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**NEUTRINO COUNTING WITH THE SLD AT THE
STANFORD LINEAR COLLIDER***

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INTRODUCTION

One of the fundamental measurements to be made at the e^+e^- colliders, SLC and LEP, is the determination of the number of neutrino families produced in Z^0 boson decays. In the event that a fourth generation of light Dirac neutrinos exists, the experimental consequences at the Z^0 resonance are easily seen; the total width will be increased by 171 MeV over its three generation value, to be compared to the ≈ 30 MeV precision that should be achievable once the systematic limit has been reached. A reasonable figure of merit for the precision of a neutrino counting measurement of 0.2 standard model generations corresponds to a Z^0 width measurement error of 35 MeV; close to the limit of anticipated experimental capability. In fact, it is highly desirable to achieve an even higher precision if possible, in order to distinguish potentially small effects due to exotic phenomena from beyond the Standard Model.

This paper will address the issue of how to obtain the best measurement of the number of neutrino generations as a function of the size of the available sample of Z^0 decays. The results presented here were obtained by our study group¹ in an attempt to understand the limitations of a realistic neutrino counting measurement with the SLD² at the Stanford Linear Collider. However, many of our findings are general enough to be applicable to any e^+e^- detector designed to take data at the Z^0 resonance.

Z^0 WIDTH MEASUREMENTS

The partial width of the Z^0 boson for the decay to a fermion antifermion pair is given by the expression:

$$\Gamma(Z^0 \rightarrow f + \bar{f}) = C \frac{M_Z^3 G_F}{6\pi\sqrt{2}} \left\{ v_f^2 + a_f^2 - \frac{2m_f^2}{M_Z^2} (2a_f^2 - v_f^2) \right\} \left\{ 1 - 4 \frac{m_f^2}{M_Z^2} \right\}^{1/2}, \quad (1)$$

where the color factor C is 1 for leptons and 3 for quarks and the vector and axial vector weak coupling parameters are listed in Table 1. For the purposes of our study we chose $M_Z = 92.2$ GeV and $\sin^2 \theta_W = 0.226$. It is perhaps now appropriate to assume that the top quark is above threshold at the SLC,³ in which case the total width of the Z can be conveniently expressed as:

$$\Gamma_Z = \{2.58 + (N_\nu - 3) \times 0.171\} \text{ GeV} \quad , \quad (2)$$

where all fermion masses have been set equal to zero, a QCD correction has been included (see Table 1) and N_ν is the number of neutrino species. From this

expression it follows that in order to determine N_ν to a precision of $0.2 N_\nu$, the measurement error on Γ_Z must be 34 MeV.

The Z^0 width measurement error is dominated by the uncertainty in the beam energy. At the SLC, pulse-by-pulse beam energy monitors will provide beam energy measurements with an absolute error of ± 20 MeV, which when combined with an estimated error of ± 30 MeV due to residual dispersion effects at the interaction point will yield an absolute measurement accuracy of the center of mass energy of ± 40 MeV.⁴ It is possible in principle to achieve a measurement error for Γ_Z comparable to the quadrature sum of the smaller *relative* beam energy errors and the residual dispersion error, for a total systematic error of about ± 30 MeV.⁵

The Mark II collaboration has estimated that the systematic limit of $\delta\Gamma_Z = 30$ MeV will be reached with a data sample of approximately 5×10^4 Z^0 events.⁶ The interpretation of the width measurement in terms of neutrino generations does, however, also require that the radiative corrections to the Z^0 lineshape be well understood. The topic of radiative corrections to the Z^0 resonance has been widely studied; our own results rely on the work of Karaev and Fadin⁷ and on Cahn.⁸ The effect on Γ_Z of initial state radiation from the incident electron and positron, the dominant process contributing to the distortion of the Z^0 lineshape, was found to be an increase of about 55 MeV. This width increase is equivalent to $\approx 1/3$ of a neutrino generation. Such an effect can be corrected for provided that the beam energy spectrum is known. The systematic errors arising from radiative corrections should be relatively small, on the order of 10 MeV.⁶ It is also expected that QCD corrections will be manageable, with associated errors comparable to those due to radiative corrections (perhaps about 1%, or 25 MeV). A reasonable estimate of the total systematic error due to the above mentioned effects is about 40 MeV, corresponding to 0.23 generations (in our notation, $0.23 N_\nu$).

The effect of vacuum polarization corrections to the Z^0 propagator has been included in our analysis, and is absorbed into a renormalization of the electromagnetic coupling constant α . However, if the top quark mass is substantially above $M_Z/2$, relatively large effects on Γ_Z are expected due to the virtual $t\bar{t}$ loops. These loop effects have been calculated^{6,9} and the results predict a width increase of about $0.09 N_\nu$ for a top mass of 100 GeV, rising to $0.26 N_\nu$ for a mass of 200 GeV. For the near future, a measurement of a heavy top mass by one of the hadron collider experiments is likely to be the only unambiguous means for reducing the systematic uncertainty of Γ_Z due to virtual top corrections.

It has been pointed out¹⁰ that a preliminary measurement of N_ν at the level of $\delta N_\nu = 0.5$ should be possible with a small sample of 2000 Z^0 's. The method used

would determine the partial width to invisible final states from a measurement of the total width and the visible partial widths;

$$\Gamma_{invis} = \Gamma_{tot} - \Gamma_{ee} - \Gamma_{\mu\mu} - \Gamma_{\tau\tau} - \Gamma_{hadronic} \quad , \quad (3)$$

where the theoretical values are used for the leptonic widths. The observed muon-pair cross section is used to determine the total width from the equation:

$$\sigma_{\mu\mu} = \frac{12\pi}{M_Z^2} \frac{\Gamma_{\mu\mu}^2}{\Gamma_{tot}^2} \quad . \quad (4)$$

The measurement of Γ_{invis} is particularly sensitive to systematic uncertainties in the muon pair and hadronic event efficiencies ($\epsilon_{\mu\mu}$ and $\epsilon_{hadronic}$) and to the error in the luminosity. However, assuming a 3% luminosity measurement, and systemic uncertainties in $\epsilon_{\mu\mu}$ and $\epsilon_{hadronic}$ of 3% and 1% respectively, Γ_{invis} can be measured to a precision of about 80 MeV with 2000 events. The systematic limit of about 50 MeV will be reached for event samples of between 10^4 and 2×10^4 Z^0 's, corresponding to $\delta N_\nu = 0.3$.

