SLAC-PUB-4869
March 27, 1989
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Perturbative QCD Analysis of the Pion Form Factor Using a Frozen Coupling Constant*

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ABSTRACT

Within a framework of perturbative QCD, we analyze the pion form factor using a frozen coupling constant and compare the results with the available experimental data.

Submitted to *Physical Review D*

⋆ Work supported in part by the Department of Energy, contracts DE–AC03–76SF00515 and DE-AS05-79ER10407.
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1. INTRODUCTION

For form factor calculation in strong interactions, it has been shown[1] that the argument of the running coupling constant should be taken as the square of the momentum transfer of the exchanged gluon in order to make the perturbation theory meaningful. This was argued from the convergence of the perturbation series and can be justified in any process which does not involve triple or quartic vertices in the lowest order. In a recent leading-order perturbative QCD analysis of the proton Dirac form factor F_1^p, we have shown[2] that it is possible to fit the data in the range of momentum transfer squared $0 < Q^2 < 30(GeV/c)^2$ by evaluating the strong coupling constant $\alpha_s(Q^2)$ at the exact gluon kinematics for each of the diagrams contributing to the leading-order process. In this paper, we extend the same considerations to the pion form factor.

The factorized[3] QCD expression for the pion form factor (see Fig. 1) is given by

$$F_\pi(Q^2) = \int_0^1 dx \int_0^1 dy \phi^*(y, \widetilde{Q}_y) T_H(x, y, Q^2) \phi(x, \widetilde{Q}_x) \tag{1}$$

where $\tilde{Q}_x = \mathrm{Min}(x, 1-x)Q, \phi(x, \tilde{Q}_x)$ is the quark distribution amplitude of the pion, and the hard scattering amplitude T_H, to the leading order in α_s is given by,[4]

$$T_H(x, y, Q^2) = \frac{64\pi}{3Q^2} \left[\frac{2}{3} \frac{\alpha_s[(1-x)(1-y)Q^2]}{(1-x)(1-y)} + \frac{1}{3} \frac{\alpha_s(xyQ^2)}{xy} \right] \quad . \tag{2}$$

In Eq. (2), the argument of α_s is the momentum transfer of the exchanged gluon as shown in diagrams of Fig. 1. While the leading order hard scattering amplitude in Eq. (2) exhibits divergence at both end points of x and y, the bound state quark distribution amplitude suppresses the end point singularities.

In this case, however, an immediate problem arises if the calculation of Eq. (1) is attempted with the usual one loop formula for the running coupling constant

$$\alpha_s(Q^2) = \frac{4\pi}{\beta \ell n\left(\frac{Q^2}{\Lambda^2}\right)} \tag{3}$$

($\beta = 2/3\ n_f$ and n_f is the number of flavors), since the integration in Eq. (1) allows α_s to be evaluated near zero momentum transfer. The same problem arises in the proton Dirac form factor analysis[2]. In Ref. 2, this problem was avoided by introducing[5] a cut-off in the formula for $\alpha_s(Q^2)$ to prevent the coupling constant from becoming infinite for vanishing gluon momenta. In particular, in Ref. 2 the following modified relation for α_s, as proposed in Ref. 6, was utilized:

$$\alpha_s(Q^2) = \frac{4\pi}{\beta \ell n\left(\frac{Q^2+m_g^2}{\Lambda^2}\right)} \tag{4}$$

where m_g is interpreted as a dynamical gluon mass with a value of typically about 0.5 GeV/c and Λ is order of 100 MeV. For $Q^2 \gg m_g^2$, it coincides with the one loop version (Eq. 3), but at very low momentum transfer, this formula "freezes" the coupling constant to some finite but not necessarily small value.

The physical meaning of frozen coupling constant[5,6] may be found in the confinement mechanism suggested by 1+1 dimensional QED.[7] If one tries to elongate a positronium (e^+e^-), it is energetically more favorable for the vacuum to create fermion and antifermion pairs so that the effective coupling between the original two charges e^+ and e^- is frozen because of screening by vacuum condensates. In fact, the color confinement does not necessarily mean the divergence of $\alpha_s(Q^2)$ at small momentum transfer. The idea of frozen coupling constant may be more natural to understand the color confinement problem. As an evidence of vacuum condensates in QCD, quark and gluon condensation order parameters are obtained by QCD sum rule from PCAC and instanton solutions:[8]

$$\langle Vac \,|: \overline{u}u :|\, Vac\rangle = \left\langle Vac \,|: \overline{d}d :|\, Vac\right\rangle \simeq -(250\ MeV) \tag{5a}$$

and

$$\left\langle Vac \,|: \frac{\alpha_s}{\pi} \quad G_{\mu\nu}G^{\mu\nu} :|\, Vac\right\rangle \simeq 0.012\ (GeV)^4 \tag{5b}$$

Using a special set of Schwinger-Dyson equations, the formation of dimensionful parameters, for example, given by Eq. (5), has been studied.[6] The numerical solution of the Schwinger-Dyson equation was consistent with the idea of frozen coupling constant given by Eq. (4) with $m_g = 500 \pm 200\ MeV$. In this way, m_g is related to Λ which is order of 100 MeV and the whole analysis still has only one QCD parameter.

