COHERENT PAIR CREATION IN LINEAR COLLIDERS^{*}

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ABSTRACT

Coherent pair creation during the collision of e^+e^- beams in linear colliders is examined. This includes contributions from both real and virtual photons, where the real photons are predominantly from beamstrahlung. The pair creation probability is shown to be orders of magnitude larger than the corresponding incoherent processes. The energy spectrum is also calculated where the effective threshold energy is shown to be inversely proportional to the beamstrahlung parameter Υ . Implications of this effect on future e^+e^- , $e\gamma$, and $\gamma\gamma$ linear colliders are discussed.

Submitted to Physical Review Letters.

^{*} Work supported by Department of Energy contract DE-AC03-76SF00515.

It is generally recognized that to avoid severe synchrotron radiation loss in storage rings, future high-energy e^+e^- colliders are necessarily linear.¹ To compensate for the much lower collision rates in linear colliders, one is forced to collide much tighter beams. This, however, generates its own kind of radiation loss problem. Each particle during collision would be bent by the strong collective macroscopic EM field provided by the oncoming beam, and radiate. This phenomenon, called beamstrahlung, has been a subject under intensive study in recent years.²

Recently, it was recognized³ that the generally high-energy beamstrahlung photons, which have to travel through the same collective field in the remainder of the oncoming beam, have a high probability of turning into e^+e^- pairs. Being lower in energies, these e^+e^- pairs will be deflected more severely than the high-energy primary particles, and will potentially cause background problems for high-energy physics experiments.

The problem of pair creation in a magnetic field is not new. Klepikov⁴ first calculated this problem in a uniform magnetic field. Several others⁵ have re-examined the problem with different formalisms. In the lowest-order approximation of perturbation theory, the matrix element for the pair creation process is essentially the same as that for beamstrahlung, except that now the initial electron momentum k_{μ} has to be replaced by $-k_{\mu}$ in the cross channel. The probability of pair creation per unit time can then be obtained to be⁵ ($\hbar = c = 1$)

$$\frac{dn}{dt} = \frac{8\alpha}{3\pi^2} \frac{m}{\omega\chi} \int_0^\infty du \int_0^\infty dv \cosh^3 u
\times \left\{ K_{1/3}^2(\eta) + \cosh^2 u \left(2\cosh^2 v - 1 \right) \left[K_{1/3}^2(\eta) + K_{2/3}^2(\eta) \right] \right\} ,$$
(1)

where

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$$\chi \equiv \frac{\omega}{m} \frac{B}{B_c}$$
, $\eta \equiv \frac{4}{3\chi} \cosh^2 v \cosh^3 u$;
 $\cosh^2 u = 1 + \gamma^2 \theta^2$, $\cosh^2 v = \frac{1}{4} \frac{\omega^2}{\varepsilon(\omega - \varepsilon)}$;

and α is the fine structure constant; *B*, the local magnetic field strength; $B_c \equiv m^2 c^3/e\hbar \sim 4.4 \times 10^{13}$ Gauss, the Schwinger critical field; ω and ε , the energies of the photon and the pair-created particle (either e^+ or e^-), respectively; and θ , the angle between the photon and the $(\vec{v}, \dot{\vec{v}})$ plane of the secondary particle. The integral can be carried out analytically in the asymptotic limits:

$$\frac{dn}{dt} = \begin{cases} 0.23 \frac{\alpha m^2}{\omega} \chi \exp(-8/3\chi) &, \quad \chi \ll 1 \\ 0.38 \frac{\alpha m^2}{\omega} \chi^{2/3} &, \quad \chi \gg 1 \end{cases}$$
(2)

For the entire range of χ , dn/dt can be well approximated by the following expression:⁶

$$\frac{dn}{dt} \simeq \frac{4}{25} \frac{\alpha m^2}{\omega} K_{1/3}^2(4/3\chi) \quad . \tag{3}$$

We see that $\chi \sim 1$ corresponds to the threshold condition for finite probability, below which the pair-creation rate is exponentially suppressed. This condition can be appreciated by the following intuitive arguments: Consider the boosted frame where the e^+e^- pair is created at rest. In this frame, there is an electric field which is $E' = (\omega/2m)B$. At the threshold, the created particle with unit charge e should acquire enough energy within one Compton wavelength to supply for its rest mass. Thus, the threshold condition is $eE'\lambda_c \sim m$, or $\chi \sim 1$.