DETERMINATION OF N_ν FROM RADIATIVE NEUTRINO PAIR PRODUCTION

It has been known for some time¹¹ that a direct measurement of Z^0 decays into neutrino pairs is possible from the observation of the process :

$$e^+ + e^- \rightarrow Z^0 \rightarrow \nu + \bar{\nu} + \gamma \quad .$$

It follows that the presence of a single unaccompanied photon in the detector signals e^+e^- annihilation into neutrinos. In the following, we discuss this process and the possible sources of background affecting this measurement. In particular, we derive the statistical and systematic limitations of the measurement of N_ν by several different methods.

The Cross Section

The process with one γ is described by the Feynman diagrams shown in Fig. 1. The major contributor to this process is the annihilation via the Z^0 channel, diagram (a). Only this diagram depends on the number of neutrino species. The other contributors to this process are the W exchange term, diagram (b), and the interference between these two. The double differential cross section has been calculated at tree level¹¹ and is given by

$$\frac{d^2\sigma}{dxdy} = \frac{G_F^2 \alpha}{6\pi^2} \frac{s(1-x)}{x(1-y^2)} [(1-x/2)^2 + x^2 y^2/4] \times \left\{ \frac{N_\nu(v_e^2 + a_e^2) + 2(v_e + a_e)[1 - s(1-x)/M_Z^2]}{[s(1-x)/M_Z^2 - 1^2] + \Gamma_Z^2/M_Z^2 + 2} \right\}, \quad (5)$$

where

$$x = E_\gamma/E_{beam}$$

$$s = 4E_{beam}^2$$

$$y = \cos \theta_\gamma$$

$$v_e = (-1/2) [1 - 4 \sin^2 \theta_W]$$

$$a_e = -1/2$$

Recently, a more detailed treatment¹² of the W exchange diagram has lead to a more accurate expression for Eq. (5). Nevertheless, the differences are no more than 2-3% at $E_{cm} = M_Z$ and even less at 96 GeV; hence, all our results have been determined using the expression above.

A few preliminary observations are:

1. There is a large increase in signal due to an additional neutrino generation; the cross section for four neutrinos is $\approx 27\%$ higher than the cross section for three neutrinos. This difference varies a few percent as one varies the center-of-mass energy.
2. The statistical errors associated with this measurement will be considerable. Assuming a one year run with an effective luminosity of $0.5 \times 10^{30} \text{ cm}^{-2} \text{ sec}^{-1}$ (including a correction factor of 0.78 for radiative corrections as discussed in the following section) the number of observed events for the case of three neutrinos is 1500 single photon events at a center-of-mass energy of 92.2 GeV and 3200 events at a center-of-mass energy of 96 GeV.
3. The energy distribution of the photon shows that most of the photons are very low energy (≤ 2 GeV) unless the center-of-mass energy of the collisions is ≈ 96 GeV.
4. The cross section for radiative neutrino production has a marked energy dependence on the center-of-mass energy between 91 and 95 GeV. Above the

Z^0 mass, at ≈ 96 GeV, the cross section becomes relatively large and less sensitive to variations in the center-of-mass energy.

The dependence of the cross section on E_{cm} , as well as on photon kinematic cuts (in this case, cuts on the photon polar angle and transverse momentum with respect to the beam direction) is illustrated in Fig. 2. The results for both three and four neutrino generations are shown. Figure 3 gives the photon energy spectrum for various running conditions. Evidently, a direct measurement of the cross section for $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ can most easily be made at a center-of-mass energy well above M_Z/c^2 , but for obvious reasons it is essential that one study the limitations of a measurement made while at the resonance peak where all other important measurements will be made.

It is clear that this measurement will make stringent demands on the apparatus; a low noise, finely segmented, high efficiency electromagnetic calorimeter with full angular coverage and good resolution is essential, as is an effective low energy trigger. In a following section we will see how the requirement for background rejection leads to the need for good calorimetry at very small angles with respect to the incident beams. The SLD detector will in principle meet these specifications. Our qualitative study of the effect of the SLD performance on systematic errors will be the subject of a later section.

The Effect of Radiative Corrections

Recently, a series of studies¹³ have pointed out that higher-order photon loop corrections to the e^+e^- annihilation process lead to sizable corrections that reduce the cross section anywhere from 10 to 40%. Nevertheless, it was pointed out¹⁴ that if one includes the corrections due to the emission of a second soft photon these corrections can be reduced. The result of these calculations leads to a correction of 22% at $E_{cm} = M_Z$. A more accurate treatment of these corrections¹² employing kinematical cuts that are more realistic concludes that these corrections reduce the cross section by 17%. It is expected that these corrections become even smaller if one includes the observation of multiphoton ($N>2$) events. Because these corrections are large at these energies, the present first- and second-order calculations may not be accurate enough. Work is now in progress to refine the calculations,¹⁵ and it should be possible to reduce the theoretical error in the radiative neutrino pair cross section to about 5%.

Background Processes

Several processes may produce single photons with characteristics similar to the γ 's from the $\nu\bar{\nu}\gamma$ final state. Such a situation can occur for the γ associated with the final states $e^+e^-\gamma$ (radiative Bhabha), $\mu^+\mu^-\gamma$, $\tau^+\tau^-\gamma$, and $\gamma\gamma\gamma$ when

the charged particles or additional photons may escape undetected down the beam pipe or into inefficient areas of the detector. The contribution to the background from the two-photon processes $e^+e^- \rightarrow e^+e^-X$ is expected to be negligible. The transverse momentum (p_T) of the observed γ must be balanced by the p_T of the unseen particles. If efficient particle detection occurs down to small angles around the beam axis, the p_T of the observed γ is limited by the maximum angle that the charged particles can have without triggering the veto counter which, in our case, is the luminosity monitor. This limit is given by the relation

$$p_T^\gamma \leq (E_{cm} - E_\gamma) \sin \theta_{veto} \quad . \quad (6)$$

The SLD detector has luminosity monitor coverage down to 22 mrad. For this veto angle and $E_{cm} = M_Z = 92.2$ GeV we get $p_T^\gamma \leq 2.02$ GeV/c. Because the photon transverse momentum is almost always balanced by either the electron or positron alone instead of by both simultaneously, most of the photons have $p_T \leq 1$ GeV/c. Also, the radiative Bhabha process has a large “t” channel contribution to the cross section at small angles which is not present in the other processes mentioned above. Hence it is by far the dominant background. In fact, given a θ_{veto} angle of 22 mrad, the $\mu^+\mu^-\gamma$ and $\tau^+\tau^-\gamma$ backgrounds are totally suppressed and are neglected here.