Therefore, it is concluded that the QCD vacuum condensate affects not only quark distribution amplitude, but also the QCD running coupling constant at small momentum transfer region.[9] Furthermore, since the value of m_g given by Ref. 6 freezes $\alpha_s(Q^2)$ (Eq. 4) to a value less than 1 even at $Q^2 = 0$, the perturbation series may be expanded in terms of frozen coupling constant.

In this paper, we present the leading-order perturbative analysis of the pion form factor using a frozen coupling constant given by Eq. (4). We follow the same method of calculations employed in Ref. 2 and use the same numerical values for m_g and Λ as introduced there. In section 2, we present the quark distribution amplitude including its QCD evolution which is used in this analysis. Numerical results and comparison with experimental data are presented in section 3 and conclusions are followed in section 4.

2. Quark Distribution Amplitude and Its Evolution

Useful constraints on the lowest moments of the distribution amplitude $\phi(x, Q)$ can be obtained using the QCD sum rule approach.[10] Although the numerical accuracy of this method is not known, the general agreement between its predictions and overall consistency with other hadron phenomenology[11] lends credence to its validity.

The distribution amplitude of the pion at $Q = 500\ MeV$ is given by[10]

$$\phi(x, \mu) = \frac{30 f_\pi}{2\sqrt{3}} \quad x(1-x)(2x-1)^2 \tag{6}$$

where $\mu = 500\ MeV$ and $f_\pi = 93\ MeV$. The normalization of $\phi(x, \mu)$ is given by the condition

$$\int_0^1 dx \phi(x, \mu) = \frac{f_\pi}{2\sqrt{3}} \tag{7}$$

Once the distribution amplitude is given at a certain value of $Q (Q = 500\ MeV$, for example), then $\phi(x, Q)$ at other values of Q can be obtained by solving a Bethe-Salpeter type evolution equation.[3] The result for valence quark distribution amplitude of the pion is expanded in terms of Gegenbauer polynomials $C_n^{3/2}(2x-1)$ and is given by

$$\phi(x, Q) = \sqrt{3} f_\pi x(1-x) \left\{ C_0^{3/2}(2x-1) + \frac{2}{3}\ C_2(2x-1) \left[\frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)} \right]^{50/81} \right\} \tag{8}$$

where $C_0^{3/2}(z) = 1$ and $C_2^{3/2}(z) = \frac{3}{2}\ (5z^2 - 1)$. At the boundary of $Q = \mu$, Eq. (8) reduces to Eq. (6) and in the limit $Q \longrightarrow \infty$, Eq. (8) reduces to the asymptotic form in Ref. 3, as expected.

3. Numerical Results and Comparison with Experimental Data

Using the quark distribution amplitude of Eq. (8) and the frozen coupling constant of Eq. (5), we evaluated the integrals given by Eq. (1). As we did in Ref. 2 for the proton Dirac from factor analysis, we included only the leading order hard scattering amplitude[4] as given in Eq. (2). The results for the pion form factor is shown in Fig. 2. In Ref. 2, it was shown that it is possible to fit the data for proton Dirac form factor F_1^p in the range of $10 < Q^2 < 30$ $(GeV/c)^2$ when one uses the distribution amplitudes proposed by QCD sum rule calculations[10,12] and a frozen coupling constant (Eq. (5)) with m_g^2 between 0.1 and 0.5 GeV^2. In the present case, as shown by Fig. 2, we have the same consistency with the available experimental data even though further comparison with future experimental data is necessary at higher Q^2 region.

Since the pion form factor $F_\pi(Q^2)$ is multiplied by Q^2 in Fig. 2(a), the numerical results seem to be sensitive to different values of m_g even at high Q^2. However, the pion form factor $F_\pi(Q^2)$ itself is much less sensitive to variation of m_g (see Fig. 2(b)). It is also interesting to note that the numerical values of m_g used in this analysis are consistent with those of an effective gluon mass in the condensed vacuum obtained by a QCD lattice calculations[13] and a recent discussion of dynamical mass generation in QCD.[6,14]

4. Conclusions

In this paper we have analyzed the pion form factor within a framework of perturbative QCD using a frozen coupling constant. While similar results should be obtained using any form of cut-off which prevents $\alpha_s(Q^2)$ from becoming infinitely large at small momentum transfers, we chose to use the formula of Eq. (4) because of its simple analytical form, and its successful application for the analysis of the proton Dirac form factor[2]. Using the quark distribution amplitude of Eq. (8) constrained by QCD sum rule[10], we obtained numerical results shown in Fig. (2). The pion form factor obtained in this analysis is in a reasonable agreement with experimental data (as was the case of proton form factor in Ref. 2). Whether the same method would work for other form factors such as $N - \Delta$ transition form factors[15] is an interesting question which necessitates the application of this technique to other processes.

