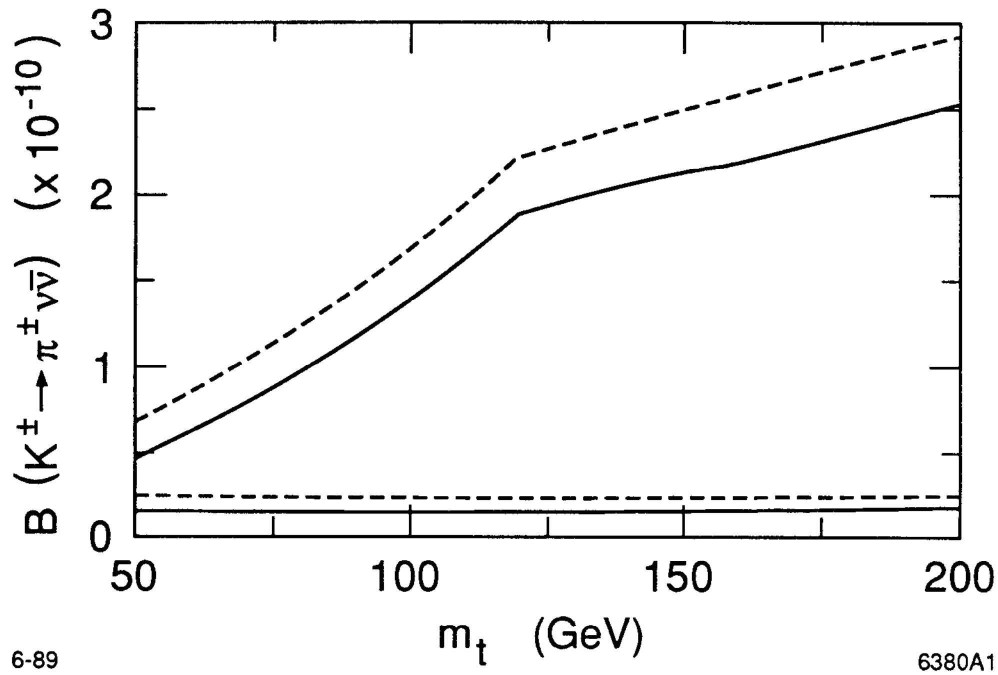


Fig. 3. The maximum and minimum of the branching ratio (per neutrino flavor) for $K^\pm \to \pi^\pm \nu\overline{\nu}$ without (dashed curve) and with (solid curve) QCD corrections ($\Lambda_{QCD} = 150$ MeV). From Ref. 26.

$\underline{K_L \to \pi^0 \ell^+ \ell^-}$

Another $K$ decay in which it is possible to observe CP violation and which has emerged as the object of concentrated theoretical and experimental study is $K_L \to \pi^0 e^+ e^-$. If we define $K_1$ and $K_2$ to be the even and odd CP eigenstates, respectively, of the neutral $K$ system, then $K_L \to \pi^0 e^+ e^-$ has three contributions:

(1) Through a two-photon intermediate state:

$$K_2 \to \pi^0 \; \gamma\gamma \to \pi^0 e^+ e^- \quad .$$

This is higher order in $\alpha$, but is CP conserving.

(2) Through the small (proportional to $\epsilon$) part of the $K_L$ which is $K_1$ due to CP violation in the mass matrix:

$$K_L \approx K_2 + \epsilon K_1 \quad ,$$

$$K_1 \to \pi^0 \ \gamma_{virtual} \to \pi^0 e^+ e^- \quad .$$

We call this "indirect" CP violation.

(3) Through the large part of the $K_L$ which is $K_2$ due to CP violation in the decay amplitude:

$$K_2 \to \pi^0 \ \gamma_{virtual} \to \pi^0 e^+ e^- \quad ,$$

We call this "direct" CP violation.

The question before us is the relative magnitude of these three contributions. Let us take them one at a time.