A Monte Carlo program by Mana and Martinez¹⁶ was optimized specifically to calculate the cross section and generate $e^+e^-\gamma$ events with the e^\pm at small angles to the beam axis and the γ satisfying the experimental detection cuts. In Fig. 4 we show the differential cross section as a function of p_T^γ for radiative Bhabhas for different photon cuts and two different veto angles at $E_{cm} = 95$ GeV (the background is rather insensitive to the value of E_{cm}). The dramatic effect of the photon cut and small angle veto is readily apparent. The energy and angular resolution of the SLD liquid Argon calorimeter $\delta E = 9\sqrt{E}$ % and $\delta\theta = 1^\circ$, will smear out the photon transverse momentum in the observed events, and the resulting effect on the size of the background was seen to be small. Figure 5 illustrates the dependence of both the Bhabha background and the $\nu\bar{\nu}\gamma$ signal on the applied photon transverse momentum cut. The signal-to-noise ratio with a cut set at 1 GeV/c is about 7:1 when running on the Z^0 peak, leading to reasonable background subtraction errors as we will see. The $\gamma\gamma\gamma$ background was studied and found to be about a third as large as the radiative Bhabha contribution.

Detector Related Effects

The SLD liquid argon calorimeter² consists of an electromagnetic section of ≈ 21 radiation lengths, with a forward longitudinal section of just over 5 radiation

lengths (hadronic calorimetry follows at larger distance from the beam). The transverse segmentation is fine, with towers of 33 by 33 mrad. Coverage is over the full azimuth, and down to about 8° in polar angle. With this system, the performance should be $\delta E = 9\sqrt{E} \%$ and $\delta\theta = 1^\circ$. Medium and small angle systems with $\delta E = 20\sqrt{E} \%$ complete the coverage down to 22 mrad. Our preliminary studies indicate that these energy and angular resolutions introduce a small ($\approx 2\%$) systematic error while running on the Z^0 peak; this error is reduced even further if data is taken at 96 GeV. We have also found that that our finite resolution for low energy photons leads to a small ($\approx 2\%$) error in our background estimates. In fact, we hope to use $e^+e^-\gamma$ and $\mu^+\mu^-\gamma$ events and kinematic fitting to better understand our response to soft photons. In an effort to estimate our ability to distinguish “noise” photons that originate from points other than the interaction point, we have found that our EM calorimeter allows us to reconstruct the photon incidence angle to $\pm 10^\circ$ for $\theta = 90^\circ$. This “pointing resolution” could be marginally useful in identifying photons scattered from our masks at ± 20 cm from the interaction point.

We have seen that the level of background is quite sensitive to the veto efficiency of our luminosity monitor. Our studies show that an increase of θ_{veto} from 22 to 28 mrad increases the background by a factor of 10, but this increase may be compensated for by raising the cut on p_T^γ to 1.5 GeV/c with only a 30% loss of signal. For significantly larger values of θ_{veto} , statistical errors become unacceptably high.

A few items warrant additional study:

1. We need to examine the effect of low efficiency regions in our calorimeter, such as the “Barrel/Endcap” overlap annulus. Inhomogeneities of this kind should be correctable.
2. Our single photon trigger will take advantage of the considerable online computational power of the SLD calorimeter, coupled with the low SLC beam crossing rate (180 Hz maximum). Local energy depositions in 2×2 tower groups will be searched for, with a threshold set somewhere between 500 and 1000 MeV. It is presently unclear how realistic such a trigger might be, both insofar as efficiency and rate are concerned. Further running experience with the SLC and with the SLD electronics should clarify the situation.

Refinements of the Measurement of $e^+e^- \rightarrow \nu\bar{\nu}\gamma$

In order to understand how an optimal measurement of N_ν might be made with the SLD detector at the SLC, our study group examined two alternative techniques for determining N_ν . It is assumed that these methods would be appropriate when large samples ($> 10^5$ Z^0 's running at the resonance peak) of events become available.

The Use of Polarization

The schedule for SLC improvements calls for the availability of longitudinally polarized electron beams near the time the SLD is installed.¹⁷ It is expected that, initially, the polarization will be $\approx 0.45 \pm 0.02$, with future improvements approaching perhaps 100% with an error of 1%. We investigated the advantages and disadvantages of using a polarized e^- beam,¹⁸ assuming that the luminosity, energy stability and reliability of the SLC will not change in the presence of accelerated polarized electron beams. The utility of polarized beams arises from the fact that the background due to the QED radiative Bhabhas and $\gamma\gamma\gamma$ processes cancels out when one takes the difference in the observed rates due to the possible longitudinal polarization states of the incident electron beam (the helicity dependent γ/Z_0 interference contribution is expected to be smaller than the dominant QED term by $\approx 10^4$). However, the disadvantages of this measurement are the greatly increased statistical errors (a factor of about 9 for $E_{cm} = 96$ GeV), increased sensitivity to the value of $\sin^2\theta_W$ and the additional systematic error due to the polarization uncertainty. Details may be found in Ref. 1; in summary we expect that the use of polarization will not be preferred.

$$\text{Measuring the Branching Ratio } R = \frac{e^+ + e^- \rightarrow \nu + \bar{\nu} + \gamma}{e^+ + e^- \rightarrow \mu^+ + \mu^- + \gamma}$$

If one normalizes the number of observed radiative neutrino pairs by the number of observed radiative muon pairs, several gains are made:

1. Due to the expected similarity of the radiative corrections to the $\mu^+\mu^-\gamma$ and $\nu\bar{\nu}\gamma$ final states, errors due to radiative corrections are made negligible.
2. Errors due to luminosity uncertainties are made negligible.
3. Errors due to photon detection efficiency and resolution are substantially reduced.

These advantages come at the price of increased statistical errors and sensitivity to $\delta\sin^2\theta_W$. As is the case with the direct cross section measurement, the measurement of R is best done at 96 GeV where one is less sensitive to variations in the center of mass energy, to the determination of the photon energy given the calorimeter resolution, and to the value of $\sin^2\theta_W$.

