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THE GOOGOLPLEXUS*

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ABSTRACT

We carry out third quantization in a minisuperspace model of quantum gravity. The scale factor of the 3-geometry plays the role of time. Wormholes turn the cosmological constant into a new kind of dynamical variable. The *a priori* probability for the cosmological constant is typically a smooth distribution.

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1. Introduction

This talk is based on joint work with W. Fischler and J. Polchinski.[1]

Recent work on quantum gravity[2–5] has demonstrated fairly convincingly that creation and annihilation of microscopic baby universes leads to no observable loss of quantum coherence.[6] Instead, the effect of these topological fluctuations is to turn the coupling parameters relevant to physics in a macroscopic universe into a peculiar kind of dynamical variables governed by a probability distribution. In a further development, Coleman[4] found that the probability to find the cosmological constant equal to zero is 1. Coleman's theory relies on the Euclidean path integral quantization of gravity which, at present, is only a formal technique. The unboundedness of the action

$$I_E = \int d^4x \sqrt{g} \left(\Lambda - \frac{1}{16\pi G} R \right) \tag{1.1}$$

makes it difficult to interpret the Euclidean path integral of quantum gravity. In particular, Coleman's mechanism for the vanishing of the cosmological constant relies heavily on the apparent instability with respect to nucleating arbitrarily large numbers of the Euclidean four-spheres each contributing $-2/3\lambda$ to the action ($\lambda = 16G^2\Lambda/9$). Unfortunately, the same instability seems to lead to a catastrophic number of macroscopically or even cosmically large wormholes in spacetime.[7] (Resolutions of this problem have been proposed in Ref. 8, but were strongly criticized in Ref. 9.)

On the other hand, there are mathematical prescriptions which eliminate the instabilities. Consider, for example, the path integral over conformally flat geometries of spherical topology: $g_{ij} = \phi^2 \delta_{ij}$. This set includes Coleman's networks of wormhole-connected spherical universes. The Euclidean path integral for Einstein gravity reduces to

$$\int [d\phi] \exp \left(\int d^4x \left(\frac{3}{8\pi G} (\partial\phi)^2 - \Lambda \phi^4 \right) \right) \tag{1.2}$$

Clearly, this expression is formal due to the unconventional sign of the kinetic term for ϕ. With the Gibbons-Hawking-Perry conformal rotation[10] $\phi \to i\phi$, it is defined to be

$$\int [d\phi] \exp\left(-\int d^4x \left(\frac{3}{8\pi G}(\partial\phi)^2 + \Lambda\phi^4\right)\right) \tag{1.3}$$

This is just the Euclidean path integral for the stable ϕ^4 theory (we assume that the cosmological constant is positive).* Such a procedure may define a consistent quantum theory of gravity, but it surely eliminates the divergences as $\lambda \to 0$ which drive Coleman's mechanism.

Other prescriptions rotate the contour about the Euclidean saddle point or approximate saddle points associated with wormhole-connected four-spheres. In this case a careful analysis of the modes of fluctuation[11] reveals a prefactor i^{D-2} in front of the Baum-Coleman-Hawking amplitude $\exp(2/3\lambda)$[12,13,4] associated with each four-sphere. The result is, once again, not favorable: in 4 dimensions ($D = 4$) the marvelous $\exp(\exp(2/3\lambda))$ becomes a disappointing $\exp(-\exp(2/3\lambda))$. Evidently, the Euclidean path integral is so ill-defined that it can be imaginatively used to produce vastly different answers. For these reasons it seems necessary to provide a different formulation of the theory of topological fluctuations.

In this paper we present a Hilbert space analysis of topology change in a minisuperspace model of quantum gravity. This model is quite simple to work with, but is rich enough to include any number of universes with spherical spatial geometry. We will find that, under some rather general assumptions, the average number of large universes is $\mathcal{O}(\exp(2/3\lambda))$. In view of the known bound $\lambda \lesssim 10^{-120}$, the average number of universes is $\gtrsim 10^{10^{120}}$. This is the origin of the name googolplexus.† We will show that all these universes are cold, empty and uninteresting. Their presence, much like the presence of the infrared photons in QED, makes no effect on any of the observable probabilities. In fact, in contrast

* This is not a conventional theory since it must be regulated in a conformally invariant way.

† This term, coined by L. Susskind, is a fusion of two words. The googolplex is the largest finite integer with a special name. It is equal to $10^{10^{100}}$. Plexus means a network. (Websters New Collegiate Dictionary.)

with Coleman's result, we will find that the *a priori* probability for the cosmological constant is a smooth function, with no enhancement of either the Baum-Hawking or Coleman type.

The Coleman double exponential does occur, but in the form $\exp\left(-e^{2/3\lambda}\right)$. Furthermore, it does not have the interpretation of a probability for λ but rather a transition amplitude from the state of the googolplexus to the out-vacuum, i. e., it is the amplitude to create no universes. The $\exp(2/3\lambda)$ universes are rather like the soft photons emitted in electrodynamics and the factor $\exp\left(-e^{2/3\lambda}\right)$ is the analogue of the soft photon factor which suppresses transitions to exclusive states with a finite number of soft photons.