<u>Contribution (1)</u>

The CP conserving amplitude has a history of some uncertainty. If we consider the absorptive part of the amplitude corresponding to Fig. 4, it involves the product of the amplitude for $K_L \to \pi^0 \ \gamma\gamma$ with the QED amplitude for $\gamma\gamma \to e^+e^-$. With two real photons, there are two possible Lorentz invariant amplitudes for $K_L \to \pi^0 \ \gamma\gamma$. One is the coefficient of $F^{(1)}_{\mu\nu} \ F^{(2)}_{\mu\nu}$, which corresponds to the two photons being in a state with total angular momentum zero. Consequently, it picks up a factor of $m_e$ when contracted with the QED amplitude, as the interactions are all chirality conserving. Its contribution to the branching ratio for $K_L \to \pi^0 e^+ e^-$ is totally negligible.[28]

The other invariant amplitude is the coefficient of a tensor which contains two more powers of momentum. One might hope for its contribution to be suppressed by angular momentum barrier factors. Because of the extra powers of momentum, in chiral perturbation

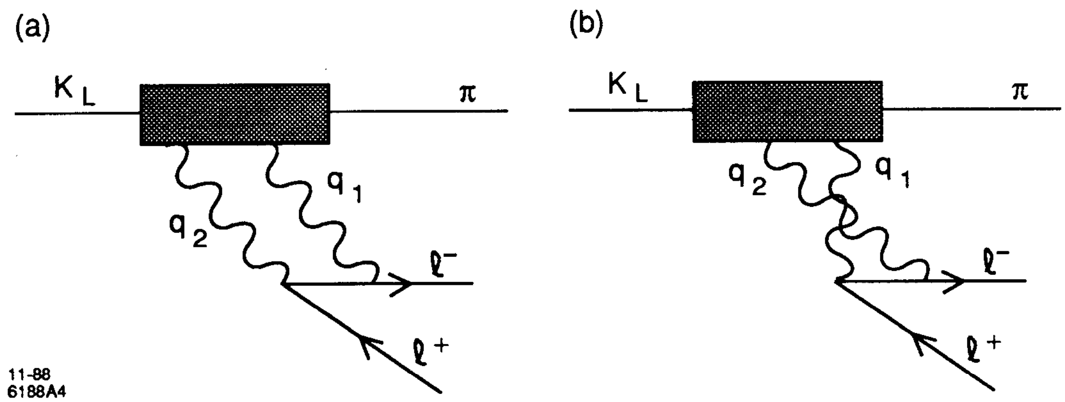


Fig. 4. Diagrams involving $K_2 \to \pi^0\gamma\gamma \to \pi^0\ell^+\ell^-$ which give a CP conserving contribution to $K_L \to \pi^0\ell^+\ell^-$.

theory this amplitude is put in by hand and its coefficient not predicted. An order of magnitude estimate may be obtained by pulling out the known dimensionful factors in terms of powers of $f_\pi$, and asserting that the remaining coupling strength should be of order one.[23] The branching ratio for $K_2 \to \pi^0\ e^+e^-$ is then of order $10^{-14}$. Again, the CP conserving amplitude would make a negligible contribution to the decay rate. However, an old fashioned vector dominance, pole model predicts[29] a much bigger invariant amplitude and a consequent much bigger branching ratio of order $10^{-11}$, roughly at the level as that arising from the CP violating amplitudes (see below). The applicability of such a model, however, can be challenged on the grounds that the low energy theorems and Ward identities of chiral perturbation theory are not being satisfied.[30] The consistent implementation of vector dominance with the chiral and other constraints may lead to an extra suppression factor. The experimental upper limit on the branching ratio for $K_L \to \pi^0\gamma\gamma$ has very recently been considerably improved,[31] and now is only a few times larger than some of the predictions.[29,23] In the future, we might have not only a measurement of the branching ratio, but a Dalitz plot distribution which could help distinguish between models. The final answer for this amplitude remains to be seen both theoretically and experimentally.

Contribution (2)

We may estimate the contribution to the decay rate from the amplitude induced by "indirect" CP violation by using the identity:

$$B(K_L \to \pi^0 e^+ e^-)_{\text{indirect}} \equiv$$
$$B(K^+ \to \pi^+ e^+ e^-)\, \frac{\tau_{K_L}}{\tau_{K^+}} \times \frac{\Gamma(K_1 \to \pi^0 e^+ e^-)}{\Gamma(K^+ \to \pi^+ e^+ e^-)}\, \frac{\Gamma(K_L \to \pi^0 e^+ e^-)_{\text{indirect}}}{\Gamma(K_1 \to \pi^0 e^+ e^-)} \quad . \tag{4}$$