In our study of the $\mu\mu\gamma$ cross section, we found that with our photon cuts the major contribution to the process is due to the radiation from the final state muons. This surprising result is due to two effects: (1) the photon emission from the muons does not change the sharp rise in the cross section due to the Z^0 pole, unlike the effect of photon emission from the incident particles; (2) the cross section has a $\sin^{-2}\theta$ dependence, where θ is the angle between the photon and the emitting particle. Since we can only observe photons with an angle greater than 10° to the

beam direction, this cut reduces by ≈ 2 the observed contribution from initial state photon emission relative to the final state emission where no such cut relative to the muon direction is applied. Hence, the number of events that we can expect from this final state can be quite substantial and it will have an energy dependence that is different from the $\nu\bar{\nu}\gamma$ channel. We found that this undesirable effect can be reduced by demanding that the angle between the photon and either muon be greater than 20° . The decrease in the contribution to the total cross section from the final state muons radiation has decreased markedly, while the initial state radiation contribution is only slightly lower.

The conclusion of our study is that the branching ratio measurement will provide the best determination of N_ν with the SLD detector at the SLC, provided data samples representing the equivalent integrated luminosity of $O(3 \times 10^5)$ Z^0 events on peak are available. The systematic limits for the cases where a measurement of R is done at $E_{cm} = M_Z$ and $E_{cm} = 96$ GeV are $\delta N_\nu = 0.11$ and 0.04 , respectively. Of course, in the later case, the statistical errors are dominant; even with an integrated luminosity equivalent to 5×10^6 Z^0 events on peak the total error would be about $0.07 N_\nu$.

Tables 2-5 summarize our results for various running conditions and experimental cuts, where we have made the following assumptions:

1. The effective average luminosity is $0.5 \times 10^{30} \text{ cm}^{-2} \text{ sec}^{-1}$.
2. The cross section for $e^+ + e^- \rightarrow \mu^+ + \mu^-$ at $E_{cm} = M_Z$ is 1.34 nb. The branching ratio is 3.27%.
3. The beam energy distribution is Gaussian and the center-of-mass energy resolution is 50 MeV. This is a conservative estimate; recent work indicates that resolutions below 30 MeV should be possible.
4. The calorimeter in the SLD detector has a photon energy resolution given by $\delta E = 9\sqrt{E}\%$.
5. The angular resolution of the calorimeter is good enough that it does not contribute to the overall error.
6. The ratio in the cross section for $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ between the case for $4N_\nu$ and $3N_\nu$ is 1.30 at $E_{cm} = M_Z$ and 1.23 at $E_{cm} = 96.0$ GeV.
7. The theoretical uncertainty in the magnitude of the cross section for $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ is 5%. This is an optimistic assumption since, at present, the radiative corrections are large ($\approx 22\%$). Nevertheless, we expect that the third order corrections will be calculated soon and will reduce the uncertainty in the magnitude of this cross section.
8. The theoretical uncertainty in the magnitude of the cross section for $e^+e^- \rightarrow e^+e^-\gamma$ (radiative Bhabha) is 1%.

9. The theoretical uncertainty in the magnitude of the cross section for $e^+e^- \rightarrow \mu^+\mu^-\gamma$ is less than 1%.
10. The uncertainty in the luminosity measurement is 3%.
11. The veto angle for radiative Bhabhas is 22 mrad.

CONCLUSIONS

We have examined the relative merits of several methods for determining the number of neutrino generations with the SLD at the Stanford Linear Collider. We conclude that in the limit of large statistics, the most precise technique will be a measurement taken at a collision energy of 96 GeV of the ratio of the cross sections for the processes $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ to $e^+e^- \rightarrow \mu^+\mu^-\gamma$, for which errors smaller than $0.10 N_\nu$ should be achievable. Alternatively, if $10^6 Z^0$ events are collected at the Z^0 resonance peak, this measurement will provide a precision of about $0.15 N_\nu$.

In Table 6, we give a summary of the relative performance of the experimental methods examined in this paper as a function of the size of the available data sample.

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Table 1. Z_0 Decay rates^a

Final State	v_f	a_f	Γ (GeV)
$e^+e^-, \mu^+\mu^-, \tau^+\tau^-$	$-\frac{1}{2} [1 - 4 \sin^2 \theta_W]$	$-\frac{1}{2}$	0.087
$\bar{\nu}_e\nu_e, \bar{\nu}_\mu\nu_\mu, \bar{\nu}_\tau\nu_\tau$	$\frac{1}{2}$	$\frac{1}{2}$	0.171
$\bar{u}u, \bar{c}c$	$\frac{1}{2} [1 - \frac{8}{3} \sin^2 \theta_W]$	$\frac{1}{2}$	0.297
$\bar{d}d, \bar{s}s, \bar{b}b$	$-\frac{1}{2} [1 - \frac{4}{3} \sin^2 \theta_W]$	$-\frac{1}{2}$	0.382
$\sum_i \bar{q}_i q_i g$			<u>0.072</u>
		Total	2.58

^aUsing the following parameters:

$$\sin^2 \theta_W = 0.226$$

$$M_Z = \left\{ \frac{\pi \alpha_R}{\sqrt{2} G_F} \right\}^{1/2} \times \frac{1}{\sin \theta_W \cos \theta_W} = 92.2 \text{ GeV}$$

$$\alpha_R^{-1} = 137.036 - 171/6\pi$$

$$G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2}$$

$$Z^0 \rightarrow \bar{q}_i q_i g = 0.04 Z^0 \rightarrow \bar{q}_i q_i \quad (\text{Ref. 19})$$

Table 2. Method used is comparison of the observed number of $\nu\nu\gamma$ events with the expected. Run conditions: $E_{cm} = M_Z$, $p_T^\gamma \geq 1$ GeV, $\theta_{\gamma beam} \geq 10^\circ$. For one additional neutrino: $\delta\sigma/\sigma = 30\%$. All errors are stated in units of N_ν .