In accelerators, the particles in a high-energy beam are normally in Gaussian distributions, with standard deviations σ_x , σ_y , and σ_z . For the sake of simplicity in discussions, one may consider the beam particles as distributing uniformly within an elliptical cylinder with dimensions $2\sigma_x$, $2\sigma_y$, and $2\sqrt{3}\sigma_z$, instead. The local field strength certainly varies in the x-y plane. But it has been shown⁷ that an effective field strength can be assigned to the entire beam, upon which the various beam-strahlung phenomena can be faithfully described. In association with the canstant effective field, the beamstrahlung parameter for a particle with energy \mathcal{E} is defined as⁷

$$\Upsilon = \frac{5}{6} \frac{\gamma r_e^2 N}{\alpha \sigma_z \sigma_y (1+R)} \quad , \tag{4}$$

where $\gamma = \mathcal{E}/m$, $R \equiv \sigma_x/\sigma_y$ is the beam *aspect ratio*, and N is the total number of particles in the bunch. The coefficient 5/6 is empirical.

It is useful to express the pair creation probability in terms of the primary particles, instead of the intermediate photons. For pair creation through the real beamstrahlung photons, the number of pairs per primary particle after collision is

$$n_b = \frac{4\sqrt{3}}{25\pi} \left(\frac{\alpha\sigma_z}{\gamma\lambda_c}\Upsilon\right)^2 \Xi(\Upsilon) \quad , \tag{5}$$

where

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$$\Xi(\Upsilon) \equiv \frac{1}{2\Upsilon^2} \int_0^1 \left[\int_q^\infty K_{5/3}(p) \, dp + \frac{y^2}{1-y} \, K_{2/3}(q) \right] K_{1/3}^2(4/3y\Upsilon) \frac{dy}{y} \\ \simeq \begin{cases} 0.5 \, \exp(-16/3\Upsilon) &, & \Upsilon \ll 1 \\ 2.6 \, \Upsilon^{-2/3} \ell n\Upsilon &, & \Upsilon \gg 1 \end{cases} ,$$
(6)

where $y \equiv \omega/\mathcal{E}$. Here, the synchrotron radiation spectrum *a la* Sokolov-Ternov is used with the beamstrahlung parameter defined in Eq. (4) and $q \equiv (2/3\Upsilon)y/(1-y)$.

The trident cascade through virtual photons, on the other hand, has been studied in the past.^{8,9} For the sake of comparison, we express it as

$$n_v = \frac{4\sqrt{3}}{25\pi} \left(\frac{\alpha \sigma_z}{\gamma \lambda_c} \Upsilon\right) \Omega(\Upsilon) \quad , \qquad (7)$$

where, according to Ritus,⁹

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$$\Omega(\Upsilon) \simeq 2.6 \,\alpha \ell n \Upsilon \quad , \qquad \Upsilon \gg 1 \quad . \tag{8}$$

The auxiliary functions $\Xi(\Upsilon)$ and $\Omega(\Upsilon)$ are introduced following the spirit of Erber.⁶ The $\Upsilon \ll 1$ limit for the trident cascade was omitted in Eq. (8), due to the subtlety of its being a negative probability as derived by Ritus. However, Ritus argues that this causes only a minor suppression to the real process, while the total probability of $n_b + n_v$ is still positive definite, and satisfies the unitary condition. The clarification of this issue is beyond the scope of this paper, and will be pursued in a separate effort.