The same assumptions on the state of the googolplexus that produce $\mathcal{O}(\exp(2/3\lambda))$ cold empty universes, typically lead to no enhancement in the number of warm, liveable universes as $\lambda \to 0$. We will show that there is a class of states which contain few cold universes but $\mathcal{O}(\exp(2/3\lambda))$ warm ones. It is not yet clear whether a theory based on such states is sensible.

2. Third Quantization in Minisuperspace

In this section we consider the path integral in a minisuperspace model of quantum gravity. This model includes the spatially spherical geometries with metric

$$ds^2 = \frac{2G}{3\pi}(-d\tau^2 + a^2(\tau)d\Omega_3^2) \tag{2.1}$$

where $d\Omega_3^2$ is the metric of a unit three-sphere. We also include a number of spatially constant matter fields $\phi_i(\tau)$. The Einstein-Hilbert action with matter couplings reduces to

$$I = \frac{1}{2}\int_0^T d\tau \left(-a\dot{a}^2 + a + a^3\left(\dot{\phi}_i^2 - \lambda - V(\phi_i)\right)\right) \tag{2.2}$$

where $\lambda = 16G^2\Lambda/9$. This action defines a hamiltonian

$$H = \frac{1}{2}\left\{-\frac{\pi_a^2}{a} - a + a^3(\lambda + V(\phi_i)) + a^{-3}\pi_i^2\right\} \tag{2.3}$$

which is the generator of translations in the parameter time τ. Due to general covariance of the theory, the hamiltonian must annihilate the physical states. This defines the Wheeler-De Witt equation for the wave function of the universe $\Phi(a, \phi_i)$

$$\left\{a^{-p}\frac{\partial}{\partial a}a^p\frac{\partial}{\partial a} - a^{-2}\frac{\partial^2}{\partial\phi_i^2} - a^2 + a^4(\lambda + V(\phi_i))\right\}\Phi(a, \phi_i) = 0 \tag{2.4}$$

where the uncertainty in p is a part of the operator ordering ambiguities inherent in the hamiltonian of Eq. (2.3). Sometimes Eq. (2.4) is regarded as gravity's Schroedinger equation, but it is obviously more like gravity's Klein-Gordon equation. Indeed many authors, beginning with De Witt,[14] have noted the similarity between the scale factor a and time. In 1+1 dimensions the correspondence is precise.[15] As in the case of the Klein-Gordon equation, the lack of a positive definite probability density makes the one-universe theory hard to interpret. In any case, we are not after formulating such a theory here. Instead, we are interested in processes where the number of spatially connected components of geometry changes. A convenient formalism involves a Hilbert space for universes. With this in mind, we will carry out "third quantization" of the Wheeler-De Witt equation,[16–21] which is a step analogous to second quantization of the Klein-Gordon equation. We will therefore consider a quantum field theory of the Wheeler-De Witt equation with a playing the role of time and Φ being the quantum field. First we will consider a theory without topology changing processes. Subsequently, we will include wormholes. One of the goals is to build a theory whose Feynman diagrams resemble Coleman's wormhole-connected bubbles. We will find that this theory has a natural probability interpretation which is quite different from the interpretation suggested by Coleman. Surprisingly, although this theory has a graph structure similar to Coleman's, the graphs do not compute the probabilities *we* measure.

To simplify the discussion, we will first ignore the scalar fields ϕ_i and return to consider their effects in Section 4. Also, we will adopt the operator ordering in the Wheeler-De Witt Eq. (2.4) which corresponds to $p = -2$. Then it is convenient to think of $a^3 = V$ as the argument of the Wheeler-De Witt wave function:

$$\left(\frac{\partial^2}{\partial V^2} - \frac{1}{9V^{2/3}} + \frac{\lambda}{9}\right)\Phi(V) = 0 \tag{2.5}$$

Although our results will not depend much on the choice of p, the advantage of this prescription is that the potential on the left side approaches a constant for large V. If we think of V as time and $\Phi(V)$ as a real coordinate, then Eq. (2.5) is just the classical equation for a harmonic oscillator with a spring constant which varies with time. In fact, the oscillator is upside down for $V < \lambda^{3/2}$ but becomes an ordinary harmonic oscillator with frequency $\omega_0 = \sqrt{\lambda}/3$ as $V \to \infty$. As explained above, it is natural to think of $\Phi(V)$ as a 0+1 dimensional Klein-Gordon field. In practical terms, third quantization of the minisuperspace Wheeler-De Witt equation amounts to solution of the time-dependent harmonic oscillator problem defined by Eq. (2.5). To make this analogy yet more explicit, we change notation and replace $\Phi(V)$ with $X(t)$. The third-quantized lagrangian is

$$L = \frac{1}{2}\dot{X}^2 - \frac{1}{2}\omega^2(t)X^2 - JX\delta(t) \tag{2.6}$$

For reasons that will be explained later, we have included a δ-function source at $t = 0$. The time-evolution is generated by the hamiltonian

$$H = \frac{1}{2}P^2 + \frac{1}{2}\omega^2(t)X^2 + JX\delta(t) \tag{2.7}$$

We emphasize that $X(t)$ is not an ordinary oscillator variable: it creates or annihilates entire universes with spatial volume t. t plays the role of time in a new space—the googolplexus —where particles are universes. In order to introduce universe creation and annihilation operators, we expand

$$X(t) = f(t)a + f^{\star}(t)a^{\dagger} \tag{2.8}$$

where f and $f^{\star}$ are the incoming and outgoing solutions of the Wheeler-De Witt equation. The conjugate momentum is