Running Time (days)	0.6	6	30	60	600
Z^0 events	1×10^3	1×10^4	5×10^4	1×10^5	1×10^6
$\nu\nu\gamma$ events	1	9	44	88	879
Radiative Bhabhas events	0.1	1	7	13	134
$\delta\sigma$ (due to number of events)	3.5	1.0	0.50	0.35	0.12
$\delta\sigma$ (due to beam energy resolution)	0.13	0.13	0.13	0.13	0.13
$\delta\sigma$ (due to photon energy resolution)	0.02	0.02	0.02	0.02	0.02
$\delta\sin^2\theta_W$	0.001	0.0003	0.0003	0.0002	0.0001
$\delta\sigma$ (due to $\delta\sin^2\theta_W$)	0.34	0.10	0.10	0.07	0.03
$\delta\sigma$ (due to radiative correction uncertainty)	0.17	0.17	0.17	0.17	0.17
$\delta\sigma$ (due to luminosity uncertainty)	0.10	0.10	0.10	0.10	0.10
$\delta\sigma$ Total	3.5	1.0	0.57	0.42	0.27

Table 3. Method used is comparison of the observed number of $\nu\nu\gamma$ events with the expected run conditions: $E_{cm} = 96.0$ GeV, $p_T^\gamma \geq 1$ GeV, $\theta_{\gamma beam} \geq 10^\circ$. For one additional neutrino: $\delta\sigma/\sigma = 23\%$. All errors are stated in units of N_ν .

Running Time (days)	0.6	6	30	60	600
Z^0 events	186	1860	9290	18600	1.86×10^5
$\nu\nu\gamma$ events	3	30	150	300	3000
Radiative Bhabhas events	0.1	1	7	13	130
$\delta\sigma$ (due to number of events)	2.6	0.81	0.36	0.25	0.080
$\delta\sigma$ (due to beam energy resolution)	0.01	0.01	0.01	0.01	0.01
$\delta\sigma$ (due to photon energy resolution)	0.01	0.01	0.01	0.01	0.01
$\delta\sin^2\theta_W$	0.001	0.0003	0.0003	0.0002	0.0001
$\delta\sigma$ (due to $\delta\sin^2\theta_W$)	0.03	0.009	0.009	0.006	0.003
$\delta\sigma$ (due to radiative correction uncertainty)	0.22	0.22	0.22	0.22	0.22
$\delta\sigma$ (due to luminosity uncertainty)	0.13	0.13	0.13	0.13	0.13
$\delta\sigma$ Total	2.6	0.85	0.45	0.36	0.27

Table 4. Method used is Comparison of the Observed number of $\nu\nu\gamma$ with $\mu\mu\gamma$ events. Run conditions: $E_{cm} = M_Z$, $p_T^\gamma \geq 1$ GeV, $\theta_{\gamma beam} \geq 10^\circ$. $R = N(\nu\bar{\nu}\gamma)/N(\mu^+\mu^-\gamma)$. For one additional neutrino: $\delta R/R = 48\%$. All errors are stated in units of N_ν .

Running Time (days)	0.6	6	30	60	600
Z^0 events	1×10^3	1×10^4	5×10^4	1×10^5	1×10^6
$\nu\nu\gamma$ events	1	9	44	88	879
Radiative Bhabhas events	0.1	1	7	13	134
$\mu\mu\gamma$ events	4	37	185	370	3700
$\delta\bar{R}$	2.4	0.82	0.37	0.26	0.08
(due to number of events)					
δR	0.06	0.06	0.06	0.06	0.06
(due to beam energy resolution)					
δR	0.02	0.02	0.02	0.02	0.02
(due to photon energy resolution)					
$\delta\sin^2\theta_W$	0.001	0.0003	0.0003	0.0002	0.0001
δR (due to $\delta\sin^2\theta_W$)	0.94	0.28	0.28	0.19	0.09
δR	0.0	0.0	0.0	0.0	0.0
(due to radiative correction uncertainty)					
δR	0.0	0.0	0.0	0.0	0.0
(due to luminosity uncertainty)					
δR Total	2.6	0.87	0.47	0.33	0.14

Table 5. Method used is comparison of the observed number of $\nu\nu\gamma$ with $\mu\mu\gamma$ events. Run conditions: $E_{cm} = 96.0$ GeV, $p_T^\gamma \geq 1$ GeV, $\theta_{\gamma beam} \geq 10^\circ$. $R = N(\nu\bar{\nu}\gamma)/N(\mu^+\mu^-\gamma)$. For one additional neutrino: $\delta R/R = 28\%$. All errors are stated in units of N_ν .

Running Time (days)	0.6	6	30	60	600
Z^0 events	186	1860	9290	18600	1.86×10^5
$\nu\nu\gamma$ events	3	30	150	300	3000
Radiative Bhabhas events	0.1	1	7	13	130
$\mu\mu\gamma$ events	1	9	45	91	914
δR (due to number of events)	4.6	1.4	0.60	0.42	0.13
δR (due to beam energy resolution)	0.03	0.03	0.03	0.03	0.03
δR (due to photon energy resolution)	0.01	0.01	0.01	0.01	0.01
$\delta \sin^2\theta_W$	0.001	0.001	0.0003	0.0003	0.0001
δR (due to $\delta \sin^2\theta_W$)	0.14	0.14	0.04	0.04	0.01
δR (due to radiative correction uncertainty)	0.0	0.0	0.0	0.0	0.0
δR (due to luminosity uncertainty)	0.0	0.0	0.0	0.0	0.0
δR Total	4.6	1.4	0.60	0.42	0.13

Table 6. Comparison of methods for the measurement of N_ν .

Measured Quantity	Total Systematic Error (in units of neutrino generations)	Systematic Limit Reached (in equivalent peak Z^0 's)
Γ_{invis}	$0.3 N_\nu$	$1 - 2 \times 10^4$
Γ_Z	$0.2 N_\nu$	5×10^4
$\sigma(\nu\bar{\nu}\gamma)$	$0.25 N_\nu$	2×10^5
$R = \frac{\sigma(\nu\bar{\nu}\gamma)}{\sigma(\mu^+\mu^-\gamma)}$	$0.1 N_\nu$	2×10^6
-At 96 GeV:		
$R = \frac{\sigma(\nu\bar{\nu}\gamma)}{\sigma(\mu^+\mu^-\gamma)}$	$0.05 N_\nu$	8×10^6

FIGURE CAPTIONS

1. The Feynman diagrams that describe the process $e^+e^- \rightarrow Z^0 \rightarrow \nu\bar{\nu}\gamma$.
2. The cross section for the process $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ for the case of three and four neutrino generations and for cuts in the photon transverse momentum and angle.
3. The photon energy distribution in the process $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ for various conditions.
4. The differential cross section of the radiative Bhabha process as a function of the photon transverse momentum for various photon cuts at a center-of-mass energy of 95 GeV.
5. A comparison of the cross section dependence on a minimum photon transverse momentum cut for radiative Bhabhas at a center-of-mass energy of 95 GeV and for the $\nu\bar{\nu}\gamma$ process at 92 and 96 GeV. Since the radiative Bhabha process only varies slowly with energy, the characteristics at 95 GeV apply equally to all energies in the vicinity.