Experimental values[8] of $2.7 \times 10^{-7}$ and 4.2 may be inserted for the first two factors on the right-hand side. The last factor is $|\epsilon|^2$ by the definition of what we mean by "indirect" CP violation in the convention where $A_0(K \to \pi\pi)$ is real. The third factor, in which $\Gamma(K_1 \to \pi^0 e^+ e^-)$ is the undetermined quantity, can be measured directly one day. As discussed previously, it has considerable theoretical uncertainties due to long-distance contributions. The ratio has a value of one if the transition between the $K$ and the $\pi$ is $\Delta I = 1/2$, as is the case for the short-distance amplitude which involves a transition from a strange to a down quark. For $\Delta I = 3/2$, the corresponding value is 4. With both isospin amplitudes present and interfering, any value is possible.[32] Eventually, an experimental measurement of $\Gamma(K_S \to \pi^0 e^+ e^-)$ will take all the present Model dependence away. For now, using a value of unity for this factor makes

$$B(K_L \to \pi^0 e^+ e^-)_{\text{indirect}} = 0.58 \times 10^{-11} \quad . \tag{5}$$

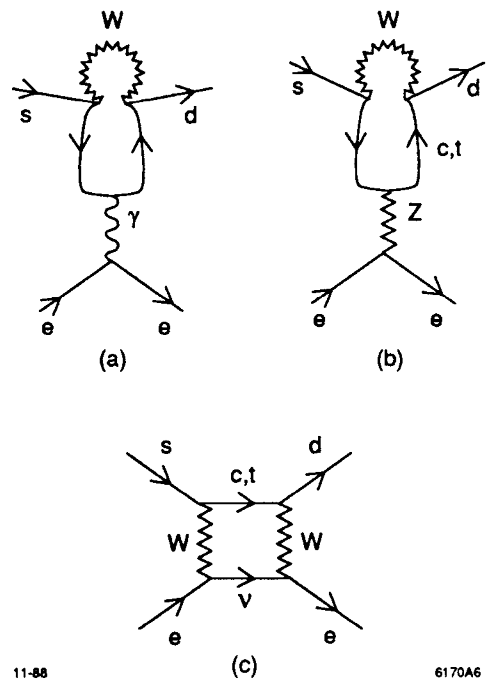


Fig. 5. Three diagrams giving a short distance contribution to the process $K \to \pi\ell^+\ell^-$: (a) the "electromagnetic penguin;" (b) the "$Z$ penguin;" (c) the "$W$ box."

Contribution (3)

The amplitude for "direct" CP violation comes from penguin diagrams with a photon or $Z$ boson replacing the usual gluon and also from box diagrams with quarks (of charge 2e/3), leptons (neutrinos) and $W$ bosons as sides, as shown in Fig. 5. For values of $m_t \ll M_W$, it is the "electromagnetic penguin" that gives the dominant short-distance contribution to the amplitude, which is summarized in the Wilson coefficient of the appropriate operator,

$$Q_{7V} = \alpha \, [\bar{s}\gamma_\mu(1-\gamma_5)d] \, (\bar{e}\gamma^\mu e) \quad ,$$

and which behaves like $\ell n(m_t^2/m_c^2)$. The $Z$ penguin and $W$ box graph contributions are "suppressed" by a power of $m_t^2/M_W^2$. Here is another example of where values of $m_t \sim M_W$ allow the "$Z$ penguin" and "W box" contributions to become comparable to that of the "electromagnetic penguin" and to bring in another operator,

$$Q_{7A} = \alpha \, [\bar{s}\gamma_\mu(1-\gamma_5)d] \, (\bar{e}\gamma^\mu\gamma_5 e) \quad .$$

The QCD corrections are substantial for the "electromagnetic penguin" contribution and have been redone for the case[33,34] when $m_t \sim M_W$. The top quark contributions from the "$Z$ penguin" and "$W$ box" live up at the weak scale and get only small QCD corrections.