$$P(t) = \dot{f}(t)a + \dot{f}^{\star}(t)a^{\dagger} \tag{2.9}$$

From the canonical commutation relations it follows that $[a, a^{\dagger}] = 1$ if the Wronskian

$$\dot{f}f^{\star} - \dot{f}^{\star}f = -i \tag{2.10}$$

This fixes the normalization of f:

$$f(t \to \infty) = (2\omega_0)^{-1/2} e^{-i(\omega_0 t + \delta)} \tag{2.11}$$

We will also fix the phase δ by requiring that

$$\dot{f}_1(0) = 0 \tag{2.12}$$

where $f(t) = f_1(t) + if_2(t)$. A nice property of the hamiltonian in Eq. (2.7) is that it loses its explicit time dependence as $t \to \infty$:

$$H(t \to \infty) = \frac{1}{2}\omega_0\{a, a^{\dagger}\} \tag{2.13}$$

The ground state at late times is simply the oscillator vacuum

$$a \mid out >= 0 \tag{2.14}$$

The quanta of the theory can now be cleanly identified: $a^{\dagger}$ and a create and annihilate the out-universes.

The key question is what determines $\Psi(X,t)$, the Schroedinger wave function of the googolplexus. The minisuperspace formulation has a peculiar feature that negative time is physically meaningless (t refers to the volume of 3-space). Therefore, it is sensible to assume that Ψ is determined near $t=0$ by a smooth match on to short distance physics. Since the Planck scale physics is almost completely insensitive to the value of λ, this assumption implies that the boundary condition at $t=0$ has no strong dependence on λ. We will consider a few such choices of the state of the googolplexus. One of them, which we denote by $\mid i>$, will be motivated by the assumption of symmetry under $t \to -t$. This suggests that

$$\dot{X}(t=0)|\, i >= P(t=0)|\, i >= 0 \tag{2.15}$$

or, in other words,

$$\Psi_i(X,0) = const \tag{2.16}$$

A general wave function consistent with our assumption is initially a wave packet of width w, which has no singular dependence on λ. For illustration, we will work with Gaussians

$$\Psi_w(X,t=0) = \left(\frac{1}{w^2\pi}\right)^{1/4} \exp\left(-\frac{X^2}{2w^2}\right) \tag{2.17}$$

We will see that some of our results are insensitive to the precise nature of the boundary conditions imposed at small t, as long as these boundary conditions have no fine-tuned dependence on λ. However, for the purposes of comparing our theory with Coleman's, it *will* be necessary to fine tune the physics near $t=0$. Alternatively, one can think of this fine-tuning in terms of imposing "generic" boundary conditions at large t. We will discuss the possible implications of this rather counter-intuitive procedure in Section 4.

In order to carry out comparison with Coleman's results, we need to calculate the dependence of the path integral on J and λ for an arbitrary state of the googolplexus $\mid in >$. If we set $J = 0$, then the path integral over all $X(t)$ for $t > 0$ is just the transition amplitude between the in-state and the out-vacuum

$$\int [DX(t)] e^{i \int_0^\infty L(J=0) dt} = < out \mid in > \tag{2.18}$$

This path integral can be thought of as the exponential of the vacuum graph which represents a sum over minisuperspace geometries with the topology of a torus. There are no toroidal solutions of $R_{ij} = 8\pi G \Lambda g_{ij}$. Therefore, on semiclassical grounds Coleman did not expect the sum over tori to have an exponential dependence on $1/\lambda$. In Coleman's approach, the crucial geometries have spherical topology. In order to introduce spheres into our model, it is necessary to turn on the source J. Treating the source term as a perturbation, the path integral is exactly given by the sum of graphs shown in Fig. 1. Each line represents a

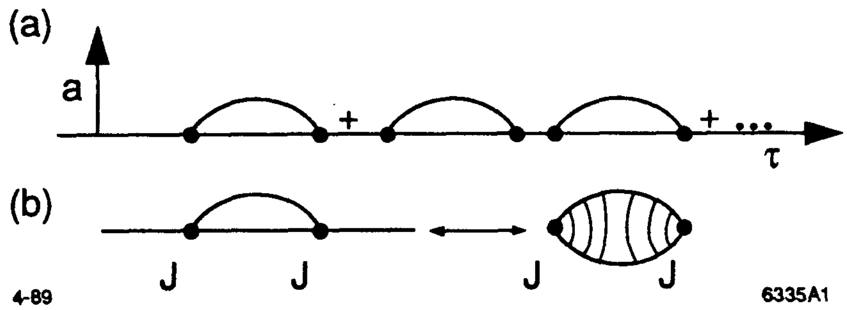

Figure 1. a) The sum of Feynman diagrams representing the exponential of Eq. (22). The vertical axis is the scale factor a, the horizontal is parameter time τ. Each line represents $G(0,0)$, the sum over all paths from $a = 0$ back to $a = 0$. b) G(0,0) is a sum over minisuperspace geometries with the topology of a sphere.

universe which is created by the source with zero radius, propagates, shrinks back to zero size and is annihilated by the source. In other words, each line stands for a sum over geometries of spherical topology. In analogy with Coleman's sum over