Acknowledgement

We would like to thank S. R. Cotanch and A. F. Sill for many helpful discussion.

REFERENCES

1. S. J. Brodsky, G. P. Lepage and P. M. Mackenzie, Phys. Rev. D22, 228 (1983).
2. C. R. Ji, A. F. Sill and R. M. Lombard-Nelsen, Phys. Rev. D36, 165 (1987).
3. S. J. Brodsky and G. P. Lepage, Phys. Scr. 23, 945 (1981); G. P. Lepage and S. J. Brodsky, Phys. Rev. D22, 2157 (1980); Phys. Rev. Lett. 43, 545 (1979); A. Duncan and A. H. Mueller, Phys. Lett. 90B, 159 (1980); Phys. Rev. D21, 1636 (1980); V. L. Chernyak, I. R. Zhitnitsky and V. G. Serbo, JETP Lett. 26, 594 (1977).
4. Next-to-leading order hard scattering amplitude has been presented in R. D. Field, G. Gupta, S. Otto and L. Chang, Nucl. Phys. B186, 429 (1981); F. M. Dittes and A. V. Radyushkin, Sov. J. Nucl. Phys. 34, 293 (1981). However, the results in these papers are not in agreement with each other.
5. G. Parisi and R. Petronzio, Phys. Lett. 95B, 51 (1980); S. J. Brodsky, T. Huang and G. P. Lepage, in Particles and Fields Z, proceedings of the Banff Summer Institute, 1981, edited by A. Z. Capri and A. N. Kamal (Plenum, New York, 1983); V. N. Pervushin et. al, E2-88-86 (preprint of the Joint Institute for Nuclear Research, Dubna).
6. J. M. Cornwall, Phys. Rev. D26, 1453 (1982). In this reference, $m_g = 500 \pm 200\ MeV$ was obtained by taking $\Lambda = 300\ MeV$. However, because we include the effect of m_g only in the running coupling constant and not in the gluon propagator, we take $\Lambda = 100\ MeV$ while keeping the same numerical value for m_g. This point was discussed in the same reference.
7. J. Schwinger, Phys. Rev. 127, 324 (1962).

8. M. A. Shifman, A. I. Vainshtein and V. T. Zakharov, Nucl. Phys. B147, 385 (1979).

9. C. R. Ji, in Nuclear and Particle Physics on the Light Cone, proceedings of the Los Alamos, 1988, edited by Johnson and Kisslinger.

10. V. L. Chernyak and I. R. Zhitnitsky, Phys. Rept. 112, 173 (1984).

11. G. Martinelli and C. T. Sachrajda, CERN-TH-4637/87 (1987); S. Gottlieb and A. S. Kronfeld, Phys. Rev. D33, 227 (1986); A. S. Kronfeld and D. M. Photiadis, Phys. Rev. D31, 2939, (1985); Z. Dziembowski and L. Manliewicz, Phys. Rev. Lett. 55, 1839 (1985).

12. V. L. Chernyak and I. R. Zhitnitsky, Nucl. Phys. B246, 52 (1984); I. D. King and C. T. Sachrajda, Nucl. Phys. B279, 785 (1987); M. Gari and N. G. Stefanis, Phys. Lett. 175B, 462 (1986); G. R. Farrar, in Nuclear and Particle Physics on the Light Cone, proceedings of Los Alamos 1988, edited by M. Johnson and L. Kisslinger.

13. C. Bernard, Phys. Lett. 108B, 436 (1982).

14. L. S. Celnza, C. R. Ji and C. M. Shakin, Phys. Rev. D36, 895 (1987).

15. C. E. Carlson and J. L. Poor, WM-88-101 (1988) to be published in Phys. Rev. D.

Figure Captions

$Fig.1$: Valence Fock state contribution to the large momentum transfer meson form factor. T_H is computed for zero mass quarks q and q parallel to the pion momentum.

$Fig.2$: Pion form factor calculation with the distribution amplitude of Chernyak and Zhitnistky (Eq. (8)) and with the argument of $\alpha_s(Q^2)$ evaluated at gluon momentum in Eq. (2). 2(a) $Q^2F_\pi(Q^2)$; 2(b) $F_\pi(Q^2)$.