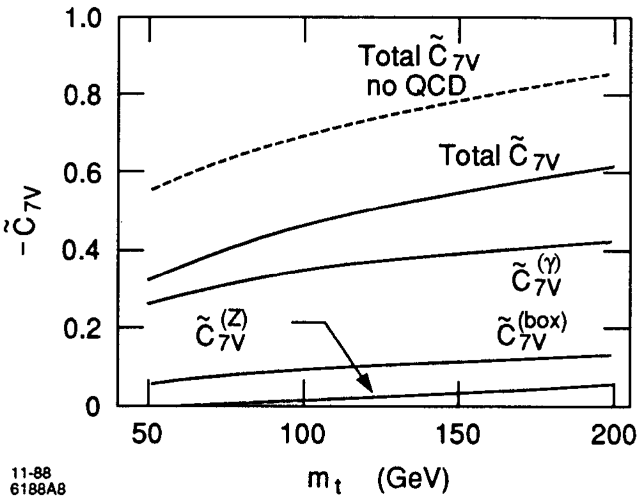


Fig. 6. Contributions to the coefficient $\widetilde{C}_{7V}$ from each of its components, the "electromagnetic penguin," the "$Z$ penguin" and the "$W$ box" diagrams and the total $\widetilde{C}_{7V}$ with QCD corrections (solid curves) with $\Lambda_{QCD} = 150$ MeV, and the total coefficient without QCD corrections (dashed curve) as a function of $m_t$. From Ref. 33.

The CP violating amplitude in which we are interested is proportional to the imaginary part of the Wilson coefficients and thence the difference of the contributions from the top and charm quarks:

$$ImC_7 \;=\; s_2 s_3 s_\delta \big(\widetilde{C}_{7,t} - \widetilde{C}_{7,c}\big) \;, \tag{6}$$

where the tilde indicates that the Kobayashi–Maskawa factor has been removed from the coefficient. As can be seen in Fig. 6, the coefficient $\widetilde{C}_{7V}$ comes largely from the "electromagnetic penguin," even after its reduction from QCD corrections. This would not be the case if the $Z$ couplings to charged leptons were not small due to the particular value for $\sin^2\theta_W$ chosen in Nature. On the other hand, the "electromagnetic penguin" cannot contribute to

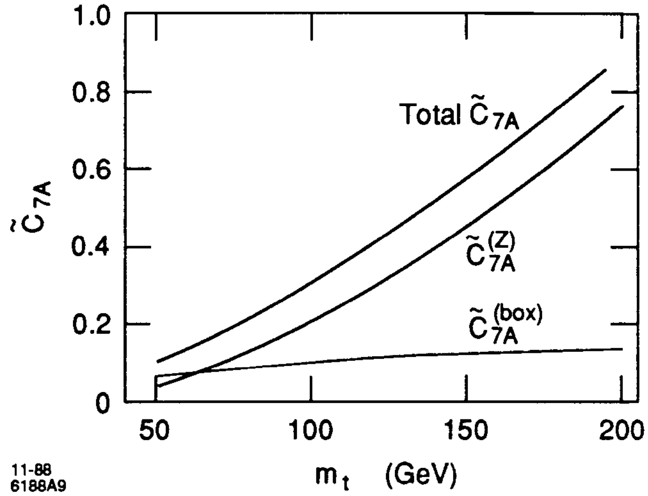


Fig. 7. Contributions to the coefficient $\widetilde{C}_{7A}$ from the "$Z$ penguin" and "$W$ box" diagrams as a function of $m_t$. From Ref. 33.

$C_{7A}$, and here it is the "$Z$ penguin" which gives the dominant contribution, as shown in Fig. 7.

The overall decay rate due to these "direct" CP violating amplitudes can be obtained by relating the hadronic matrix elements of the operators $Q_{7V}$ and $Q_{7A}$ to that which occurs in $K_{e3}$ decay. Then, we find that

$$B(K_L \to \pi^0 e^+ e^-)_{direct} = 1.0 \times 10^{-5} \, (s_2 s_3 s_\delta)^2 \left[ |\widetilde{C}_7|^2 + |\widetilde{C}_{7A}|^2 \right] \quad . \tag{7}$$

The last factor ranges[33] between about 0.1 and 1.0, and as $s_2 s_3 s_\delta \leq 2.5 \times 10^{-3}$ and is typically of order $10^{-3}$, the corresponding branching ratio induced by this amplitude alone for $K_L \to \pi^0 e^+ e^-$ is around $10^{-11}$. Note that when $m_t \gtrsim 150$ GeV, the contribution from $C_{7A}$ overtakes that from $C_{7V}$, and it is the "$Z$ penguin" and "$W$ box," coming from the top quark with small QCD corrections, which dominate the decay rate.