Euclidean bubbles, the sum of graphs in Fig. 1 exponentiates. To see this, we note that

$$Z = \int [DX(t)] e^{i\int_0^\infty L(J)dt} = e^{-\frac{1}{2}J^2 G(0,0)} < out \mid in > \qquad (2.19)$$

where

$$G(t_1, t_2) = \frac{< out \,|T\big(X(t_1)X(t_2)\big)|\, in >}{< out \mid in >} \qquad (2.20)$$

If we specialize to $\mid in >= \mid i >$, then

$$G(t_1 < t_2) = 2f_1(t_1)\ f(t_2) \qquad (2.21)$$

As expected, $G(t_1, t_2)$ is an outgoing wave as a function of the bigger argument t_2 and also satisfies

$$\frac{\partial}{\partial t_1} G(t_1 = 0, t_2) = 0 \qquad (2.22)$$

Let us examine in more detail the factor $-\frac{1}{2}J^2 G(0,0)$ associated with each line in Fig. 1. Naively it appears that for agreement with Coleman's analysis $G(0,0)$ must be $\sim \exp(2/3\lambda)$. It is easy to see that this is not generally the case in our model. First, the real part of $f(t)$ satisfies $\dot{f}_1(0) = 0$. Since the boundary condition is specified at $t = 0$, the generic behavior of f_1 is to increase exponentially with t until it enters the oscillatory region $t > \lambda^{-3/2}$, where it must oscillate with the amplitude fixed by Eq. (2.11). Standard WKB techniques indicate that $f_1(0) \sim \exp(-1/3\lambda)$. On the contrary, the imaginary part of f, $f_2(t)$, which is out of phase with $f_1(t)$ in the oscillatory region, exponentially grows toward $t = 0$. In the WKB approximation, $f_2(0) \sim \exp(1/3\lambda)$. As a result,

$$G(0,0) \sim A + iB \qquad (2.23)$$

where A is $\mathcal{O}(\exp(-2/3\lambda))$ and B is $\mathcal{O}(1)$. It seems that there is an immediate disagreement with Coleman's analysis. However, as explained in Ref. 22, there is no disagreement. In the first-quantized theory, specified by Eq. (2.2), $G(0,0)$ is the path integral over all trajectories $a(\tau)$ with $a(0) = a(T) = 0$ integrated over T. If one performs the Euclidean continuation by sending $\tau \to i\tau$, then the resulting Euclidean action

$$I_E = -\frac{1}{2} \int_0^T d\tau (a\dot{a}^2 + a - \lambda a^3) \tag{2.24}$$

is not bounded from below. Within this ill-defined Euclidean path integration, how would one "approximate" $G(0,0)$? In the spirit of Coleman's theory, we need to sum over all the Euclidean stationary paths, including those with multiple bounces off the point $a = 0$. (The bounces occur because the problem is defined on the half-line $a > 0$.) Therefore, we include all the trajectories of the form

$$a(\tau) = \frac{1}{\sqrt{\lambda}} |\sin(\sqrt{\lambda}\tau)| \tag{2.25}$$

with duration $T = n\pi/\sqrt{\lambda}$ (see Fig. 2). Geometrically, these are simply linear

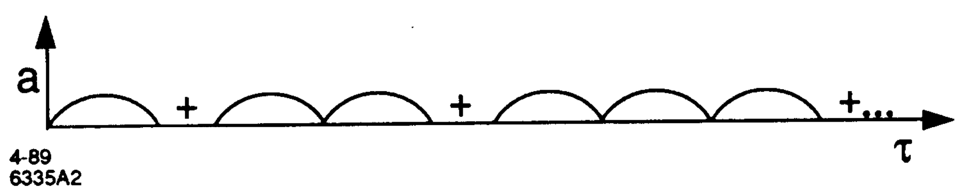

Figure 2. The euclidean trajectories of the form $a(\tau) = |\sin(\sqrt{\lambda}\tau)|/\sqrt{\lambda}$, with $0 < \tau < n\pi/\sqrt{\lambda}$, that need to be included in the semiclassical approximation for $G(0,0)$. The reflections off the point $a = 0$ are the minisuperspace wormholes that attach to the north and south poles of the large four-spheres.

chains of n four-spheres glued at their poles. The action of each four-sphere is $I_1 = -2/3\lambda$. The saddle point "approximation" to $G(0,0)$ is then

$$G(0,0) \approx e^{2/3\lambda} + \mu e^{4/3\lambda} + \mu^2 e^{6/3\lambda} + \ldots = \frac{e^{2/3\lambda}}{1 - \mu e^{2/3\lambda}} \approx -\frac{1}{\mu} + \mathcal{O}(e^{-2/3\lambda}) \tag{2.26}$$

where we have, of course, used an analytically continued definition of the geometric sum. The relative factor of μ in the subsequent bounces depends on the boundary conditions at $a = 0$. Semiclassically, $\mu \sim e^{-S_w}$ where S_w is the Euclidean wormhole action which typically has only weak dependence on λ. The agreement between Eq. (2.26) and Eq. (2.23) is quite detailed. The formal sum over bounces of Eq. (2.26) is capable of reproducing even the exponentially suppressed term $\mathcal{O}(\exp(-2/3\lambda))$ in Eq. (2.23). Inclusion of the multiple bounces is therefore crucial to obtaining the correct normalization of the Green function $G(0,t)$ from the Euclidean saddle point analysis. It comes as a bit of a surprise that the minisuperspace model contains a certain subset of wormhole configurations in Coleman's sense. Due to the asymmetry of the model, we have inadvertently included the wormholes that can couple to the north and the south poles of the four-spheres. Thus, each line in Fig. 1 generally corresponds not just to one four-sphere but to a geometric sum over linear chains of four-spheres. Is there a way to turn off these wormholes built into a generic minisuperspace model? If we succeed in turning them off by effectively setting $\mu = 0$ in Eq. (2.26), then each propagator in Fig. 1 would be associated with just one four-sphere and would carry a factor $e^{2/3\lambda}$.