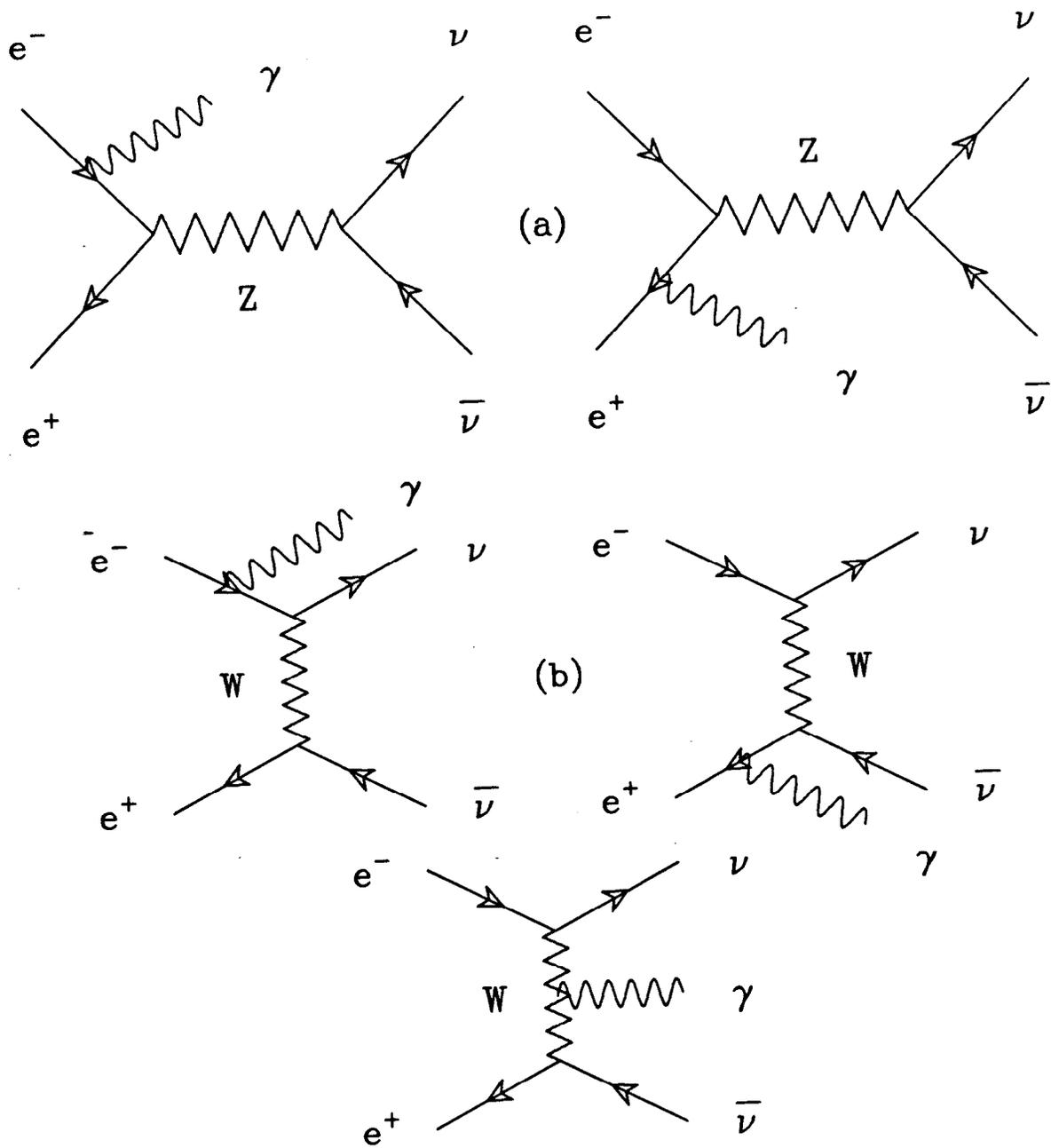


Fig. 1

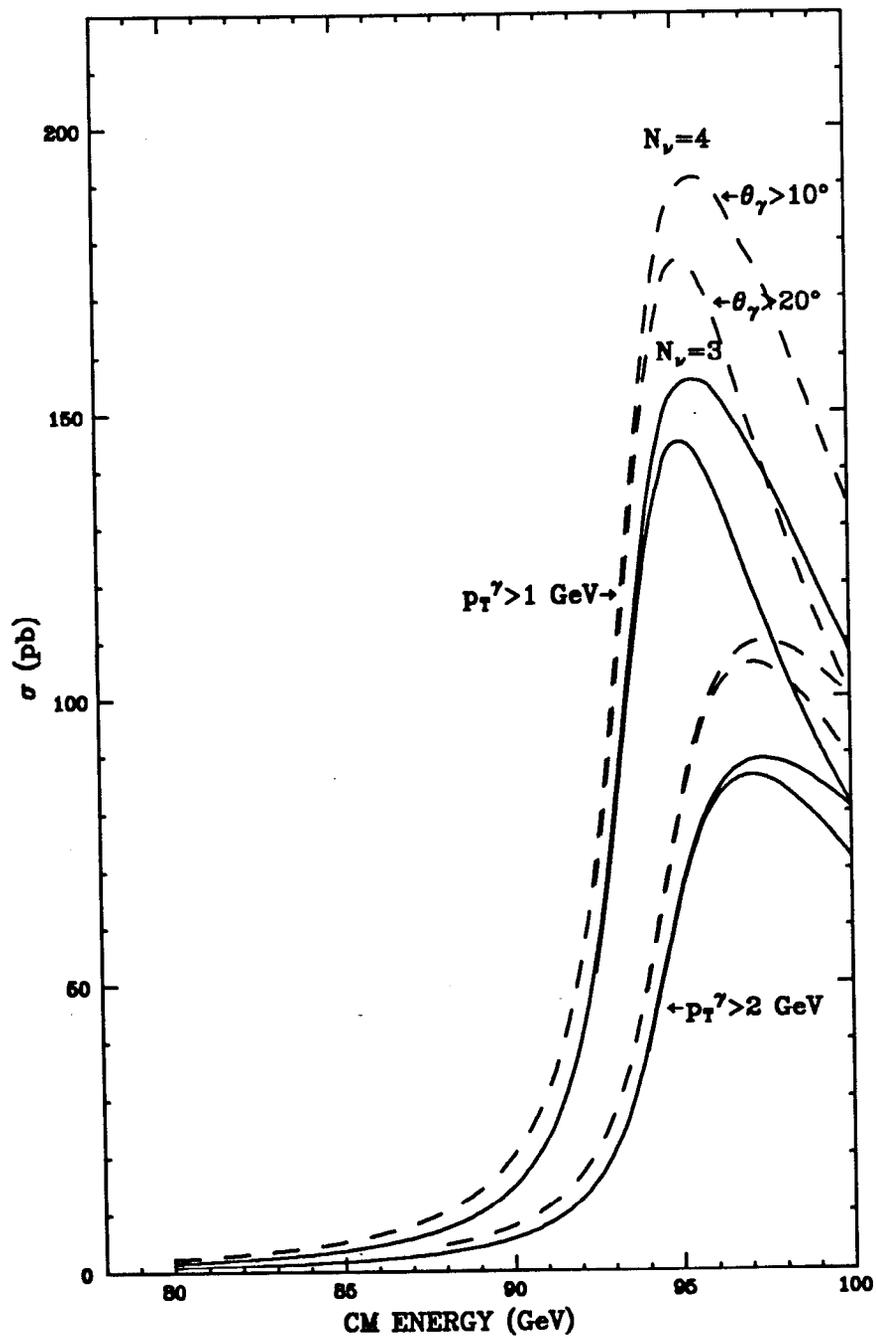