Thus it appears at this point that the contributions from the CP conserving, "indirect" CP violating, and "direct" CP violating amplitudes could all be comparable. The weighting of the different pieces in this decay is entirely different from that in $K \to \pi\pi$. The present experimental upper limit[35,36] is $4 \times 10^{-8}$, with prospects of getting to the Standard Model level of around $10^{-11}$ in the next several years. Hopefully, over the next few years the CP conserving and "indirect" CP violating amplitudes will be pinned down much better, permitting an experimental measurement of this decay to be interpreted in terms of the magnitude of the "direct" CP violating amplitude.

$\underline{K_L \to \pi^0 \nu\bar{\nu}}$

Having descended to miniscule branching ratios, we now add the impossible in detection: the decay $K_L^0 \to \pi^0 \nu_\ell \bar{\nu}_\ell$ is an even more striking example of a process in which the relative size of various contributions to the decay rate are totally different[37] than in $K \to \pi\pi$. There is, of course, neither an "electromagnetic penguin" nor a two-photon, CP conserving, contribution to the amplitude. Furthermore, the "indirect" CP violation arising from the neutral $K$–mass matrix gives a negligible contribution to the decay rate. That leaves us with just the "$Z$ penguin" and "$W$ box," and the V–A character of the gauge boson couplings to neutrinos allows only the operator:

$$Q_\nu = \frac{e^2}{4\pi} \; [\bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\alpha] \; [\bar{\nu}_\ell \gamma^\mu (1-\gamma_5) \nu_\ell] \quad . \tag{8}$$

Being CP violating, it is the imaginary part of $C_\nu$ that is required:

$$Im\ C_\nu \;=\; (s_2 s_3 s_\delta)\ (\widetilde{C}_{\nu,t} - \widetilde{C}_{\nu,c}) \quad , \tag{9}$$

which is totally dominated by the top quark contribution. The branching ratio (per neutrino flavor) is

$$B(K_L^0 \to \pi^0 \nu_\ell \bar{\nu}_\ell) \;\approx\; 2.1 \times 10^{-5} (s_2 s_3 s_\delta)^2 \; |\widetilde{C}_{\nu,t} - \widetilde{C}_{\nu,c}|^2 \quad , \tag{10}$$

with the latter quantity shown in Fig. 8. Again, as $s_2 s_3 s_\delta$ is of order $10^{-3}$, the branching ratio with three-generations of neutrinos is of order $10^{-11}$. The QCD corrections to the $t$–quark contribution should be small, making this theoretically an ideal decay in which to study CP violation in the decay amplitude. Experimentally, the problems are perhaps best represented by the statement that nobody has yet shown that a measurement of this decay is absolutely impossible.

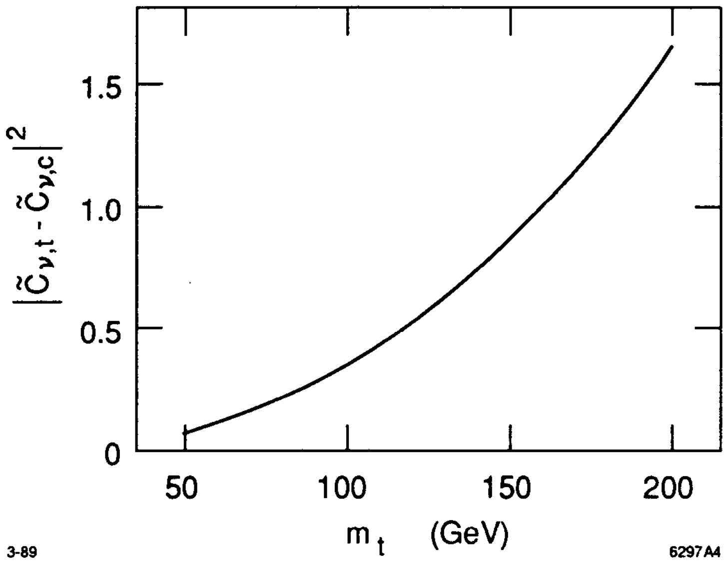


Fig. 8. The quantity $|\widetilde{C}_{\nu,t} - \widetilde{C}_{\nu,c}|^2$, which enters the branching ratio for the CP violating decay $K_L \to \pi^0 \nu_\ell \bar{\nu}_\ell$, as a function of $m_t$. From Ref. 26.

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