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MULTIBUNCH BEAM BREAKUP IN HIGH ENERGY LINEAR COLLIDERS*

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ABSTRACT

The SLAC design for a next-generation linear collider with center-of-mass energy of 0.5 to 1.0 TeV requires that multiple bunches (~ 10) be accelerated on each RF fill. At the beam intensity ($\sim 10^{10}$ particles per bunch) and RF frequency (11 to 17 GHz) required, the beam would be highly unstable transversely. Using computer simulation and analytic models, we have studied several possible methods of controlling the transverse instability: (1) using damped cavities to damp the transverse dipole modes; (2) adjusting the frequency of the dominant transverse mode relative to the RF frequency, so that bunches are placed near zero crossings of the wake; (3) introducing a cell-to-cell spread in the transverse dipole mode frequencies; and (4) introducing a bunch-to-bunch variation in the transverse focusing. The best cure(s) to use depend on the bunch spacing, intensity, and other features of the final design.

1. INTRODUCTION

In this paper, we address the problem of transverse instability of a train of bunches in a high energy linac. The main motivation for accelerating multiple bunches per RF fill is to obtain higher luminosity for a given expenditure of RF energy. The optimal bunch spacing, bunch charge, and number of bunches depend upon many factors other than just the need to be able to control the beam breakup. There is a serious constraint on charge per bunch, due to pair creation at the interaction point.[1,2] Another strong constraint is imposed by the need to keep the bunch-to-bunch energy variation sufficiently small.[3] Our general approach has been to look for the most feasible cure (or combination of cures) for the instability, given bunch spacing and charge that are largely determined by such factors as these.

2. METHODS OF ALLEVIATING THE INSTABILITY

We have examined the following possible cures for the transverse instability:

Damped cavities. Theoretical and experimental studies show that it is possible to construct damped acceleration cavities that significantly reduce the $Q's$ of the transverse dipole wake modes.[4] One way to construct such cavities is to cut axial slots through the irises of structure and couple these slots to radial waveguides. Transverse mode Q's as low as 10 can be obtained in this way, although very precise machining of the slots will be required. Measurements have shown that there is no significant adverse effect of such slots on the accelerating mode. Another type of damped cavity has side-coupled slots that go into the cavity without cutting the irises. These slots perturb the accelerating mode to some extent, but do not transmit it. The Q's of the transverse modes can be as low as about 40 in this case.

Placing the bunches near wake zero crossings. If the transverse wake is strongly dominated by its fundamental mode, then it has zero crossings that are approximately equally spaced. Therefore it is possible to place all the bunches in a train near zero crossings of the wakefield, if the ratio of the frequency of the fundamental dipole mode to the frequency of the accelerating RF is tuned to satisfy:

$$\frac{1}{2}\, n\lambda_{W_\perp} = m\lambda_{rf} = \ell \quad , \tag{1}$$

where ℓ is the bunch spacing, m and n are integers, and λ_{rf} and $\lambda_{W_\perp}$ are the wavelengths of the RF and the fundamental dipole wake mode.

Spread in frequency of each transverse dipole mode. One might also consider an RF structure in which the frequencies of corresponding transverse dipole modes differ from cell to cell. This is the case, for example, in the existing SLAC linac, where the mode frequency spread is a few percent.[5] The frequency spread results in a reduction of the effective Q of each mode.

Bunch-to-bunch variation of transverse focusing. Using a system of time-varying quadrupoles, one could introduce a small spread in the focusing functions k_n of the bunches, so as to partially cancel the wake force due to preceding bunches. This is essentially the BNS damping mechanism[6] applied to multiple bunches. However, the focusing spread needed to produce a significant effect is large; we have discussed this cure elsewhere[7] as a possible adjunct to other methods.