The answer to the above question is positive. The minisuperspace wormholes can be turned off by a careful tuning of either the boundary conditions or the spring constant $\omega^2(t)$ near $t = 0$.* For instance, if we make the oscillator spring in this region sufficiently attractive (say, by adding to $\omega^2(t)$ a term $\sim \delta(t)$), then we can fine tune $f_1(t)$ to become exponentially decreasing with t and simultaneously maintain the boundary condition (2.12). The precise requirement on $\omega^2(t)$ is that the Wheeler-De Witt equation have a zero-energy bound state when $\lambda = 0$, provided the wave function satisfies boundary condition (2.12). Then, for small λ there exists a metastable state with lifetime $\mathcal{O}(\exp(2/3\lambda))$. Since $f_1(t)$ is normalized to oscillate with amplitude $(2\omega_0)^{-1/2}$ at large t then, in the fine-tuned case, $f_1(0) \sim \exp(1/3\lambda)$. Substituting this into Eq. (2.21), we find $G(0,0) \sim \exp(2/3\lambda)$.

* The precise form of the Wheeler-De Witt equation is uncertain in this region due to the lack of knowledge of short-distance behavior of quantum gravity.

If we compare this result with Eq. (2.26), we find agreement provided we set $\mu = \exp(-S_w) \sim \exp(-2/3\lambda)$. Somehow we have succeeded in making the minisuperspace wormholes very costly. At the expense of fine-tuning $\omega^2(t)$ in the unknown microscopic region near $t = 0$, we have constructed a minisuperspace model which is free of wormholes defined in Coleman's sense. This special adjustment is probably not a true fine-tuning of physical parameters but a procedure necessary to remove the asymmetry built into a generic minisuperspace model. Without this adjustment the model would have picked two special points on each stationary Euclidean geometry (the two poles of the four-sphere), and would have contained wormholes that can attach to these special points. Now that these asymmetrically coupled wormholes have been removed, we can add by hand wormholes that can create contacts between arbitrary pairs of points, such as those considered by Coleman.[4] This will be done in Section 3.

We now offer a somewhat different, perhaps simpler prescription for turning off the asymmetric wormholes contained in the minisuperspace models. For that purpose, we need to elucidate the crucial difference between the fine-tuned case and the general case. Let us solve for the shape of the Schroedinger wave function of the out-vacuum $\Psi_{out}(X,t)$ at early times. By integrating the Schroedinger equation backwards in time, we obtain

$$\Psi_{out}(X,t) = \left(\frac{Re\{\alpha(t)\}}{\pi}\right)^{1/4} e^{-\frac{1}{2}\alpha(t)X^2} \tag{2.27}$$

where

$$\alpha(t) = -i\frac{\dot{f}^\star(t)}{f^\star(t)} \tag{2.28}$$

Without fine-tuning, $Re\{\alpha(0)\}$ is $\mathcal{O}(\exp(-2/3\lambda))$ while $Im\{\alpha(0)\}$ is $\mathcal{O}(1)$. This means that $\Psi_{out}(X,0)$ is an oscillating function with a broad envelope. The special property of the fine-tuned case is that $Im\{\alpha(0)\}$ is $\mathcal{O}(\exp(-2/3\lambda))$ and the oscillations are eliminated. In fact, in the limit $\lambda \longrightarrow 0$, $\Psi_{out}(X,0)$ becomes flat and therefore indistinguishable from $\Psi_i(X,0)$. Strictly speaking, we are only interested in this limiting situation: only in the limit $\lambda \to 0$, where four-spheres are infinitely large, can we make a clean separation between two bubbles connected by a wormhole, and one deformed bubble. Working with very small λ, we will therefore replace the boundary condition of Eq. (2.16) with the boundary condition

$$\Psi_{in}(X,0) = \Psi_{out}(X,0) \tag{2.29}$$

which is exponentially close to it. Thus, the requirement of the absence of wormholes can be fulfilled by choosing the wave function of the googolplexus at $t = 0$ which evolves into the ground state at late times, and leads to no production of de Sitter universes. In the Heisenberg picture, this choice is simply

$$| \, in >= | \, out > \tag{2.30}$$

This leads to

$$G(0,0) = f(0)f^{\star}(0) \sim \exp(2/3\lambda), \tag{2.31}$$

in agreement with the claim that the asymmetric wormholes have been turned off. Perhaps, it is no coincidence that, in the absence of a source of small geometries, a suppression of the number of large universes amounts to a suppression of wormholes. The process of production of a pair of large universes can be pictured as a wormhole geometry where the radius of the universe first contracts to a Planckian value and then reexpands.