A numerical plot of the two functions is given in Fig. 1. Notice that the crossover between the two functions occurs at $\Upsilon \sim 10^3$. In addition to the different scalings between Ξ and Ω , the beamstrahlung pair creation increases quadratically with the quantity $(\alpha \sigma_z / \gamma \lambda_c) \Upsilon$, while the trident cascade scales linearly. This is simply because the former necessarily involves a real intermediate process and thus a double integration in time.

It turns out that in e^+e^- linear colliders, the quantity $(\alpha\sigma_z/\gamma\lambda_c)\Upsilon$ cannot be arbitrary. It has been shown^{7,10} that the average energy loss δ of the primary particles due to beamstrahlung behaves differently in the *classical* ($\Upsilon \leq 0.1$), the *transition* $(0.1 \leq \Upsilon \leq 100)$, and the *quantum* ($\Upsilon \gtrsim 100$) regimes:

$$\delta \approx \begin{cases} \frac{3}{10\sqrt{\pi}} \frac{\alpha \sigma_z}{\gamma \lambda_c} \Upsilon , & 0.1 \leq \Upsilon \leq 100 ; \\ \frac{6}{5\sqrt{\pi}} \frac{\alpha \sigma_z}{\gamma \lambda_c} \Upsilon^{2/3} , & \Upsilon \gtrsim 100 . \end{cases}$$
(9)

In designing linear colliders, one usually chooses a reasonable value of δ as a constraint to the choices of other beam parameters, such that the energy resolution of the colliding beam is adequate for meaningful high-energy experiments. Combining Eqs. (6), (8), and (9), we find that:

$$\frac{n_b}{n_v} \sim \frac{5\sqrt{\pi}}{6\alpha} \delta \Upsilon^{-1/3} \quad , \qquad \Upsilon \gtrsim 100 \quad . \tag{10}$$

Take, for example, $\delta \sim 0.2$, then the quantity $(\alpha \sigma_z / \gamma \lambda_c) \Upsilon$ is of order unity, and $n_b \sim \mathcal{O}(10^{-2})$ for the most part of the transition regime ($\Upsilon \gtrsim 1$). The typical

number of particles in each bunch is $\mathcal{O}(10^{10})$, so one expects to find ~ $\mathcal{O}(10^8) e^+e^$ pairs per collision. In the extreme quantum regime, if we again fix $\delta \sim 0.2$, then $n_b/n_v \sim 40\Upsilon^{-1/3}$ and ~ $\mathcal{O}(1)$ at $\Upsilon \sim 10^4$.

For our purpose, it is important to study the energy spectrum of the pair-created secondary particles. From Eq. (1), we find that:¹¹

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$$\frac{d^2n}{dxdt} = \frac{1}{2\sqrt{3}\pi} \frac{\alpha m}{\gamma} \frac{1}{x(y-x)} \left\{ (1 + \tanh^2 v) K_{2/3}(a) + \frac{1}{2\cosh^2 v} \int_a^\infty dz K_{1/3}(z) \right\} ,$$

$$= \frac{1}{4\sqrt{3}\pi} \frac{\alpha m}{\gamma} \begin{cases} \frac{\sqrt{\pi}}{2} (3\Upsilon)^{1/2} \frac{3 + (1 - 2x/y)^2}{[xy(y-x)]^{1/2}} \exp\left\{-\frac{2}{3\Upsilon} \frac{y}{x(y-x)}\right\}, & y\Upsilon \ll 1 \\ \Gamma\left(\frac{2}{3}\right) (3\Upsilon)^{2/3} \frac{1 + (1 - 2x/y)^2}{[xy^2(y-x)]^{1/3}} , & y\Upsilon \gg 1 \end{cases},$$
(11)

where $x \equiv \varepsilon/\mathcal{E}$ and $a \equiv 8 \cosh^2 v/(3y\Upsilon)$. For $\Upsilon \ll 1$, the pair tends to equally share the photon energy, while for $\Upsilon \gg 1$, the spectrum becomes much broader, with two peaks located at x and $(1-x) \sim 1.6/\Upsilon$, respectively. A numerical plot of the spectral function is shown in Fig. 2, where the pair-created particle energy is normalized as $x\Upsilon$. For a given value of Υ , the threshold energy x_{th} is independent of the intermediate photon energy y. In addition, for different values of Υ 's, x_{th} scales as $1/\Upsilon$. This can be explained by the following qualitative arguments:

In the Lorentz frame, where the pair is created at rest, the invariant mass of the system is $W = 2eE'\lambda_c$. The Lorentz factor for the boost is obviously the photon energy ω devided by the invariant mass. Thus, we have $W^2 = 2eB\omega\lambda_c$. On the other hand, from the final state we have $W^2 = \omega^2 m^2/\varepsilon_+\varepsilon_-$, where $\varepsilon_+, \varepsilon_-$ are the energies of the pair particles. In the case where one particle is at very low energy, e.g., $\varepsilon_+ \ll \varepsilon_- \sim \omega$, we have $W^2 \sim \omega m^2/\varepsilon_+$. Thus, $\varepsilon_{th} \sim \gamma m/2\Upsilon$, or $x_{th} \sim 1/2\Upsilon$, and is independent of the photon energy. Indeed, we see from Fig. 2 that the fall-off of the spectrum starts universally around $1/2\Upsilon$. The "half-maximum" of the peak value, however, tends to locate at a smaller value of x. For practical purposes, it should be reasonable to assume the minimum energy to be $x_{min} \sim 1/5\Upsilon$.

If the e^+e^- pair is created in a field-free space, their outgoing angles will be simply the final angles from the creation process. In the case of beam-beam interaction, however, these low-energy secondary particles will be strongly deflected by the same macroscopic collective field. The nature of the deflection differs between the pair. Consider, for example, a pair created by a primary electron moving against the positron beam. In this case, the secondary electron sees a focusing field, while the positron sees a defocusing field. Since both particles are generally low in energy, the electron tends to be confined by the potential and oscillate on its way out, whereas the positron would be deflected without bound. In general, for *flat* beams, *i.e.*, $R \gg 1$, the most effective deflection occurs for positrons in the vertical direction. This is because for flat beams the vertical field extends beyond the beam height $2\sigma_y$ to a distant $\sim 2\sigma_x \gg 2\sigma_y$, with a fairly uniform strength $E + B \sim 2eN/\sqrt{3}\sigma_x\sigma_z$, whereas in the horizontal direction, the field strength inceases only linearly to the same value at $2\sigma_x$.

Define the diagonal angle of the field to be $\theta_d \equiv 2\sigma_x/\sqrt{3}\sigma_z$; then the vertical deflection angle can be shown to be approximately:

$$\theta_{y} = \begin{cases} \frac{\sqrt{3}Nr_{e}\sigma_{z}}{\gamma x \sigma_{x}^{2}} \theta_{d} , & \theta_{y} \leq 2\theta_{d} ; \\ \\ \left[\frac{2\sqrt{3}Nr_{e}\sigma_{z}}{\gamma x \sigma_{x}^{2}}\right]^{1/2} \theta_{d} , & \theta_{y} \geq 2\theta_{d} . \end{cases}$$
(12)

If an e^+e^- collider is designed such that the beams are colliding head-on, then the above consideration imposes a severe constraint on its design. The typical distance between the final focusing magnet and the interaction point (IP) is ~ $\mathcal{O}(10^2)$ cm, while the aperture of the final focusing magnet is $\leq \mathcal{O}(10^{-1})$ cm. This means that any particle with outcoming angle larger than ~ $\mathcal{O}(1)$ mrad will necessarily hit the magnet and generate backgrounds.

One obvious way to alleviate the problem is to sufficiently suppress these coherent processes. In principle, this can be achieved by imposing a constraint on the value of Υ through the requirement $n_b N \simeq 1$. From Fig. 1, the condition is realizable only if $\Upsilon \leq 0.3$, where the pair creation rates are exponentially suppressed to $n_b \sim \mathcal{O}(10^{-10})$. To achieve this goal, one needs to reduce the charge density of the beams by either lengthening the bunch, or reducing the number of particles per bunch with a corresponding increase in the number of bunches, or both, in order not to sacrifice the luminosity. In practice, however, these leverages can be handeled only within certain limits due to various other accelerator constraints. Furthermore, there are still other sources of pair creations even if the coherent ones can be entirely suppressed.