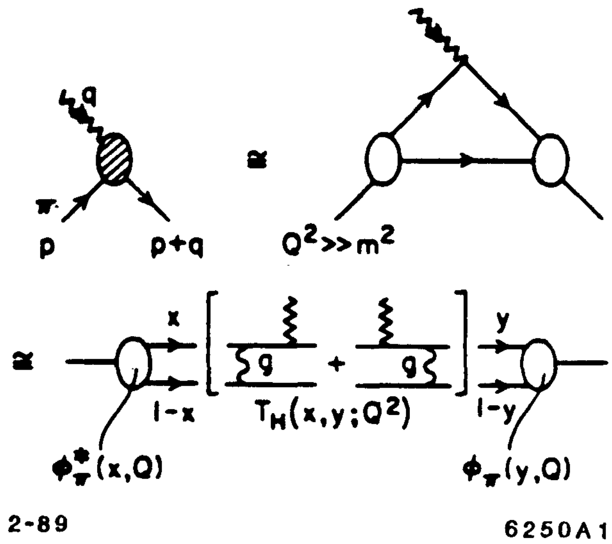
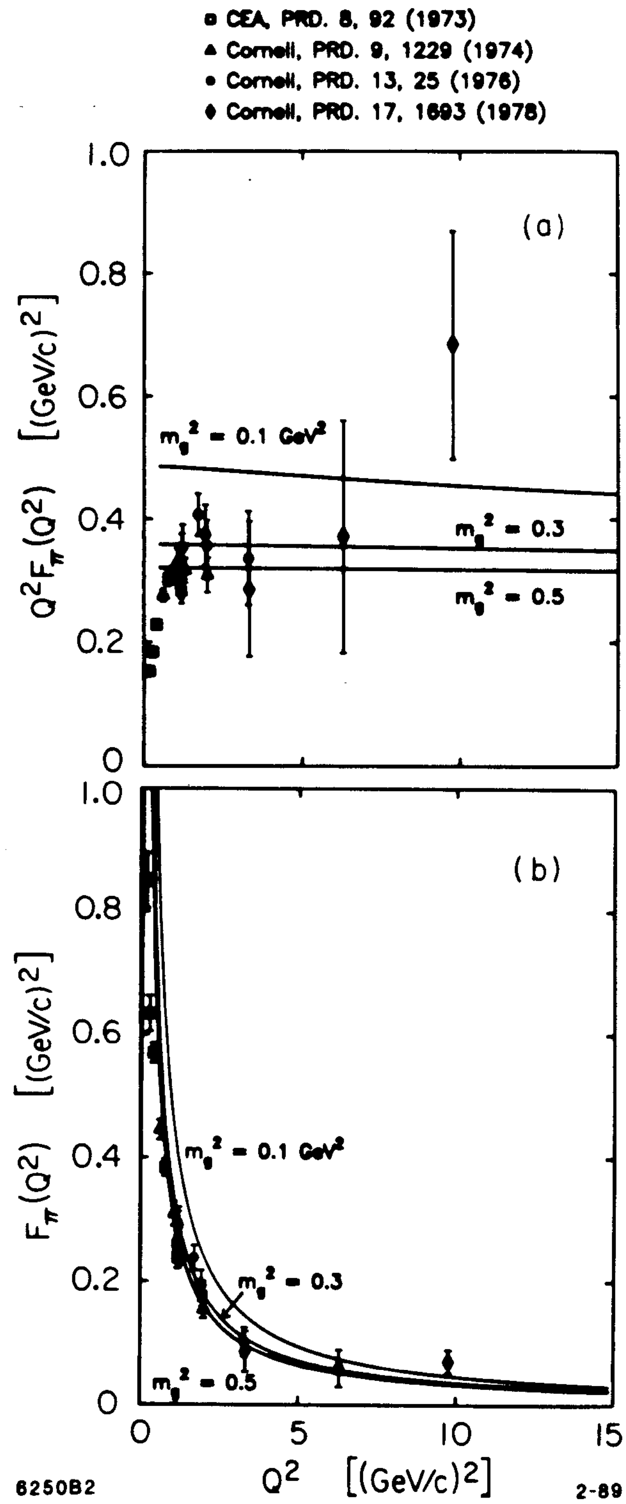


Fig 1



CEA, PRD. 8, 92 (1973)
Cornell, PRD. 9, 1229 (1974)
Cornell, PRD. 13, 25 (1976)
Cornell, PRD. 17, 1693 (1978)
(a)
$Q^2F_\pi(Q^2)$ $[(GeV/c)^2]$
$m_g^2 = 0.1$ GeV^2
$m_g^2 = 0.3$
$m_g^2 = 0.5$
(b)
$F_\pi(Q^2)$ $[(GeV/c)^2]$
$m_g^2 = 0.1$ GeV^2
$m_g^2 = 0.3$
$m_g^2 = 0.5$
Q^2 $[(GeV/c)^2]$
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Fig. 2