Let us now compare the sum of disconnected diagrams of Fig. 1 with Coleman's sum over disconnected four-spheres. Substituting Eq. (2.31) into Eq. (2.19), we find the path integral to be

$$Z(J, \ \lambda) = e^{-\frac{1}{2} J^2 f(0) f^{\star}(0)} \tag{2.32}$$

where $f(0)f^{\star}(0) \sim \exp(2/3\lambda)$. Thus, we have reproduced Coleman's double exponential with one important change: each bubble carries a minus sign in addition to the large factor $\exp(2/3\lambda)$. In the limit $\lambda \to 0$ this leads to an enormous suppression of the path integral instead of enhancement. The origin of this suppression lies in the fact that the average number N of outgoing universes produced by the action of the current J diverges in the limit $\lambda \to 0$:

$$N =< out\ |e^{iJX(0)}a^\dagger a e^{-iJX(0)}|\ out >= J^2\ f(0)f^\star(0) \tag{2.33}$$

It is also easy to see that the de Sitter universes produced by the current are Poisson distributed. The path integral Z with no insertions measures the amplitude to produce no universes in the final state. The Poisson distribution implies

$$|Z|^2 = e^{-N} \tag{2.34}$$

in agreement with Eqs. (2.32) and (2.33). The entire situation is similar to QED with a time dependent external current, which is well known to produce a divergent number of soft photons. The exponential factor in the amplitude (2.32) to produce no out-going universes is analogous to the infinite suppression of exclusive amplitudes, in which no infrared photons are emitted. The analogy with QED suggests the following probability interpretation: since we cannot detect the other universes, we are interested in inclusive probabilities, which are in no way sensitive to the infrared divergences. We will postpone a detailed discussion of the probability interpretation until Section 4.

3. Topology Change In Minisuperspace.

In order to complete our construction, we now include topology changing processes in our third-quantized model. This will have the effect of turning some of the disconnected diagrams of Fig. 1 into diagrams connected by wormhole lines. Following the method of Refs. 2, 3, 4, 18, 22, 23, we introduce into the action a variable α, which is essentially the field for universes of Planckian size:

$$S = \frac{1}{2}\alpha^2 + \int_0^\infty dt \left\{ \frac{1}{2}\dot{X}^2 + \frac{1}{2}\left(\frac{1}{9t^{2/3}} - \frac{\lambda + g\alpha}{9}\right) X^2 - JX\delta(t) \right\} . \tag{3.1}$$

The role of α is to create contact between arbitrary pairs of points on the macroscopic universes. This is accomplished through a repeated application of the interaction vertex, which describes creation of a baby universe by a macroscopic universe. Since the scale factor is time, this interaction is non-local. In order to construct a hamiltonian treatment, we will make the action local in time by formally promoting α into a function of t, and by introducing a lagrange multiplier $\beta(t)$:

$$S = \frac{1}{2}\alpha^2 + \int_0^\infty dt \left\{ \frac{1}{2}\dot{X}^2 + \frac{1}{2}\left(\frac{1}{9t^{2/3}} - \frac{\lambda + g\alpha(t)}{9}\right) X^2 - JX\delta(t) + \beta(t)\dot{\alpha}(t) \right\} \tag{3.2}$$

The argument in the first term is arbitrary since there is a constraint $\dot{\alpha} = 0$. The fields α and β are a pair of conjugate variables. Therefore, the wave functions can be taken to be functions of X and α, with initial conditions of the form $\Psi_{in}(X,\ \alpha,\ t = 0)$. Since $\dot{\alpha} = 0$ is obeyed as an operator equation, the Hilbert space breaks up into sectors labeled by α which is independent of time. Specifically, a general state of the googolplexus can be expanded in a complete set of states with definite values of α:

$$|\Psi >= \int d\alpha e^{\frac{1}{2}i\alpha^2} F_\Psi(\alpha)|\Psi;\alpha > |\alpha > . \tag{3.3}$$

The Schroedinger wave function corresponding to $|\Psi;\alpha>$ evolves in time according to the hamiltonian with a shifted cosmological constant $\lambda_{eff} = \lambda + g\alpha$. $F_\Psi(\alpha)$ in Eq. (3.3) is the weighting function for different α-sectors, which is completely determined by the boundary condition at $t = 0$. Thus, it is natural to assume that

F_Ψ is a smooth function. In general, the out-state is

$$|out>= \int d\alpha F_{out}(\alpha)|out;\alpha> |\alpha> \tag{3.4}$$

where $|out;\alpha>$ is the out-vacuum in each α-sector and $F_{out}(\alpha)$ is assumed to be smooth. Then the path integral, which is the transition amplitude from the in-state to the out-state, is given by

$$\int d\alpha e^{\frac{1}{2}i\alpha^2} F^\star_{out}(\alpha) F_{in}(\alpha) Z(\lambda + g\alpha). \tag{3.5}$$

$$Z(\lambda + g\alpha) =< out;\alpha|in;\alpha> \tag{3.6}$$

is to be computed in a theory with a shifted cosmological constant $\lambda + g\alpha$. This expression closely resembles the wormhole summation formula of Refs. 2-4. If, as Coleman claimed,[4] $Z(\lambda) \sim \exp(\exp(2/3\lambda))$, then the path integral is dominated by the value of α such that the observed cosmological constant is zero. Recall that, after setting

$$| \, in >= e^{-iJX(0)}| \, out > ^{\star} \tag{3.7}$$

we have found $Z(\lambda) \sim \exp(-\exp(2/3\lambda))$, which differs by a crucial sign from Coleman's result.[†] It could then appear that the zero of the cosmological constant is strongly suppressed. Before we jump to such a drastic conclusion, however, we need to analyze perhaps the most confusing issue of quantum gravity: the issue of probability interpretation. As we will argue in the next section, the probability interpretation suggested by the third quantization is quite different from the lore of the Euclidean quantum gravity.