3. SIMULATION PROGRAM FOR BEAM BREAKUP IN LINACS

As the basis for our computer simulations, we have derived an integral representation for the transverse offset of each bunch, assuming adiabatic acceleration and smooth focusing. We consider point bunches of charge N particles per bunch, with interbunch spacing ℓ. The acceleration is assumed to be linear: $\gamma = \gamma_0 + Gs$, where γ is the particle energy divided by the rest energy mc^2, and G is a constant. We use the smooth-focusing approximation $k(s) = 1/\beta(s)$ for the focusing function, where $\beta(s)$ is the "average" betatron function at longitudinal position s. The smooth focusing function of bunch n is taken to be:

$$k_n(s) = \left[\frac{\gamma_0}{\gamma(s)}\right]^p k_n(0) \quad . \tag{2}$$

For the main linacs of the collider, we will assume $p = 1/2$, but for other linac subsystems[8] (*e.g.*, injector linacs) it may be desirable to maintain more uniform k, namely $p \approx 0$.

The transverse dipole wake function is a sum of modes of the following form:

$$W_\perp(z) = \sum_m W_m \, \sin(K_m z) \, \exp\left(-\frac{K_m z}{2Q_m}\right) \quad , \tag{3}$$

where z is the distance behind the exciting bunch; K_m is the wavenumber and Q_m is the quality factor of mode m, and the W_m's are constant coefficients. Units of $W_\perp(z)$ are V/Coul/m^2. The equation of motion (in one transverse plane) for the offset x_n of bunch n is:

$$\begin{aligned} \gamma(s)\, x_n'' \;+\; \gamma'(s)\, x_n' \;+\; \gamma\,(s)\, k_n^2\,(s)\, x_n \\ = \frac{Ne^2}{mc^2} \sum_{j=1}^{n-1} W_\perp\,[(n-j)\ell]\, x_j\,(s) \quad . \end{aligned} \tag{4}$$

Here primes denote derivatives with respect to s. If we assume

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the WKB solution

$$x_1(s) = x_1(0)\left[\frac{\gamma_0}{\gamma(s)}\right]^{\frac{1-p}{2}} \exp\left[\psi_1(s,0)\right] , \qquad (5)$$

as the motion for the first bunch, and drop a term with rapidly-oscillating integrand, one can show (Ref. 7) that the solution for the transverse motion of bunch n may be written:

$$x_n(s) = \left\{x_n(0) + \frac{Ne^2}{2i\gamma_0 mc^2 k_n(0)}\int_0^s \left[\frac{\gamma_0}{\gamma(s')}\right]^{\frac{1-p}{2}} \exp[-i\psi_n(s',0)]\right.$$

$$\left.\times \sum_{j=1}^{n-1} W_\perp[(n-j)\ell]x_j(s')ds'\right\}\left[\frac{\gamma_0}{\gamma(s)}\right]^{\frac{1-p}{2}} \exp[+i\psi_n(s,0)] \quad , \qquad (6)$$

where

$$\psi_n(s,s') \equiv \int_{s'}^{s} k_n(s'')ds'' \qquad (7)$$

is the phase advance of bunch n between s' and s. A computer program LINACBBU was written to numerically integrate the equations for $x_n(s)$.

4. VERY STRONGLY DAMPED WAKE

If the wake is so strongly damped that a bunch only sees a significant wake from the immediately preceding bunch, we can use a simple "daisy chain" model to estimate the transverse blowup of each bunch in the train. Let us assume that the focusing function is the same for all bunches and scales according to Eq. (2) with $p = 1/2$. Then, one may show that the equations of motion can be written as if there were no acceleration (see Ref. 7):

$$\begin{aligned} x_1'' + k_0^2 x_1 &= 0 \\ x_n'' + k_0^2 x_n &= \frac{Ne^2 W_\perp(l)}{E_0} x_{n-1} \qquad (n > 1) \quad , \end{aligned} \qquad (8)$$

where k_0 and E_0 are the focusing function and energy at the beginning of the linac, and the longitudinal coordinate s is to be interpreted as an "effective length"

$$s_{eff} \equiv \frac{1}{k_0}\int_0^s k(s)ds \quad \approx \quad 2\left(\frac{\gamma_0}{\gamma}\right)^{1/2} \quad \text{for } \gamma \gg \gamma_0 \quad . \qquad (9)$$