Fig. 2

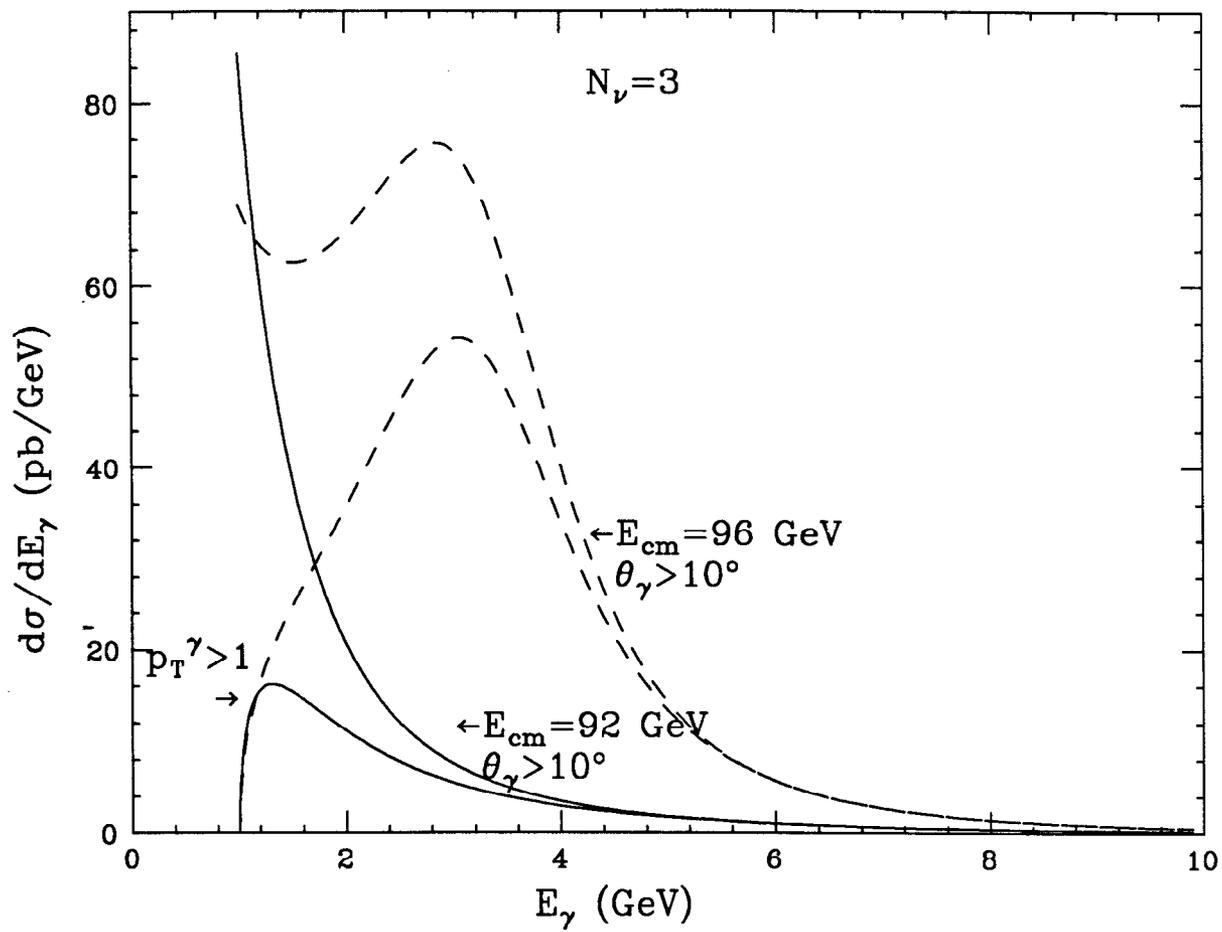


Fig. 3