⋆ There is no discrepancy with Eq. (2.30). We have simply absorbed the effects of the source at early times into the definition of the in-state.

† In 4 dimensions our sign agrees with the sign found by Polchinski (Ref. 11).

4. The Probability Interpretation.

As shown by Coleman[2,4] and Giddings and Strominger,[3] the magical effect of the wormhole sum is to introduce integration over coupling constants into the path integral. Coleman went further and interpreted the weight in the integration as the *a priori* probability distribution for the coupling constants.[4] This formed the basis for his claim that the observed cosmological constant is zero in the wormhole theory. Implicit in these arguments is the assumption, first advocated by Hawking, that the Euclidean path integral with insertions computes expectation values in quantum gravity. We will see, however, that this assumption does not generally hold in the theory of the googolplexus.

Our methods rely on thinking of the scale factor as the time in the third-quantized dynamics. Then, as demonstrated by Eq. (2.7), the third-quantized hamiltonian has explicit time dependence. In such a theory, the path integral with operator insertion Θ_i is a transition matrix element of the from

$$< out\, |\Theta_i|\, in > \tag{4.1}$$

On the other hand, the expectation value of Θ_i in a state of the googolplexus $|\, in >$ is given by

$$< in\, |\Theta_i|\, in > \tag{4.2}$$

which cannot in general be calculated using conventional path integrals. For instance, the probability distribution for the cosmological constant is given by

$$\rho(\lambda_{eff}) = g \int_{-\infty}^{\infty} dX \left|\Psi_{in}\left(X, \frac{\lambda_{eff}-\lambda}{g}, t=0\right)\right|^2 = g\left|F_{in}\left(\frac{\lambda_{eff}-\lambda}{g}\right)\right|^2 \tag{4.3}$$

which, by assumption, is a smooth function. The infrared factor $Z(\lambda_{eff})$ which strongly depends on the cosmological constant does not appear in Eq. (4.3). The wormhole calculus, which is intimately connected with the third-quantized Feynman rules, is not suited for the calculation of *a priori* probabilities. Instead, the calculation of *a priori* probabilities bears a close analogy to the calculation of inclusive cross-sections in QED. On the other hand, Coleman's definition appears to be related to exclusive cross-sections, which are well-known to contain large infrared suppression factors. In fact, the properties of $\rho(\lambda_{eff})$ in Eq. (4.3), the *a priori* probability for the cosmological constant, depend entirely on the unknown mechanisms of quantum gravity at Planck scale. Coleman's precognition, which provides the connection between the Planckian and cosmic physics, is absent from our model.

Equations (4.1) and (4.2) show that the path integral computes the expectation values only if $\mid in>=\mid out>$. This corresponds to setting $J=0$ in Eq. (3.7). However, as explained above, this state of the googolplexus leads to no sharp peaks in the probability distribution, just like almost any other state.

Where does the example of the previous section fit in this discussion? There, in order to introduce a diagrammatic structure into the path integral we worked with the state of Eq. (3.7). As evidenced by Eq. (2.33), this state contains a large number of de Sitter universes with $\lambda \to 0$ (from here on we drop the subscript on λ_{eff}). As a result, the path integral, which is the amplitude to produce no outgoing quanta, is strongly suppressed. On the other hand, the *a priori* probability distribution for the cosmological constant in such a googolplexus, obtained by substituting Eq. (3.7) into Eq. (4.3), displays no sharp suppression or enhancement of any value.

If the coupling constants we measure are actually distributed according to the *a priori* probabilities defined by Eq. (4.2), then our conclusions can not be more disappointing. From the point of view of a low-energy observer, wormholes seem to turn all the fundamental parameters into random variables. However, we should not be hasty to accept this unpleasant state of affairs. Probability interpretation is one of the most confusing issues in quantum gravity. Our definition of *a priori* probabilities is perfectly reasonable from the point of view of an outside observer who can watch entire universes being created and destroyed. But how does the world appear to *us*, the observers confined to one universe? An analogous hypothetical question concerns the meaning of probabilities observed by an electron participating in a scattering process. Within a second-quantized framework such a question is confusing and may not have a unique answer.