Assuming $x_1(s) = a_1 e^{iks}$ where a_1 is a constant, one finds solutions $x_n(s) = a_n(s)e^{iks}$, where

$$a_n(s) = \sum_{j=0}^{n-1} \frac{(-i\sigma s)^j}{j!} a_{n-j}(0) \quad , \qquad (10)$$

and we have defined

$$\sigma \equiv \frac{Ne^2 W_\perp(l)}{2k_0 E_0} \quad . \qquad (11)$$

If the initial conditions are $a_n(0) = 1$, this is just the first n terms of the Taylor series for $\exp(-i\sigma s)$. Thus, if σL is of order 1, (where L is the effective length of the linac), there is no significant blowup of bunches beyond the first few in the train. We show an example of this behavior in the next section.

5. EXAMPLES

5.1. Linac Parameters

For illustration, we consider a main linac with accelerating frequency 17.1 GHz, length 3 km, initial energy 18 GeV, and final energy 500 GeV. The beta function is taken to be 3.2 m at the beginning of the linac and scales as $\gamma^{1/2}$. Keeping the bunch-to-bunch energy variation as small as possible imposes a relation between the number of particles per bunch, N, and the bunch spacing ℓ (Ref. 3):

$$\ell = cT_f \frac{\eta_0}{2} e^{\tau} \quad , \qquad (12)$$

where T_f is the filling time and τ is the ratio of the filling time to the attenuation time of the RF structure. The single-bunch loading is

$$\eta_0 \equiv \frac{4Ne\kappa_0}{E_z} \quad , \qquad (13)$$

where κ_0 is the loss parameter of the accelerating mode and E_z is the acceleration gradient. Taking $T_f = 60$ nsec, $\tau = 0.6$, $\kappa_0 = 436$ V/pC/m, and $E_z = 186$ MeV/m gives

$$\ell \approx (0.246\text{m})\frac{N}{10^{10}} \quad . \qquad (14)$$

We shall take ℓ to be 24 RF cycles (about 42 cm) and $N = 1.67 \times 10^{10}$ in our examples.

5.2. Highly Damped Wake

The number of e-foldings of the wake between bunches is about:

$$\frac{K_0 \ell}{2Q} \approx \frac{100}{Q} \quad , \qquad (15)$$

for $\ell = 0.42$ m and wavenumber K_0 of the fundamental transverse mode about 470 to 480 m^{-1}. Let us take as an example $Q = 35$ and $K_0 = 475$ m^{-1}, so that there are about 3 e-foldings between bunches.

In Fig. 1, the result of the daisy chain model is compared with the result of the full simulation program LINACBBU.

5.3. Damped Wake Combined With Tuning of Wake Zero-Crossings

The combination of lowering the Q's and tuning the frequency of the fundamental transverse dipole mode has been discussed in detail in Refs. 7 and 8.

Figure 2 shows an example of "tuning curves", where the maximum transverse blow-up factor of any of the bunches is plotted versus the frequency of the fundamental transverse mode, for various values of Q (assumed for simplicity to be the same for all transverse modes). The numbers plotted along the curves show the bunch that had the maximum blow-up. Thus, for this example, the curves are independent of the number of bunches in the train, provided there are at least four bunches; current TLC designs have at least 10 bunches per train. Note that even for the higher Q's, the tolerance on tuning the fundamental mode frequency is at least ±0.1%, which should not be too difficult to achieve.

5.4. Use of a Spread in Tranverse Mode Frequencies

We give an example similar to the previous one, except that we also introduce a frequency spread in each of the transverse modes.