In addition to the coherent processes, both beamstrahlung photons and virtual photons can also turn into e^+e^- pairs through individual scattering, or incoherent, processes. These are the well-known Bethe-Heitler and Landau-Lifshitz processes. In the context of linear collider backgrounds, these processes have been studied in sufficient detail.^{12,13} The cross sections from standard calculations are¹⁴

$$\sigma_{_{BH}} \;=\; (28/9) \; \alpha \; r_e^2 \; \ell n (4 y \gamma^2) \quad ,$$

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$$\sigma_{LL} = (1/\pi) \alpha^2 r_e^2 \left[(28/27) \ln^3(4\gamma^2) - 6.59 \ln^2(4\gamma^2) - 11.8 \ln(4\gamma^2) + 104 \right] .$$
(13)

At $\pm + 1$ TeV, they are ~ 5 and 3×10^{-26} cm², respectively. For high-energy physics purposes, a collider at this energy range would have luminosity per collision $\mathcal{L} \sim 10^{31}$ cm⁻². Thus, the number of incoherent pairs is ~ $\mathcal{O}(10^6)$ per collision, which is about two orders of magnitude smaller than the coherent ones if $\Upsilon \gtrsim 1$, and is equivalent to the coherent yields at $\Upsilon \sim 0.4$.

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It has been pointed out,¹⁵ however, that in the context of e^+e^- beam-beam collisions, these cross sections will be reduced by a factor of 2 or 3 when the transverse beam size is smaller than the maximum impact parameter. In addition, when $\Upsilon \gg 1$, the incoherent processes will be further suppressed by the very strong external field.¹⁶ Therefore, one expects the incoherent cross section in future e^+e^- colliders to be substantially smaller than the standard calculations.

In light of the potential dificulties with the pair creation backgrounds when Υ is unavoidably large for future e^+e^- colliders beyond the 1-TeV range, one may concieve a variant approach to linear colliders. It has been proposed¹³ that the high-energy experiments be performed through either $\gamma\gamma$ or $e\gamma$ collisions. As a hindsight, this idea may have the advantage over the e^+e^- scheme, as far as reducing the beam-beam macroscopic field is concerned. As it turns out, however, the $\gamma\gamma$ and $e\gamma$ schemes are not free from the influence of coherent pair creation for technical reasons.

In the $\gamma\gamma$ collision scheme of Ref. 13, each of the two primary electron beams first collides with a low-energy photon beam and converts into a high-energy photon beam through Compton back scatterings. The scattered electron beams are then swept aside by bending magnets. Although there is no direct collective field in $\gamma\gamma$ collision, photons may still be influenced by the residual field of the swept beams and create e^+e^- pairs. In addition to possible background pollutions, in this scheme, pair creation also reduces the $\gamma\gamma$ luminosity. It is therefore necessary that the electron beams be bent sufficiently far from the IP of the photons.

Let the conversion efficiency be κ ; then the number of electrons that have not been Compton scattered is $N' = N(1 - \kappa)$. Let the location of the conversion be a distance *d* upstream from the IP, and the horizontal separation between the electron beam and the photon beam at the IP be x_0 . It can be verified that the effective χ which the photon beam experiences at the IP due to the residual field of the electron beam is

$$\chi = \frac{2\gamma r_e^2 (1-\kappa)N}{\sqrt{3} \alpha \sigma_z x_0} \quad , \tag{14}$$

where the Compton-scattered electrons are not contributing, since they are much softer and assumed to be bent farther away.