One interesting modification of the definition of probabilities was proposed in Ref. 19. There it was suggested that we observe the *a priori* probabilities weighted by the number of created de Sitter universes. This number is infrared-sensitive and seems to offer a new possibility for solution of the cosmological constant problem. To show this we come back to the example considered in Section 2. If the in-state is taken to be of the form (3.7), then the source J produces a large number of quanta at late times. According to Eq. (2.33), this number is $\mathcal{O}(\exp(2/3\lambda))$ as $\lambda \to 0$. If this number is included in the weighting of probabilities for different values of the cosmological constant, then we find a single exponential peak at $\lambda = 0+$, similar to the peak of the Baum-Hawking approach.[12,13] This peak actually occurs for a broad class of the wave functions of the googolplexus, provided that some "generic" boundary conditions are specified at early times. The reason is quite simple. Before $t = \lambda^{-3/2}$ the harmonic oscillator is upside-down, and the wave packet spreads. As a result, at late times the wave function is in a highly excited state and contains a large number of quanta, which rapidly grows with a decreasing λ. For instance, consider the gaussian boundary conditions of Eq. (2.17). Calculating the number of quanta at late times as a function of the width w, we find*

$$N(w) =< w \; |a^\dagger \; a| \; w >= \frac{1}{2w^2}|f(0)|^2 + \frac{w^2}{2}|\dot{f}(0)|^2 - \frac{1}{2} \tag{4.4}$$

We see that, without any fine tuning near $t = 0$, $N(w) \sim \exp(2/3\lambda)$. (This was first pointed out by Rubakov.[19]) The crucial question is whether it is justified to

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* From here on we set $J = 0$. Turning on the source does not modify any of the conclusions.

identify the number of produced de Sitter universes with the probability to find a given value of λ? We believe that the answer is negative. In order to explain our point of view, it is necessary to add some matter content to our model.

The simplest model which contains some of the crucial features includes one periodic scalar field ϕ, which is an angle ranging from 0 to 2π. If the field has no potential, then the problem splits into sectors labeled by the discrete values of the momentum conjugate to ϕ. The third-quantized Wheeler-De Witt field can be expanded as

$$X(t,\ \phi) = \sum_{k=-\infty}^{\infty} \left(e^{ik\phi}\ f_k(t) a_k + e^{-ik\phi}\ f_k^{\star}(t) a_k^{\dagger}\right) \tag{4.5}$$

where f_k is an incoming solution of

$$\left(\frac{\partial^2}{\partial t^2} + \frac{k^2}{9t^2} - \frac{1}{9t^{2/3}} + \frac{\lambda}{9}\right) f_k(t) = 0 \tag{4.6}$$

For a value of $|k| \ll \frac{2}{3\sqrt{3}\lambda}$, this equation has two classical solutions: a Friedmann-Robertson-Walker (FRW) universe, which starts from a singularity, expands to a maximum volume $\approx |k|^{3/2}$, and recontracts to a singularity; and a de Sitter universe of minimum volume $\approx \lambda^{-3/2}$. For $|k| > \frac{2}{3\sqrt{3}\lambda}$ the barrier separating the FRW and de Sitter regions disappears, and there is only one classical solution: a universe expanding forever.

We will assume that, in this toy model, a universe similar to ours is described by a FRW solution with

$$1 \ll |k| \ll \frac{2}{3\sqrt{3}\lambda} \tag{4.7}$$

In other words, we assume that our universe has a small positive spatial curvature which will eventually force it to recontract. We will further suppose that the probabilities *we* observe must be weighted by the average number of the universes contained in the googolplexus, which resemble our universe. Then it is clearly wrong to count the de Sitter universes which contain virtually no heat. (The energy of the scalar field is negligible in the de Sitter region $t > \lambda^{-3/2}$ provided condition (4.7) is satisfied.)

Our goal, instead, will be to count the FRW universes with a given large value of k when the cosmological constant is so small that the condition (4.7) holds. The discussion of boundary conditions for a given value of k only concerns the oscillators a_k and $a_k^\dagger$ and reduces to the quantum mechanical system of Section 2. As we have found, a generic boundary condition imposed at $t = 0$ leads to production of $\mathcal{O}(\exp(2/3\lambda))$ de Sitter universes. However, as we argued, this number should not be included in the definition of the probabilities measured by us. In order to count the FRW universes, we introduce their creation and annihilation operators through

$$X(t,\ \phi) = \sum_{k=-\infty}^{\infty} \left(e^{ik\phi}\ h_k(t) b_k + e^{-ik\phi}\ h_k^\star(t) b_k^\dagger\right) \tag{4.8}$$

where

$$h_k(t) \to \sqrt{t/k}\exp(-ik\log t), \qquad\qquad t \to 0 \tag{4.9}$$

Generic boundary conditions at $t = 0$ do not lead to a sharp dependence of the number of FRW universes on λ, since the Schroedinger evolution at early times is very weakly sensitive to λ. As explained in Section 2, it seems natural to impose boundary conditions at $t = 0$ in order to mimic the short-distance effects of quantum gravity. Unfortunately, such boundary conditions do not lead to a semiclassical solution of the cosmological constant problem.

Much more interesting results follow if, as in the example of Section 2, we impose boundary conditions at late times, such as in Eq. (2.30). If condition (4.7) holds, then there is a thick barrier separating the de Sitter region from the FRW region. As shown in Eqs. (2.27) and (2.28), the wave function which tends to the ground state gaussian at late times must be a wave packet of width $\sim \exp(2/3\lambda)$ at early times. Such a state contains $\mathcal{O}(\exp(2/3\lambda))$ FRW universes! The same conclusion follows if the wave function at late times is not in the ground state, but in any finitely excited state. Thus, if we work with generic boundary conditions specified at late times, we find that the number of FRW universes which reach some specified maximum volume V_{max} is $\mathcal{O}(\exp(2/3\lambda))$ as $\lambda \to 0$. This sharp peak is relevant if we weigh the probability for λ by the number of universes which are similar to ours. Such a theory may offer new prospects for solution of the cosmological constant problem.

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