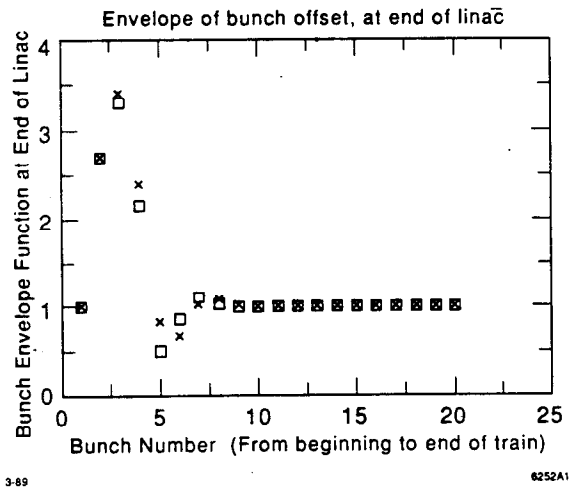

Fig. 1. Comparison of the results of daisy chain model (plotted as □'s) with the results of the program LINACBBU (plotted as X's). In each case, the value of the envelope function $|a_n(s)|$ at the end of the linac, for each bunch number n, is plotted. The transverse offset $x_n(s) = a_n(s)\exp(iks)$. The focusing function, k, is assumed the same for all bunches.

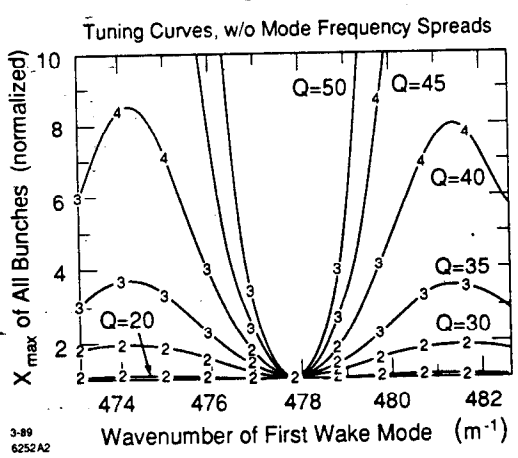

Fig. 2. Maximum transverse amplitude x_{max} (normalized) of all bunches as a function of the frequency of the fundamental transverse dipole mode, for values of Q = 20 to 50. The central value of the fundamental transverse mode wavenumber, where $\lambda_{rf}/\lambda_{W\perp} = 4/3$, is 477.85 m^{-1}. The range shown about K_0 is ± 1%.

Figure 3 shows tuning curves for $Q = 40$ to 70, with a total spread of 2% in the frequency of each transverse mode distributed uniformly over 200 values; other parameters are as in the preceding example. For $Q = 40$, the blow-up is a factor 2 or less, even with the fundamental transverse mode frequency not tuned to place bunches near wake zero crossings. For the higher values of Q shown, some tuning would be required. Note that Q's of 40 or so are obtainable without slotting the irises, and that according to Figs. 2 and 3, an acceptable solution could be obtained by either tuning the fundamental transverse mode frequency or introducing a 2% spread in the transverse mode frequencies, with $Q \sim 40$.

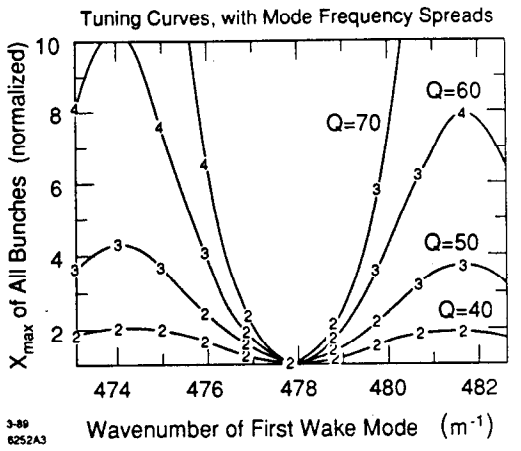

Fig. 3. Maximum transverse amplitude x_{max} (normalized) of all bunches as a function of the frequency of the fundamental transverse dipole mode, for values of Q = 40 to 70, with a spread in each transverse mode frequency of 2%. The central value of the fundamental transverse mode wavenumber, where $\lambda_{rf}/\lambda_{W\perp} \doteq 4/3$, is 477.85 m^{-1}. The range shown about K_0 is ± 1%.

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