The luminosity in this case is¹³ $\mathcal{L}_{\gamma\gamma} = f\kappa^2 N^2/4\pi a_{\gamma}^2$, where a_{γ} is the spot size of the photon beam at the interaction point and f the collision rate. The spot size is dominated essentially by the stochastic nature of Compton scattering, which has a typical open angle $\sim 1/\gamma$, and thus $a_{\gamma} \sim d/\gamma$. But d and x_0 are related by $x_0/d^2 =$

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 $eB_{e}/2\gamma m$, where B_{e} is the external bending field with length d. Therefore, $\mathcal{L}_{\gamma\gamma} = (f\kappa^{2}N^{2}/8\pi\lambda_{c}x_{0})(\gamma B_{e}/B_{c})$. However, x_{0} is not a free parameter in our consideration.

For our concerns, we would like to limit the number of e^+e^- pairs per photon during collision. This determines χ through Eq. (3), which, in turn, determines x_0 . For multi-TeV colliders, we take the $\chi \gg 1$ limit of Eq. (3), and eventually get

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$$\mathcal{L}_{\gamma\gamma} \simeq \frac{fN}{4\pi r_e^2} \frac{B_e}{B_c} \left(\frac{\kappa^2}{1-\kappa}\right) \left(\frac{n^3 \gamma^3 \lambda_c}{\sqrt{3} \alpha \sigma_z}\right)^{1/2} \quad . \tag{15}$$

Consider, for example, a 5 + 5 TeV $\gamma\gamma$ collider, with f = 1000, $N = 10^{10}$, $\sigma_z = 0.1 \ \mu\text{m}$, $\kappa = 0.5$, and $B_e = 3$ Tesla. If we further choose the pair creation probability to be n = 0.1, then we find $\chi \sim 4.5 \times 10^4$, and the attainable luminosity is $\sim 6 \times 10^{34} \text{ cm}^{-2} \text{ sec}^{-1}$.

For $e\gamma$ collisions, the primary contribution to coherent pair creation comes from the direct interaction of the high-energy photon beam with the collective field in the electron beam, where $\chi \sim \gamma r_e^2 N/\alpha \sigma_z \sigma_x$ as in Eq. (4). The luminosity is now $\mathcal{L}_{e\gamma} = f\kappa N^2/4\pi\sigma_x a_\gamma$. Through similar arguments for $\gamma\gamma$ collisions, with a minimal possible horizontal seperation $x_0 \gtrsim \sigma_x$, we find

$$\mathcal{L}_{e\gamma} \lesssim \frac{f\kappa}{\sqrt{2} \pi \alpha r_e^2} \left(\frac{N\gamma B_e}{B_c}\right)^{1/2} \left(\frac{n^3 \gamma \lambda_c}{\sqrt{3} \alpha \sigma_z}\right)^{3/4} \quad . \tag{16}$$

With the same parameters as above, we find $\mathcal{L}_{e\gamma} \leq 8 \times 10^{33} \text{ cm}^{-2} \text{ sec}^{-1}$. If one insists on larger $e-\gamma$ separation, e.g., $x_0 \gtrsim 5\sigma_x$, then the luminosity reduces to $\mathcal{L}_{e\gamma} \leq 3.5 \times 10^{33} \text{ cm}^{-2} \text{ sec}^{-1}$, and is about 20 times smaller than that of the $\gamma\gamma$ case.

ACKNOWLEDGEMENTS

We appreciate helpful discussions with K. Yokoya and K. Oide of KEK, Japan; R. Blankenbecler, S. D. Drell, and B. Richter of SLAC, USA; and V. N. Baier of Novosibirsk State University, USSR.

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FIGURE CAPTIONS

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Fig. 1. Auxiliary functions Ξ and Ω of the real and virtual coherent pair creation probability, respectively, as a function of the beamstrahlung parameter Υ .

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Fig. 2. Normalized spectrum for coherent pair creation, in units of $\alpha m/\gamma$, as a function of $x\Upsilon$. One sees clearly that $x_{th}\Upsilon$ is independent of the photon energy for a given Υ , and is also independent of Υ itself.



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Fig. 1



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