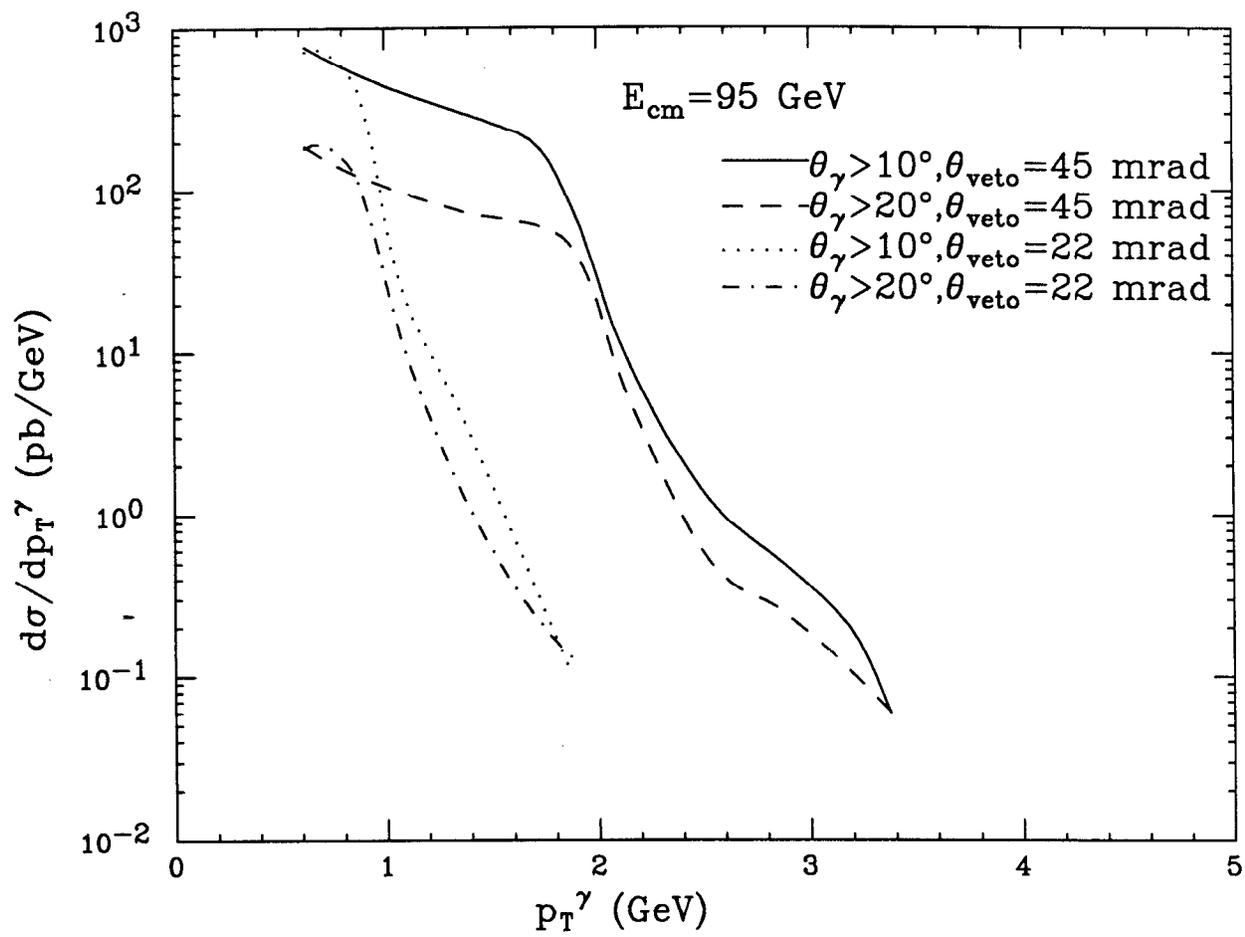


Fig. 4

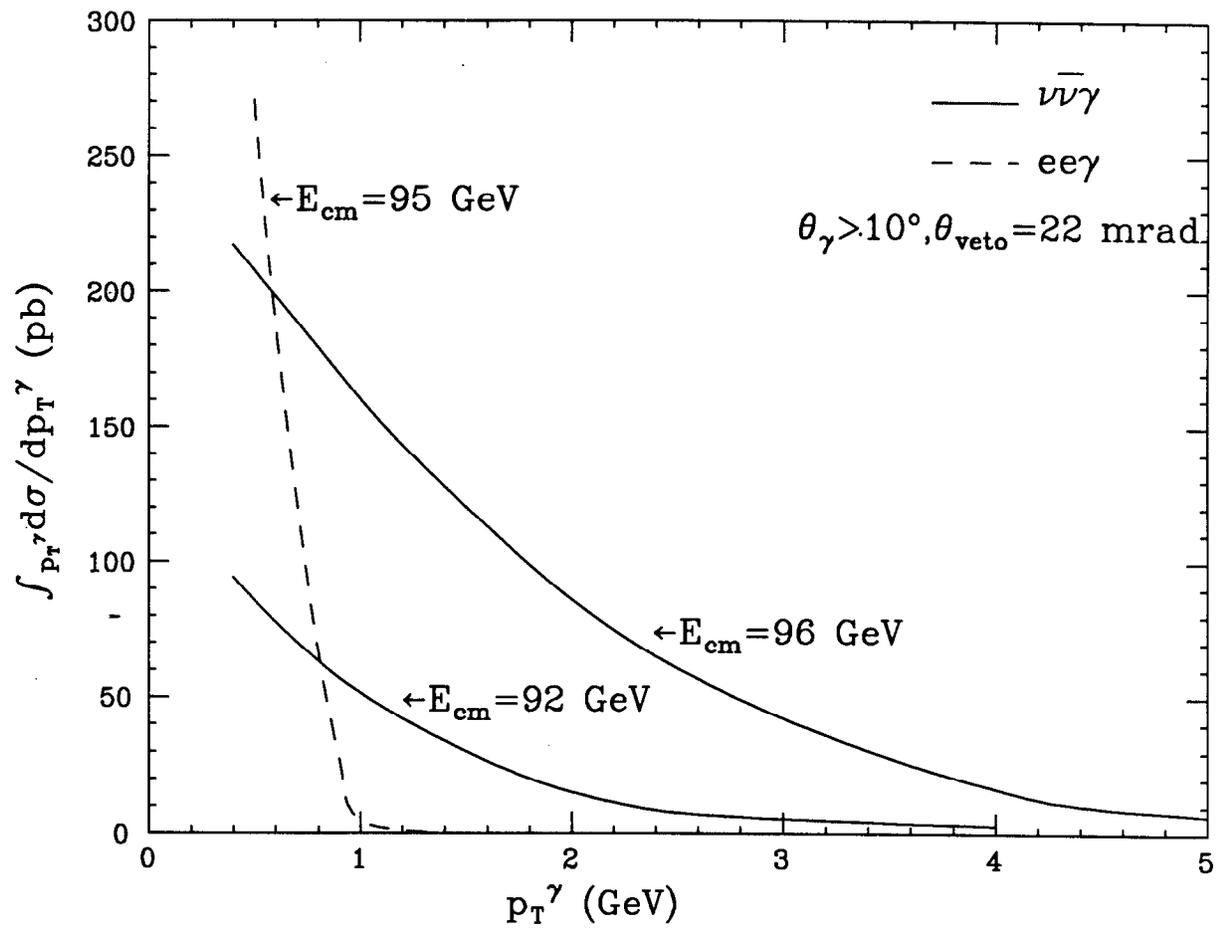


Fig. 5